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Investment and Disclosure: 
The Disciplinary Role of Periodic Performance Reports

CHANDRA KANODIA* AND DEOKHEON LEE†

1. Introduction

Many important management decisions are made in the light of information that is not available to the public but is privately held by managers. The decisions themselves become public, but the information underlying them is not directly visible to investors who trade the firm's shares in a capital market. How, then, is the firm valued? How do market perceptions and inferences affect the decisions of value-maximizing managers?

To assess the valuation consequences of management decisions, in such settings, investors use other available signals, including the observed decisions themselves to make inferences about the hidden information. Management decisions thus acquire an informational value apart from the real value that derives from the future cash flow consequences of those decisions. In equilibrium, the informational and real values must coincide, but the possibility of misalignment creates a severe incentive problem that could result in large distortions from the socially optimal decisions. For example, if the market reacts more favorably to decision A than to decision B, the firm's managers would be motivated

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to take decision $A$, even though $A$ is optimal only under a particular set of circumstances which may not hold at the time (see Brandenburger and Polak [1996] for an example of such market-driven perverse incentives). Value-maximizing managers would be encouraged to substitute the market's expectations for their own superior judgments, resulting in inefficient decisions.

In this paper we argue that periodic performance reports (such as earnings statements) go a long way in alleviating the perverse incentives that arise when the capital market is less informed than corporate managers. Performance reports are used by investors to update their assessments of the firm's future cash flows and price the firm accordingly. Managers concerned with the market's expected response to the performance report will assess the distribution of the performance report in the light of their own private information and the decisions they make. We show that price pressure leads to performance reporting pressure, which, in turn, disciplines managerial decisions in such a way that these decisions reveal management's hidden information to the market. This disciplinary role of performance reporting arises through the equilibrium behavior of capital market prices, rather than through explicitly designed compensation contracts that reward or penalize managers contingent on measured performance.

Specifically, we study a setting where the firm's management has superior private information about the profitability of new investment opportunities, but the investments undertaken are publicly observed. Direct disclosure of management's information is not credible. The discipline imposed by performance reporting works in the following way. In equilibrium, investors in the capital market extract information from both the firm's observable investment and the performance report $\tilde{y}$. The observed investment is used to make inferences about management's prior beliefs, and the performance report is used to update the distribution of future cash flows given these inferred prior beliefs. Thus the release of $\tilde{y}$ triggers price reactions in the capital market; ceteris paribus, the lower the realized value of $\tilde{y}$, the lower the firm's equilibrium price. From management's perspective the performance report $\tilde{y}$, to be released subsequent to their choice of investment, is a random variable whose distribution is affected by the true profitability of investment. Thus, given investment, the more unfavorable managers' information is, the lower their private expectation of $\tilde{y}$ must be. Now suppose management has unfavorable information but contemplates a large investment to induce overly optimistic beliefs in the capital market. Since managers expect low realizations of $\tilde{y}$, they must also anticipate a market price that is low relative to the large investment. If the investment needed to induce successively more optimistic beliefs in the market is increasing rapidly enough relative to the expected market price contingent on $\tilde{y}$, deception becomes unattractive. Thus, the anticipation of performance reports disciplines managers' investment incentives and allows the firm's
observable investment to emerge as a credible signal of management’s prior information.

We show that equilibrium expectations in the market are such that managers must overinvest to credibly signal the firm’s type. The extent of overinvestment depends on the informativeness (precision) of the performance report with respect to future cash flows. Higher precision increases the sensitivity of the equilibrium price to realizations of $\tilde{y}$ which, in turn, enhances the disciplinary role of performance reports and relieves the signaling burden on the firm’s investment schedule. Thus, a more precise performance report allows the firm to decrease its overinvestment and move closer to first-best.

However, increasing the precision of the performance report is costly in the following sense. Trading between current and prospective shareholders redistributes the risk associated with the firm’s investment. A performance report that conveys information about future returns before trade takes place inhibits this risk transfer and causes the firm’s current shareholders to bear too much risk. Consequently, periodic performance reports that serve only to update the distribution of future cash flows, without any disciplinary effects, result in suboptimal investment and a suboptimal distribution of risk. This result, first developed by Hirshleifer [1971] and Diamond [1985] for trading economies, extends naturally to economies with endogenous investment. We show how these endogenous costs are traded off against the beneficial disciplinary effects to determine the optimal precision of the performance report.

We show that, given the information asymmetry, optimal allocations require a particular bundling of disclosure and investment that is inconsistent with an unconstrained signaling equilibrium. Therefore, we depart from the usual techniques, developed in Spence [1974] and Riley [1979], for characterizing signaling equilibria. Optimal allocations are characterized via the mechanism design methodology with the constraint that capital market prices are sequentially rational and market clearing. We then show that these optimal allocations can be attained as a signaling equilibrium if disclosure decisions are appropriately regulated.

Disclosure of unverifiable information has also been studied by Newman and Sansing [1993] and Gigler [1994], who model disclosure made simultaneously to two audiences. Managers have an incentive to exaggerate the profitability of their operations to one audience, but this imposes proprietary costs because of the reaction of the second audience. This tension results in credible disclosure. In contrast, we are concerned with disclosure to a single audience, the capital market, and we focus on disclosure of postdecision performance, not ex ante information of strategic importance to competitors. Bhattacharya [1979] and Miller and Rock [1985] have shown how signaling via dividends can communicate the future prospects of a firm, and Leland and Pyle [1977] demonstrated that the fraction of equity retained by insiders can play a similar role. This literature acknowledges, but does not investigate, the possible role
of accounting disclosure. We find that periodic performance reports provide the same discipline as the fraction of equity retained by insiders in the Leland and Pyle analysis. However, there are also subtle differences that we elaborate later.

Section 2 describes the economy and the assumptions underlying our analysis. Section 3 discusses equilibrium expectations and market pricing and section 4 characterizes the firm’s optimal investment and disclosure schedules. Section 5 motivates disclosure regulation and shows how the optimal allocations can be attained as a signaling equilibrium. Section 6 concludes. Proofs of all theorems are contained in Appendix A.

2. Structure of the Economy

At date zero a firm, owned by an entrepreneur with constant absolute risk aversion $\rho$, invests $k$ units of capital in a risky technology whose return $\tilde{\Theta}$ is described by:

$$\tilde{\Theta} = k[\tilde{\mu} + \tilde{\gamma}].$$  

The random variable $\tilde{\gamma}$ is Normally distributed independently of $\tilde{\mu}$, with mean zero and variance $\sigma$. The expected return per unit of capital $\mu$ is drawn from a distribution with strictly positive density $f(.)$ on the interval $[\mu_L, \mu_H]$, where $\mu_H > \mu_L > 1$. The specification in (1) describes a constant returns to scale technology, where the aggregate expected return is linearly increasing with investment but the variance of return increases disproportionately. Conditional on any fixed $\mu$, the expected return from investing $k$ units is $k\mu$ and the variance of return is $k^2\sigma$. The assumption $\mu_L > 1$ guarantees a positive level of investment.

The firm (i.e., its managers or current owners) privately observes the realization of $\tilde{\mu}$ before it chooses its investment and any other decision, so that the firm has superior information about future returns to its investment. Direct disclosure of $\mu$ is not credible since there is no way to verify the truth of the disclosure and, as will be apparent shortly, there are obvious incentives to lie. We assume that the choice of investment, $k$, is publicly observed.

