Hedge Disclosures, Future Prices, and Production Distortions

CHANDRA KANODIA,∗ ARIJIT MUKHERJI,† HARESH SAPRA,‡ AND RAGHU VENUGOPALAN∗

ABSTRACT

In this paper, we identify social benefits to hedge accounting disclosures that have not previously been examined. We show that from the perspective of price efficiency in the futures market the key information that is provided by hedge accounting is information about firms’ underlying risk exposures. Without this information, the futures price confounds information regarding firms’ hedge-motivated trades with their speculative trades, making the futures price inefficient. Our model shows that an inefficient futures price causes significant externalities by distorting the production choices of an entire industry. In the presence of hedge disclosures, the futures price appropriately informs production decisions in the whole industry. In addition to distortion in production choices, we also investigate the effect of an inefficient futures price on the risk-sharing role of the futures market. We find that

Arijit Mukherji passed away in October 2000. He was our colleague, mentor, advisor, and friend. We dedicate this paper to his memory and his contributions to the field of accounting. He will be missed.

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lack of appropriate information about hedge disclosures also distorts the risk-sharing role of the futures market, thereby resulting in an increase in risk premium embedded in the futures price. Using numerical calculations, we demonstrate that the magnitude of the distortions in expected industry output can be substantial.

[KEYWORDS: hedge disclosures; future prices; information efficiency; production distortions.]

1. Introduction

The controversy surrounding Statement of Financial Accounting Standards No. 133: Accounting for Derivative Financial Instruments and Hedging Activities [1998] (hereafter SFAS No. 133) has centered on its effect on the volatility of reported income induced by marking to market firms' derivative positions. The Financial Accounting Standards Board (FASB) seems to have adopted the position that such volatility reflects the true economic state of the firm, that derivatives create new risks that are poorly understood, and therefore that providing information on the market value changes of firms' derivative positions will make firms' risk characteristics more transparent to investors in the capital market. On the other hand, industry leaders argue that the reported volatility is misleading and that, rather than creating new risks, derivative instruments are used to manage and reduce the risks inherent in their business (see Greenspan [1997] and Wolfe [1997]).

In this paper, we shed new light on the need for hedge-related disclosures by developing a perspective quite different from that centered on the induced volatility of reported income. This new perspective is consistent with the argument that derivatives are used to manage risks, but rather than focusing on the informational needs of the capital market, we examine the effect of disclosure on price efficiency in the futures market. From the perspective of a futures market, the volatility of a firm's reported income is unimportant; what is important is the aggregation of beliefs regarding spot prices in commodity markets. Agents trade in the futures market on the basis of their beliefs regarding the uncertainties that affect future spot prices. Thus, as developed in Danthine [1978], Grossman [1976; 1977], and Bray [1981], a futures market aggregates and communicates information, that may initially be dispersed over a large number of agents in the economy, about demand and supply conditions in future commodity spot markets.

The informational properties of a futures price has strong implications for the production decisions of every producer in the industry. The link between the futures price and aggregate industry output is due to an important separation result, first derived in Danthine [1978]. Danthine established that, given a futures market, the production decisions of individual firms in the industry are guided solely by the current price in the futures market and are independent of any information that a firm may have regarding future demand and supply conditions in commodity spot markets. Such information affects a firm's production
plans only to the extent that it affects the equilibrium futures price; and if it affects the equilibrium futures price, it affects the production of every other firm in the industry. Thus, there are industry-wide externalities to disclosure. A futures price that is informationally inefficient will misguide the production decisions of every producer in the industry, and this real effect could potentially be very significant.

To illustrate the significance of Danthine’s separation result, consider a wheat farmer who has private information that the demand for wheat at the date of harvest will be booming and, consequently, the future spot price for wheat will be high. It might seem that such beliefs would induce the farmer to plant a large amount of wheat. The separation result indicates that this intuition is false. The wheat farmer’s production would be guided entirely by the futures price for wheat, and the farmer’s beliefs would be used only to take a position in the futures market. Similarly, a building contractor, negotiating an upfront price for a home to be built over the next six months, should be guided solely by the prevailing futures price in the lumber market rather than by his/her beliefs regarding the uncertainties in future lumber prices.

Given Danthine’s separation result, we study how equilibrium prices in futures markets are affected by public disclosure of firms’ hedge-related transactions. We demonstrate that a producer’s optimal position in the futures market consists of two components. The first component is motivated purely by hedging considerations, in the sense that it depends solely on the firm’s production plans. The second component is informationally motivated and depends exclusively on the firm’s beliefs regarding conditions in future commodity spot markets. We show that if the information in the futures market is inadequate to disentangle these two components, then the equilibrium futures price will misguide production decisions. We show that, absent appropriate hedge disclosures, such confounding is inevitable.

Futures prices that are informationally inefficient for this reason are biased downward relative to an economy where all information is transparent. This downward bias leads to a potentially significant reduction in aggregate industry output. To provide some sense of magnitudes, we compute the percentage loss in expected industry output for a wide range of parameter values and examine the sensitivity of this loss to each of the key parameters in our model. We find that for the range of parameters examined, the decline in expected industry output ranges from 2.5% to 22.5%. We establish that the absence of hedge disclosures causes industry output to be imprecisely tuned to fluctuations in consumer demand for the commodity, inducing overproduction when consumer demand is low and underproduction when consumer demand is high. This insensitivity to consumer demand increases the volatility of commodity spot prices.

Futures markets provide a venue for risk sharing. A derivatives trade that offsets some inherent risk faced by an individual firm is viewed by
that firm as an instrument for reducing and managing its risk exposure. But, from a social perspective, this risk management is simply a reassignment of risk to others in the economy, making a futures market an instrument for risk sharing. We demonstrate that, in addition to the downward distortion in production decisions, the absence of hedge disclosures hinders the efficient allocation of risk, thereby increasing the risk premium that is embedded in the futures price. To understand how information asymmetries influence risk sharing and risk premiums, we use Wilson’s [1968] theory of syndicates to calculate the risk premium from the “surrogate” preferences and beliefs of a representative individual.

Surprisingly, the consideration of futures markets is completely absent from the current public debate surrounding SFAS No. 133 and has also been ignored in the academic literature. The emphasis, in the popular press, has been exclusively on the informational needs of the capital market. The disclosure effects we study are more fundamental, in the sense that industry output and risk sharing in commodity markets must surely affect capital market prices. Thus, any analysis of hedge disclosures that does not consider its impact on futures markets is substantially incomplete.

Most of the academic literature on hedge disclosures has focused on the incentive issues surrounding a manager’s hidden talents or efforts. DeMarzo and Duffie [1995] analyze a model of hedging where profits serve as a signal of the manager’s ability. They show that if hedge positions are disclosed, a risk-averse manager would forgo desirable hedge opportunities because hedging would make profits a more informative signal of his/her ability. On the other hand, if hedge positions are not disclosed, the manager would fully hedge. Jorgensen [1997] compares deferral hedge accounting to mark-to-market hedge accounting in an agency setting where the manager may or may not be informed about the future spot price. He shows that hedging alleviates the moral hazard problem if the manager is uninformed. However, if the manager is privately informed, the moral hazard problem is exacerbated, although shareholders benefit from the profits arising from the manager’s futures trades. Jorgensen also shows that under deferral hedge accounting, managers always prefer the first-best hedge position, but under mark-to-market hedge accounting, the manager’s hedge position may be distorted.

Melumad, Weyns, and Ziv [1999] study the effect of alternative hedge disclosures on the hedging choices of a firm that is concerned with the price at which it is traded in the capital market at an interim date. In

1 Production effects of disclosure have also been studied in the information-sharing literature (Gal-Or [1985] and Kirby [1988]). In this literature, firms directly exchange information to strategically influence the production of rivals. The informational efficiency of prices is not an issue in this literature, and it is unclear how this literature could be used to examine the economic consequences of hedge disclosures.
their model, the firm is endowed with a random quantity of a risky asset whose return can be hedged in a futures market. They show that information about the firm’s asset endowments, revealed at the interim date, affects the firm’s incentives to hedge at the initial date. Comprehensive fair value hedge accounting reveals sufficient information at the interim date about the firm’s asset endowments to sustain first-best hedging policies, while deferral hedge accounting leads to less than first-best hedging policies.

Our study differs significantly from the previous accounting literature on hedge disclosures. Although previous studies have examined the private benefits or costs to individual firms or managers under different hedge accounting methods, the effect of hedge disclosures on the functioning of markets, and thereby on the economy-wide allocation of real resources, has not previously been examined. Private benefits and costs result in private incentives for disclosure and usually do not call for disclosure regulation. The aggregate effects, social consequences, and externalities associated with disclosure must surely be of greater concern to regulators. Our analysis focuses on the externalities associated with hedge disclosures. We show that hedge disclosures affect the informational properties of futures prices which, in turn, affect the aggregate output of the industry and thereby affect both producers and consumers.

