Classifications Manipulation and Nash Accounting Standards

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ABSTRACT

This paper studies a model of “classifications manipulation” in which accounting reports consist of one of two binary classifications, preparers of accounting reports prefer one classification over the other, an accounting standard designates the official requirements that have to be met to receive the preferred classification, and preparers may engage in “classifications manipulation” in order to receive their preferred accounting classification. The possibility of classifications manipulation creates a distinction between the official classification described in the statement of the accounting standard and the de facto classification, determined by the “shadow standard” actually adopted by preparers. The paper studies the selection and evolution of accounting standards in this context. Among other things, the paper evaluates “efficient” accounting standards, it determines when there will be “standards creep,” it introduces and analyzes the notion of a Nash accounting standard, and it compares the standards set by sophisticated standard-setters to those set with less knowledge of firms’ financial reporting environments.

1. Introduction

Financial reporting is, at its roots, a process of classification: firms are “going concerns” or not; transactions are “recognized” in a firm’s financial...
statements or not; leases are capital leases or operating leases; expenditures are assets or expenses; financial claims are liabilities or equity, etc. Many analytical studies acknowledge this central role of classification in financial reporting explicitly by representing the financial reporting process as the production of a partition on some underlying state space (see, e.g., Christensen and Demski [2003], Demski [1980], Dye [1985], Ijiri [1975]).

When this classification process works well, financial reporting helps investors predict firms’ future cash flows, which in turn affects investors’ share purchase decisions. To the extent the capital investors supply to firms through their share purchases is used to finance the firms’ subsequent investments, it follows that the classification process underlying financial reporting has real resource allocational effects. This paper contains an examination of the relationship between this classification process and the economy’s productive efficiency in a model that captures several features of the financial reporting environment.

First, the representation of accounting standards adopted in this paper adheres to the conventional binary classification often found in GAAP and GAAS, and the binary classification suppresses many probabilistic nuances. Each of the examples of accounting classification presented in the opening paragraph exhibits both this binary structure and the suppression of probabilistic details. For example, whether a firm will remain a going concern over a particular time interval is uncertain until the time interval passes, yet a firm’s auditors either issue a going concern qualification or they do not. Whether an expenditure will generate future benefits to a firm is often uncertain in the period the expenditure is made, yet the firm must report the expenditure either as an asset or as an expense. And so on.

Second, one of the binary classifications is unambiguously perceived more favorably by investors than the other, and uncertainty regarding which of the two classifications is deemed appropriate is resolved in the financial reporting process by requiring that some (typically context-dependent) threshold be exceeded in order to receive the more favorable classification. The examples of GAAP and GAAS in the opening paragraph illustrate this, too. Obviously, avoiding a going concern qualification is preferred to the alternative, and the likelihood of financial distress influences an auditor’s decision to issue the qualification. Most firms prefer recognizing revenue today to not recognizing it (or recognizing it with a delay), and whether revenue gets recognized following a sale depends on whether the earnings process is considered to be sufficiently complete and whether the proceeds from sale are measurable with a reasonable degree of certainty, etc.

Third, the analysis considers the incentives of firms to engage in “classifications manipulation” in their attempt to secure the preferred accounting classification. The financial reporting process is replete with examples of such behavior: firms often structure lease agreements that are essentially purchase transactions so as to skirt GAAP criteria for capital leases; firms try to include one-time gains in income from continuing operations; firms try to convince their auditors that whatever contingent liabilities they have
outstanding are insufficiently probable so as not to warrant inclusion among their estimated liabilities, etc.

In the presence of these features of the financial reporting process, the analysis emphasizes the distinction between the “official” (or de jure) partitioning of firms (or their activities) designated by GAAP and the “effective” (or de facto) partitioning of firms (or their activities) induced by firms’ classifications manipulation. Classifications manipulation results in more firms receiving the favorable accounting classification than a strict application of GAAP would warrant. And, sophisticated investors adjust their interpretation of a firm’s financial statements accordingly. They recognize that the boundary delimiting the more and less favorable classifications encoded in a GAAP (or GAAS) standard, will—as a consequence of classifications manipulation—be shifted to a lower threshold, a shadow standard, demarking the boundary between those firms who are or are not willing to pay the cost of classifications manipulation to secure the more favorable accounting classification.

While official standards can be viewed as either exogenous or endogenous—depending on whether one perceives the process defining GAAS and GAAP as exogenous or endogenous—a shadow standard is always endogenous: it is the outcome of an equilibrium process involving the specification of mutually consistent behavior for both firms and investors, and it depends on each of: the prevailing official accounting standards, the differential amounts investors attach to firms that receive the more or less preferred accounting classifications, the costs of classifications manipulation, and investors’ understanding (or lack thereof) of the firms’ production technologies.

The analysis demonstrates that, as investors learn more about firms’ production technologies over time, the relationship between the official and shadow standards changes. As a consequence of this learning, accounting standard-setters face a fundamental trade-off: they can choose to hold the official standard constant over time, which causes the shadow standard to change over time, or they can hold the shadow standard constant by repeatedly changing the official standard, but they cannot hold constant both the official and the shadow standards. This trade-off appears to be new to the accounting literature. Moreover, the paper establishes that, if standard-setters choose to hold the shadow standard constant, then on average they will have to increase the official standard over time. To the extent that increases in the official standard represented in this paper proxy for an expansion in the set of financial reporting standards in GAAP, the paper provides an explanation for the perpetual increase in GAAP standards.

When the official standard is considered endogenous, the question arises: what are GAAP/GAAS standards designed to maximize? We posit that standards are chosen to maximize that expected value of the firms subject to the standards net of all relevant costs, including the costs of classifications manipulation. What standards emerge as optimal are shown to depend on the extent to which the distribution of available projects changes as accounting
standards change, on how well standard setters anticipate the financial market’s reaction to changes in standards, and relatedly, on how well standard setters know the parameters of the economy.

Since, in practice, standard-setters may not have good knowledge of some dimensions of the economy, it is important to understand how robust the firms’ values are to standards that depart from those set by fully-informed standard-setters. The paper evaluates the effects of such errors in two ways. First, it contains explicit calculations of the loss in value from incorrectly set standards. Second, it compares the standards chosen by a sophisticated standard setter who is fully aware of how preparers react to a change in standards to the standards chosen by a naive standard setter who selects standards assuming that the actions taken by preparers this year will be the same as their actions last year, and hence who assumes that there will be no reaction to a change in standards. While the naive standard setter’s beliefs will often be wrong—that is, preparers’ actions this year often will be different from the actions they chose in previous years—over time, we show that in stable environments, the standards chosen by naive standard setters will converge to a standard, dubbed a “Nash” standard in the following, in which preparers repeat their actions over time. Upon comparing the Nash standard to the optimal standard chosen by a sophisticated standard setter (dubbed a “Stackelberg” standard), we find that the Nash standard is below the Stackelberg standard. That is, we show that if standard setters do not anticipate the reaction of preparers to a change in standards, then insufficiently stringent standards emerge. Additional comparisons between Nash and Stackelberg standards are also made in the paper.

The contemporary accounting literature related to this paper is sparse. Demski [1973], [1974] made the accounting profession aware of the difficulties in choosing among accounting standards in general environments where the information supplied by accounting reports had wealth redistribution effects and where the information supplied under alternative accounting standards could not be Blackwell-ranked. Demski’s work suggested the development of both narrower specifications of accounting standards and the adoption of narrower efficiency criteria than Pareto-optimality; the present work represents one attempt at developing a theory of standards responsive to these concerns. Arya, Fellingham, Glover, and Schroeder [1998] have recently proposed evaluating depreciation policies designed to achieve efficient investment selection, just as the present work evaluates financial reporting standards from an efficiency perspective.

The paper proceeds as follows: a description of the base model is presented in section 2. Following that, the notion of a financial reporting equilibrium is presented in section 3. The characteristics and evolution of a financial reporting equilibrium are given in section 4. Value-maximizing standards that assume the distribution of firms subject to the standards is fixed are described in section 5. Section 6 presents the effects of introducing errors in the formulation of accounting standards. In section 7, characteristics of value-maximizing standards are considered again, but in this section,
the distributions of firms subject to the standards can change as the standards themselves change. The notions of Nash and Stackelberg standards are introduced and analyzed in that section. Section 8 summarizes the results. The appendix contains all relevant proofs.

2. Model Description

In the base model, entrepreneur $i$ has a “stand-alone” project or production technology which, for life cycle or cash flow reasons, he wishes to sell. If the entrepreneur’s production technology is viable, then by investing $I$ in the technology after the sale, the purchasers of this technology (also known as “investors” in what follows) receive $\hat{\beta}_i \times I^\alpha / \alpha + \tilde{\epsilon}_i$ in cash flows in the period following the sale. Here, $\hat{\beta}_i$ denotes a random “productivity parameter” with prior mean $\bar{\beta}_i$, and $\tilde{\epsilon}_i$ is a mean zero error term independent of $\hat{\beta}_i$. When there are multiple entrepreneurs (indexed by $i$), we assume $\tilde{\epsilon}_i$ is independently distributed across $i$. The scalar $\alpha \in (0, 1)$ indicates the rate at which decreasing returns to investment occur. If the entrepreneur’s production technology is nonviable, then by investing $I$ after the sale, investors receive no cash flows in the period following the sale.

Production technologies differ from each other in terms of the probability they are viable. The probability $\phi_i$ that the entrepreneur’s technology is viable is the realization of some random variable $\hat{\phi}_i$, with density $f(\phi_i)$ and sample space $[\phi_l, \phi_u]$. This probability is not observable to external investors, nor is it capable of being communicated directly by the entrepreneur. Consistent with the remarks made in the Introduction, we model reports produced in compliance with an accounting/auditing standard as providing information about the realized $\hat{\phi}_i$ through a binary partition on $\hat{\phi}_i$’s sample space. Specifically, we define an accounting/auditing standard as a threshold probability $\phi^*$ that partitions the sample space into two sets $W \equiv [\phi_l, \phi^*)$ and $B \equiv [\phi^*, \phi_u]$ ($W$ = “worse”; $B$ = “better”). The accounting/auditing standard is “bright line” in so far as it designates a particular probability threshold that must be attained to achieve the better ($B$) classification.\footnote{Some prevailing accounting standards are explicitly bright line (e.g., accounting for capital and operating leases); others are nearly so (e.g., where the “more likely than not” criterion is employed in the valuation allowance for deferred tax assets, and in the accounting for investments, where the 20% and 50% thresholds are focal points, if not dispositive); and others are somewhat more vague (e.g., the criteria distinguishing between contingent and estimated liabilities). Why there are such variations in the precision of the statement of accounting/auditing thresholds is not addressed in this paper.}

We assume that the entrepreneur’s auditor, or some similar party, administers the bright-line standard: the auditor evaluates which of the classifications $W$ or $B$ is consistent with the appearance of the production technology, and then the auditor issues a report of its findings. Given the report ($W$ or $B$), investors must make inferences about $\hat{\phi}_i$. This inference process

I wish to thank Lenny Soffer for suggesting the deferred valuation allowance example mentioned above.
is complicated because the entrepreneur has the capability to engage in *classifications' manipulation*, that is, the ability to alter the appearance of the production technology so as to achieve the better classification. We posit that an entrepreneur with a technology \( \phi_i < \phi^s \) can alter, at cost \( c \times (\phi^s - \phi_i) \) its appearance to qualify for the better classification. With this specification, the farther the realized \( \phi_i \) is below the official standard \( \phi^s \), the more costly it is for the entrepreneur to obtain the better classification.