We assume the investment horizon exceeds the consumption horizon of the firm’s current owners, so that ownership changes hands before the final return to investment is realized.1 We operationalize this ownership change by assuming the final return is realized at date two, but current owners sell their holdings inelastically at date one at a competitive market price $\tilde{P}$. Thus the firm’s current owners collectively consume $\tilde{P} - k$, while the new owners collectively consume $\tilde{\Theta} - \tilde{P}$. Because we assume that sales by current owners convey no information to prospective investors.

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1 Similar assumptions are made in Bhattacharya [1979], Miller and Rock [1985], and most of the financial signaling literature. Absent ownership changes, the capital market price is irrelevant and there is no need to disclose information to investors.
buyers, we have ruled out the Leland and Pyle [1977] type of signaling. However, we will show that an interim performance report released prior to the sale has the same qualitative effect as the fraction of equity retained by the entrepreneur in the Leland and Pyle analysis.

We now introduce an interim performance report, released after the investment decision but before ownership changes. Our focus is the informational features of the performance report and not its underlying events and measurements. Specifically, because of managerial discretion over various aggregations, allocations, valuations, anticipation of future cash flow consequences of past events, and other accrual adjustments, a performance report can be made more or less informative about the firm’s future cash flows. Thus, the informativeness of performance reports is, to some degree, a variable of choice for the firm’s managers.

The following specification of the performance report, which we denote $\tilde{y}$, is an abstraction that is intended to capture only its informational features:

$$\tilde{y} = k[\mu + \tilde{\gamma} + \tilde{\epsilon}] = \tilde{\theta} + k\tilde{\epsilon}$$

(2)

where $\tilde{\epsilon}$ is Normally distributed with zero mean and variance $\text{var}(\tilde{\epsilon})$, and $\tilde{\epsilon}$ is independent of $\tilde{\gamma}$. Conditional on $\mu$, the distribution of $\tilde{y}$ is Normal with $E(\tilde{y}) = k\mu$ and $\text{var}(\tilde{y}) = k^2[\sigma + \text{var}(\tilde{\epsilon})]$. The following informational features of the specification in (2) are important to our analysis: (i) The performance report communicates a noisy but unbiased signal of the future return to investment. (ii) The distribution of the performance report is affected by the true expected return $\mu$. This appears reasonable because it is the true profitability of investment that affects the past events underlying the performance report. (iii) Once the report $\tilde{y}$ is realized, it cannot be distorted. In most cases, this is a reasonable approximation because of the existence of audits.

The firm’s management chooses the informativeness (precision) of the performance report by choosing $\text{var}(\tilde{\epsilon})$ at the same time the firm chooses its investment, i.e., after observation of $\mu$. It is convenient to express the choice of precision in a slightly different way. Define $\beta = \sigma(\sigma + \text{var}(\tilde{\epsilon}))$. Since $\sigma$ is an exogenous parameter, there is a one-to-one correspondence between $\text{var}(\tilde{\epsilon})$ and $\beta$ (greater precision implies a higher $\beta$). No disclosure is equivalent to $\beta = 0$, and perfect information is equivalent to $\beta = 1$. With some abuse of terminology, we use $\beta$ to describe the precision of the performance report and think of the firm as choosing $\beta$ rather than $\text{var}(\tilde{\epsilon})$. Realistically, there is a technological upper bound on $\beta$ since it is difficult to imagine that an interim performance report could perfectly predict the final cash flow. However, we will show that there is an endogenous upper bound on $\beta$ arising from the trade-offs faced by managers. We assume that the choice of $\beta$ and the statistical structure of the performance report are common knowledge.

If the price $\hat{P}$ in the capital market is Normally distributed (as will be the case), the expected utility of current owners can be represented in
terms of the mean and variance of their collective consumption. To focus on the external role of performance reports, we assume there are no conflicts of interest between managers and current shareholders (or, equivalently, the firm is owner managed). Therefore, we assume management makes all decisions in the best interests of the firm’s current owners, and its objective function is:

$$\text{Max} - k + E(\hat{P}|\mu) - 1/2 \rho_e \text{ var}(\hat{P}|\mu).$$  \hspace{1cm} (3)$$

The firm observes $\mu$, then chooses its investment $k$ and disclosure policy $\beta$ to maximize the above, treating the pricing rule $\hat{P}(.)$ as given. This determines the optimal investment and disclosure schedules $k(\mu)$ and $\beta(\mu)$.

Turning now to the capital market, we assume that all traders have constant absolute risk aversion, with $\rho_i$ being the risk aversion of trader $i$. Let $\lambda = [\Sigma(1/\rho_i)]^{-1}$ be the aggregate risk aversion in the capital market. Since traders assess the distribution of $\hat{\theta}$ conditional on their observations of $\bar{y}$, $k$, and $\beta$, the equilibrium price is described by some function $P(y,k,\beta)$. If this conditional distribution of $\hat{\theta}$ is Normal (as will be the case), then standard results in finance\(^2\) imply:

$$P(y,k,\beta) = E(\hat{\theta}|y,k,\beta) - \lambda \text{ var}(\hat{\theta}|y,k,\beta).$$  \hspace{1cm} (4)$$

We assume that $\rho_e > 2\lambda$, i.e., the risk aversion of the current owners exceeds the aggregate risk aversion in the capital market.\(^3\) This assumption induces an endogenous cost to any disclosure that conveys information about the final return to the firm’s investment. To see this, consider the extreme cases of $\beta = 1$ and $\beta = 0$. When $\beta = 1$, the performance report perfectly reveals the value of $\theta$ to prospective buyers, and the equilibrium price must be $\theta$. In this case, all investment risk is borne by the firm’s current owners and the sale of the firm transfers no risk to the new owners. On the other hand, when $\beta = 0$, i.e., there is no performance report, all the risk is transferred to the new owners. In general, a higher $\beta$ implies a greater fraction of the total risk will be borne by the current owners. But, since the aggregate risk aversion in the capital market is lower than the current owners’ risk aversion, it is more efficient to transfer risk to the capital market.\(^4\) Thus disclosure of

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\(^2\)See Stapleton and Subrahmanyam [1978].

\(^3\)We do not require that each individual current owner is more risk averse than each individual trader. Since aggregate risk aversion generally declines with the number of traders in the market, this assumption is likely to be satisfied when the number of current owners is small relative to the number of traders who demand the firm’s shares.

\(^4\)The situation is analogous to that of a wheat farmer whose crop is uncertain. The farmer may be the best person to grow the wheat because he possesses superior information on wheat farming, but he is not necessarily the best person to bear the associated risks. The farmer transfers his risks to traders by selling wheat futures.
a performance report inhibits the efficient transfer of risk, making disclosure costly.

In a first-best world, where \( \mu \) is common knowledge, performance reports would also inhibit the firm’s investment. Since \( \mu \) is known, the choice of \( k \) and \( \beta \) conveys no information, and the performance report \( \bar{y} \) merely updates the distribution of future cash flows. Since the conditional distribution of \( \bar{\theta} \) given \( y \) is Normal with \( E(\bar{\theta}|y) = \beta y + (1-\beta)\mu \), and \( \text{var}(\bar{\theta}|y) = (1-\beta)\sigma^2 k^2 \), the equilibrium capital market price (4) becomes \( P(y,k,\beta) = \beta y + (1-\beta)k\mu - \lambda(1-\beta)\sigma k^2 \). Inserting this pricing rule into (3), the firm’s objective function becomes \( \text{Max} - k + k\mu - \sigma k^2 \left[ \beta (1/2) \rho_e + (1-\beta)\lambda \right] \). Thus, for any fixed \( \beta \), optimal investment is described by:

\[
    k = \frac{\mu - 1}{2\sigma \left[ \beta (1/2) \rho_e + (1-\beta)\lambda \right]}.
\]

Given \( (1/2)\rho_e > \lambda \), (5) indicates that a more precise performance report (i.e., a larger \( \beta \)) results in a smaller investment. This is because a performance report with precision \( \beta \) divides the total risk of investment, \( \text{var}(\bar{\theta}) = \sigma k^2 \), between current and prospective owners; the fraction \( \beta \) is assigned to the current owners and the remaining fraction \( (1-\beta) \) to prospective owners. Since the current owners’ risk aversion exceeds the aggregate risk aversion in the capital market, more precise performance reports induce a more risk-averse investment choice. It is obvious from the firm’s objective function that the expected utility of current owners is strictly decreasing in \( \beta \) for any fixed \( k \). Therefore, the firm optimally makes no disclosure. A performance report, in this symmetrically informed setting, results in both a suboptimal assignment of risk and suboptimal investment.