In section 2 we describe the setting that we study and derive the separation result. Section 3 characterizes equilibrium futures prices in the absence of hedge disclosures and section 4 shows how the equilibrium changes in the presence of hedge disclosures. In section 5, we compare the equilibrium futures price in the two regimes and relate this to differences in industry output. The distortions in risk sharing and the increase in risk premium due to the absence of hedge disclosures are characterized in section 6. Section 7 provides numerical calculations to illustrate the magnitude of distortions in industry output and the sensitivity of these distortions to various parameters. Finally, in section 8 we indicate some limitations to our analysis and describe possible extensions. Proofs of propositions are contained in Appendix A.

2. The Model

Consider an industry with $N + 1$ producers indexed by $i = 0, 1, \ldots, N$. Resources are committed to production at date 1, but the output from production is available for sale only at date 2. At date 1, when production decisions are made, the date 2 commodity spot price $\hat{p}$ at which output is sold is uncertain because of uncertainties in both industry supply and consumer demand. However, there is a futures market at date 1, where producers can hedge against the uncertain spot price or take any speculative position they choose. Let $p_f$ be the price in the futures market, where $p_f$ represents the price of a contract that promises delivery of
one unit of the commodity at date 2. Thus, ex ante, the profit of a producer, who has known production of \( q \) units and sells \( z \) units of futures contracts, is a random variable described by:

\[
\tilde{\pi} = zp_f + (q - z)\tilde{p} - c(q), \tag{1}
\]

In (1) the quantity \((q - z)\), which could be positive or negative, represents the net trade of the producer in the date 2 spot market, and \(c(q)\) is her production cost, assumed to be increasing and strictly convex.

We assume that the production plans of producers 1, ..., \( N \) are deterministic and publicly known, with production quantities \( q_1, \ldots, q_N \). However, producer \( i = 0 \) (hereafter called the informed producer) has private information about her production. We model this by assuming that the resources committed to production by the informed producer determines her expected output, but her actual output fluctuates randomly around the mean due to yield uncertainties. Formally, the informed producer’s output is \( q_0 + \theta \), where \( \theta \) is a normally distributed random variable with zero mean and variance \( \sigma_\theta^2 \), while her production cost \( c(q_0) \) depends only on her expected output. The informed producer knows the value of \( \theta \), but others in the industry know only the informed producer’s expected output \( q_0 \) and the distribution of \( \theta \).

All producers are risk averse, with strictly concave utility functions \( U_i(\pi_i) \). Producer \( i, i \in [1, \ldots, N] \) solves:

\[
\text{Max}_{q_i} E_i[U_i(z_ip_f + (q_i - z_i)\tilde{p} - c(q_i))], \tag{2}
\]

where \( E_i \) is the expectation operator over the random variable \( \tilde{p} \), conditional on producer \( i \)'s information. The information available to producer \( i \) will be specified later. The informed producer solves:

\[
\text{Max}_{q_0, z_0} E_0[U_0(z_0p_f + (q_0 + \theta - z_0)\tilde{p} - c(q_0))]. \tag{3}
\]

2.1 CONDITIONAL SPOT MARKET EQUILIBRIUM

Demand conditions in the spot market are described by the linear downward-sloping demand function:

\[
d = \tilde{\eta} + \tilde{\gamma} - \tilde{p} \tag{4}
\]

where \( d \) is the quantity demanded, \( \tilde{p} \) is the spot price, \( \tilde{\eta} \) represents random shifts in the demand function, and \( \tilde{\gamma} \) is to be interpreted as noise in consumer demand. The random variables \( \tilde{\eta} \) and \( \tilde{\gamma} \) are normally distributed with \( E(\tilde{\eta}) = \mu > 0 \) and \( \text{Var}(\tilde{\eta}) = \sigma_\eta^2 \), \( E(\tilde{\gamma}) = 0 \) and \( \text{Var}(\tilde{\gamma}) = \sigma_\gamma^2 \). Further \( \tilde{\eta} \), \( \tilde{\gamma} \), and \( \tilde{\theta} \) are independently distributed. Spot market clearing implies that conditional on equilibrium production decisions, \( q_0^*, \ldots, q_N^* \), the equilibrium spot price is described by:

\[
\tilde{p} = \tilde{\eta} + \tilde{\gamma} - \sum_{i=0}^{N} q_i^* - \tilde{\theta}. \tag{5}
\]
All producers in the industry correctly believe that the distribution of \( \hat{p} \) is governed by (5). Since they are all price takers, their beliefs regarding \( \hat{p} \) are independent of their own production choices, implying that the equilibrium aggregate output in (5) is viewed as a constant perturbed by the random variable \( \theta \).

2.2 PRODUCTION DECISIONS: A SEPARATION RESULT

We now derive a key result (due to Danthine [1978]) regarding the effect of the futures price on the production decisions of each producer in the industry.

**PROPOSITION 1.** Producers' choices of production quantities, \( q_0, q_1, \ldots, q_N \), depend only on the futures price \( p_f \) and are independent of their risk aversion and their expectations of the spot price \( \hat{p} \).

**Proof.** First consider the informed producer, whose maximization problem is described in (3). The first-order conditions with respect to \( z_0 \) and \( q_0 \) are:

\[
E_0[U_0'(\pi_0)(p_f - \hat{p})] = 0
\]  
and:

\[
E_0[U_0'(\pi_0)(\hat{p} - c'(q_0))] = 0.
\]  
From equation (6), we get:

\[
p_f E_0[U_0'(\pi_0)] = E_0[U_0'(\pi_0)\hat{p}].
\]  
Inserting (8) into (7) gives:

\[
p_f E_0[U_0'(\pi_0)] = c'(q_0) E_0[U_0'(\pi_0)],
\]  
which implies:

\[
c'(q_0) = p_f.
\]  

The intuition for the result in Proposition 1 is as follows. Suppose an individual producer has information indicating that demand conditions are very favorable and therefore that the spot price will be very high. It might seem that this belief would lead her to produce a large quantity of the commodity. However, this intuition is false; what is true is that she would want to enter the spot market with a large amount of the commodity on hand. Now, given the existence of a futures market, she has two sources from which she can acquire the commodity: production and purchase in the futures market. As long as \( c'(q) < p_f \), it is...
cheaper to produce, but beyond this point it is cheaper to buy the commodity by taking a long position in the futures market. Therefore, she produces only to the point where \( c'(q) = p_f \). Conversely, suppose the producer has unfavorable information regarding demand conditions and expects the spot price to be lower than the futures price in the market. She is better off selling at the futures price than waiting to sell at the spot price. Therefore, she produces till the point where \( c'(q) = p_f \) and sells all her production and perhaps more at \( p_f \) by taking a short position in the futures market.

Notice the production is determined entirely by the futures price; beliefs about the commodity spot market are irrelevant for production purposes. These beliefs are rationally used only to take a position in the futures market. Of course, when producers take short or long positions in the futures market based on their information, the futures price will change. However, the important implication of Proposition 1 is that the information that any individual producer may have regarding demand conditions in the spot market affects her production only to the extent that this information is impounded and communicated by the futures price. Additionally, if her own production is affected by her information, then the production of others in the industry is also affected, because all production is guided solely by the futures price. This sets up a strong externality. If the futures price is a sufficient statistic of all the relevant information in the economy, then the production decision of all producers is better informed and the production efficiency of the industry is enhanced. Conversely, if the futures price does a poor job at aggregating and communicating information, the production efficiency of the entire industry suffers. This is why regulators concerned with hedge disclosures ought to be fundamentally concerned with how such disclosures enhance the informational efficiency of futures prices.

We now proceed to demonstrate how, in the absence of appropriate hedge disclosures, the equilibrium futures price will be informationally inefficient and thereby misguide the production decisions of firms in the industry. We then identify the crucial hedge disclosures that would mitigate this inefficiency.

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2Danthine's separation result depends only on the ability to make a perfect hedge and not on price-taking behavior. Even an oligopolistic producer faces the same trade-off between production and futures trades as a means of reaching an optimal position in the spot market. Given a marginal cost schedule for production and a upward-sloping marginal futures price schedule, the producer would trace out the lower envelope of these two cost schedules and produce till the point where the marginal cost of production equals the marginal futures price. The separation result does not hold when the presence of basis risk or the presence of production uncertainty precludes a perfect hedge. Even in these cases, the equilibrium futures price will influence the choice of production.
3. Equilibrium Futures Prices in the Absence of Hedge Disclosures

In order to obtain closed-form characterization of futures market equilibria, we make additional parametric assumptions regarding preferences and technology. Hereafter, we assume that all producers have identical constant absolute risk aversion $\rho$, i.e., $U_i(\pi) = -e^{-\rho \pi_i}$, $i \in \{0, 1, \ldots, N\}$. We also assume that producers’ cost functions are quadratic, i.e., $c(q) = q^2/2k$, $k > 0$. Given Proposition 1, the latter assumption implies that the optimal production quantities of all producers are $q_i = k\bar{p}_f$, $i \in \{0, 1, \ldots, N\}$. We now turn to the calculation of optimal futures trades, given these optimal production quantities.