An entrepreneur bent on classifications manipulation will do so by just enough to qualify for the preferred accounting treatment, since the investing public only observes the reported classification. An implication of this is that if we—researchers—could observe the fraction of production technologies/projects that just met the standard \( \phi^s \) for the preferred classification, then we would see an atom (i.e., a positive fraction) of projects there, even if the underlying distribution of projects \( \tilde{\phi}_i \) were atomless (i.e., no single point has a positive probability of occurrence). Such “bunching” would be direct evidence of classifications manipulation.3

Whether an entrepreneur will choose to engage in classifications manipulation depends upon whether the benefits from it exceed the costs. If the market value of a project classified as \( W \) (resp., \( B \)) is \( m_W \) (resp., \( m_B \)), then the calculus of classifications manipulation leads to the following optimizing behavior by preparers:

\[
\text{classify } \phi_i \text{ as } B \text{ if } m_B - c \times \max \{\phi^s - \phi_i, 0\} \geq m_W; \\
\text{otherwise, classify } \phi_i \text{ as } W.
\]

Let \( \phi \) be the minimum of the probabilities \( \phi_i \) for which the entrepreneur finds it advantageous to engage in classifications manipulation,4 i.e.,

\[
\phi = \phi^s - \frac{m_B - m_W}{c}.
\]

This \( \phi = \phi(\phi^s) \) is what was referred to in the Introduction as the *shadow standard*, since it determines the effective partitioning of projects induced

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2 Two points should be made here: first, obviously, no entrepreneur with a production technology that is viable with probability \( \phi \geq \phi^s \) must incur a cost to secure the better classification. Second, if an entrepreneur with production technology \( \phi < \phi^s \) does engage in enough classifications manipulation so that his firm qualifies for the better classification, I assume that the manipulation is “form rather than substance,” i.e., the actual probability the production technology is viable remains at its original level \( \phi \).

3 This is consistent with the “bunching” documented in Degeorge, Patel, and Zeckhauser [1999] and Burgstahler and Dichev [1997].

4 This equation is governing only when the threshold \( \phi \) is inside the support of \( \phi \). If \( \phi \) has support \([\phi_l, \phi_u]\), then a complete specification of \( \phi \) is: if \( \phi^s - \frac{m_B - m_W}{c} \in [\phi_l, \phi_u] \), then \( \phi = \phi^s - \frac{m_B - m_W}{c} \); if \( \phi^s - \frac{m_B - m_W}{c} > \phi_u \), then \( \phi = \phi_u \); if \( \phi^s - \frac{m_B - m_W}{c} < \phi_l \), then \( \phi = \phi_l \).

In practice, the boundary cases \( \phi \in [\phi_l, \phi_u] \) will be unimportant, since any official standard that induced \( \phi \) to assume one of these boundaries is tantamount to a standard in which all projects are classified in the same way, in which case the standard (and associated classification of projects) serves no allocative function. In such cases, a policy of having no standard will be weakly superior to having any standard.
by the interaction between the official standard $\phi^i$ and preparers’ optimizing classifications manipulation.\(^5\) Thus, a given official standard $\phi^i$ induces two partitions on projects, the “official” one $\{[\phi_i, \phi^i], [\phi^i, \phi_u]\}$, and the “effective” one $\{[\phi_i, \phi], [\phi, \phi_u]\}$.\(^6\) In the following, considerable attention is devoted to the shadow standard and its associated effective partition, because the economics of an accounting standard are not embodied in the official standard or partition, but rather in the shadow standard and effective partition induced by the official standard. This occurs because the allocation of resources, and the market prices of projects, are determined by the shadow standard, not the official standard.

3. A Financial Reporting Equilibrium

So far, we have discussed how an official standard gets transformed into a shadow standard through an entrepreneur’s optimizing classifications’ manipulating behavior, taking as given the market values $m_B$ and $m_W$ associated with the classifications $B$ and $W$. But, these market values are endogenous and depend upon how much classifications’ manipulation investors believe the entrepreneur has engaged in. To define $m_B$ and $m_W$ in an internally consistent fashion based on exogenous variables, we must describe and characterize a financial reporting equilibrium relative to a given official standard. We do this presently.

We start by expanding on the time line governing the sequence of events. First, an entrepreneur learns the probability $\phi_i$ that his project is viable. Anticipating the market values $m_B$ and $m_W$ associated with the classifications $B$ and $W$, the entrepreneur then decides whether to engage in classifications manipulation. The financial report ($B$ or $W$) for the project is then released to investors. Competition among the, presumed risk-neutral, investors results in the project being sold for the expected value of the future cash flows anticipated to be generated by the project’s stochastic production technology, net of the cost of anticipated future investment they must make in the project, given the reported classification. After the sale, investors proceed with their previously anticipated investments. Finally, the project’s realized cash flows are generated and consumed by the investors. We now discuss the endogenous components of this event sequence in detail.

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\(^5\) I wish to thank my colleague Sri Sridhar for suggesting the name “shadow standard” for this threshold.

\(^6\) While, as the Introduction noted, accounting classifications in practice are often binary, there is nothing in the model that would prevent consideration of partitions with more than two elements. I would like to thank an anonymous referee for this observation.

An expanded analysis could endogenize the optimal number of elements in the partition. In such an expanded analysis, it would typically be undesirable to have a large number of elements in the partition, even if the costs of writing the extra classifications were zero, in order to economize on the costs of classifications manipulation. (These costs typically increase as the number of elements in the partition increase, since the extra elements create more opportunities for classifications manipulation.)
We first describe how a project’s expected cash flows are linked to its classification and the level of investment made in the project. Suppose investors conjecture that the official standard \( \phi^i \) will generate the shadow standard \( \phi \). If the released report is \( B \) (resp., \( W \)), then investors calculate the probability that the project’s stochastic production technology is viable to be 
\[
\Pr(\text{viable} \mid B, \phi) = E[\phi_i \mid \phi_i \geq \phi] \quad \text{and} \quad \Pr(\text{viable} \mid W, \phi) = E[\phi_i \mid \phi_i < \phi].
\]
Thus, from outsiders’ perspective, the net expected cash flows from an investment of \( $I \) on a project that receives the classification \( R \in \{B, W\} \) is:
\[
\Pr(\text{viable} \mid R, \phi) \times \frac{I^o}{\alpha - I} .
\]
(This requires recalling that investment in a nonviable technology produces no return.)

Next, we discuss how the investment of \( $I \) in a project gets determined. Since the project is sold to investors, and it is these investors (not the entrepreneur) who make the investment decision, the investment level \( $I \) chosen must be based on information available to them. Investors know which of the classifications \( B \) or \( W \) is associated with the project they purchase, and so they can predicate their choice of \( $I \) at least on whether \( \phi_i < \phi \) or \( \phi_i \geq \phi \). The following analysis proceeds by assuming that this is all the information about \( \hat{\phi}_i \) on which investors base their choice of \( $I \). However, it is important to digress briefly to consider what qualitative differences would emerge in extensions of the analysis were investors to obtain additional post-sale information about \( \hat{\phi}_i \).

**Digression on when accounting standards have economic consequences**

For any such extension, for arbitrary \( \phi \), either investors’ ultimate knowledge of \( \hat{\phi}_i \) varies with what they knew about \( \hat{\phi}_i \) at the time of sale—that is, on whether \( \hat{\phi}_i < \phi \) or \( \hat{\phi}_i \geq \phi \) (call this case I)—or else their ultimate knowledge of \( \hat{\phi}_i \) is independent of what they knew about \( \hat{\phi}_i \) at the time of sale (call this case II). An example of case I is this: after the sale of a project that received the classification \( B \) (resp., \( W \)), investors additionally learn whether \( \hat{\phi}_i \)’s realization belongs to the upper or lower half of the interval \( [\phi, \phi_u] \) (resp., \( [\phi_l, \phi] \)), i.e., they learn which element of the partition \( \{[\phi, \phi_i + \phi/2], \ldots, [\phi_i + \phi/2, \phi] \} \) the realized \( \hat{\phi}_i \) belongs to. Generically, the only example of case II of which we are aware involves investors learning the exact realization of \( \hat{\phi}_i \).

\[\text{Note that this refined partition varies with } \phi \text{ and hence is consistent with case I.}\]

\[\text{Clearly, if following the sale, investors learn the exact realization of } \hat{\phi}, \text{ then their knowledge of } \hat{\phi} \text{ at the time of sale (namely, whether } \phi < \hat{\phi} \text{ or } \hat{\phi} \geq \phi \text{) does not affect their ultimate knowledge of } \phi. \text{ Hence, learning the exact realization of } \hat{\phi} \text{ is consistent with case II.}\]

We now argue that, generically, the converse is also true: that is, unless investors ultimately learn the exact value of \( \phi \), then their knowledge of \( \phi \) will vary with what they know about \( \phi \) at the time of sale.

The argument runs as follows. For a fixed accounting standard \( \phi^i \) and associated shadow standard \( \phi \), as a consequence of the accounting report \( R \in \{B, W\} \), investors know which...
In extensions of the present analysis, the economic consequences of an accounting standard vary significantly depending on which of case I or case II is applicable. If case I is applicable, then—as in the analysis that follows—selecting an accounting standard $\phi$ has both distributional and allocational consequences: the choice of $\phi'$ affects $\phi$, which in turn affects investors’ ultimate knowledge of $\tilde{\phi}_i$, which in turn affects their choice of $I$. So, both the selling price of the project (a distributional effect) and the amount of investment in the project (an allocational effect) result from the specification of $\phi'$ in case I. In contrast, when case II is applicable, there are no allocational effects associated with $\phi$, although there are distributional effects. In that case, the specification of $\phi'$ affects $\phi$ and hence affects the project’s selling price, but it has no allocational effects, since investors’ ultimate knowledge of $\tilde{\phi}_i$ is, by definition of case II, independent of $\phi$ (and hence independent of $\phi'$), and so the investment $I$ does not vary with $\phi'$. Thus, in case II, there are no allocational consequences of choosing among accounting standards.

While some of our main results\(^9\) will hold for extensions of our analysis involving both cases I and II, the results below regarding efficient accounting standards are suggestive of the kinds of results that would obtain were investors to acquire additional post-sale information about $\tilde{\phi}_i$ only when, as in case I, the information gleaned from the accounting reports remains incrementally informative relative to whatever additional post-sale information investors acquire.\(^10\)

3.1 THE DEFINITION OF EQUILIBRIUM

Returning to the model, the efficient allocation of capital, based on investors’ knowledge of the returns on investment, requires maximizing the net expected cash flows displayed in (2). The net expected cash flows from the project based on this anticipated investment then determine the market prices $m_W$ and $m_B$. Finally, these market prices are linked to the equilibrium amount of classifications manipulation through (1). When all of these

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\(^9\) In particular, each of the following continues to be valid: the distinction between a shadow standard and the official standard, the importance of classifications manipulation, and the drift over time in the relationship between the official and shadow standards depicted in Theorem 2 below.