The above result can be strengthened. Prospective buyers would also prefer that there be no performance reporting,\(^5\) because they earn a return only through risk bearing and disclosure implies they have a smaller risk to purchase. Thus, viewed purely as a postdecision signal, there would be no demand for external performance reporting. If performance reporting serves a social role, it must do something more than merely provide better assessments of future cash flows.

### 3. Expectations and Incentive Compatibility

To develop this additional disciplinary role, we now turn to the setting in which the ex ante profitability of investment \( \mu \) is privately known to management. In this setting, the equilibrium price function \( P(y,k,\beta) \) depends on the information that traders extract from the performance report \( \bar{y} \) as well as management’s choice of investment \( k \) and precision \( \beta \). Since management chooses \( k \) and \( \beta \) after observing the realization of \( \bar{\mu} \),

\(^5\) A formal proof of this claim can be obtained from the authors.
these observed choices potentially contain information on management’s superior knowledge. To extract this information, traders must have beliefs about the firm’s decision policies \( \{k(\mu), \beta(\mu)\} \). On the other hand, the firm’s actual choice of investment and disclosure schedules depends on the equilibrium pricing rule, which, in turn, depends on the beliefs and inferences of traders. An equilibrium must satisfy the rational expectations condition that the investment and disclosure policies implied by capital market beliefs are the same policies on which these beliefs are based.

If, in equilibrium, the firm’s decision policies are such that either \( k(\mu) \) or \( \beta(\mu) \) or some function of \( k \) and \( \beta \) is invertible in \( \mu \), then \( \mu \) will be revealed to the capital market. Because the distribution of the performance report \( \tilde{y} \) is also affected by \( \mu \), in principle, the performance report could also be used to update the prior distribution of \( \tilde{\mu} \). However, if traders \( \text{perfectly} \) infer \( \mu \) from their observation of \( (k, \beta) \), they must necessarily use the information in \( \tilde{y} \) only to update the inferred prior distribution of \( \tilde{\theta} \). Specifically, suppose that \( \mu_r \) is the inferred value of \( \tilde{\mu} \) from observation of some \( (k, \beta) \). Then traders must necessarily believe that the \( \text{prior} \) distribution of \( \tilde{\theta} \) is Normal, with mean \( k \mu_r \) and variance of \( \sigma k^2 \), and the \( \text{posterior} \) distribution of \( \tilde{\theta} \) conditional on \( y \) is Normal, with \( E(\tilde{\theta} | y) = \beta y + (1 - \beta) k \mu_r \) and \( \text{var}(\tilde{\theta} | y) = (1 - \beta) \sigma k^2 \). Whether inferences made in this way can be sustained in equilibrium is an open question. We will show that the performance report plays a crucial role in sustaining a fully revealing equilibrium of this type.

Before characterizing a fully revealing equilibrium, we discuss management’s incentives when expectations in the market are formed in the manner described above. For any \( (k, \beta) \) that results in the capital market belief that \( \tilde{\mu} = \mu_r \), the equilibrium price as a function of \( y \) must be:

\[
P(y; k, \beta, \mu_r) = \beta y + (1 - \beta) k \mu_r - \lambda (1 - \beta) \sigma k^2.
\]

(6)

When management chooses \( k \) and \( \beta \), the performance report \( \tilde{y} \) is a random variable. Management, knowing the true value of \( \mu \), must assess \( E(\tilde{y}|\mu) = k \mu \), regardless of the market’s belief \( \mu_r \). Thus the firm knows that if the true value of \( \mu \) is low, the value of \( \tilde{y} \) is also likely to be low. Given (6) the firm must expect a price of \( E[P(\tilde{y}, k, \beta)|\mu_r, \mu] = \beta k \mu + (1 - \beta) k \mu_r - \lambda (1 - \beta) \sigma k^2 \), and \( \text{var}(P(\tilde{y}, k, \beta)) = \beta^2 \text{var}(\tilde{y}) = \beta \sigma k^2 \). Thus, management will seek to maximize:

\[
W(k, \beta, \mu_r, \mu) = -k + \beta k \mu + (1 - \beta) k \mu_r - \lambda (1 - \beta) \sigma k^2 - (1/2) \sigma^2 k^2
\]

(7)

which can be rewritten as:

\[
W(k, \beta, \mu_r, \mu) = \beta [\mu k - k - (1/2) \sigma^2 k^2] + (1 - \beta) [\mu_r k - k - \lambda \sigma k^2].
\]

The firm’s objective function is a weighted average of two expected utilities. The expression multiplying \( \beta \) is the expected utility of current
shareholders if they held the firm to liquidation. The expression multiplying \((1-\beta)\) is the expected utility of prospective shareholders if they had been the original shareholders and if their assessment of \(\mu_x\) was \(\mu_r\).

In effect, a performance report issued prior to sale of the firm makes the firm’s current shareholders share in the consequences of their decisions even though the firm is sold before those consequences are realized. It is as if the current shareholders retained the fraction \(\beta\) of the firm and sold only the remaining fraction \((1-\beta)\). There is thus an analogy between the precision of the performance report and the fraction of equity retained by insiders in the Leland and Pyle [1977] analysis.\(^6\)

However, in Leland and Pyle’s analysis, investment is exogenous and therefore communicates no information to outsiders. The fraction of equity retained by insiders is the only signal. In our analysis, investment and disclosure interact to communicate information. We will show that investment alone serves as the signal, while the performance report plays a disciplinary role that sustains the investment signal.

In choosing its investment and disclosure, \(k\) and \(\beta\), the firm takes into account their effect on the market’s inference \(\mu_r\). To see how the firm’s investment incentives vary with its type, examine the marginal rate of substitution between \(\mu_r\) and \(k\) for any fixed \(\beta\).

\[
\frac{\partial \mu_r}{\partial k} = -\frac{W_k}{W_{\mu_r}} = -\frac{\beta \mu + (1-\beta)\mu_r - 1 - 2\sigma k[\beta 1/2 \bar{x} + (1-\beta)k]}{(1-\beta)k}.
\]

If \(0 < \beta < 1\) and \(k > 0\), the marginal rate of substitution is strictly decreasing in \(\mu\), so the single crossing property required for sorting of types is satisfied. If in addition \(\partial \mu_r/\partial k > 0\), then higher \(\mu\) types would be willing to invest more than lower \(\mu\) types to influence market perceptions, and the firm’s investment would signal its true type. But \(\partial \mu_r/\partial k > 0\) only if \(W_k < 0\), which implies that firms must overinvest to credibly signal their types. Underinvestment will not sustain a signaling equilibrium, because in this case \(\partial \mu_r/\partial k < 0\) and greater investment will signal lower profitability. Notice that the existence of a performance report \((\beta > 0)\) is essential for the sorting condition to hold. Intuitively, in the absence of a performance report, any belief that higher types invest more than lower types invites deception, since all types would be tempted to invest large amounts, induce optimistic beliefs in the market, and sell out with impunity.