We have already assumed that the informed producer privately knows $\theta$ which represents the uncertainty in her production. We additionally assume that the informed producer obtains private information about future demand conditions. Specifically, she observes the realization of $\eta$ which is one of the parameters in the spot demand schedule. The other producers in the industry have no private information but can extract information from the observable equilibrium futures price $\bar{p}_f$. Thus, absent hedge disclosures, the informed producer conditions her beliefs on $\theta, \eta, p_f$, and uninformed producers condition their beliefs on $p_f$. There is no public information on individual traders’ positions in the futures market, i.e., $\{z_i\}$ is not disclosed.

Since for each of the uninformed producers, $i \in \{1, 2, \ldots, N\}$, $\bar{\pi}_i = z_i\bar{p}_f + (q_i - z_i)\bar{p} - q^2/2k$, $\bar{\pi}_i$ is normally distributed if the conditional distribution of $\bar{p}$ given $p_f$ is normal. We show later that this is indeed the case. Anticipating this result, the maximization of each producer’s expected utility is equivalent to the maximization of $E(\bar{\pi}_i) - \frac{p}{2}\text{Var}(\bar{\pi}_i)$. Thus, each uninformed producer chooses her futures trade as the solution to:

$$\text{Max}_{z_i} z_i p_f + (q_i - z_i)E(\bar{p} | p_f) - \frac{q_i^2}{2k} - \frac{p}{2} (q_i - z_i)^2 \text{Var}(\bar{p} | p_f),$$

which yields the first-order condition:

$$z_i = q_i - \frac{E(\bar{p} | p_f) - \bar{p}_f}{\rho \text{Var}(\bar{p} | p_f)}.$$  

Equation (11) indicates that an individual producer arrives at her optimal trade $z_i$ in the futures market, as if she first hedges her total risk exposure $q_i$ and then adjusts away from this perfect hedge via a speculative trade. Thus her recorded trade of $z_i$ consists of two components: a pure hedge component and a speculative component. For example, suppose a wheat farmer’s anticipated production is 100 bushels of wheat (i.e., $q_i = 100$), and her recorded trade in the futures market, $z_i$, is a sale
of 60 bushels of wheat. The farmer arrives at this net futures position of 60 bushels by implicitly selling her entire anticipated output of 100 bushels at the futures price and then repurchasing 40 bushels. Naive observation of her recorded trade would suggest that the farmer has taken a short position of 60 bushels in the futures market and therefore must be pessimistic about the spot price of wheat. However, (11) indicates that the correct interpretation is that the farmer is long to the extent of 40 bushels of wheat and is indeed optimistic about the spot price. The farmer’s information about supply and demand conditions for wheat is reflected neither in the amount she produced, nor in the amount she sold in the futures market, but in the amount she did not sell. The hedge component (100 bushels) of the farmer’s net trade in the futures market is independent of her information about \( \bar{p} \), and only the speculative component (40 bushels) is informationally motivated. The farmer’s beliefs about \( \bar{p} \) are more accurately reflected in this speculative position of 40 bushels, than in the futures trade of 60 bushels that is actually recorded.

The informed producer chooses her futures position, \( z_0 \), to solve:

\[
\begin{align*}
\max_{z_0} & \quad z_0 p_f + (q_0 + \theta - z_0)E(\bar{p}|p_f, \eta, \theta) \\
& \quad - \frac{q_0^2}{2k} - \frac{p}{2} (q_0 + \theta - z_0)^2 \text{Var}(\bar{p}|p_f, \eta, \theta),
\end{align*}
\]

which yields the first-order condition:

\[
z_0 = q_0 + \theta - \frac{E(\bar{p}|p_f, \eta, \theta) - p_f}{\rho \text{Var}(\bar{p}|p_f, \eta, \theta)}.
\]

Equation (13) indicates that the informed producer’s optimal trade \( z_0 \) has both a hedge component and a speculative component similar to the trades of uninformed producers. The shock \( \theta \) in the informed agent’s production quantity plays a dual role. It affects her hedge motivated trade, which is \( q_0 + \theta \), but it also affects her informationally motivated trade through the assessed distribution of \( \bar{p} \). This characteristic of producers’ trades is inevitable whenever they have private information about their anticipated production or risk exposure, leading to a confounding of hedge-motivated trades with informationally motivated trades. Such confounding has potent implications for the efficiency of futures prices.

Let \( p_f^0 \) represent the equilibrium futures price, determined through market clearing, in this economy with no hedge disclosures. Market

\[\text{We have assumed price-taking behavior for all firms, including the informed firm. In some situations this may be unrealistic, especially for informed firms. It is likely that an informed firm would trade strategically, taking into account the informational effect of its trades on the equilibrium futures price. The possibility of such strategic trading makes it even more desirable to mandate public disclosure of firms’ inherent risk, since strategic trading would surely make the equilibrium futures price more inefficient.}\]
clearing requires that $\Sigma_{i=0}^{N} z_i = 0$. Let $Q^0 = \Sigma_{i=0}^{n} q_i^0$ be the expected equilibrium industry output, where each $q_i^0 = kp_i^0$. Then, from (11) and (13), market clearing implies that the equilibrium futures price must satisfy:

$$Q^0 + \theta = \frac{E(\tilde{p}|p_f^0, \eta, \theta) - p_f^0}{\rho \text{Var}(\tilde{p}|p_f^0, \eta, \theta)} + N \frac{E(\tilde{p}|p_f^0) - p_f^0}{\rho \text{Var}(\tilde{p}|p_f^0)}. \tag{14}$$

Equation (14) is similar to the standard market clearing condition for any risky asset (see Grossman [1977]). There is only one risky asset in this economy, namely, the commodity whose production is being studied. In equilibrium, all of this risky asset must be held until the spot market opens. The market price at which the risky asset is purchased is the futures price $p_f^0$ and $E(\tilde{p})$ is the expected return on the risky asset. The right-hand side of (14) then represents the aggregate demand for the risky asset, while the left-hand side represents the aggregate supply. The presence of $\theta$ in the left-hand side of (14) represents the kind of supply noise that Grossman [1977] uses to preclude a fully revealing rational expectations price. Here, it arises naturally as a consequence of uncertainties in output leading to uncertainties in hedge-motivated trades, rather than through irrational trades by liquidity traders. As in Grossman [1977], one would expect that this supply noise would prevent the revelation of the informed producer’s knowledge of consumer demand, $\eta$, via the equilibrium futures price. This intuition is verified in the analysis to follow.

Before the equilibrium futures price can be calculated, the distribution of $\tilde{p}$ conditional on the information of the informed and uninformed producers needs to be specified. This requires the information content of $p_f^0$ to be known. As in Grossman’s [1978] “artificial economy” construction, we resolve this problem by making a conjecture about the information revealed by $p_f^0$, calculating the equilibrium trades conditional on this conjecture, and then confirming that the equilibrium $p_f^0$ does, indeed, reveal the conjectured information.

Recall from (5) that the equilibrium spot price is described by:

$$\bar{p} = (\bar{\eta} - \bar{\theta}) + \tilde{\gamma} - Q^0. \tag{15}$$

Now, $Q^0$ is publicly known, the informed producer knows the values of $\eta$ and $\theta$, and no agent in the economy observes the value of $\tilde{\gamma}$. Therefore, it is reasonable to conjecture that the informed producer learns nothing from the equilibrium futures price beyond what she already knows. Given this conjecture:

$$E(\tilde{p}|\eta, \theta, p_f^0) = \eta - q - Q^0 \tag{16}$$

4 As with any normally distributed shocks to price, spot prices, futures prices, and therefore production quantities could become negative. This “absurdity” does not, however, affect the interpretation of our results.
and:
\[
\text{Var}(\tilde{p} \mid \tilde{p}^0 \theta) = V_{\gamma}. \tag{17}
\]

Inserting these beliefs for the informed producer in (14), the market clearing condition becomes:
\[
Q^0 + \theta = \frac{\eta - \theta - Q^0 - \tilde{p}^0}{\rho V_{\gamma}} + \frac{E(\tilde{p} \mid \tilde{p}^0) - \tilde{p}^0}{\rho \text{Var}(\tilde{p} \mid \tilde{p}^0)}. \tag{18}
\]

Define \( \tilde{y} \) as the following statistic of \( \tilde{\eta} \) and \( \tilde{\theta} \):
\[
\tilde{y} \equiv \tilde{\eta} - \tilde{\theta}(1 + \rho V_{\gamma}) \tag{19}
\]
so that (18) can be written as:
\[
Q^0 = \frac{y - Q^0 - \tilde{p}^0}{\rho V_{\gamma}} + \frac{E(\tilde{p} \mid \tilde{p}^0) - \tilde{p}^0}{\rho \text{Var}(\tilde{p} \mid \tilde{p}^0)}. \tag{20}
\]

We conjecture that the equilibrium price \( \tilde{p}^0 \) reveals the value of the statistic \( \tilde{y} \). This would imply that \( E(\tilde{p} \mid \tilde{p}^0) = E(\tilde{p} \mid y) = E(\tilde{\eta} - \tilde{\theta} \mid y) - Q^0 \). Since \( \tilde{y} \) is normally distributed, \( E(\tilde{\eta} - \tilde{\theta} \mid y) \) is strictly increasing in \( y \) and \( \text{Var}(\tilde{p} \mid y) = \text{Var}(\tilde{\eta} - \tilde{\theta} \mid y) + V_{\gamma} \) is independent of the value of \( y \). Therefore, both terms on the right-hand side of (20) will be strictly increasing in \( y \). This implies that the equilibrium futures price \( \tilde{p}^0 \) will reveal the statistic \( \tilde{y} \), conforming our conjecture.\(^5\) The following proposition exploits this intuition to provide an exact characterization of \( \tilde{p}^0 \).