\(^10\) Case I is the practically relevant case when investors cannot acquire exact information about their production technology (and hence $\phi'$’s realization) following the technology’s sale.

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conditions hold concurrently we obtain a financial reporting equilibrium. Formally:

**Definition 1.** A financial reporting equilibrium relative to accounting standard $\phi^s$ consists of a pair of investment levels $I^*(B)$ and $I^*(W)$ and market values $m_B$ and $m_W$ and a shadow standard $\hat{\phi}$ such that:

(i) An entrepreneur whose project is viable with probability $\phi_i$ engages in classifications manipulation if and only if $\phi_i \geq \hat{\phi}$;

(ii) $\Pr(viable \mid B, \phi) = E[\tilde{\phi}_i \mid \tilde{\phi}_i \geq \phi]$ and $\Pr(viable \mid W, \phi) = E[\tilde{\phi}_i \mid \tilde{\phi}_i < \phi]$;

(iii) For $R \in \{B, W\}$, the investment $I^*(R)$ attains the maximum of:

$$\max_i \Pr(viable \mid R, \phi) \tilde{\beta}_i I^{\alpha} - I.$$  

(iv) For $R \in \{B, W\}$, the market value $m_R$ is given by:

$$m_R = \Pr(viable \mid R, \phi) \tilde{\beta}_i (I^*(R))^{\alpha} - I^*(R);$$

(v) $\phi^s = \phi^t - m_B - m_W c$.

In brief, a financial reporting equilibrium consists of a pair of investments and market values and a shadow standard determining how projects are classified, given the prevailing accounting standard, so that:

- investment is efficient given investors’ knowledge;
- prices of projects are set correctly;
- preparers optimize when engaging in classifications manipulation.


We start the formal analysis with a more explicit description of the relationships among the equilibrium investment levels, market values, and the shadow standard. We then follow this with an analysis of how the financial reporting equilibrium evolves over time.

A simple calculation shows that, when the accounting classification is $B$, the optimal level of investment, $I^*(B)$ is given by:

$$I^*(B) = (\tilde{\beta}_i E[\tilde{\phi}_i \mid \tilde{\phi}_i \geq \phi])^{\frac{1}{\alpha}},$$

which implies that the project has market value$^{11}$

$$m_B = E[\tilde{\phi}_i \mid \tilde{\phi}_i \geq \phi] \tilde{\beta}_i I^*(B)^{\alpha} - I^*(B) = \frac{1 - \alpha}{\alpha} (\tilde{\beta}_i E[\tilde{\phi}_i \mid \tilde{\phi}_i \geq \phi])^{\frac{1}{\alpha}}.$$  

$^{11}$ The computation of the market value $m_B$ runs as follows. The first-order condition for $I^*(B)$ is 

$$E[\tilde{\phi} \mid \tilde{phi} \geq \phi] \tilde{\beta} I^*(B)^{\alpha - 1} = 1.$$  

So,
Similar calculations for a project that receives the classification $W$ yield:

$$I^*(W) = (\bar{\beta}_i E[\hat{\phi}_i | \hat{\phi}_i < \phi])^{1/\alpha}$$ (5)

and

$$m_W = \frac{1 - \alpha}{\alpha} (\bar{\beta}_i E[\hat{\phi}_i | \hat{\phi}_i < \phi])^{1/\alpha}.$$ (6)

The equation describing the shadow standard (1) links these two market prices together with the cost of classifications manipulation:

$$\phi = \phi^* - \frac{1 - \alpha}{\alpha c} \times \bar{\beta}_i \times \left( E[\hat{\phi}_i | \hat{\phi}_i \geq \phi]^{1/\alpha} - E[\hat{\phi}_i | \hat{\phi}_i < \phi]^{1/\alpha} \right).$$ (7)

We shall show below that, even when this equation cannot be solved explicitly, much can be learned about the relationship between the official and shadow standards. However, to exhibit a financial reporting equilibrium explicitly, it is useful to parameterize the model in a way which admits an explicit solution to this equation. We parameterize the problem by assuming $\alpha = .5$ and that $\hat{\phi}_i$ is uniform on $[l, u]$, i.e., $f(\hat{\phi}_i) = \frac{1}{u - l}, \hat{\phi}_i \in [l, u].$ 12

For this parameterization, we conclude:

**THEOREM 1 (The Relationship Between Official and Shadow Standards).**

Assume $\alpha = .5, \hat{\phi}_i \sim \text{Uniform}[l, u]$, and that the official standard $\phi^*$ is such that the shadow standard $\phi$ is interior (i.e., $\phi \in (l, u)$). Then, a financial reporting equilibrium relative to accounting standard $\phi^*$ and distribution $\hat{\phi}_i$ exists, is unique, and is (partially) described by the shadow standard:

$$\phi = \frac{\phi^* - \frac{1}{\alpha} \frac{u - \frac{l}{\bar{\beta}_i \alpha}}{4c}}{1 + \frac{1}{\bar{\beta}_i^2 (u - l)}}$$ (8)

and the market prices:

$$m_W = \bar{\beta}_i^2 \left( \frac{l + \phi}{2} \right)^2 \quad \text{and} \quad m_B = \bar{\beta}_i^2 \left( \frac{u + \phi}{2} \right)^2.$$ (9)

The expression (8) relates the threshold $\phi$ to the standard $\phi^*$, the cost of classifications manipulation, the parameters of the distribution of $\hat{\phi}_i$, and the unknown productivity parameter $\bar{\beta}_i$. Notice that $\phi < \phi^*$ unless the cost of classifications manipulation is infinite, so the set of projects that receive the

$$m_B = E[\hat{\phi} | \hat{\phi} \geq \phi] \bar{\beta} I^*(B)^{\alpha - 1} - I^*(B)$$

$$= (E[\hat{\phi} | \hat{\phi} \geq \phi] \bar{\beta} I^*(B)^{\alpha - 1} - 1) \times I^*(B)$$

$$= (1/\alpha - 1) \times I^*(B) = \frac{1 - \alpha}{\alpha} (\bar{\beta} E[\hat{\phi} | \hat{\phi} \geq \phi])^{1/\alpha}.$$ (11)

The computations for $m_W$ are similar.

12 We write this in the following as $\hat{\phi} \sim \text{Uniform}[l, u]$. 


better classification \( B \) is always overstated relative to what a strict application of GAAP would warrant, as suggested above.

While comparative statics can be performed directly on the shadow standard \( \phi \), the comparative statics that are of most interest employ this shadow standard to calculate the probability that a project will receive a particular classification. We state these comparative statics in the next corollary, after rewriting the shadow standard in terms of the mean and standard deviation of \( \phi_i' \)'s distribution. With \( \mu = \frac{u + l}{2} \) and \( \sigma = \frac{u - l}{2\sqrt{3}} \), the shadow standard can be written as

\[
\phi = \frac{\phi_i' - \bar{\beta}_i \sigma \sqrt{3} \mu}{1 + \bar{\beta}_i \sigma \sqrt{3} c}.
\]

(10)

**COROLLARY 113 (The Comparative Statics of Shadow Standards).** Maintain the assumptions of the preceding theorem. The probability that a project will receive the classification \( B \) (resp., \( W \)) is

(i) decreasing in the standard \( \phi_i' \) and the cost of classification manipulation \( c \), and

(ii) increasing in \( \bar{\beta}_i \) and \( \mu \).

Most of these results are intuitive: as the standard \( \phi_i' \) goes up, or as the cost \( c \) of classifications manipulation increases, then the probability of getting the more favorable accounting treatment declines. Also, as the expected value of the productivity parameter \( \bar{\beta}_i \) increases, or as the prior probability \( \mu \) that a project is viable increases, then the returns to the preferred classification also increase (since \( \frac{\partial}{\partial \mu} (m_B - m_W) > 0 \) and \( \frac{\partial}{\partial \mu} (m_B - m_W) > 0 \) and so preparers will engage in classifications manipulation more often (i.e., for a bigger set of realized \( \phi_i' \)'s). No comparative static is reported involving changes in the standard deviation of \( \phi_i' \) on the probability that a project is classified as \( B \), since that effect is generally ambiguous.

Next, we consider the preceding model and associated equilibrium as a snapshot of a multi-period model in which, in any given period, the (current generation) entrepreneur and (current generation) investors get to witness and learn from what happened in prior periods when other projects were transferred between previous generations of entrepreneurs and investors. In this setting, it is natural to inquire how the financial reporting equilibrium evolves over time. We study the stationary case, where the distribution of \( \phi_i' \) is independently and identically distributed over time, and—with the possible exception of the productivity parameter \( \bar{\beta}_i \)—all other parameters of the model (the cost of classifications manipulation \( c \), and the preferences of the entrepreneurs and investors) remain constant. The following theorem

---

13 The proof of the corollary involves simple algebra and is not included in the appendix.

14 But, if the standard \( \phi_i' \) is at or below the mean probability \( \mu \) that a firm is viable, one can conclude that the probability a firm will be classified as viable is decreasing in the standard deviation \( \sigma \).
applies in both the “completely” stationary case where $\tilde{\beta}_i$ is constant over time, as well as in the “partially” stationary case where $\tilde{\beta}_i$ varies over time but has some stationary component. To be specific, in the partially stationary case, it suffices that there be no information available at time $t$ better than the time $t$ value of productivity parameter that is helpful in predicting the productivity parameter at any time $t' > t$ and that the time series $\tilde{\beta}_it$ be a martingale. More formally, if $\vartheta_t$ represents all that can be known about the time series $\tilde{\beta}_it$ up to time $t$, then $E[\tilde{\beta}_it'|\vartheta_t,\tilde{\beta}_it]=\tilde{\beta}_it$ for all $t' > t$.

The following theorem demonstrates that, even with all this stationarity, the relationship between the shadow standard and the official standard is not constant over time. The theorem shows that if standard setters want to hold the shadow standard constant over time (perhaps because they prefer consistency in the classification of projects across time), then on average they will have to increase the official standard over time. That is, there must be “standards creep.”

**THEOREM 2 (Standards Creep).** If the official standard $\phi^s$ is chosen in each period so as to induce the same shadow standard $\phi$ and the environment is (partially or completely) stationary, then the expected value of the official standard $\phi^s$ must be increasing over time.

This result is suggestive of a fundamental trade-off in designing accounting standards: accounting standards can be chosen so that the “face” value of the standard $\phi^s$ is intertemporally constant, or accounting standards can be chosen so that the induced shadow standard $\phi$ is intertemporally constant, but not both.

The result is somewhat surprising given the assumed stationarity in the environment. To obtain intuition for the result, we will consider in the text the special case where $\alpha = .5$ and $\tilde{\beta}_i$ is “completely” stationary (though the result also holds for any $\alpha \in (0, 1)$ and for the case of time-varying $\tilde{\beta}_it$’s too). In this case, the equation (7) defining the shadow standard reduces to:

$$\phi = \phi^s - \frac{1}{c} \times \tilde{\beta}_i^2 \times \left( E[\phi_i | \phi_i \geq \phi]^2 - E[\phi_i | \phi_i < \phi]^2 \right).$$  

(11)

It is apparent from (11) that the shadow standard $\phi$ prevailing in a period is affected by the market’s perceptions of $\tilde{\beta}_i$ only through $\tilde{\beta}_i^2$. In particular, if $\tilde{\beta}_i^2$ increases, then the official standard must be raised to keep the shadow standard constant.\(^\text{15}\)

Now, consider what happens with the passage of time. As time evolves, successive generations of investors will acquire more and more precise information about the value of the unknown productivity parameter $\tilde{\beta}_i$. This learning about $\tilde{\beta}_i$ will occur since the cash flows generated by each viable project, $\tilde{\beta}_i \times I^\alpha/\alpha + \tilde{\varepsilon}_i$, provide investors with indirect information about

\(^{15}\) Since $E[\phi_i | \phi_i \geq \phi]^2 - E[\phi_i | \phi_i < \phi]^2 > 0$.  