We now derive incentive compatibility constraints that must necessarily be satisfied by any fully revealing equilibrium. Suppose the schedules \(k(\mu)\), \(\beta(\mu)\) are fully revealing. For these schedules to be sustained in equilibrium, they must satisfy:

---

\(^6\) The firm’s objective function induced here by performance reporting is similar to the objective function in Miller and Rock’s [1985] analysis of dividend signaling.
where $W(.)$ is the objective function specified in (7). In specifying (9) we have used the fact that, since the investment and disclosure schedules are perceived as fully revealing, any type that chooses the pair $(k(\mu'), \beta(\mu'))$ will be assessed by the market to be of type $\mu'$.

It is convenient to express (9) in a slightly different form. Given fully revealing schedules $(k(\mu), \beta(\mu))$, let $V(\mu) = W(k(\mu), \beta(\mu), \mu; \mu)$, and let $\alpha = (1/2)\rho_c - \lambda > 0$. Then from (7):

$$V(\mu) = k(\mu)[\mu - 1] - \sigma k(\mu)^2 \left[ \lambda + \alpha \beta(\mu) \right].$$

Let:

$$V^*(\mu', \mu) = W(k(\mu'), \beta(\mu'), \mu'; \mu)$$

$$= -k(\mu') + \beta(\mu') k(\mu') \mu + (1-\beta(\mu')) k(\mu') \mu'$$

$$- \sigma k(\mu')^2 \left[ \lambda + \alpha \beta(\mu') \right]$$

$$= V(\mu') - \beta(\mu') k(\mu') [\mu' - \mu].$$

Then, the incentive compatibility constraints (9) are equivalent to:

$$V(\mu) \geq V(\mu') - \beta(\mu') k(\mu') [\mu' - \mu], \forall \mu, \mu'.$$

To see the role of the performance report in these incentive compatibility constraints, examine the market’s pricing rule given the belief that the firm chooses its investment and disclosure in accordance with the fully revealing schedules $(k(\mu), \beta(\mu))$. Since this pricing rule must be sequentially rational and market clearing, a choice of $k' = k(\mu')$ and $\beta' = \beta(\mu')$ will be priced at $P(y, k', \beta') = \beta'y + (1-\beta') k'[\mu' - \lambda(1-\beta') \sigma k'^2]$, regardless of the firm’s type. Thus for each realization of $y$, the market price is independent of the firm’s type and depends only on its choice of investment and disclosure. But, on average, the value of the performance report $\hat{y}$ is lower if the firm’s type is $\mu$ than if its type is $\mu' > \mu$ (because the performance report is affected by the firm’s true type, not by the type it pretends to be). Specifically, a firm of type $\mu$ that chooses the investment and disclosure of a higher type $\mu'$ expects to be sold at the price $E[P(y, k', \beta') | \mu] = \beta' k'[\mu + (1-\beta') k'[\mu' - \lambda(1-\beta') \sigma k'^2] + E[P(y, k', \beta') | \mu'] - \beta' k'[\mu' - \mu]$. If $k(\mu') > k(\mu)$ and $\beta(\mu') \geq \beta(\mu)$, as will turn out to be the case, a type $\mu$ firm that chooses the higher investment and disclosure of a type $\mu'$ firm bears the full cost of these choices but does not receive the full benefit, since the market price it expects is lower than the type $\mu'$ price. This disciplining effect of the performance report arises even though the report is used only to update the distribution of the firm’s future cash flows.

4. Characterization of the Optimal Investment and Disclosure Schedules

Since traders consciously extract information from observed choices of investment and disclosure and trade in a sequentially rational man-
ner, and since the firm tries to influence the market’s perception of its profitability through these choices, the appropriate equilibrium concept for this setting is that of a signaling equilibrium. However, the incentive compatibility constraints described in (9) and (11) assume that disclosure and investment can be bundled, in the sense that choosing \( k' = k(\mu') \) necessarily implies choosing \( \beta' = \beta(\mu') \). Such bundling, though consistent with a mechanism design approach, is inconsistent with the unconstrained choice of any \((k, \beta)\) combination in signaling equilibria. Also, the incentive compatibility conditions are necessary but not sufficient for signaling equilibria, since there is no assurance that some function of investment and disclosure will be invertible in \( \mu \). Nevertheless, we proceed to characterize the optimal investment and disclosure schedules subject only to the incentive compatibility constraints. We will show later how the bundling implicit in the incentive compatibility constraints can be implemented, and how the allocations derived from this mechanism design approach can be supported as a signaling equilibrium. The following result characterizes investment and disclosure schedules that are incentive compatible in the sense of (11).

**Theorem 1.** The schedules \( \{k(\mu), \beta(\mu)\} \) are incentive compatible if and only if \( V'(\mu) = \beta(\mu)k(\mu), \forall \mu \) and \( \beta(\mu)k(\mu) \) is increasing in \( \mu \).

Theorem 1 is a standard result in adverse selection settings with a continuum of types. It establishes that the continuum of incentive compatibility constraints can be replaced by a tractable differential equation combined with a montonicity requirement.

In order to determine the optimal investment and disclosure schedules among those that satisfy (11), consider the optimal control problem. Choose \( k(\mu) \) and \( \beta(\mu) \) to maximize \( \int_{\mu_L}^{\mu_H} V(\mu)f(\mu)\,d\mu \), subject to:

\[
V'(\mu) = \beta(\mu)k(\mu)
\]

(12)

where \( f(.) \) is the probability density function over \([\mu_L, \mu_H]\). In specifying this program, we have ignored the requirement that \( \beta k \) must be increasing in \( \mu \). We will verify that the solution to this relaxed program possesses this characteristic and also the characteristic that \( k(\mu) \) is strictly increasing. Let \( L(\mu) \) be the Lagrange multiplier associated with constraint (12). Then differentiating the Hamiltonian with respect to \( k(\mu) \) and \( \beta(\mu) \), respectively, yields the necessary conditions:

\[
\begin{align*}
|\mu - 1 - k(\mu)2\sigma [\lambda + \alpha \beta(\mu)] |f(\mu) + L(\mu)\beta(\mu) &= 0, \\
- k(\mu)^2 \sigma \alpha f(\mu) + L(\mu)k(\mu) &= 0.
\end{align*}
\]

In specifying the latter equation, we have claimed that the solution for \( \beta \) is interior. This is verified later. Dividing the first equation by the second yields the following condition on the marginal rate of substitution between \( \beta \) and \( k \):

\[
\mu - 1 - k(\mu)2\sigma [\lambda + \alpha \beta(\mu)] \\ -k(\mu)^2 \sigma \alpha
\]

(12)

Solving gives the
optimal investment schedule as a function of the optimal disclosure schedule:

$$k(\mu) = \frac{\mu - 1}{\sigma [2\lambda + \alpha \beta (\mu)]}.$$  \hspace{1cm} (13)

In the absence of the incentive compatibility constraints and given a fixed $\beta$, a type $\mu$ firm would invest the first-best amount $k = \frac{\mu - 1}{2\sigma [\lambda + \alpha \beta]}$.

Thus the need to signal the profitability of its investment causes the firm to overinvest relative to its disclosure. This kind of result usually arises in signaling models because overinvestment in the signal raises the cost for lower types to mimic higher types. However, in our setting there is an additional reason for overinvestment. The firm must choose an efficient combination of two signals $k$ and $\beta$. At first-best investment levels, $\partial V / \partial k = 0$, while $\partial V / \partial \beta < 0$. Thus an increase in investment has at most a second-order effect on the expected utility of current shareholders, while a decrease in disclosure has a first-order effect. Therefore, efficient signaling requires the firm to increase its investment above first-best and decrease its disclosure.