**Proposition 2.** In the economy with no hedge disclosures, the equilibrium futures price is strictly increasing in \( \tilde{y} \equiv \tilde{\eta} - \tilde{\theta}(1 + \rho V_{\gamma}) \) and is characterized by:
\[
A^0 \tilde{p}^0 = \left( \frac{V + \alpha NV_{\gamma}}{V + NV_{\gamma}} \right) y + \left( \frac{(1 - \alpha) NV_{\gamma}}{V + NV_{\gamma}} \right) \mu \tag{21}
\]

where:
\[
\alpha = \frac{V_{\eta} + (1 + \rho V_{\gamma}) V_{\theta}}{V_{\eta} + (1 + \rho V_{\gamma})^2 V_{\theta}}, \quad 0 < \alpha < 1 \tag{22}
\]
\[
V \equiv \text{Var}(\tilde{\eta} - \tilde{\theta} \mid y) + V_{\gamma} \equiv V_{\gamma} + V_{\eta} + V_{\theta} - \left[ \frac{V_{\eta} + (1 + \rho V_{\gamma}) V_{\theta}}{V_{\eta} + (1 + \rho V_{\gamma})^2 V_{\theta}} \right]^2 \tag{23}
\]

and:
\[
A^0 = 1 + (N + 1) k + \left( \frac{\rho V}{V + NV_{\gamma}} \right) V_{\gamma}(N + 1) k. \tag{24}
\]

\(^5\)There may be other equilibriums with self-fulfilling beliefs. We have not investigated this possibility, but the equilibrium we characterize seems intuitive and plausible.
Proposition 2 indicates that, in the absence of hedge disclosures, the futures price is informationally inefficient. The information that is relevant to production decisions is \((\eta - \theta)\). When this quantity is high, the spot price will also be high and producers ought to respond by increasing their production. However, production responds to variations in the statistic \(\bar{\eta}\), which is not equivalent to variations in \((\eta - \bar{\theta})\). In order to understand the source of this inefficiency, examine (18). If there were no \(\bar{\theta}\) on the left-hand side of (18), then, indeed, the equilibrium futures price would reveal \((\eta - \bar{\theta})\) and the uninformed producers would have the same information as the informed producer. The presence of \(\bar{\theta}\) on the left-hand side of (18) arises from the hedging needs of the informed producer, and this component of her trade is independent of her beliefs regarding \(\tilde{p}\). Only the first term on the right-hand side, which depends on \((\eta - \bar{\theta})\), reflects her beliefs about \(\tilde{p}\). Unfortunately, the hedging need of the informed producer is not publicly known; so \((\eta - \bar{\theta})\) cannot be disentangled from \(\bar{\theta}\). This is why the futures price fails to transmit all the relevant information in the economy. The precise nature of the induced production inefficiency will be elaborated in the analysis to follow.

4. Equilibrium in the Futures Market with Hedge Disclosures

We have shown how the futures price fails to appropriately inform production decisions when producers’ hedging needs are confounded with their speculative demands. Although we have derived this result in a very simple model, where this confounding exists for only one producer, the same result will hold when many, or all, producers have private information about their hedging needs. This suggests that the crucial hedge disclosure needed is public disclosure of the inherent risk exposure of producers, which, in our model, is captured by the production quantity \(q_0 + \theta\) for the informed producer. The Securities and Exchange Commission (SEC [1997, n. 58]) refers to such risk as “primary market risk exposures” and suggests its disclosure on a voluntary basis.

---

\^We have shown that the crucial information that needs to be publicly disclosed in order to enhance the informational efficiency of the futures price is the firm’s inherent risk exposure. In our model, where firms are hedging the uncertainty in output price, this inherent risk exposure is described by the firm’s output \((q + \theta)\). Clearly, there are alternative ways in which information about a firm’s output could be disclosed. For example, sales forecasts or earnings forecasts could provide similar information. However, if firms were hedging the uncertainty in the price of some important raw material, a sales forecast would not necessarily be adequate. Since inherent risk could take many different forms, such as foreign currency exposure, interest rate exposure, commodity price exposure, etc., a direct disclosure of firms’ inherent risk is the most effective way to make the relevant information public. It seems that SFAS No. 133 emphasizes the mark-to-market valuation of hedge positions, and its inclusion or noninclusion in income statements, rather than the disclosure of inherent risk. Such measurement issues may be important for providing information to capital markets. However, our analysis indicates that they are irrelevant to price efficiency in a futures market.
The FASB’s SFAS No. 133 gives firms the option to disclose their inherent risk exposure, in the sense that such disclosure is required only if firms opt to use comprehensive fair value hedge accounting.7

To see why inherent risk disclosures would be effective, let us examine the informed producer’s first-order condition described in (13). Although not essential, let us assume that in addition to disclosure of inherent risk, \( q_0 + \theta \), the informed producer’s net futures trade \( z_0 \) is also disclosed, as required by SFAS No. 133.8 Given these disclosures, the informed producer’s beliefs, \( E(\tilde{p}|\eta,\theta,p_f) \), can be immediately inferred without even inverting the equilibrium futures price. This is all the information that is needed by all the other producers in the industry. Specifically, for the model under consideration, since \( E(\text{fin}_0,p_f) = (\eta - \theta) - (N + 1)kp_f \), the information \((\eta - \theta)\), which was hidden in the regime without hedge disclosures, can be readily inferred. Alternatively, if \( z_0 \) is not required to be disclosed, the value of \( \theta \) can be inferred from inherent risk disclosures; and given this direct inference, we will show that the equilibrium futures price can be inverted to infer the value of \( \eta \). Informationally, both disclosure regimes are equivalent, but the inference problem is less demanding when inherent risk exposures as well as futures positions are disclosed.

We proceed now to the calculation of the new futures price given that only inherent risk disclosures are mandated. We conjecture that the equilibrium futures price is invertible in \( \eta \), given that \( \theta \) can be directly inferred from inherent risk disclosures. If this is the case, the expectations of informed and uninformed producers coincide, so the market clearing condition becomes:

\[
Q^* + \theta = (N + 1) \left( \frac{\eta - \theta - Q^* - p_f^*}{\rho V_f} \right)
\]

where \( p_f^* \) and \( Q^* \) are the equilibrium futures price and equilibrium expected industry output in the economy with hedge disclosures. Inserting \( Q^* = (N + 1)kp_f^* \) in (25) and solving for \( p_f^* \) yields:

**PROPOSITION 3.** In the economy with hedge disclosures, the equilibrium futures price is characterized by:

\[
A^* p_f^* = \eta - \theta - \left( \frac{\rho}{N + 1} \right) V_f \theta
\]

7 Given this choice, an informed firm may choose not to use comprehensive fair value hedge accounting, since doing so would destroy its informational advantage and likely result in lower profits. However, even though the disclosing firm may reduce its profits, there are significant social benefits associated with the disclosure, as shown by our analysis. Considering such externalities, in a futures market context, indicates that allowing such a choice is undesirable.

8 It can be shown that disclosure of producers’ net futures trades, \( z_0 \), with no disclosure of inherent risk, would have no effect on equilibrium futures prices.
where:

\[ A^* = 1 + (N + 1)k + \left( \frac{\bar{\theta}}{N + 1} \right) V \gamma (N + 1)k. \]  

(27)

It is immediate from Proposition 3 that the equilibrium futures price reveals \( \eta \), given that \( \theta \) is known directly from inherent risk disclosures.