\( \hat{\beta}_i \)'s realized value. To describe this formally, we recall the notation \( \vartheta_t \) introduced above. It represents all that will be known about the productivity parameter \( \hat{\beta}_i \) up through period \( t \). If "today" is period 0, then \( \hat{\vartheta}_0 \) is uncertain for any \( t > 0 \) whereas \( \vartheta_0 \) is known. In this notation, today's (resp., period \( t \)')s expected value of the productivity parameter \( \hat{\beta}_i \) is denoted by \( E[\hat{\beta}_i | \vartheta_0] \equiv \beta_i \) (resp., \( E[\hat{\beta}_i | \hat{\vartheta}_i] \)).

In any period \( t > 0 \), the official standard \( \phi_i^t \) and the shadow standard \( \phi^t \) prevailing in period \( t \) will be connected to each other through the following counterpart to (11):

\[
\phi_i = \phi_i^t - \frac{1 - \alpha}{\alpha c} (E[\hat{\beta}_i | \hat{\vartheta}_i])^2 (E[\hat{\phi}_i | \phi_i > \phi_i^t])^2 - E[\hat{\phi}_i | \phi_i < \phi_i^t])^2. \tag{12}
\]

Analogous to what (7) demonstrated in period 0, equation (12) demonstrates that the shadow standard \( \phi_i \) prevailing in period \( t \) is affected by the market's perceptions of \( \hat{\beta}_i \) at that time only through the expectation \( (E[\hat{\beta}_i | \hat{\vartheta}_i])^2 \). In fact, (12) tells us more: it tells us that to hold the shadow standard constant as \( (E[\hat{\beta}_i | \hat{\vartheta}_i])^2 \) increases, the official standard \( \phi_i^t \) must increase. This fact drives the theorem: since more and more information accumulates about \( \hat{\beta}_i \) as time passes, the expected conditional variance of \( \hat{\beta}_i \) (calculated today), \( E[Var(\hat{\beta}_i | \hat{\vartheta}_i) | \vartheta_0] \), will decrease the further into the future we look (i.e., the higher \( t \) is). That is,

\[
E[Var(\hat{\beta}_i | \hat{\vartheta}_i) | \vartheta_0] = E[E[\hat{\beta}_i^2 | \hat{\vartheta}_i] - E[\hat{\beta}_i | \hat{\vartheta}_i]^2 | \vartheta_0]
= E[\hat{\beta}_i^2 | \vartheta_0] - E[E[\hat{\beta}_i | \hat{\vartheta}_i]^2 | \vartheta_0]
\]

decreases as \( t \) increases, and so \( E[Var(\hat{\beta}_i | \hat{\vartheta}_i) | \vartheta_0] \) increases as time passes, and hence the official standard \( \phi_i^t \) must be adjusted upwards to keep \( \phi \) constant. Thus, the anticipation of learning about \( \hat{\beta}_i \) is responsible for an increase in the expected value of the official standard over time.

Moreover, notice that this effect will be reinforced if the cost \( c \) of classifications manipulation were to decline over time, perhaps because of the information diffused through investment bankers about how a preferred accounting treatment can be achieved.\(^{16}\)

\(^{16}\) I would like to thank Mary Barth for the observation that the cost \( c \) of classifications manipulation might, under some circumstances, increase over time. While the diffusion of innovations by investment bankers across firms would tend naturally to reduce \( c \) over time (as the text mentions), increased scrutiny by SEC officials of practices that they considered to be violative of the spirit of some standards could cause these costs to increase over time. Of course, if this happened, then the interaction between the learning effects discussed in Theorem 2 and changes in \( c \) would lead to indeterminant effects on the time series evolution of the official standard \( \phi \).

I also want to thank Lenny Soffer for his observation that, apart from the behavior of investment bankers, one might expect \( c \) to decline over time because of firms' constantly “pushing the envelope” to determine what accounting treatments qualify as being in accordance with GAAP. Once a firm, or a collection of firms, discovers that no objection gets raised to what previously was regarded as an aggressive accounting treatment of some transaction, then that
5. Value-Maximizing Standards When the Distribution of Available Projects Is Fixed

The analysis so far has taken the official standard $\phi^s$ as exogenously given. In this section, as well as throughout much of the remainder of the paper, we consider endogenizing the choice of the official standard. In making the official standard endogenous, we must endow the standard setters empowered to choose the standard with an objective function. In this section, we presume that standards are chosen to maximize the expected value of a project, before the project is classified, net of the expected cost of classifications manipulation, while holding the distribution of projects fixed. Choosing a standard to maximize this objective in part involves trading off the costs of two kinds of errors: misclassifying a viable project as nonviable, and misclassifying a nonviable project as viable. The standard setters must also account for the relative frequency of viable and nonviable projects in the population, as well as the cost of classifications manipulation.

Once again we restrict attention to the parameterizations $\alpha = .5$ and $\tilde{\phi}_i \sim \text{Uniform}[l, u]$. The expected value of a project net of the cost of classifications manipulation is given by:

$$
\Pr (\tilde{\phi}_i \leq \phi (\phi^s)) \times m_W + \Pr (\tilde{\phi}_i > \phi (\phi^s)) \times m_B - \int_{\phi (\phi^s)}^{\phi^s} c(\phi^s - \phi_i) f(\phi_i) \, d\phi_i. 
$$

The last term is the expected cost of classifications manipulation. When $\phi (\phi^s) \in (l, u)$, these expected costs integrate to:\(^{18}

$$
\frac{(u - l) \tilde{\beta}_1^4}{8c} \left( \phi (\phi^s) + \frac{u + l}{2} \right)^2. 
$$

Since the shadow standard $\phi (\phi^s)$ increases in $\phi^s$, we see that the expected costs of classifications manipulation increase as the standard $\phi^s$ increases. It

aggressive treatment subsequently becomes the benchmark against which even more aggressive reporting of transactions will be compared in the future.

\(^{17}\) In later sections, we allow the distribution of projects to change as the accounting standard changes.

\(^{18}\) Recall from Theorem 1 that $\phi^s$ and $\phi$ are related through the equation

$$
\phi \left( 1 + \frac{\tilde{\beta}_1^2}{2c} (u - l) \right) + \frac{\tilde{\beta}_1^2}{4c} (u^2 - l^2) = \phi^s.
$$

Hence, we can express the expected cost of transactions manipulation as:

$$
\frac{1}{u - l} \int_{\phi}^{\phi^s} c(\phi^s - \phi) \, d\phi = \frac{c}{2(u - l)} (\phi^s - \phi)^2 = \frac{c}{2(u - l)} \left( \phi \left( \frac{\tilde{\beta}_1^2}{2c} (u - l) \right) + \frac{\tilde{\beta}_1^2}{4c} (u^2 - l^2) \right)^2
$$

$$
= \frac{(u - l) \tilde{\beta}_1^4}{8c} \left( \phi + \frac{u + l}{2} \right)^2.
$$
can be shown that these costs are also always increasing in the productivity parameter $\beta_i$.

Further, the expected cost of classifications manipulation is never monotonic in $c$. The graph in figure 1 below illustrates how these costs change as the parameter $c$ changes.

The official standard $\phi^*$ and the associated shadow standard $\phi(\phi^*)$ that maximize the expected value of a project net of the expected cost of classifications manipulation are detailed in the following theorem.

**THEOREM 3 (Value-Maximizing Standards when the Distribution of Projects is Fixed).** When $\alpha = .5$ and $\beta_i^*$ Uniform[$l, u$], and the standard $\phi^*$ is chosen to maximize the expected value of a project net of the expected cost of classifications manipulation, then

(i) when $c > \frac{(3l + u)\beta_i^2}{2}$, the optimal shadow standard is

$$\phi = \frac{u + l}{2} \times \frac{2 - \frac{(u - l)\beta_i^2}{c}}{2 + \frac{(u - l)\beta_i^2}{c}} = \mu \times \frac{1 - \frac{\sqrt{3\sigma_i \beta_i^2}}{c}}{1 + \frac{\sqrt{3\sigma_i \beta_i^2}}{c}},$$

and the official standard associated with this shadow standard is

$$\phi^* = \frac{u + l}{2} = \mu;$$

(ii) the net expected value of the project is strictly increasing in the cost $c$ of classifications manipulation, for $c > \frac{(3l + u)\beta_i^2}{2}$.

(iii) When $c \leq \frac{(3l + u)\beta_i^2}{2}$, the optimal standard is no standard, i.e., $\phi = \phi^* = l$.

Several parts of the theorem are of interest. First, one might expect that, since the probability a project is viable is uniformly distributed, the optimal standard would result in half the projects receiving the classification $B$ and half receiving the classification $W$. But, according to the theorem, even though $\phi^* = \mu$, more than 50% of all projects are classified as $B$. This follows since the reported classifications are determined by the shadow standard $\phi$, not the official standard $\phi^*$, and $\phi < \mu$. The explanation for this asymmetry is the presence of, and the accounting for, the costs of classifications manipulation in the standard setters’ objective function. Since the costs of classifications manipulation are incurred only when the better ($B$) classification is received, the optimal standard that “nets out” these costs of classifications manipulation is set below the mean. It can be shown that, were the objective

---

19 That is, $\frac{(u - l)\beta_i^4}{8u} - \left( \frac{\phi^* - \beta_i^2 \sigma_i^2}{1 + \beta_i^2 \sigma_i^2} \right)^2 + \frac{u + l}{2}$ is increasing in $\beta_i$.

20 This last claim is easiest to see by writing the expected cost of transactions manipulation directly in terms of $\phi^*$. Omitting the algebra, these costs can be shown to be expressable as $\frac{(u - l)\beta_i^4}{8u} - \left( \frac{\phi^* + \phi - \beta_i^2}{1 + \beta_i^2 \sigma_i^2} \right)^2$. Now, for any positive finite $c$, under the preceding conditions, these expected costs are positive. Since, as $c$ approaches $0$, the costs approach zero, and as $c$ approaches infinity, the costs also approach zero, the nonmonotonicity follows.

21 I would like to thank Madhav Rajan for pointing out an error in the construction of this graph in a previous version of this manuscript.
designed to maximize the expected value of a project gross of these expected costs of classifications manipulation, then the optimal shadow standard is set at the mean, and the official standard is set above the mean. But, since the costs of classifications manipulation are real costs, standard setters should account for them when designing standards, and so, other things equal, standards should be set in part to economize on these costs.