Before characterizing the optimal disclosure schedule, we verify that the monotonicity requirements for $\beta(\mu)k(\mu)$ and for $k(\mu)$ are satisfied.

**Theorem 2.** The optimal investment and disclosure schedules satisfy:

$$K'(\mu) = \frac{\beta(\mu)}{2\lambda \sigma}.$$  \hspace{1cm} (14)

$$\frac{\partial \{ \beta(\mu) k(\mu) \}}{\partial \mu} = \frac{1 - \beta(\mu)}{\sigma \alpha}.$$  \hspace{1cm} (15)

Thus the optimal investment schedule is strictly increasing at each point where $\beta(\mu) > 0$, which will turn out to be the case for each $\mu > \mu_L$. This result implies that the investment schedule characterized in (13) is fully revealing, justifying the price function we have used. Additionally, we will show that $\beta(\mu) < 1$ at each $\mu$, implying that $\beta(\mu)k(\mu)$ is also strictly increasing.

We now proceed to an explicit characterization of the optimal disclosure schedule. First, a preliminary result.

**Theorem 3.** (i) $\beta(\mu_L) = 0$, (ii) $\beta(\mu) > 0$, $\forall \mu > \mu_L$.

The intuition underlying this result is that disclosure is essential to deter low types from mimicking higher types; investment alone cannot perform this role. However, no deterrence is needed for communicating the lowest type since there is no incentive to pretend that profitability is low.
Since (13) specifies the optimal investment schedule as a function of the optimal disclosure schedule, the latter is the particular $\beta(\mu)$ schedule that reconciles (13) with incentive compatibility. Differentiating the expression for $V(\mu)$, as defined in (10), yields $V'(\mu) = k'(\mu)[\mu-1] + k(\mu) - 2\sigma k(\mu)k'(\mu) + \sigma(k(\mu))^2 \beta'(\mu)$. Thus, the necessary incentive compatibility condition that $V'(\mu) = \beta(\mu)k(\mu)$ is equivalent to

$$k'(\mu)\left[\frac{\mu-1}{k(\mu)} - 2\sigma[\lambda + a\beta(\mu)]\right] = \beta(\mu) - 1 + \sigma(k(\mu))\beta'(\mu).$$

In the left-hand side of the above equation, replace $k(\mu)$ by its optimal value characterized in (13) and replace $k'(\mu)$ by the expression derived in (14). This yields:

$$\beta'k\sigma a2\lambda = 2\lambda - 2\lambda\beta - a\beta^2$$ (16)

where the arguments of functions have been suppressed. The right-hand side of (16) is strictly decreasing in $\beta$, strictly positive at $\beta = 0$, and strictly negative at $\beta = 1$. Therefore, the equation $2\lambda - 2\lambda\beta - a\beta^2 = 0$ has a unique solution $\beta_F$ that satisfies $0 < \beta_F < 1$. For each $\beta < \beta_F$, the right-hand side of (16) is strictly positive, indicating that $\beta(\mu)$ is strictly increasing whenever $\beta(\mu) < \beta_F$. Additionally, if $\beta(\mu') = \beta_F$ for some $\mu'$, $\beta(\mu) = \beta_F$ for each $\mu > \mu'$. Thus there is an endogenous upper bound to the precision of the performance report arising from the self-interested behavior of firms. Firms with more favorable ex ante information will disclose more precise performance reports until the upper bound is reached, and thereafter the disclosure precision is constant.

Inserting the expression for $k(\mu)$ from (13) into (16) yields the first-order nonlinear differential equation:

$$\beta' a(\mu - 1) = \left(\frac{2\lambda + a\beta}{2\lambda}\right)(2\lambda - 2\lambda\beta - a\beta^2).$$ (17)

The optimal disclosure schedule is the solution to this differential equation, with initial condition $\beta(\mu_L) = 0$. The solution is characterized in Theorem 4 below.

**Theorem 4.** The equilibrium disclosure schedule $\beta(\mu)$ satisfies: (i) $\beta(\mu_L) = 0$, (ii) $0 < \beta(\mu) < \beta_F$ for $\forall \mu > \mu_L$, and (iii) $\beta(\mu)$ is nondecreasing and is implicitly characterized by:

$$\log\left[\frac{2\lambda + a\beta}{\sqrt{2\lambda} \sqrt{2\lambda - 2\lambda\beta - a\beta^2}}\right] + \frac{\lambda}{\sqrt{\lambda^2 + 2\alpha}} \left[\tanh^{-1}\frac{\lambda + a\beta}{\sqrt{\lambda^2 + 2\alpha}} - \tanh^{-1}\frac{\lambda}{\sqrt{\lambda^2 + 2\alpha}}\right] = \log\left[\frac{\mu - 1}{\mu_L - 1}\right].$$ (18)

The optimal investment and disclosure schedules are found by first solving the above equation for $\beta(\mu)$, inserting this solution in (13), and
then solving for $k(\mu)$. While these describe the optimal policies in principle, it is not immediately apparent how these allocations can be implemented via the rational beliefs of traders. We now turn to this issue.

5. Mandatory Disclosure and a Signaling Equilibrium

In constructing the optimal investment and disclosure schedules, we have not characterized any explicit function of $(k, \beta)$ that is used by the market to make its inferences. We have assumed, instead, that investment and disclosure are bundled so that when the firm chooses the bundle $(k(\mu'), \beta(\mu'))$ the market infers that the firm’s type is $\mu'$ and when the firm chooses an alternative bundle $(k(\mu''), \beta(\mu''))$ the market’s inference is $\mu''$. Since the optimal investment schedule is strictly increasing and the optimal disclosure schedule is nondecreasing in $\mu$, these market inferences are feasible and consistent with the firm’s choices. However, we have not considered all choices. To see this suppose there are only three possible values of $\mu$, $[\mu_1, \mu_2, \mu_3]$. The equilibrium choices of the firm can be characterized in terms of three combinations of $(k, \beta)$, say $(k_1, \beta_1, k_2, \beta_2, k_3, \beta_3)$. However, the firm is not restricted to choosing among these three combinations of $(k, \beta)$; in fact there are nine different choices available. For example, in principle, the firm could choose $(k_3, \beta_1)$. Similarly, in the continuum of types setting the firm could, in principle, choose $(k(\mu''), \beta(\mu'))$. Market inferences and therefore market prices are undefined for such choices.\(^7\) If the firm’s choices are to be guided entirely by the equilibrium pricing rule $P(y, k, \beta)$ in the capital market, as is the case in a signaling equilibrium, then this pricing rule needs to be well defined for all feasible combinations of $(k, \beta)$.

One way to solve this implementation problem is to regulate disclosure decisions, as is in fact done by bodies like the SEC and the Financial Accounting Standards Board. Since the optimal investment schedule characterized above is strictly increasing in $\mu$, it is possible to restate the disclosure schedule as a function of investment alone. This implies that a regulator can mimic the disclosure schedule we have calculated by mandating a disclosure schedule $D(k)$ which bundles investment and disclosure.\(^8\) Such disclosure regulation would require firms with higher investment to disclose their performance with greater precision than firms with lower investment, until the upper bound $\beta_F$ is reached.

We show how the mandatory disclosure schedule $D(k)$ can be calculated directly without knowledge of the optimal investment schedule.

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\(^7\)This is not merely an issue of beliefs off the equilibrium path. The bundling used in our analysis makes the incentive compatibility constraints much less restrictive and, therefore, has the potential to change the equilibrium.