5. Industry Output with and without Hedge Disclosures

Having characterized the equilibrium futures price in each of the two regimes, we can now examine the implication of hedge disclosures on industry output. Since in each regime, industry output is the same multiple of the equilibrium futures price, i.e., industry output equals \( (N + 1)k\gamma \theta + \theta \), we need only examine how the futures price differs in the two regimes. In order to carry out this comparison, it is useful to think of \( (\eta - \theta) \) and \( \theta \) as two distinct random variables. The market clearing condition for the futures price, characterized in (18), indicates that \( \theta \) plays a dual role. The presence of \( \theta \) on the left-hand side of (18) represents the shock to the hedging demand of producers. On the other hand, the same \( \theta \) on the right-hand side of (18) describes the effect of \( \theta \) on the equilibrium distribution of \( \bar{\theta} \). This latter effect is similar to the effect of \( \eta \) on the equilibrium distribution of \( \bar{\theta} \). Therefore, from an informational point of view, the relevant random variable on the right-hand side of (18) should be thought of as the aggregate variable \( (\eta - \theta) \).

Hereafter, we refer to the regime with hedge disclosures as the first-best economy, since in this regime all the information possessed by an individual agent is publicly revealed. The first-best equilibrium futures price characterized in (26) can be expressed as:

\[ p^*_f = \beta^*_1 + \beta^*_2 (\eta - \theta) - \beta^*_3 \theta. \]  

(28)

Inserting \( \gamma = (\eta - \theta) - \rho V \gamma \theta \) in (21), the equilibrium price in the economy without hedge disclosures can be expressed as:

\[ p^0_f = \beta^0_1 + \beta^0_2 (\eta - \theta) - \beta^0_3 \theta \]  

(29)

where:

\[ \beta^*_1 = 0, \beta^0_1 = \frac{1}{A^0} \left( \frac{(1 - \alpha)NV\gamma}{V + NV\gamma} \right) \mu \]  

(30)

\[ \beta^*_2 = \frac{1}{A^*}, \beta^0_2 = \frac{1}{A^0} \left( \frac{V + \alpha NV\gamma}{V + NV\gamma} \right) \]  

(31)

\[ \beta^*_3 = \frac{1}{A^*} \left\{ \frac{\rho V\gamma}{N + 1} \right\}, \beta^0_3 = \frac{1}{A^0} \left( \rho V\gamma \right) \left( \frac{V + \alpha NV\gamma}{V + NV\gamma} \right). \]  

(32)
PROPOSITION 4. In the economy without hedge disclosures, industry output is relatively insensitive to fluctuations in the spot market price and overly sensitive to fluctuations in hedge-motivated futures trades, i.e., $\beta_2^0 < \beta_2^*$ and $\beta_3^0 > \beta_3^*$.

To see why these results hold, let us return to the interpretation of the futures price provided in the context of equation (14). The futures price $p_f$ can be thought of as the price of a risky asset with return $\tilde{p}$ in a trading economy which has been endowed with some quantity $Q + \theta$ units of the risky asset. Given a mean-variance setting, the well-known pricing rule derived in the finance literature is:

$$p_f = \frac{E(\tilde{p} | \eta, \theta) - V(\eta, \theta)}{\rho V_\gamma}$$

where $\lambda$ is the aggregate risk aversion in the economy. In our analysis, in the first-best economy, all agents condition their beliefs on $(\eta, \theta)$, so $E(\tilde{p}) = E(\tilde{p} | \eta, \theta)$ and $V(\eta, \theta) = V_\gamma$. Given that $Q + \theta$ is exogenous, the market clearing condition determining $p_f^*$ is:

$$Q + \theta = (N + 1) \left( \frac{E(\tilde{p} | \eta, \theta) - p_f^*}{\rho V_\gamma} \right).$$

Solving for $p_f^*$ yields:

$$p_f^* = \frac{E(\tilde{p} | \eta, \theta) - \left( \frac{\rho}{N+1} \right) V_\gamma (Q + \theta)}{\rho V_\gamma}.$$  

Since $E(\tilde{p} | \eta, \theta) = (\eta - \theta) - Q$, and keeping in mind that $(\eta - \theta)$ and $\theta$ are conceptualized as two distinct random variables, the coefficient of $\theta$ in (35) is $\left( \frac{\rho}{N+1} \right) V_\gamma = \Lambda^* \beta_3^*$. Comparing (35) to (33), it is clear that the factor $\left( \frac{\rho}{N+1} \right)$ in the coefficient of $\theta$ represents the aggregate risk aversion in the first-best economy. The factor $\Lambda^*$ contained in $\beta_3^*$ is simply an adjustment for the fact that in an economy with endogenous production, the production quantity and, therefore, the amount of the risky asset to be traded also depends on $p_f^*$.

We repeat the above analysis for a trading economy without hedge disclosures, holding constant the aggregate endowment of the risky asset at $Q + \theta$. The market clearing condition determining $p_f^0$ is:

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9 One could also examine the total sensitivity of the futures price to fluctuations in $\theta$. This total sensitivity is described by $\beta_2^0 + \beta_3^0$ in the regime with no hedge disclosures and by $\beta_2^* + \beta_3^*$ in the regime with hedge disclosures. It can be established that $\beta_2^0 + \beta_3^0 > \beta_2^* + \beta_3^*$. However, it is difficult to interpret this result. Algebraically it implies that, in the regime with no hedge disclosures, the effect on the futures price of shocks to hedge-motivated trades dominates the effect of production shocks on the equilibrium spot price.
\[ Q + \theta = \frac{E(\hat{p} | \eta, \theta) - \rho_f^0}{\rho V^\gamma} + \frac{N(E(\hat{p} | y) - \rho_f^0)}{\rho V}. \]  

Solving for \( \rho_f^0 \) yields:

\[ \rho_f^0 = \frac{V}{V + \rho V^\gamma} E(\hat{p} | \eta, \theta) + \frac{NV^\gamma}{V + \rho V^\gamma} E(\hat{p} | y) \]

\[ \quad - \frac{\rho V}{V + \rho V^\gamma} V^\gamma (Q + \theta). \]  

The equilibrium futures price is a precision-weighted average of the expected spot price as perceived by the informed and uninformed producers less a risk adjustment term. Now:

\[ E(\hat{p} | y) = ay + (1 - \alpha)\mu - Q = a(\eta - \theta) - \alpha \rho V^\gamma \theta + (1 - \alpha)\mu - Q. \]  

Therefore, the coefficient of \( \theta \) in (37) is

\[ \frac{\rho V}{V + \rho V^\gamma} V^\gamma + \frac{\alpha NV^\gamma}{V + \rho V^\gamma} \rho V^\gamma = A^0p^y_0. \]  

It follows from this analysis that there are two reasons the futures price without hedge disclosures is overly sensitive to variations in \( \theta \). First, the factor \( \frac{\rho V}{V + \rho V^\gamma} \) is greater than the factor \( \frac{\rho}{N+1} \), since \( V > V_\gamma \). This, by itself, increases the risk premium embedded in the futures price, which, in turn, makes the futures price overly sensitive to variations in the quantity to be hedged. Second, \( (\eta - \theta) \) is confounded with \( \theta \) in the statistic \( \hat{y} \), so \( \theta \) acquires an unwarranted informational role in the assessment of the spot price. This informational role, operating through the statistic \( \hat{y} \), results in the addition of \( \frac{\alpha NV^\gamma}{V + \rho V^\gamma} \rho V^\gamma \) to the coefficient of \( \theta \), which additionally increases the sensitivity of \( \rho_f^0 \) to fluctuations in \( \theta \).

Proposition 4 also indicates that, absent hedge disclosures, the futures price and, therefore, industry output is relatively insensitive to fluctuations in \( (\eta - \theta) \), in comparison to the first-best economy. The reason for this is the following. A change in \( (\eta - \theta) \) is perceived only through a change in the statistic \( \hat{y} \). Since \( \hat{y} \) is a noisy representation of \( (\eta - \theta) \), the economy responds to fluctuations in \( \hat{y} \) with caution, assigning a weight less than unity to \( \hat{y} \) and putting some weight on the prior mean of \( (\eta - \theta) \). The economy cannot distinguish between fluctuations in \( (\eta - \theta) \) and fluctuations in \( \theta \) and therefore takes a middle road, underreacting to \( (\eta - \theta) \) and overreacting to \( \theta \). This relative insensitivity to \( (\eta - \theta) \) has implications for the volatility of the spot price. An increase in \( \eta \) represents an upward shift in the demand function of the commodity being produced. Industry output responds inadequately to such demand increases, inducing spot prices that are unduly high. Conversely, industry output does not shrink adequately in response to a decrease in \( \eta \),
making spot prices unduly low in this case. Ceritis paribus, this implies that the spot price of the commodity will be more volatile in the economy without hedge disclosures.

Next, we compare the level of industry output across the two regimes. In general, the futures price will be lower in some states and higher in some other states in the regime without hedge disclosures than in the regime with hedge disclosures. We first identify the states in which the futures price and industry output are unambiguously lower.