Second, it is interesting to note from (i) that, even though the standard is chosen to maximize the expected value of a project net of the cost of classifications manipulation, the cost \(c\) does not enter into the determination of the optimal standard \(\phi^t\), as long as the cost \(c\) exceeds \( (3l + n)\bar{\beta}^2 \). However, as part (iii) reports, when the cost \(c\) is low \((c \leq (3l + n)\bar{\beta}^2)\), standards serve no function. For such low \(c\)'s, nontrivial standards \((\phi^t > l)\) generate costs of classifications manipulation, yet they fail to discriminate among projects: all entrepreneurs, regardless of their project’s realized \(\phi^t\), choose to have their project classified as \(B\). This may be the economics underlying SFAS 2 and other standards that purposefully do not discriminate between economically different expenditures.

Part (ii) observes that increases in the cost of classifications manipulation always increases the net expected value of a project. While, holding projects’ market prices fixed, entrepreneurs may prefer to reduce the cost of obtaining the better classification, ex ante they are always worse off by having these costs decline, since the market will anticipate their subsequent exploitation of the reduced costs of getting the preferred classification. High costs of classifications manipulation (i.e., high \(c\)'s) commit entrepreneurs to engage in less manipulation, which is \textit{ex ante} efficient.

The theorem also yields several comparative statics. According to the theorem, the optimal standard \(\phi^t\) equals the expected probability that a project is viable, and so it increases as this mean probability increases. \(\phi^t\) is otherwise independent of the parameters of the financial reporting environment, a
fact that will be important in the analysis of errors in standard setter’s beliefs in the next section. Finally, the optimal shadow standard $\phi$ is: below the mean probability that a project’s production technology is viable; increasing in the mean and decreasing in the standard deviation of the distribution of $\phi_i$; and decreasing in the expected value of the productivity parameter $\tilde{\beta}_i$. These are all testable implications of the theorem.

Before concluding this section, we discuss the robustness of the results in this section to variations in the distribution of $\tilde{\phi}_i$. Since the uniform distribution is a special case of the beta distribution, one way to evaluate the robustness of the above results is by comparing them to corresponding results generated by other beta distributions. While members of the beta family other than the uniform distribution are difficult to deal with analytically, numerical plots of these cases provide some insight into these robustness questions. Figures 2a–2d plot four densities drawn from the beta class, and Figures 3a–3d plot the net expected value of a project (net of the expected cost of classifications’ manipulation) as a function of the shadow standard that de facto partitions the better ($B$) and worse ($W$) accounting classifications.

The beta distributions depicted in Figures 2a and 2b are symmetric, and as careful scrutiny of Figures 3a and 3b reveals, the optimal shadow standards corresponding to these symmetric distributions are close to, but slightly below, the (common) mean (.5) of these distributions. In contrast, the beta distribution in Figure 2c (resp., Figure 2d) is skewed right (resp., left), and upon careful scrutiny, Figure 3c (resp., Figure 3d) reveals that the optimal value of the shadow standard is close to but to the left (resp., right) of the distribution’s mean (which is 5/7 (resp., 2/7) for the distribution in Figure 3c (resp., Figure 3d)). Generalizing from these figures, it appears that a useful heuristic is to have the shadow standard close to the mean of $\tilde{\phi}_i$’s distribution, and that if $\tilde{\phi}_i$ is either symmetric or skewed right, then the optimal value of the shadow standard is slightly below the distribution’s mean, consistent with the results presented above: in these cases, the cost of classifications’ manipulation “pulls down” the shadow standard below the mean of $\tilde{\phi}_i$’s distribution. But, if $\tilde{\phi}_i$’s distribution is skewed sufficiently left, then, notwithstanding the extra cost of classifications manipulation it induces, the optimal shadow standard (and, a fortiori, the optimal official standard) may be to the right of the distribution’s mean.

6. Errors in Specifying Standards

The analysis in the preceding section assumed that standard setters know the economic environment in which the sale of projects was conducted: they know the parameters of the distribution ($u$ and $l$) generating viable projects; they know the cost of classifications manipulation; they have the same beliefs about the expected value of the productivity parameter as do other market participants, etc. While the actual participants in an economy have strong financial incentives to acquire such a sophisticated understanding of the environment in which they work, it is not clear that standard
setters have corresponding incentives. Indeed, one might argue that many lobbying efforts are designed to prevent standard-setters from acquiring such knowledge. It is important, consequently, in any discussion of standards, to consider the robustness of the standards with respect to errors. 

Fig. 2.—Plots of beta densities for various parameterizations.
in the standard setters’ knowledge. This section discusses these robustness questions.

To evaluate errors in standards, we continue to assume that market participants (preparers and investors) know the correct values of the economy’s
parameters, and we consider what happens when standards are set incorrectly. That is, regardless of how standard setters arrive at the specified official standard $\phi^*$, the shadow standard $\phi$, and the selling prices $m_W$ and $m_B$ are set rationally according to (8) and (9). We further assume that, as before, standard setters choose standards that, based on their information, are perceived to maximize the expected value of a project net of the expected cost of classifications manipulation. That is, if the standard setters believe $\tilde{\phi}_i \sim \text{Uniform}[l^*, u^*]$, then they set $\phi_i = \frac{u^* + l^*}{2}$.

In this setting, notice that standard setters can remain ignorant—or wrongheaded—concerning both the cost $c$ of classifications manipulation and the expected value $\bar{\beta}_i$ of the productivity parameter $\tilde{\beta}_i$, and it makes no difference to the construction of the standard. Implementation of the optimal standard is robust with respect to these kinds of errors. And, if the standard setters are wrong about the support of the distribution, i.e., about $l^*$ and $u^*$, the only sense in which their error matters to the formation of the standard is the error in the mean: $e = \frac{u^* + l^*}{2} - \frac{u + l}{2}$. To evaluate the consequences of the error $e$, we calculate the difference in the maximum expected value of a project had no error in standards’ specification occurred to the maximum expected value of a project in the presence of this erroneously set standard. We refer to this difference as the absolute robustness of a standard subject to error $e$. (The relative robustness of a standard subject to error $e$ is the ratio of this difference to the maximum value of a project when the standard is constructed with no error.)

**THEOREM 4.**

(1) **The Robustness of Standards to Errors in Standard Setters’ Beliefs.)** Suppose $\alpha = .5$ and $\tilde{\phi}_i \sim \text{Uniform}[l, u]$. If the objective in setting a standard is to maximize the expected value of a project net of the expected cost of classifications manipulation, then the robustness of a standard with error $e = \frac{u^* + l^*}{2} - \frac{u + l}{2}$ is given by:

\[
\begin{array}{ccc}
\text{error} & \text{absolute robustness} & \text{relative robustness} \\
e & \frac{\beta_i^2 e^2}{4c + 2\beta_i^2(u - l)} & \frac{8c e^2}{2c(5l^2 + 6lu + 5u^2) - \beta_i^2(u - l)^3} \\
\end{array}
\]

Notice that the robustness of a standard is quadratic in the error $e$. This has two implications. First, the first-order effect of small errors is zero in both absolute and relative terms (i.e., $\frac{d}{de} \frac{\beta_i^2 e^2}{4c + 2\beta_i^2(u - l)} \big|_{e=0} = 0$ and $\frac{d}{de} \frac{8c e^2}{2c(5l^2 + 6lu + 5u^2) - \beta_i^2(u - l)^3} \big|_{e=0} = 0$). Second, the direction of the error does not matter: overestimates of the mean $\frac{u + l}{2}$ are equally costly as underestimates (and vice versa).\(^{23}\) It can also be shown that the absolute (resp.,

\(^{22}\) The proof of this theorem, which involves simple, but lengthy, algebraic manipulations, is omitted.

\(^{23}\) Prior to obtaining this result, one might have guessed that the presence of—and concern over—classifications costs might have induced an asymmetry between the effects of overstated errors and understated errors: errors of overstatement lead to higher standards which in turn would seem to lead to higher classifications costs, which would seem to make overestimates of standard setters more costly than underestimates.
relative) robustness of a standard is decreasing (resp., increasing) in $\beta$, so larger values for the productivity parameter accentuate the loss in the expected value of a project due to an erroneously set standard, but this loss decreases in percentage terms. In contrast, it can be shown that increases in the classifications cost parameter $c$ adversely affect both the absolute and relative loss due to errors in a standard’s specification.

7. Value-Maximizing Standards When the Distribution of Available Projects Is Endogenous

So far, the paper has examined determinants of efficient accounting standards, the evolution of standards, and the effects of errors in standards in a setting where the real resource allocation effects of standards derive from their impact on the size of the investments in the set of available existing projects. This might be considered an analysis of the ex post effects of standards, since the set of available projects is taken as given—that is, not responsive to changes in the accounting standards. But, in practice, we might expect there to be ex ante effects of changes in standards as well, since the behavior of entrepreneurs and others involved in the production of projects, IPOs, new ventures, and the like may respond to changes in accounting standards. This happens in practice: What deals (spin-offs, takeovers, investments, leases, etc.) are completed are affected by prevailing accounting standards, and as standards change, both the kind and structuring of these deals change. We now formally account for these responses by allowing the distribution of projects to change as accounting standards change.

The standards that emerge in this setting depend on the extent to which standard setters anticipate the dependence of the (now) endogenous distribution of projects on the chosen standards. We will consider two types of standard setters: “sophisticated” (also known as “Stackelberg leaders”) and “naive” (also known as, “followers”). A sophisticated standard setter correctly anticipates how preparers will respond to changes in standards. Necessarily, a sophisticated standard setter knows all the details of the economic environment in which standards are set. In contrast, a naive standard setter is presumed to know nothing about the economic environment other than what can be deduced from observing preparers’ past behavior. Moreover, a naive standard setter believes that preparers do not alter the distribution of their projects in reaction to a change in standards. More detail concerning the behavior of both naive and sophisticated standard setters will be presented below.

Given these assumed differences in sophistication, it is clear that if the only purpose of the analysis were to compare the expected performance of these two types of standard setters, then there would be no need for a formal analysis. Whatever the objective used to calculate the performance of standards, a naive standard setter will never select a better standard than
a sophisticated standard setter. It is nevertheless worthwhile to evaluate formally the behavior of both sophisticated and naive standard setters because, in practice, standard setters may not have sufficient information to be able to act as “our” sophisticated standard setters do. So, it is desirable to compare the standards adopted by the two standard-setting types, and see what parameters of the model are key to explaining the differences in their promulgated standards. To identify these differences, we turn to a formal analysis.

We start by describing how the distribution of projects is endogenized. In our model, projects differ from each other in terms of the probability \( \phi_i \) they are viable, and an accounting standard is a threshold probability that a project is viable. So, it seems natural to posit that a change in an accounting standard induces the entrepreneur to change the probability distribution of \( \tilde{\phi}_i \).

We concentrate on mean effects, and assume that the cost of increasing the mean probability a project is viable is quadratic in the mean. Specifically, we assume that at cost \( k \frac{\Delta^2}{I} \), the entrepreneur can shift the mean of \( \tilde{\phi}_i \) to \( \mu = E[\tilde{\phi}_i] = \mu + \Delta \), where \( \mu \in (0, 1) \) is some base probability and \( k \) is some positive constant.\(^{24}\)

As discussed above, a sophisticated standard setter correctly anticipates how preparers (i.e., entrepreneurs) respond to a selected standard. To describe this response precisely, we must adapt the previously given definition of a financial reporting equilibrium to the present situation in which the distribution \( \tilde{\phi}_i \) is endogenous. (In this definition, we let \( f(\phi_i | \mu) \) denote the probability density that a project is viable, given that the mean probability of viability is \( \mu \)).