\(^8\)We are not seeking a socially optimal regulation policy that would balance the interests of current and future owners of the firm. We ask the more limited question of how regulation can help firms implement disclosure policies that are in their self-interest.
This mandatory disclosure schedule supports a signaling equilibrium that avoids implementation problems and achieves the same allocations as that prescribed by the optimal mechanism characterized above. Hereafter, we use the notation \( \{ k^*(\mu), \beta^*(\mu) \} \) to refer to the optimal investment and disclosure schedules that firms would like to implement.

A mandatory disclosure schedule \( D(k) \) implements the policies \( \{ k^*(\mu), \beta^*(\mu) \} \) as a signaling equilibrium if there exists a pricing rule \( \varphi(y,k) \) and an inference rule \( \mu_r(k) \) such that for each \( \mu \in [\mu_L, \mu_H] \):

\[
k^*(\mu) = \text{Arg Max} \,- k + E[\varphi(\bar{y},k)|\mu,D(k)] - 1/2 \rho \text{var}[\varphi(\bar{y},k)], \tag{19}
\]

\[
D(k^*(\mu)) = \beta^*(\mu), \tag{20}
\]

\[
\varphi(y,k) = D(k)y + [1 - D(k)]k\mu_r(k) - \lambda (1 - D(k))\sigma k^2, \tag{21}
\]

\[
\mu_r(k^*(\mu)) = \mu. \tag{22}
\]

Condition (19) states that the investment schedule maximizing the expected utility of its current owners is the same as that prescribed by the optimal mechanism, given the required disclosure schedule. Condition (20) states that, along the equilibrium investment schedule, the firm’s mandatory disclosure is the same as that prescribed by the optimal mechanism. Condition (21) states that, given the inference that outsiders make from observation of the firm’s investment, the pricing rule is sequentially rational and market clearing. Finally, (22) requires that the inferences made by outsiders are consistent with the firm’s equilibrium investment policy.

**Theorem 5.** The optimal allocations \( \{ k^*(\mu), B^*(\mu) \} \) are implemented as a signaling equilibrium by the mandatory disclosure schedule \( D(k) \) characterized by:

\[
\log \frac{k}{k_L} = \frac{1}{2} \log \left[ \frac{2\lambda}{2\lambda - 2\lambda D - aD^2} \right] - \frac{\lambda}{\sqrt{\lambda^2 + 2\lambda a}} \left[ \tanh^{-1} \left( \frac{\lambda + aD}{\sqrt{\lambda^2 + 2\lambda a}} \right) - \tanh^{-1} \left( \frac{\lambda}{\sqrt{\lambda^2 + 2\lambda a}} \right) \right] \tag{23}
\]

for \( \forall \ k \geq k_L, \ D(k) = 0 \) for \( k \leq k_L \), where \( k_L = k^*(\mu_L) = (\mu_L - 1) / 2\lambda a \). In this signaling equilibrium, the inference schedule is:

\[
\mu_r(k) = k\sigma[2\lambda + aD(k)] + 1 \tag{24}
\]

and the pricing rule is:

\[
\varphi(y,k) = D(k)y + [1 - D(k)]\left[ \sigma k^2(\lambda + aD(k)) + k \right]. \tag{25}
\]

In this signaling equilibrium, market inferences, prices, and disclosure requirements are defined for all feasible investment levels, even those that are off the equilibrium path. The firm makes its own choice
of investment and is guided only by the capital market’s pricing rule and the disclosure requirements. The information needed by the regulator to calculate the mandatory disclosure schedule consists only of the aggregate risk aversion in the capital market $\lambda$, the aggregate risk aversion of the firm’s current owners $\rho_e$, and the smallest investment $k_L$ that the firm would undertake.

6. Conclusion

We have identified and studied a disciplinary role for periodic performance reports released to the capital market. Doubtless, performance reports are also used for the control of operating decisions and compensation contracting. However, internal performance measurements are different from the external reporting that is the focus of this study. Consistent with the FASB’s Conceptual Framework [1978], we have modeled investors as the primary users of external financial reports and firms’ investments as the key decision that could be affected by these reports.

Our analysis has shown that periodic performance reports, such as earnings statements, do more than provide information for predicting future cash flows. The anticipation of future performance reports disciplines management’s investment choices so that these choices themselves transmit information to the capital market. Since the precision of the report and the firm’s investment are chosen simultaneously, and the former choice disciplines the latter, there is value to bundling the two choices. This bundling creates a need for regulating the precision of performance reports; without regulatory intervention the economy would face a difficult implementation problem.

Rajan and Sarath [1996] study a complementary bundling problem in which there are two pieces of information to be communicated and only one signal. They show that if prices are constrained to be sequentially rational given the information in the signal, then full disclosure is impossible and the two pieces of information must be aggregated in a particular way. In our setting, the information to be communicated is one-dimensional but there are multiple signals whose interaction is essential for full disclosure.

In our model, trade between prospective and current shareholders occurs only after the performance report is released. If prospective shareholders made an inference from the firm’s observed investment and traded before seeing the performance report, they would be deceived because the firm’s incentives would change and the equilibrium would unravel. The empirical implication here is that large price movements and trading volume would be observed when performance reports were released and not when investment choices were observed.

The financial and real sides of a decentralized economy interact in complex ways to value firms and allocate investment. When the financial side is ill-informed, frictions are introduced and investment is misallocated (see Kanodia [1980]). Accounting measurements and disclosure
play an important role in reducing such frictions. Kanodia and Mukherji [1996] demonstrated that firms would underinvest if accounting measurements failed to distinguish between operating and investment expenditures. In that setting, periodic performance reports alleviate the underinvestment problem. In the setting studied here, the presence of performance reports allows other credible signals to emerge and communicate management's private information about the firm's profitability. Thus performance reports have real consequences, in the sense that they alter the very cash flow they help to predict.

APPENDIX A

Proof of Theorem 1

First, we prove the necessity part of the theorem. Consider any two types, $\mu$ and $\mu'$. The incentive compatibility (IC) constraint for type $\mu$ is $V(\mu) \geq V(\mu') - \beta(\mu') k(\mu') [\mu' - \mu]$, while the IC constraint for type $\mu'$ is $V(\mu') \geq V(\mu) - \beta(\mu) k(\mu) [\mu - \mu']$. Thus the IC constraints are equivalent to:

$$\beta(\mu) k(\mu) [\mu' - \mu] \leq V(\mu') - V(\mu) \leq \beta(\mu') k(\mu') [\mu' - \mu], \quad \forall \mu, \mu'. \quad (A1)$$

It follows immediately from (A1) that $\mu' > \mu \implies \beta(\mu) k(\mu) \leq \beta(\mu') k(\mu')$. In turn, this monotonicity implies that $\beta(\mu) k(\mu)$ is continuous almost everywhere. Dividing (A1) by $[\mu' - \mu]$ and taking the limit as $\mu' \to \mu$ yields $V'(\mu) = \beta(\mu) k(\mu)$.