**PROPOSITION 5.** \( p_f^0 < p_f^* \) and \( Q_f^0 < Q^* \) when \( (\eta - \theta) \geq E(\eta - \theta) = \mu \).

Proposition 5 is a sufficient, but not necessary, condition, indicating that when spot demand is sufficiently high, industry output in the economy without hedge disclosures is unambiguously lower than first-best. Using equations (A4) and (A5), in Appendix A, we can similarly characterize the set of states \( (\eta - \theta, 0) \) in which the industry overproduces relative to first-best. These are the states in which industry output is generally low. Thus, if industry output \( Q_f^0 \) is rank ordered, an outside observer could conclude that when industry output is in the right-hand tail of this ranking, output would have been even greater if hedge disclosures had been required, and when observed industry output is in the left-hand tail of this ranking, output would have been even lower if hedge disclosures had been required. This is consistent with Proposition 4 which describes the insensitivity of industry output to variations in consumer demand.

We now examine the expected level of industry output and show that the lack of hedge disclosures results in a downward bias in the futures price and therefore in a downward bias in industry output. To see this, we calculate the average futures price in each of the two regimes by taking expectations over the states \( (\eta - \theta) \) and \( \theta \). It follows from (A4) and (A5) in Appendix A that:

\[
E_{\eta, \theta}(p_f^0) = \frac{\mu}{A^0}, \quad E_{\eta, \theta}(p_f^*) = \frac{\mu}{A^*}.
\]

Since \( A^0 > A^* \), it follows immediately that:

**PROPOSITION 6.** On average, industry output is strictly lower in the regime without hedge disclosures than in the regime with hedge disclosures.

In section 7, we provide numerical calculations that demonstrate that the decline in expected industry output due to lack of hedge disclosures could be very significant.

### 6. Distortions in Risk Sharing

A futures market facilitates risk sharing among risk-averse individuals when there are risks associated with production. Risk sharing generally increases the risk-bearing ability of the group of agents constituting the economy, allowing risks that would otherwise be forgone to be undertaken. In the current setting, higher risk taking is equivalent to higher in-
dustry output. Since the instrument that regulates industry output is the futures price, it must be the case that the combination of how risk is shared and the heterogeneity of beliefs among the informed and uninformed producers influences the level of the futures price. We have shown that absent hedge disclosures, the futures price is biased downward and, in equilibrium, the beliefs of informed and uninformed producers do not coincide. In this section, we analyze (i) the distortions in risk sharing caused by the heterogeneity of equilibrium beliefs and (ii) how the downward bias in equilibrium futures prices is caused by a higher risk premium due to the manner in which these heterogeneous beliefs are aggregated.

To examine risk sharing, we calculate the equilibrium holdings of the risky asset by informed and uninformed producers in both regimes. Since all producers are equally risk averse, it is intuitive that efficient risk sharing would imply that, in equilibrium, all producers hold an equal amount of the risky asset. We show that this is indeed the case in the first-best economy. However, in the economy without hedge disclosures, the informed producer bears a disproportionate amount of risk.

From (11), the demand for the risky asset by each producer in the first-best economy is:

\[ s_i^* = \frac{E(\hat{p}|\eta, \theta) - p_f^*}{\rho V_\gamma}, \quad i = 0, \ldots, N. \]  

Inserting the expression for \( p_f^* \) from equation (32) into the above expression and simplifying yields:

\[ s_i^* = \frac{Q^* + \theta}{N + 1}, \quad i = 0, \ldots, N \]  

confirming the intuition that each producer holds an equal fraction of the risky asset.

In the economy without hedge disclosures, the demand of the informed producer for the risky asset is:

\[ s_0^0 = \frac{E(\hat{p}|\eta, \theta) - p_f^0}{\rho V_\gamma}, \]  

and the demand of an uninformed producer is:

\[ s_i^0 = \frac{E(\hat{p}|\gamma) - p_f^0}{\rho V}, i = 1, \ldots, N. \]  

Inserting the expression for \( p_f^0 \) from (37) into the demand functions described in (42) and (43) and simplifying yields:

\[ s_0^0 = \frac{V}{V + NV_\gamma} (Q^0 + \theta) \]

\[ + \frac{NV_\gamma}{V + NV_\gamma} \frac{(\eta - \theta) - E(\eta - q|\gamma)}{\rho V_\gamma} \]  

(44)
\[ s_i^0 = \frac{V_i}{V + NV_i} (Q^0 + \theta) \]
\[ - \frac{V_i}{V + NV_i} \frac{(\eta - \theta) - E(\eta - \theta|y)}{\rho V_i} , \quad i = 1, \ldots, N. \]  

Since \( E[(\eta - \theta) - E(\eta - \theta|y)] = 0 \), equation (44) indicates that, on average, the informed producer holds the fraction \( \frac{V}{V + NV_i} \) of the aggregate supply of the risky asset, while equation (45) indicates that the uninformed producers collectively hold the remaining fraction \( \frac{NV_i}{V + NV_i} \).

Since \( \frac{V}{V + NV_i} > \frac{1}{N+1} \), on average, the informed producer bears more than her proportionate share of the aggregate risk in the economy. The difference in the informed and uninformed producers' beliefs is reflected in the term \( (\eta - \theta) - E(\eta - \theta|y) \) and in the assessed variances \( V_i \) and \( V \), respectively, of the spot price. The second component in (44) and (45) that arises from this difference in beliefs is analogous to the "side bets" in the syndicate sharing rules derived in Wilson [1968].

We now analyze how the aggregation of beliefs affects the risk premium in futures prices. The risk premium in the first-best equilibrium futures price described in (35) is \( \frac{\rho}{N+1} V_i(Q + \theta) \), while the risk premium in the economy without hedge disclosures is \( \frac{\rho V}{V + NV_i} V_i(Q + \theta) \).

Since \( \frac{V}{V + NV_i} > \frac{1}{N+1} \), it follows that the risk premium is higher in the economy with no hedge disclosures. Wilson's [1968] analysis of group decision making can be used to obtain insights into why the risk premium is higher. Wilson shows that given exponential utility functions and normally distributed payoffs, a "representative individual" with surrogate beliefs and surrogate risk aversion can be constructed such that the choices of this representative individual would correspond to the "equilibrium" choices of the group. While there is no notion of equilibrium prices in Wilson's analysis, we show that the risk premium inherent in the price that this representative individual would pay to acquire a unit of the risky commodity, in our setting, corresponds to the risk premium in the futures price determined in a competitive market.

For the first-best economy, the representative individual, as constructed by Wilson, would have a risk aversion of \( \frac{\rho}{N+1} \) and would assess the expected return from the industry output as \( E(\hat{\theta}|\eta, \theta)(Q + \theta) \) and the vari-
ance of returns from the industry output as \( V_y(Q + \theta)^2 \) (see Wilson [1968, sec. 7]). This implies that this representative individual would value industry output at:

\[
\phi^*(Q + \theta) = E(\tilde{p} | \eta, \theta)(Q + \theta) - \frac{\rho}{N+1} V_y(Q + \theta)^2.
\]  

(46)

In a competitive market, valuation rules are linear, so \( \phi^*(Q + \theta) \) has the dot product form \( p_j^* \cdot (Q + \theta) \). Dividing both sides of (46) by \( Q + \theta \) yields the per unit futures price \( p_j^* \) characterized in (35). Thus, the risk premium in the competitively determined futures price \( \frac{\rho}{N+1} V_y(Q + \theta) \) is the same as the risk premium in the per unit price that Wilson’s representative individual would pay to acquire the risky asset.

We now construct Wilson’s representative individual for the economy without hedge disclosures. In this economy, the equilibrium beliefs of individual producers do not coincide. Given heterogeneous beliefs, Wilson shows that the representative individual would have surrogate beliefs about \( \tilde{p} \) characterized by the mean \( m \) and precision \( h \) described below:

\[
h = \frac{\sum_{i=0}^{N} h_i}{N+1}
\]  

(47)

and:

\[
m = \frac{1}{h} \cdot \frac{1}{N+1} \sum_{i=0}^{N} h_i m_i
\]  

(48)

where \( h_i \) is producer \( i \)'s precision of \( \tilde{p} \) and \( m_i \) is her assessed mean of \( \tilde{p} \) given her information. In our setting, \( h_i = \frac{1}{V} \) for each of the \( N \) uninformed producers and \( h_0 = \frac{1}{V} \) for the informed producer. Further, \( m_i = E(\tilde{p}|y) \) for each of the \( N \) uninformed producers and \( m_0 = E(\tilde{p}|\eta, \theta) \) for the informed producer. Inserting these specifications into (47) and (48) yields:

\[
m = \frac{V}{V + NV_y} E(\tilde{p}|\eta, \theta) + \frac{NV_y}{V + NV_y} E(\tilde{p}|y)
\]  

(49)

and:

\[
\frac{1}{h} = \frac{(N+1)VV_y}{V + NV_y}.
\]  

(50)
This implies that this representative individual would value industry output at:

\[ \phi^0(Q + \theta) = m \times (Q + \theta) - \frac{\rho}{N+1} \frac{1}{h} (Q + \theta)^2. \]  

(51)

Dividing both sides of (51) by \( Q + \theta \) and imposing a linear pricing rule yields the per unit futures price \( p_f^0 \) characterized earlier in (37). Comparing (51) to (46), the aggregate risk aversion, \( \frac{\rho}{N+1} \), in the economy remains unchanged, but the “average” precision of surrogate beliefs described by \( h \) is lower. In a competitive market, each individual trader assesses the risk of the asset in light of his/her own information. These assessments are aggregated by the market to determine the risk premium in the equilibrium price. The lower precision of beliefs held by Wilson’s representative individual is equivalent to a higher “average” assessment of risk by the market, which, in turn, leads to a higher risk premium in \( p_f^0 \). This is the cause of the downward bias in the equilibrium futures price in the absence of hedge disclosures.