**Definition 2.** A financial reporting equilibrium relative to accounting standard \( \phi_s \) when the mean of the distribution of projects \( \tilde{\phi}_i \) is endogenous consists of a pair of investment levels \( I^*(B) \) and \( I^*(W) \) and market values \( m_B \) and \( m_W \), a shadow standard \( \tilde{\phi} \), and a mean \( \mu^* \) such that:

1. An entrepreneur whose project is viable with probability \( \phi_i \) engages in classifications manipulation if and only if \( \phi_i \geq \phi \);
2. \( Pr(viable \mid B, \phi, \mu^*) = E[\tilde{\phi}_i \mid \tilde{\phi}_i \geq \phi, \mu^*] \) and \( Pr(viable \mid W, \phi, \mu^*) = E[\tilde{\phi}_i \mid \tilde{\phi}_i < \phi, \mu^*] \);
3. For each \( \mu \), \( Pr(W \mid \tilde{\phi}, \mu) = Pr(\tilde{\phi}_i < \phi \mid \mu) \) and \( Pr(B \mid \phi, \mu) = Pr(\tilde{\phi}_i \geq \phi \mid \mu) \);
4. For \( R \in \{B, W\} \), the investment \( I^*(R) \) attains the maximum of:
   \[
   \max_I Pr(viable \mid R, \phi, \mu^*) \times \tilde{\beta}_i I^a/\alpha - I.
   \]
5. For \( R \in \{B, W\} \), \( m_R = Pr(viable \mid R, \phi, \mu^*) \tilde{\beta}_i (I^*(R))^a/\alpha - I^*(R) \);

\(^{24}\)Adopting a quadratic functional form for the cost of production is often done in the agency and disclosure literatures when explicit closed form solutions are sought. See, e.g., Baiman and Sivaramarkrishnan [1991], Holmstrom [1979], and Kirby [1988].
(vi) Taking $\phi, m_W, \text{and } m_B$ as given, $\mu^*$ attains the maximum of
\[
\max_{\mu} \Pr(W|\phi, \mu) \times m_W + \Pr(B|\phi, \mu) \times m_B - \frac{k}{2}(\mu - \mu^*)^2 \\
- \int_{\phi}^{\phi'} c(\phi - \phi_i) f(\phi_i|\mu) d\phi_i.
\]

(vii) $\phi = \phi_s - m_B - m_W c$.

There are two principal differences between this definition and the financial reporting equilibrium previously defined. The first is condition (vi), in which preparers maximize with respect to the choice of distribution $\tilde{\phi}_i$ of projects. The first two terms in (vi) relate to the expected selling price of a project. The third term $-\frac{k}{2}(\mu - \mu^*)^2$ is the cost of selecting a distribution with a mean probability $\mu$, as discussed above. The last term is the expected cost of classifications manipulation.25 The second principal difference is in condition (iii), in which preparers anticipate how their choice among distributions affects how projects are subsequently classified.

Notice that this definition presumes that, although investors reading financial statements get to infer what distribution preparers choose (through condition (vi)), investors do not get to observe this choice directly. That is, preparers take investors’ beliefs about this distribution as given (and therefore they also take as given the market prices associated with the classification of projects) when they choose what distribution $\tilde{\phi}_i$ to select. In equilibrium, however, preparers’ selection of a distribution of projects and investors’ inferences about the distribution of projects coincide.

7.1 THE STACKELBERG STANDARD CHOSEN BY A SOPHISTICATED STANDARD SETTER

We call the mapping, implicit in the previous definition of a financial reporting equilibrium, from an accounting standard $\phi_s$ to the induced equilibrium probability $\mu^*$ that a project is viable the equilibrium Nash response function, and we denote it by $\mu^* = \mu^*(\phi_s)$. Associated with the equilibrium Nash response function are the equilibrium market values $m_W = m_W(\phi_s)$ and $m_B = m_B(\phi_s)$ and equilibrium shadow standard $\phi(\phi_s)$. A sophisticated standard setter knows all of these mappings at the time he selects among accounting standards. Consequently, a sophisticated standard setter also knows the expected value of an investment, net of both the cost of producing the project and the expected cost of classifications manipulation, as a function of his chosen standard:

\[
\Pr(W|\phi(\phi_s), \mu^*(\phi_s)) \times m_W(\phi_s) + \Pr(B|\phi(\phi_s), \mu^*(\phi_s)) \times m_B(\phi_s) \\
- \frac{k}{2}(\mu^*(\phi_s) - \mu^*)^2 - \int_{\phi(\phi_s)}^{\phi'} c(\phi - \phi_i) f(\phi_i|\mu^*(\phi_s)) d\phi_i.
\]  

25 In the appendix, this expected cost appears in the form $\frac{1}{2\sigma \sqrt{\phi}} \int_{\phi}^{\phi'} c(\phi - \phi) d\phi$.  

We shall assume that the objective of all standard setters, sophisticated and naive, is to select a standard $\phi$ that maximizes the expected value of a project net of all relevant costs, based on their beliefs about the market's response to the standards they choose. In particular, a sophisticated standard setter is assumed to maximize (15). Given this objective, it is clear why in the above we referred to a sophisticated standard setter alternatively as a Stackelberg leader: In analogy with the corresponding concept from the literature on industrial organization, a sophisticated standard setter moves first, correctly anticipating how the rest of the market will “follow” his chosen standard.26 With $\phi^{STACKELBERG}$ denoting the standard that attains this maximum, we have the following result.

**Lemma 1 (The Stackelberg Standard).** Assume $\alpha = 0.5$ and $\tilde{\beta}_i \sim Uniform[\overline{l}, \overline{u}]$. The standard chosen by a sophisticated standard setter when the distribution of projects is endogenous exists and is unique. It is given by

$$\phi^{STACKELBERG} = \mu \times \frac{1 + \frac{\alpha \beta_i^2 \sqrt{3}}{\varepsilon} + \frac{\bar{\beta}^2}{2k}}{1 + \frac{\alpha \beta_i^2 \sqrt{3}}{\varepsilon} - \frac{3\bar{\beta}^2}{2k}}.$$

By analyzing $\phi^{STACKELBERG}$, it is easy to show that the Stackelberg standard is increasing in $\mu$ and $\bar{\beta}_i$ and decreasing in $k$. These are intuitive comparative statics: increasing $\mu$ or $\bar{\beta}_i$, or decreasing $k$ increases the expected productivity of a project in so far as it either reduces the cost of producing the project with a given expected probability of being viable (for $\mu$ and $k$) or else increases the output of a viable project (for $\bar{\beta}_i$). As is shown in the appendix, the equilibrium Nash response function is given by

$$\mu^*(\phi^s) = \frac{\mu^s(1 + \frac{\alpha \beta_i^2 \sqrt{3}}{\varepsilon}) + \frac{\bar{\beta}^2}{2k} \phi'}{1 + \frac{\alpha \beta_i^2 \sqrt{3}}{\varepsilon} - \frac{3\bar{\beta}^2}{2k}}. \quad (16)$$

Since, as this expression shows, the equilibrium Nash response function is increasing in the value of the standard $\phi^s$, it follows that a sophisticated standard setter bent on maximizing the ex ante expected value of a project will exploit improvements in the project’s production technology attending changes in $\mu$, $\bar{\beta}_i$, and $k$ by increasing the threshold required to get the more favorable accounting treatment.

Perhaps the most interesting feature of the Stackelberg standard is how it compares to the standard chosen by a naive standard setter. We make this comparison in the next subsection.

**7.2 The Nash Standard Chosen by a Naive Standard Setter**

As noted above, the knowledge a sophisticated standard setter must possess to calculate the Stackelberg standard identified above is really advanced: he must know, and anticipate, how preparers respond to a change in standards. One of the advantages of having this sophisticated knowledge is that,

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26 I wish to thank an anonymous referee for suggesting this analogy.
once a standard is set by such a sophisticated standard setter, there is no reason for the standard setter to change it. In this respect, at least, the assumed sophistication of such standard setters seems to exceed that possessed by the FASB and similar bodies, because practicing standard setters repeatedly modify, amend, or rescind previously issued standards. For some recent examples, note that SFAS 140 on the transfer and servicing of financial assets and liabilities amends SFAS 125, which in turn amends SFAS 76 and 77; SFAS 138 amends the “derivatives” standard, SFAS 133, which in turn amended SFAS 119; SFAS 132 on pensions amends SFAS 87 and 106; SFAS 111 rescinds SFAS 32, etc. While some of these changes and corrections may have been designed to provide clarification of points that were ambiguous in the original standards, these standards are also modified in response to firms’ exploitation of loopholes and/or other unanticipated consequences of previously formulated standards.

In this section, we investigate the polar opposite of a sophisticated standard setter by examining the long run behavior of a naive standard setter who, as noted above, erroneously assumes that preparers are completely unresponsive to changes in standards, and whose only knowledge of the economy is based on preparers’ past actions.

We begin by expanding on the description of a naive standard setter’s behavior. The behavior of a naive standard setter can be described only in a dynamic context. At the start of, say, period \( n + 1 \), the standard setter engages in a comprehensive survey of projects sold in the previous period \( n \). The survey reveals the average frequency, say \( \mu_n \), of viable projects produced in period \( n \). (The number of projects undertaken is assumed to be sufficiently large so that the mean of the empirical distribution is indistinguishably close to the mean of the ex ante distribution.) Armed with this knowledge, the naive standard setter selects the standard \( \phi_{n+1}^s \) in period \( n + 1 \) to be the mean \( \mu_n \) of the distribution of viable projects he observed in period \( n \):

\[
\phi_{n+1}^s = \mu_n.
\]

There are two justifications for having a naive standard setter behave in this way. First, if standards are to “catch up” with the actual distribution of projects, then in any period they should be based on the most recent distribution of past projects. Second, given that a naive standard setter does not believe that the distribution of projects will change as standards change, a naive standard setter bent on maximizing what he perceives to be the expected value of a project will be guided by Theorem 3: that theorem states that, when the distribution of projects is taken as fixed, the expected value-maximizing standard is given by the standard setter’s perceived mean of the distribution of \( \hat{\phi}_i \). Thus, this theorem, combined with the naive standard setter’s beliefs and survey of period \( n \) projects, compels the naive standard setter to select \( \phi_{n+1}^s = \mu_n \).

\[\text{27 The preceding comment applies when } \hat{\beta}_i \text{’s realized value is known, or more generally, when no additional learning about } \beta_i \text{ takes place.}\]
After the standard setter has chosen this standard for period $n+1$, preparers proceed to select the—now endogenous—distribution of projects for this period in accordance with the previously given definition of a financial reporting equilibrium. This results in the actual distribution of period $n+1$ projects having mean probability of viability of $\mu_{n+1} = \mu^*(\phi_{n+1})$. At the start of period $n+2$, the naive standard setter canvases period $n+1$’s distribution of projects—and learns $\mu_{n+1}$—and the game is then repeated. Thus, this is a “cobweb” game in which, at the start of each period, the naive standard setter anticipates that what will happen in that period is what did happen in the previous period.

To distinguish the reasons for the evolution of standards in this section from that due to learning about the productivity parameter demonstrated in Theorem 2 above, in this section we will assume that the productivity parameter is a fixed, known constant.