Next, we prove the sufficiency part of the theorem. Integrating (12) yields:

$$V(\mu) = V(\mu_L) + \int_{\mu_L}^{\mu} k(t) \beta(t) dt. \quad (A2)$$

Now:

$$V(\mu) - V^*(\mu, \mu) = [V(\mu) - V(\mu')] - [V^*(\mu, \mu) - V(\mu')]$$

$$= \int_{\mu'}^{\mu} k(t) \beta(t) dt - k(\mu') \beta(\mu') [\mu - \mu']$$

$$= \int_{\mu'}^{\mu} [k(t) \beta(t) - k(\mu') \beta(\mu')] dt.$$

If $\mu' < \mu$, the above integral is nonnegative given that $k(t) \beta(t) \geq k(\mu') \beta(\mu')$ for each $t > \mu'$. If $\mu' > \mu$, the above integral becomes:

$$\int_{\mu'}^{\mu} [k(\mu') \beta(\mu') - k(t) \beta(t)] dt$$

which is again nonnegative.
Proof of Theorem 2

We first prove (14). Inserting (13) into (10) and simplifying yields $V(\mu) = \lambda \sigma [k(\mu)]^2$, which implies $V'(\mu) = 2\lambda \sigma k(\mu) k'(\mu)$. But $k(\mu)$, as derived in (13), satisfies the IC constraint $V'(\mu) = \beta(\mu) k(\mu)$. Therefore:

$$k'(\mu) = \frac{\beta(\mu)}{2\lambda \sigma}. \quad (A3)$$

We now prove (15). From (13):

$$k(\mu) [2\lambda + \alpha \beta(\mu)] = \frac{\mu - 1}{\sigma}.$$

Differentiating the above with respect to $\mu$ yields:

$$k'(\mu) [2\lambda + \alpha \beta(\mu)] + k(\mu) \alpha \beta'(\mu) = \frac{1}{\sigma}.$$

Thus:

$$k'(\mu) \beta(\mu) + k(\mu) \beta'(\mu) = \frac{1}{\sigma \alpha} - \frac{k'(\mu) 2\lambda}{\alpha}. \quad (A4)$$

But (A3) implies $k'(\mu) 2\lambda = \beta(\mu) / \sigma$. Inserting this in (A4) yields

$$\frac{d}{d\mu} \beta(\mu) k(\mu) = \frac{1}{\sigma \alpha} [1 - \beta(\mu)].$$

Proof of Theorem 3

(i) From (A2) it is clear that $\beta(\mu_L)$ and $k(\mu_L)$ affect $V(\mu)$ only through $V(\mu_L)$, and the bigger the value of $V(\mu_L)$, the bigger is $V(\mu)$ for each $\mu > \mu_L$. Therefore, $[\beta(\mu_L), k(\mu_L)]$ must maximize $V(\mu_L)$. This implies that $\beta(\mu_L) = 0$ and $k(\mu_L) = [\mu_L - 1]/2\lambda \sigma$.

(ii) We now establish that $\beta(\mu) > 0$, $\forall \mu > \mu_L$. Consider any pair of types, $(\mu_1, \mu_2)$ such that $\mu_1 < \mu_2$. The IC constraints require $V(\mu_1) \geq V(\mu_2) - \beta(\mu_2) k(\mu_2) [\mu_2 - \mu_1]$. Inserting (12) into (10), we find that for each $\mu$:

$$V(\mu) = \lambda \sigma \frac{[\mu - 1]^2}{[\sigma (2\lambda + \alpha \beta(\mu))]^2}.$$

If $\beta(\mu_2) = 0$, we have:

$$V(\mu_2) - \beta(\mu_2) k(\mu_2) [\mu_2 - \mu_1] = \lambda \sigma \frac{[\mu_2 - 1]^2}{[\sigma (2\lambda)]^2} > \lambda \sigma \frac{[\mu_1 - 1]^2}{[\sigma (2\lambda)]^2}$$

$$\geq \lambda \sigma \frac{[\mu_1 - 1]^2}{[\sigma (2\lambda + \alpha \beta(\mu))]^2} = V(\mu_1).$$
Therefore, $\beta(\mu_2) = 0$ violates the IC constraint specified above; and therefore, $\beta(\mu_2) > 0$.

**Proof of Theorem 4**

Parts (i) and (ii) of the theorem have already been established. We proceed to part (iii). State (17) in its equivalent differential form:

$$\alpha(\mu - 1)\, d\beta - \left[\frac{2\lambda + a\beta}{2\lambda}\right]\left[2\lambda - 2\lambda\beta - a\beta^2\right]d\mu = 0.$$  

Note that the above is a separable equation. In the region where $2\lambda - 2\lambda\beta - a\beta^2 > 0$, i.e., when $\beta < \beta_F$, the above expression can be multiplied through by its integrating factor to yield the following equivalent equation:

$$\frac{d\mu}{\mu - 1} - \frac{2\lambda\alpha\, d\beta}{(2\lambda + a\beta)\left(2\lambda - 2\lambda\beta - a\beta^2\right)} = 0.$$  

Integrating yields the one-parameter family of solutions:

$$\log(\mu - 1) - \int \frac{2\lambda\alpha\, d\beta}{(2\lambda + a\beta)\left[2\lambda - 2\lambda\beta - a\beta^2\right]} = C \quad (A5)$$

where $C$ is an arbitrary constant. The integral in this expression can be written as:

$$\alpha \int \left(\frac{\beta}{2\lambda - 2\lambda\beta - a\beta^2} + \frac{1}{2\lambda + a\beta}\right)\, d\beta = \log(2\lambda + a\beta) - \frac{1}{2} \int \left(\frac{-2\lambda - 2\lambda\beta}{2\lambda - 2\lambda\beta - a\beta^2} + \frac{2\lambda}{2\lambda - 2\lambda\beta - a\beta^2}\right)\, d\beta$$

$$= \log(2\lambda + a\beta) - \frac{1}{2} \log(2\lambda - 2\lambda\beta - a\beta^2) - \lambda \int \frac{d\beta}{2\lambda - 2\lambda\beta - a\beta^2}.$$  

Carrying out the last integration and inserting in (A5) yields:

$$\log \left[\frac{2\lambda + a\beta}{\sqrt{2\lambda - 2\lambda\beta - a\beta^2}}\right] - \frac{\lambda}{\sqrt{\lambda^2 - 2\lambda\alpha}}$$

$$\tanh^{-1} \left[\frac{\lambda + a\beta}{\sqrt{\lambda^2 - 2\lambda\alpha}}\right] = \log(\mu - 1) - C. \quad (A6)$$

The value of $C$ is calculated from the initial condition $\beta(\mu_L) = 0$. Inserting $\beta = 0$ and $\mu = \mu_L$ in (A6) yields:

$$C = \log(\mu_L - 1) - \log(\sqrt{2\lambda}) + \frac{\lambda}{\sqrt{\lambda^2 - 2\lambda\alpha}} \tanh^{-1} \left[\frac{\lambda}{\sqrt{\lambda^2 - 2\lambda\alpha}}\right].$$

Inserting this value of $C$ in (A6) and simplifying yields the desired result.
The left-hand side of (18) is strictly increasing in $\beta$ over the open interval $(0, \beta_F)$, and the right-hand side of (18) is strictly increasing in $\mu$. Therefore, $\beta(\mu)$ is strictly increasing over the interval where (18) has a solution.

At $\beta = \beta_F$ equation (18) is no longer valid, since $2\lambda - 2\lambda \beta_F - a\beta_F^2 = 0$.

Also, $\beta_F = \frac{\sqrt{2\lambda a + \lambda^2} - \lambda}{a}$, so that $\tanh^{-1}\left[\frac{\lambda + a\beta_F}{\sqrt{\lambda^2 + 2\lambda a}}\right] = \tanh^{-1}(1) = \infty$.

Thus when $\mu$ becomes sufficiently large, $\beta' = 0$ and $2\lambda - 2\lambda \beta - a\beta^2 = 0$, so $\beta = \beta_F$ is the only solution to (17).

**Proof of Theorem 5**

We start with a conjectured investment policy for the firm, derive market beliefs and market clearing prices from it, then go on to show that if the mandatory disclosure schedule is as constructed in (23), the firm’s actual investment policy will coincide with the conjectured investment policy. Finally, we show that in this signaling equilibrium the investment and disclosure schedules are the same as that prescribed by the optimal mechanism.