7. Magnitude of Distortions in Industry Output

We have shown that, on average, industry output is lower than first-best in the economy without hedge disclosures. In this section, we provide numerical calculations that shed some light on the magnitude of this loss and the sensitivity of this loss to various parameters. We are unable to make welfare comparisons because the consumers of the commodity under production are not explicitly modeled.\(^\text{10}\)

\(^\text{10}\)We have refrained from making explicit welfare comparisons, since consumers of the commodity are represented only in terms of the induced demand function in the spot market. As suggested by the referee, some insight into consumer welfare can be obtained by comparing consumer surplus across regimes. These calculations are presented below.

In any market with a linear demand function, say \( P(q) = a - bq \), consumer surplus \( C = bq \). In our setting, for any fixed state \((r,0)\), consumer surplus is:

\[ (N + 1)k^2 E(f_j^2) + V_0 - 2(N + 1)k(\beta_2 + \beta_3) V_0 \]

This indicates that in those states (identified in Proposition 5) where the equilibrium futures price is larger with than without hedge disclosures, consumer surplus is higher. Calculation of the expected consumer surplus yields:

\[ 2E(C(\eta,0)) = (N + 1)^2k^2 E(\beta_j^2) + V_0 + 2(N + 1)k E(\beta_j^2) \]

\[ = (N + 1)^2k^2 \left[ \frac{1}{2} b^2 V_1 + (\beta_2 + \beta_3)^2 V_0 \right] + (N + 1)^2k^2 E(\beta_j^2) + V_0 - 2(N + 1)k(\beta_2 + \beta_3) V_0 \]

\[ = (N + 1)k(\beta_2 + \beta_3) - 1)^2 V_0 + (N + 1)^2k^2\beta_2^2 V_1 + (N + 1)^2k^2 E(\beta_j^2). \]

The above expression is true for both regimes, the only difference between the two regimes being the values of \( \beta_2, \beta_3 \), and \( E(\beta_j) \). In Proposition 6, we establish that \( E(\beta_j^2) > E(\beta_j) \), and in Proposition 4, we establish that \( \beta_2^2 > \beta_3^2 \). However, \( \beta_2^2 + \beta_3^2 < \beta_4^2 + \beta_5^2 \). Therefore, the ordering of expected consumer surplus across regimes will depend on the exogenous parameter values. However, from the last expression, it is clear that if \( V_0 \) is sufficiently large relative to \( V_0 \), then expected consumer surplus is larger in the regime with hedge disclosures.
The loss in expected industry output is:

\[ L = E(Q^*) - E(Q^0) = (N + 1)kE(p^*) - (N + 1)kE(p^0). \]

Inserting (36) gives:

\[ L = (N + 1)k\mu \frac{1}{A^*} - \frac{1}{A^0}. \quad (52) \]

The statistic \( L \) measures the absolute magnitude of the loss in physical units of production. We convert this absolute magnitude into a percentage, \( L_r \), by dividing \( L \) by \( E(Q^*) \). Thus:

\[ L_r = \frac{A^0 - A^*}{A^0}. \]

The parameter \( \mu \), which describes how high consumer demand is on average, does affect the absolute magnitude of the loss but does not affect the percentage loss. Recall that:

\[ A^0 = 1 + (N + 1)k + \frac{\rho V}{V + NV_\gamma} V_\gamma(N + 1)k \]

and:

\[ A^* = 1 + (N + 1)k + \frac{\rho}{N + 1} V_\gamma(N + 1)k \]

where:

\[ V = \text{Var}(\bar{\eta} - \bar{\theta} | y) + V_\gamma. \]

From the above expressions, it is clear that the key parameters that affect \( L_r \) are: \( V_\eta, V_\theta, V_\gamma, \rho, \) and \( k \). The variance parameters, \( V_\eta \) and \( V_\theta \), influence the degree of information asymmetry between the informed and uninformed producers, while \( V_\gamma \) and the risk aversion parameter \( \rho \) affect the risk premium embedded in equilibrium futures prices. The parameter \( k \) affects the marginal cost of producers and, therefore, determines the sensitivity of production to the futures price. It can be shown that \( L_r \) is strictly increasing in \( \text{Var}(\bar{\eta} - \bar{\theta} | y) \). This variance is important because it captures the degree of information asymmetry in the economy. In turn, it can be shown that \( \text{Var}(\bar{\eta} - \bar{\theta} | y) \) is strictly increasing in each of the parameters, \( V_\eta, V_\theta, V_\gamma, \) and \( \rho \).

Since the sensitivity of \( L_r \) with respect to each parameter depends on the values of the other parameters, it is difficult to analytically compare individual sensitivities. Therefore, we first calculate the value of \( L_r \) for a benchmark set of parameter values and then vary each parameter around its benchmark value while holding the other parameter values fixed at the benchmark. This approach sheds light on the relative importance of each parameter and identifies the key drivers of the percentage loss in industry output.
FIG. 1.—Sensitivity of the loss in expected industry output to fundamental uncertainties. This figure describes the percentage loss in expected industry output ($L_R$) as a function of the uncertainties in consumer demand ($V_\eta$), production uncertainty ($V_\theta$), and residual noise in consumer demand ($V_\gamma$), caused by the absence of hedge disclosures. [$V_\eta = V_\theta = V_\gamma = 2, \rho = 2, k = 1, N = 11.$]

The benchmark set of parameter values that we used are:

$$V_\eta = V_\theta = V_\gamma = 2, \rho = 2, k = 1, N + 1 = 12.$$  

We make no claim about the realism of these parameter values since only a careful industry-specific “calibration” exercise, of the type undertaken in the calibration literature in macroeconomics (see Kydland and Prescott [1982]), would provide realistic data. For these benchmark parameter values, $A^0 = 19.15$, $A^* = 17.00$, and $L_R = 11.21\%$. Varying one parameter at a time yields the data displayed in figures 1 and 2.

It is apparent from figure 1 that the percentage loss industry output, $L_R$, increases rapidly in $V_\eta$ but is relatively insensitive to $V_\theta$ and $V_\gamma$. As $V_\eta$ ranges over the interval [0,5], the percentage loss in industry output increases from 0% to 22.5%. For most reasonable values of $\mu$, these values of $V_\eta$ seem conservative. Figure 2 indicates that producers’ risk aversion could also be a significant factor in determining the percentage loss in industry output. We also investigated the sensitivity of the percentage loss in industry output to variations in producers’ marginal cost of production. We did not find much sensitivity for plausible parameter values ($k > 0.2$). It is difficult to trace these results to the primitive forces captured in our model. Additionally, the robustness of these results to parameter values is unknown. However, the numerical exercise presented above is indicative of where a more careful calibration study should focus: the degree of uncertainty in consumer demand and the extent of
information asymmetry among individual producers regarding this uncertainty. Such a study would provide important guidance to regulators about the magnitude of potential losses associated with the absence of appropriate hedge disclosures.

8. Extensions and Limitations

We have focused our study of hedge disclosures on effects in commodity futures markets even though most of the discussion in the popular press centers on the capital market. Given that the motive to hedge arises from the uncertainty in commodity prices, an understanding of how producers manage such uncertainty through their hedging and production activities is essential to the debate regarding the economic consequences of hedge disclosures. By studying the link between futures prices and production decisions, we have shed light on an important aspect of hedge disclosures which has been missing both in public discussions and in the academic literature. We have shown that prices in futures markets are key determinants of producers’ short-term operating decisions, such as production decisions. If these prices are inefficient, producers’ operating decisions fail to appropriately respond to all the information that agents possess about the underlying states of the economy. We have shown that, absent appropriate hedge disclosures, futures prices will fail to disentangle hedge-motivated trades from informationally motivated trades, making these futures prices inefficient. This type of confounding depresses expected equilibrium futures prices, which, in turn, reduces industry output. We have shown that there are strong externalities associated

![Figure 2](image-url)
with hedge disclosures, in the sense that disclosures by a relatively small set of informed producers affect the operating decisions of every firm in the industry. Given such externalities, it is unlikely that incentives for voluntary disclosure will be adequate to ensure price efficiency.