From this description, it is clear that in any given period, the standard chosen by a naive standard setter will depend upon the past behavior of preparers, which in turn depends upon previously chosen standards. To obtain predictions about the standards chosen by a naive standard setter, we examine environments in which the long run evolution of standards is independent of the history of previously chosen standards. In the following, we refer to such environments as stable, which we define formally as follows.

**Definition 3.** A financial reporting environment is stable if, for any initial standard $\phi_0$, the sequence of standards $\{\phi_n\}_{n \geq 0}$ defined by $\phi_{n+1} = \mu_n = \mu^*(\phi_n)$, converges to some limiting standard $\phi_\infty \in [0, 1]$ that is independent of $\phi_0$.

In stable financial reporting environments, a naive standard setter sets standards that eventually converge to the limiting standard $\phi_\infty$. Moreover, since $\mu^*(\bullet)$ is continuous, if a naive standard setter ever selects the limiting standard $\phi_\infty$, the standard setter will continue to choose that standard indefinitely, as:

$$\mu^*(\phi_\infty) = \mu^*(\lim_{n \to \infty} \phi_n) = \lim_{n \to \infty} \mu^*(\phi_n) = \lim_{n \to \infty} \phi_{n+1} = \phi_\infty.$$  

In the following, we refer to standards that have this self-repeating characteristic as Nash standards.

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28 This is unlike what happens for a sophisticated standard setter: given the economy’s parameters, the standard expected to be chosen by a sophisticated standard setter is independent of what standards were chosen previously.

29 $\mu^*(\bullet)$ is continuous since, as noted in (16) above, $\mu^*(\bullet)$ is linear.

30 Referring to these self-repeating standards as Nash standards is appropriate when we consider the incentives of preparers and the beliefs of naive standard setters. Recall that the strategies that define a Nash equilibrium of any game have the property that the strategy of each player in the game maximizes that player’s utility, taking that player’s beliefs and the strategies of other players as given. Since naive standard setters believe that the market does not respond to their chosen standard, as noted above, Theorem 3 states that such a standard setter should choose a standard $\phi^*$ that coincides with his perception of the mean of $\phi$. Thus,
DEFINITION 4. An accounting standard $\phi^{NASH}$ is a Nash standard if $\mu^*(\phi^{NASH}) = \phi^{NASH}$.

The next theorem indicates that a Nash standard exists and is unique; it determines when an environment is stable; and it shows how, in stable environments, standards selected by a naive standard setter evolve over time. Finally, the theorem documents that the Nash standard is always below the Stackelberg standard in every stable environment, and it assesses the import of this result for evaluating the behavior of preparers.

THEOREM 5 (The Nash Standard and its Comparison to the Stackelberg Standard when $\alpha = .5$ and $\hat{\phi}_i$: Uniform$[l, u]$).

(a) There exists exactly one Nash standard. It is given by

$$\phi^{NASH} = \frac{\mu \times (1 + \frac{\sigma \bar{\beta}^2 \sqrt{3}}{c})}{1 + \frac{\sigma \bar{\beta}^2 \sqrt{3}}{c} - \frac{\bar{\beta}^2}{k}}.$$ 

(b) The financial reporting environment is stable if and only if

$$\bar{\beta}_i^0 < k \times \left(1 + \frac{\sigma \bar{\beta}^2 \sqrt{3}}{c}\right).$$ 

When the financial reporting environment is stable,

(c) if accounting standard $\phi_0^s$ is initially set below the Nash standard $\phi^{NASH}$, then the sequence of successive standards $\phi_{n+1}^s = \mu^*(\phi_n^s)$ increases over time $\phi_{n+1}^s > \phi_n^s$ for all $n$;

(d) the Nash standard is below the Stackelberg standard:

$$\phi^{NASH} < \phi^{STACKELBERG};$$

dii) the mean probability that a project is viable under the Nash standard is lower than under the Stackelberg standard:

$$\mu^*(\phi^{NASH}) < \mu^*(\phi^{STACKELBERG});$$

and

diii) the mean probability that a project is viable under the Stackelberg standard is lower than the Stackelberg standard itself:

$$\mu^*(\phi^{STACKELBERG}) < \phi^{STACKELBERG}.$$ 

a naive standard setter who observes that the empirical distribution of viable projects in the previous period had mean $\mu = \phi^{NASH}$ will respond by setting the next period’s standard to (also) be $\phi^{NASH}$. Since preparers will maximize their expected profits in responding to this standard by choosing $\mu^*(\phi^{NASH}) = \phi^{NASH}$, their optimizing behavior is in conformity with the naive standard setter’s expectations about their behavior.

The fact that a Nash standard is a fixed point of the equilibrium Nash response function is simply a summary way of expressing that a Nash standard defines a Nash equilibrium (in the usual sense).
Part (a) of the theorem indicates that the Nash standard exhibits many intuitive characteristics. As the project’s production technology improves ($\beta_i$ increases), as the base probability that a project is viable increases ($\mu$ increases), and as the cost of classifications manipulation increases ($c$ increases), the Nash standard increases. Also, as the marginal cost of increasing the probability that a project is viable goes up ($k$ increases), and as the standard deviation of the distribution of projects increases ($\sigma$ increases), the Nash standard declines.

To explain (b), recall the form of the equilibrium Nash response function $\mu^* = \mu^*(\phi^*)$ given in (16) above. Since this function is linear in $\phi^*$ with slope $\frac{\bar{\beta}_i}{\sigma^2(1 + \frac{\bar{\beta}_i^2}{2\sigma^2})}$, a financial reporting environment is stable provided preparers’ response to a change in a standard is less extreme than the change in the standard itself, i.e., if $\frac{d\mu^*(\phi^*)}{d\phi^*} < 1$. The condition for stability given in part (b) of the statement of the theorem is this inequality, rearranged.

Part (c) explains why standards can be expected to increase over time based on a different principle from that employed in Theorem 2 above. In the present theorem, in a stable environment, if a standard were initially set below the Nash standard, preparers’ mean response to the standard is sufficiently dampened so as to remain below the Nash standard. So, if a standard starts out “low” relative to the Nash standard, it will stay “low,” but—since all standards eventually converge to the Nash standard in a stable environment—the sequence of standards must gradually increase over time.

Part (d) compares the Nash standard to the Stackelberg standard, as well as the behavior the Nash standard induces to the behavior the Stackelberg standard induces. Part (di) states that, in stable environments, the Stackelberg standard is higher than the Nash standard. That is, standard setters who anticipate correctly the response of preparers to changes in standards will choose a higher threshold for the standard than standard setters who fail to anticipate such responses. As a consequence, part (dii) notes that preparers select a lower mean probability that a project is viable when standards are set by naive standard setters than when standards are set by sophisticated standard setters. Finally, part (diii) implies that, when standards are set optimally by a sophisticated standard setter, more than half of the induced projects will fail to achieve a probability of viability equal to the threshold designated by the official standard.

Parts (di) and (diii) combined indicate some of the difficulties involved in evaluating the performance of both accounting standard setters and those whose reporting behavior is governed by accounting standard setters. It might seem that a standard setter who sets a standard equal to the mean probability that a project is viable would be doing a good job, since such a standard setter is sending an accurate signal to investors about the mean characteristics of the projects they are purchasing. Related, it would seem that an entrepreneur who selected projects whose average probability of being viable equals the threshold specified by the prevailing accounting standard
would be doing a good job, too. But, the standard that has these characteristics is the Nash standard, and the standard setter who chooses this standard is the naive standard setter. Since (from (di)) the Nash standard is known to be below the optimal Stackelberg standard, and since (from (diii)) the mean probability that an entrepreneur’s project is viable under a Stackelberg standard is below the threshold designated by the Stackelberg standard, these seemingly reasonable criteria for judging a standard setter’s performance to be high—or the performance of an entrepreneur whose accounting reports are governed by accounting standards to be high—are in fact inappropriate. According to results (di) and (diii), optimal (Stackelberg) accounting standards set reporting thresholds above the mean probability of viability of those projects whose accounting reports are governed by the standards.

7.3 GENERAL COMPARISONS BETWEEN NASH AND STACKELBERG STANDARDS

Finally, we consider the relationship between Stackelberg and Nash standards in a very general setting. To that end, let \( g(\mu, \hat{\mu}, \phi_s) \) denote the objective function to be maximized by preparers, when the preparers privately choose actions \( \mu \), investors/purchasers conjecture that the preparers have chosen actions \( \hat{\mu} \), and accounting standard \( \phi_s \) is in effect. Unlike the set-up described in the earlier sections, here we allow the actual and conjectured actions (\( \mu \) and \( \hat{\mu} \)) to belong to a space, say \( \mathbb{R}^n \), of different dimension from the space \( \mathbb{R} \) that the accounting standard \( \phi_s \) occupies. For expositional convenience, we continue to assume in the following that the standard \( \phi_s \) is one-dimensional; the following results apply to multi-dimensional standards on a component-by-component basis.

While this generalization requires no modification of the notion of an equilibrium Nash response function \( \mu^*(\phi^*) \) (apart from replacing \( \mu^*(\phi^*) \in \mathbb{R} \) by \( \mu^*(\phi^*) \in \mathbb{R}^n \)), it does call for a change in the definition of a Nash standard chosen by a naive standard-setter. Now, a Nash standard derives from the “standard component” of a Nash equilibrium, defined as follows: it is a pair \( \phi_s^{NASH}, \mu_s^{NASH} \) such that \( \mu_s^{NASH} = \mu^*(\phi_s^{NASH}) \) and \( \phi_s^{NASH} \in \arg \max_{\phi_s} g(\mu_s^{NASH}, \mu_s^{NASH}, \phi_s) \).

The fundamental premise underlying the behavior of the naive standard-setter—as well as a Nash equilibrium—is that the standard-setter assumes preparers do not alter their behavior in response to his choice of standards. Thus, assuming the choice among standards can be characterized by first-order conditions, \( \phi_s^{NASH} \) is determined as follows (here, the subscript 3 refers to the partial derivative with respect to the third argument):

\[
g_3(\mu_s^{NASH}, \mu_s^{NASH}, \phi_s^{NASH}) = 0. \tag{17}
\]

Contrast this to the characterization of a Stackelberg standard. To describe the Stackelberg standard, first consider the total effect of any (small) change in a standard \( \phi^s \) on the preparer’s objective function. In equilibrium, we must evaluate such changes at an equilibrium point, i.e., at triples (\( \mu, \hat{\mu}, \phi^s \))
of the form \( (\mu^*(\phi^*), \mu^*(\phi^*), \phi^*) \). At any such point, the marginal effect of a change in a standard \( \phi^* \) on the objective function \( g(\bullet) \) is

\[
(g_1(\bullet) + g_2(\bullet)) \times \frac{d\mu^*(\phi^*)}{d\phi^*} + g_3(\bullet) = g_2(\bullet) \frac{d\mu^*(\phi^*)}{d\phi^*} + g_3(\bullet).
\] (18)

(The equality follows since the initial owners maximize with respect to their choice of \( \mu \) while taking \( \hat{\mu} \) as given, so \( g_1(\bullet) = 0 \) at any equilibrium point.)