Suppose that, given a mandatory disclosure schedule $D(k)$, the firm’s investment policy is:

$$k = \frac{\mu - 1}{\sigma[2\lambda + aD(k)]}.$$  \hfill (A7)

Then the inferred value of $\mu$ consistent with this investment policy is:

$$\mu_r = k\sigma[2\lambda + aD(k)] + 1,$$ \hfill (A8)

as specified in (24). Given any inferred value $\mu_r$, the price that clears the capital market, for each realization of $\tilde{y}$ is $\phi(y,k) = D(k)y + [1-D(k)]k\mu_r(k) - \lambda[1-D(k)]\sigma k^2$. Inserting the particular inference described in (A8) yields:

$$\phi(y,k) = D(k)y + [1-D(k)]\left[\sigma k^2(2\lambda + aD(k)) + k\right] - \lambda[1-D(k)]\sigma k^2$$ \hfill (A9)

$$= D(k)y + [1-D(k)]\left[\sigma k^2(\lambda + aD(k)) + k\right],$$

as specified in (25). Calculating $E[\phi(\tilde{y},k)|\mu, D(k)]$ and $\text{var}[\phi(\tilde{y},k)]$ from (A9), and inserting them into (19) yields the firm’s objective function:

$$\text{Max} -k + D(k)ku + (1-D(k))[\sigma k^2(\lambda + aD(k)) + k]$$

$$- 1/2 \rho_e D(k)\sigma k^2.$$ \hfill (A10)

Using $\lambda + aD(k) = D(k)1/2 \rho_e + [1-D(k)]\lambda$, the firm’s objective function reduces to:

$$\text{Max} D(k)k[\mu - 1] - \sigma k^2[D(k)^2 1/2 \rho_e - (1-D(k))^2 \lambda].$$ \hfill (A11)
The first-order condition with respect to \( k \) yields:
\[
D(k) [\mu - 1] - 2\sigma k[D(k)^2 1/2 \rho_e - (1 - D(k))^2 \lambda] + D'(k) [k(\mu-1) - 2\sigma k^2 (1 - D(k))\lambda + D(k) 1/2\rho_e] = 0. \tag{A12}
\]

Using \( \alpha = 1/2 \rho_e - \lambda \), (A12) can be written as:
\[
D(k) [\mu - 1 - \sigma k[2\lambda + \alpha D(k)]] - D(k)\sigma k [2\lambda + \alpha D(k)] + 2\lambda \sigma k + D'(k) [k(\mu-1) - 2\sigma k^2 (\lambda + \alpha D(k))] = 0. \tag{A13}
\]

Now evaluate (A13) along the investment schedule \( k(\mu) \) specified in (A7). Since \( D(k(\mu)) [\mu - 1 - \sigma k(\mu)[2\lambda + \alpha D(k(\mu))]] = 0, k(\mu) \), as specified in (A7), satisfies (A13) if:
\[
D(k(\mu))\sigma k(\mu) [2\lambda + \alpha D(k(\mu))] - 2\lambda \sigma k(\mu) = D'(k(\mu))(\mu)(\mu-1 - 2\sigma k(\mu)^2 (\lambda + \alpha D(k(\mu))). \tag{A14}
\]

Inserting \( k(\mu) \) from (A7), the left-hand side of (A14) is:
\[
\frac{\mu - 1}{2\lambda + \alpha D(k(\mu))} [2\lambda D(k(\mu)) + \alpha D(k(\mu))^2 - 2\lambda], \text{ and the right-hand side of (A14) =}
D'(k(\mu))k(\mu) \left[ \mu - 1 - \frac{2(\mu - 1) (\lambda + \alpha D(k(\mu))}{2\lambda + \alpha D(k(\mu))} \right] = - D'(k(\mu))k(\mu)\alpha D(k(\mu)) \left[ \frac{\mu - 1}{2\lambda + \alpha D(k(\mu))} \right]. \text{ Therefore, if } D(k) \text{ satisfies:}
\]
\[
D'(k)k \alpha D(k) = 2\lambda - 2\lambda D(k) - \alpha D(k)^2, \forall k > k_L, \tag{A15}
\]

then \( k(\mu) \) must satisfy (A13) for \( \forall \mu > \mu_L \). Solving (A15) as a differential equation in the region \( k > k_L, D > 0 \) yields the one-parameter family of solutions:
\[
\log(k) = A - \frac{1}{2} \log(2\lambda - 2\lambda D - 2\alpha D^2) - \frac{\lambda}{\sqrt{\lambda^2 + 2\lambda \alpha}} \tanh^{-1} \left[ \frac{\lambda + \alpha D}{\sqrt{\lambda^2 + 2\lambda \alpha}} \right].
\]

The constant \( A \) is found by inserting the initial condition \( D(k_L) = 0 \). This yields equation (23) of the theorem.

We have shown, so far, that if \( k(\mu) \) satisfies (23), the investment schedule \( k(\mu) \) specified in (A7) is self-sustaining. When traders use this investment schedule to infer \( \mu \) from the observed investment, the pricing rule that results in the capital market is such that this same schedule \( k(\mu) \) is optimal for the firm. It remains to show that \( D(k(\mu)) = \beta^*(\mu), \) and \( k(\mu) = k^*(\mu). \)

We first examine the firm’s investment at \( \mu_L \), and show that the first-order condition (A13) is satisfied at \( k = k_L \). Since \( D(k_L) = 0 \), evaluate (A13) at \( D = 0, \) and \( \mu = \mu_L \). Then (A13) becomes \( 2\lambda \sigma k + D'(k)(\mu)(\mu - 1 - 2\sigma k) = 0, \) which yields \( k = \frac{\mu_L - 1}{2\lambda \sigma} + \frac{1}{D'(k)}. \)
To evaluate $D'(k)$, note that for $D(k) > 0$ (A15) can be rewritten as:

$$D'(k)k = \frac{2\lambda}{\alpha D(k)} - \frac{2\lambda}{\alpha} - D(k).$$

Clearly, $D'(k) \to \infty$ as $D(k) \to 0$, or equivalently as $k \to k_L$ from above. Therefore, $k_L = (\mu - 1)/2\lambda\sigma = k^*(\mu_L)$ is optimal when $\mu = \mu_L$.

Now consider $\mu > \mu_L$. From (A7):

$$\frac{k(\mu)}{k_L} = \frac{\mu - 1}{\mu - 1} \frac{2\lambda}{2\lambda + \alpha D(k(\mu))}.$$

Therefore:

$$\log \frac{k(\mu)}{k_L} = \log \frac{\mu - 1}{\mu - 1} + \log \frac{2\lambda}{2\lambda + \alpha D(k(\mu))}.$$

Inserting this expression in (23) yields:

$$\log \frac{2\lambda + \alpha D(k(\mu))}{\sqrt{2\lambda} \sqrt{2\lambda - 2\lambda D(k(\mu))} - \alpha D(k(\mu))^{\frac{1}{2}}} = \frac{\lambda}{\sqrt{\lambda^2 + 2\lambda\alpha}}$$

$$[\tanh^{-1} \frac{\lambda + \alpha D(k(\mu))}{\sqrt{\lambda^2 + 2\lambda\alpha}} - \tanh^{-1} \frac{\lambda}{\sqrt{\lambda^2 + 2\lambda\alpha}}]$$

$$= \log \frac{\mu - 1}{\mu - 1}. \quad (A16)$$

Comparing (A16) to (18) it is clear that $D(k(\mu)) = \beta^*(\mu)$, and therefore $k(\mu) = k^*(\mu)$.

REFERENCES