Our study could be extended in several ways. In our analysis, the futures market is modeled in a static way, in the sense that futures prices do not change over time. Hence we are unable to shed light on the mark-to-market feature of SFAS No. 133. Extending the analysis to encompass such dynamic issues would be a valuable addition to the literature. This would permit an integration of capital market and futures market effects of hedge disclosures.

We have not considered how incentives for costly information acquisition would be affected by mandatory hedge disclosures. In our model, information about consumer demand is obtained free by the informed producer. If, in fact, this information is costly to acquire, then mandatory disclosure of hedge transactions might make futures prices so informative that firms would lose the incentive to acquire such costly information. Such dampening of incentives for costly information acquisition, induced by hedge disclosures, could possibly make the futures price less rather than more informative. This is a valid concern that we have not addressed, and it is left for future work. Intuitively, the factors that need to be taken into account to address this concern are the amount of noise in futures prices, the cost of information gathering, the effect of some individuals’ information gathering on the incentives of others to gather information, as well as effects on social efficiency and on wealth redistribution. The issue here is analogous to that of optimal patent length, and it is not clear where the line on mandatory disclosures should be drawn.

The wealth redistribution caused by hedge disclosures is a stark issue in our model, since only one producer is informed. If many producers were to privately receive noisy information about consumer demand, the equilibrium futures price would aggregate and potentially reflect the information of all informed producers. In this case, even informed producers might benefit from hedge disclosures since each would have the opportunity of learning from the information of others. In this case, it is plausible that hedge disclosures would result in a strict Pareto improvement.

We have modeled the futures market as consisting of only those traders who are actually engaged in production. In practice, speculators who trade on private account and do not produce any of the commodity are also present in futures markets. Any disclosure mandated by the FASB or the SEC would not apply to such speculators. However, the conclusions of our study would still be valid since all of such speculators’ trades are informationally motivated, so there is no confounding of hedge-motivated trades with informationally motivated trades for these speculators.

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11 We thank Thomas Hemmer, Rachel Hayes, and Rick Lambert for bringing this issue to our attention.
12 We thank Rick Lambert for bringing this to our attention.
The price-taking assumption in our model is questionable. Hellwig [1980] argues that in an economy with a finite number of privately informed agents, the assumption of price-taking behavior presents a technical quandary. In such an economy agents are "schizophrenic" in the sense that they are aware of the covariance between the noise in their private signals and the equilibrium price, yet they behave as price takers. Hellwig [1980] and Admati [1985] show how this schizophrenia problem goes away in a "large economy" where price-taking behavior is a more reasonable assumption. In such a large economy, the covariances that give rise to the schizophrenia problem converge to zero and the equilibrium price is a function of only the fundamentals and the aggregate supply noise. In our model, too, the equilibrium futures price is a function only of the fundamentals ($\eta - \theta$) and the "supply noise," $\theta$. Technically, this feature of our results is due to the assumption in our model that there is a single informed producer who perfectly learns the values of $\eta$ and $\theta$, and acts on this information rather than extracting information from the equilibrium futures price. If instead we had assumed that there were many informed producers, each of whom had noisy information about fundamentals, we would have obtained the same results by considering a sufficiently large economy, as did Hellwig and Admati.

APPENDIX A

Proof of Proposition 2

Given the conjecture that the equilibrium futures price reveals the statistic $\tilde{y}$, the market clearing condition characterizing $p^0$ is:

$$ Q^0 = \frac{y - Q^0 - p^0_y}{\rho V_\gamma} + \frac{E(\tilde{p}|y) - p^0_y}{\rho \text{Var}(\tilde{p}|y)}. \quad (A1) $$

Now from (15), $E(\tilde{p}|y) = E((\tilde{\eta} - \tilde{\theta})|y) - Q^0 = E(\tilde{\eta} - \tilde{\theta}) + \frac{\text{Cov}((\tilde{\eta} - \tilde{\theta}), \tilde{y})}{\text{Var}(\tilde{y})}$.

Let $\alpha = \frac{\text{Cov}((\tilde{\eta} - \tilde{\theta}), \tilde{y})}{\text{Var}(\tilde{y})} = \frac{V_\eta + (1 + \rho V_\gamma) V_0}{V_\eta + (1 + \rho V_\gamma)^2 V_0}$.

Then, using $E(\tilde{\eta} - \tilde{\theta}) = E(\tilde{y}) = \mu$:

$$ E(\tilde{p}|y) = \alpha y + (1 - \alpha)\mu - Q^0. \quad (A2) $$

Also from (15):

$$ \text{Var}(\tilde{p}|y) = V_\gamma + \text{Var}((\tilde{\eta} - \tilde{\theta})|y) $$

$$ = V_\gamma + (V_\eta + V_\theta)(1 - \frac{\text{Cov}^2((\tilde{\eta} - \tilde{\theta}), \tilde{y})}{\text{Var}(\tilde{\eta} - \tilde{\theta})\text{Var}(\tilde{y})}) $$

$$ = V_\gamma + (V_\eta + V_\theta)(1 - \frac{\alpha\text{Cov}((\tilde{\eta} - \tilde{\theta}), \tilde{y})}{V_\eta + V_\theta}). $$
Therefore:
\[ \text{Var}(\tilde{p}|y) = V\eta + V\theta + V\gamma - \alpha\text{Cov}((\bar{\eta} - \bar{\theta}),\bar{y}) = V \quad (A3) \]

Inserting (A2) and (A3) into (A1) and rearranging terms yields:
\[ \rho V\gamma VQ^0 = V(y - Q^0) + NV\gamma(\alpha y + (1 - \alpha)\mu - Q^0) - (V + NV\gamma) p_j^0 \]

which is equivalent to:
\[ \rho_j^0 + Q^0 + \frac{\rho V\gamma V}{V + NV\gamma} Q^0 = \frac{V + \alpha NV\gamma}{V + NV\gamma} y + \frac{(1 - \alpha)NV\gamma}{V + NV\gamma} \mu. \]

Inserting \( Q^0 = (N + 1)k\rho_j^0 \) gives (21), completing the proof. □

**Proof of Proposition 4**

Since \( V > V\gamma \), \( \frac{V}{V + NV\gamma} > \frac{1}{N + 1} \), which implies that \( A^0 > A^* \). Since \( A^0 > A^* \) and \( \frac{V + \alpha NV\gamma}{V + NV\gamma} < 1 \), \( \beta_2^0 < \beta_2^* \).

To prove that \( \beta_3^0 < \beta_3^* \) we need to establish that:
\[ \frac{1}{A^0} \frac{V + \alpha NV\gamma}{V + NV\gamma} > \frac{1}{A^*} \frac{1}{N + 1} \quad \text{or,} \quad A^* \frac{V + \alpha NV\gamma}{V + NV\gamma} - \frac{A^0}{N + 1} > 0. \]

Using the expressions for \( A^0 \) and \( A^* \), the last inequality is equivalent to:
\[ \frac{A^* \alpha NV\gamma}{V + NV\gamma} + \frac{V + (N + 1)kV + \rho V\gamma kV}{V + NV\gamma} \\
1 + (N + 1)k + \frac{\rho V}{V + NV\gamma} V\gamma (N + 1)k \\
\frac{N + 1}{N + 1} > 0. \]

The left-hand side of the above inequality can be expressed as:
\[ \frac{A^* \alpha NV\gamma}{V + NV\gamma} + [(N + 1)k + 1] \frac{V}{V + NV\gamma} - \frac{1}{N + 1} + \rho V\gamma kV \\
\frac{V}{V + NV\gamma} \frac{N + 1}{(N + 1)(V + NV\gamma)}. \]

The first two terms in the above expression are strictly positive, while the last term is equal to zero, which completes the proof. □
Proof of Proposition 5

Let \( t = \frac{V + \alpha NV_y}{V + NV_y} \). Clearly, \( 0 < t < 1 \), since \( 0 < \alpha < 1 \). From (21):

\[
A^0 p_f^0 = t(\eta - \theta) + (1 - t)\mu - \rho V_y t \theta 
\]

and from (26):

\[
A^* p_f^* = (\eta - \theta) - \rho V_y \frac{1}{N+1} 
\]

Since \( A^0 > A^* \), a sufficient condition for \( p_f^0 < p_f^* \), is:

\[
t(\eta - \theta) + (1 - t)\mu - \rho V_y t \theta \leq (\eta - \theta) - \rho V_y \frac{1}{N+1} 
\]

Now \( (\eta - \theta) \geq \mu \Rightarrow t(\eta - \theta) + (1 - t)\mu \leq (\eta - \theta) \), so that the desired result follows if \( t > \frac{1}{N+1} \).

We verify this last inequality below:

\[
t > \frac{1}{N+1} \quad (N + 1)(V + \alpha NV_y) > V + NV_y \\
(N + 1) V - V > NV_y - \alpha N(N + 1) V_y \\
V > V_y - \alpha (N + 1) V_y, \text{ which follows from } V > V_y. \quad \Box
\]

REFERENCES