In particular, since a Stackelberg standard maximizes \( g(\mu^*(\phi^*), \mu^*(\phi^*), \phi^*) \), a Stackelberg standard satisfies the first-order condition:

\[
g_2(\bullet) \times \frac{d\mu^*(\phi^*)}{d\phi^*} \bigg|_{\phi^* = \phi^*_{\text{Stackelberg}}} + g_3(\bullet) = 0.
\] (19)

A comparison of this last first-order condition (19) to the Nash first-order condition (17) exposes the cost of the naive standard-setter’s failure to recognize preparers’ reactions to his choice of standards. Recalling the derivations that led to (17) and (18) above, the total marginal impact of a change in standards on the preparers’ objective function, when evaluated at the Nash standard, is given by

\[
g_2(\bullet) \frac{d\mu^*(\phi^*)}{d\phi^*} \bigg|_{\phi^* = \phi^*_{\text{Nash}}} + g_3(\bullet) = g_2(\bullet) \frac{d\mu^*(\phi^*)}{d\phi^*} \bigg|_{\phi^* = \phi^*_{\text{Nash}}}.
\]

This observation leads immediately to the following pair of results.

**THEOREM 6 (A Comparison of the Nash and Stackelberg Standards in General Environments).**

(i) If \( g_2(\bullet) > 0 \) and \( \frac{d\mu^*(\phi^*)}{d\phi^*} > 0 \), then the preparers’ objective function can be increased by setting standards higher than that specified by the Nash standard.

(ii) If \( g_2(\bullet) > 0, \frac{d\mu^*(\phi^*)}{d\phi^*} > 0, \) and \( g(\mu^*(\phi^*), \mu^*(\phi^*), \phi^*) \) is strictly concave in \( \phi^* \), then the Stackelberg standard is strictly greater than the Nash standard.

Part (i) of this theorem asserts that under the (reasonable) conditions that whenever the preparers’ objective function is increasing in investors’ perceptions of the preparers’ actions, and the equilibrium response of preparers to a change in standards involves increasing the value(s) of their actions, then the objective function of preparers can be increased by pushing the standard above the Nash standard. Part (ii) adds that, provided there are globally diminishing returns to increasing the standard (i.e., the objective function is concave), the Stackelberg standard is above the Nash standard. These results are consistent with those reported in Theorem 5, but they rely on none of Theorem 5’s parametric assumptions.

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31 When \( \mu \) is a vector, \( g_1(\bullet) \) should be interpreted as a vector of derivatives (i.e., a gradient), and products such as \((g_1(\bullet) + g_2(\bullet)) \times \frac{d\mu^*(\phi^*)}{d\phi^*}\) should be interpreted as dot products.
8. Summary

This paper studies a model that captures several features of the financial reporting environment: Accounting standards consist of binary classifications that suppress probabilistic details; preparers prefer one of the binary classifications to the other; exceeding some context-dependent threshold is required to receive the more favorable classification; preparers evaluate the costs and benefits of engaging in classifications manipulation to obtain the preferred accounting classification. The possibility of classifications manipulation creates a distinction between the official classification described in the statement of an accounting standard and the de facto classification, determined by the “shadow standard” actually adopted by preparers. The paper demonstrates that learning in this environment implies that it is impossible to hold constant simultaneously both the official standard and the shadow standard. A further implication of this is that if standards are designed to keep the actual partitioning of firms constant over time, then there must be expected “standards creep,” i.e., an increase in the expected official standard over time.

The paper also considers how the accounting standard chosen by a standard setter varies with the standard setter’s knowledge of the economic environment in which standards are set. Two types of standard setters are singled out for attention: sophisticated (also known as Stackelberg) standard setters who can anticipate correctly how preparers respond to changes in standards, and naive standard setters who are unable to calculate preparers’ responses to changes in accounting standards and, hence, who choose accounting standards that are value-maximizing for the distribution of projects preparers were observed to select in the past. The paper demonstrates that, in stable environments, the standard naive standard setters adopt over time converges to a limiting Nash standard that is independent of the history of previously chosen standards, and this limiting standard is less stringent than the Stackelberg standard chosen by a sophisticated standard setter. Moreover, under the Stackelberg standard, preparers produce projects that have both higher expected value and higher mean probability of success than under the Nash standard.

APPENDIX: PROOFS OF THEOREMS

Proof of Theorem 1. It is clear from (4) and (6) that \( m_W \) and \( m_B \) are given by

\[
    m_W = \bar{\beta}_i^2 \left( \frac{l + \phi}{2} \right)^2 \quad \text{and} \quad m_B = \bar{\beta}_i^2 \left( \frac{u + \phi}{2} \right)^2.
\]
Substituting these values into (7) implies that \( \phi \) is given implicitly by:

\[
\phi = \phi^* - \beta^*_i \left( \frac{u + \phi}{2} \right)^2 - \left( \frac{t + \phi}{2} \right)^2.
\]

i.e.,

\[
\phi \left( 1 + \beta^*_i \frac{u - l}{2c} \right) = \phi^* - \beta^*_i \frac{u^2 - l^2}{4c}.
\]

Solving this equation for \( \phi \) produces the expression that appears in Theorem 1’s statement. Since this derivation is constructive, and yields only one solution for \( \phi \), this proves both existence and uniqueness. ■

**Proof of Theorem 2.** Pick two periods \( t \) and \( t' > t \). Let \( \phi^*_t \) be the accounting standard prevailing in period \( t \), and let \( \tilde{\phi}^*_t \) be the accounting standard prevailing in period \( t' \). As of period \( t \), the accounting standard \( \tilde{\phi}^*_t \) is a random variable. Also, as in the text, let \( \theta_t \) be whatever information is known about the productivity parameter \( \tilde{\beta}_i \) up to the beginning of period \( t \), and similarly, let \( \tilde{\theta}_t \) be whatever information is known about the productivity parameter \( \tilde{\beta}_i \) up to the beginning of period \( t' \). Notice that \( \tilde{\theta}_t \) is random as of period \( t \).

Since \( 0 < \alpha < 1 \) and \( \tilde{\beta}_i \) is a martingale, Jensen’s inequality applies to yield:

\[
E \left[ (E[\tilde{\beta}_{it} \mid \tilde{\theta}_t])^{\frac{1}{\alpha}} \mid \theta_t \right] \geq (E[\tilde{\beta}_{it} \mid \tilde{\theta}_t])^{\frac{1}{\alpha}} = (E[\tilde{\beta}_{it} \mid \theta_t])^{\frac{1}{\alpha}}.
\]

Define \( \Delta \equiv \left( E[\hat{\phi}_i \mid \hat{\theta}_i > \phi]^{\frac{1}{\alpha}} - E[\hat{\phi}_i \mid \hat{\theta}_i \leq \phi]^{\frac{1}{\alpha}} \right) \times \frac{\frac{1 - \alpha}{\alpha}}{\alpha} \). Of course, \( \Delta > 0 \) always holds. So, we conclude by (7) that

\[
E[\hat{\phi}_i \mid \theta_t] = \phi + \frac{E \left[ (E[\tilde{\beta}_{it} \mid \tilde{\theta}_t])^{\frac{1}{\alpha}} \mid \theta_t \right] \times \Delta \geq \phi + \frac{E[\tilde{\beta}_{it} \mid \theta_t])^{\frac{1}{\alpha}} \times \Delta = \phi^*_t.\]

**Proof of Theorem 3.** Appealing to (14) for the expression for the expected cost of classifications manipulation, we see that the objective function (13) is given by:

\[
\frac{\phi(\phi^*) - l}{u - l} \times \left( \beta_i \frac{\phi(\phi^*) + l}{2} \right)^2 + \frac{u - \phi(\phi^*)}{u - l} \times \left( \beta_i \frac{\phi(\phi^*) + u}{2} \right)^2
- \frac{(u - l)\beta_i}{8c} \left( \phi(\phi^*) + \frac{u + l}{2} \right)^2.
\]
Maximizing this is equivalent to maximizing:

$$\frac{1}{2}((\phi - l) \times (\phi + l)^2 + (u - \phi)(\phi + u)^2 - \frac{(u - l)^2 \beta_i^2}{c} (\phi + \frac{u + l}{2})^2, \quad (20)$$

and the first-order condition derived from maximizing this latter function with respect to $\phi$ is given by:

$$\frac{1}{2}((\phi + l)^2 + 2(\phi - l) \times (\phi + l) - (\phi + u)^2 + 2(u - \phi)(\phi + u)$$

$$- \frac{(u - l)^2 \beta_i^2}{c} (\phi + \frac{u + l}{2}) = 0.$$

This simplifies to:

$$\phi = \frac{u + l}{2} \times \frac{2 - \frac{(u - l)\beta_i^2}{c}}{2 + \frac{(u - l)\beta_i^2}{c}}.$$

The second order condition is satisfied $\left(-2 - \frac{(u - l)\beta_i^2}{c} < 0\right)$, so this is indeed a maximum, provided $\phi \in (l, u)$. Since $\frac{2 - \frac{(u - l)\beta_i^2}{c}}{2 + \frac{(u - l)\beta_i^2}{c}} < 1$, we only have to worry about whether $\phi > l$. The latter holds if and only if

$$\frac{u + l}{2} \times \left(2 - \frac{(u - l)\beta_i^2}{c}\right) > l \left(2 + \frac{(u - l)\beta_i^2}{c}\right),$$

i.e., if and only if

$$c > \frac{\beta_i^2}{2}(u + 3l). \quad (21)$$

This proves the first part of the theorem.

Recall the relationship between $\frac{\phi}{2}$ and $\phi^s$ established in Theorem 1:

$$\phi = \frac{\phi^s - \frac{\beta_i^2}{4c} (u^2 - l^2)}{1 + \frac{\beta_i^2}{2c}(u - l)}.$$

Substituting from the just-derived optimal value for $\phi$, we get:

$$\frac{u + l}{2} \times \frac{1 - \frac{(u - l)\beta_i^2}{2c}}{1 + \frac{(u - l)\beta_i^2}{2c}} = \frac{\phi^s - \frac{\beta_i^2}{4c} (u^2 - l^2)}{1 + \frac{\beta_i^2}{2c}(u - l)}$$

which implies

$$\phi^s = \frac{u + l}{2}.$$

This proves part i of the theorem.

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32 Note that it makes no difference whether the maximization proceeds by directly determining the optimal $\phi^s$, or rather indirectly by determining the cutoff $\phi$ associated with $\phi^s$: by the envelope theorem, $\frac{d}{d \phi} OBJ(\phi(\phi^s)) = \frac{\partial}{\partial \phi} \{OBJ(\phi)\} \frac{d \phi}{d \phi^s} = 0$ implies, since $\frac{d \phi}{d \phi^s} > 0$, that the optimum is determined by that $\phi^s$ for which the associated $\phi$ satisfies $\frac{d}{d \phi} OBJ(\phi) = 0$. 
Part ii follows from an application of the envelope theorem to (20).
Part iii follows from (21). When the inequality in (21) is reversed, the shadow standard $\hat{\phi}$ falls below the left endpoint $l$ of $\hat{\phi}'s$ distribution. This means that the classifications cost parameter $c$ is sufficiently low so that all projects are classified as viable. Obviously, the standard serves no allocational function in this case, and so results in the incurrence of unnecessary classifications' costs.

PROOF OF LEMMA 1. We prove the lemma constructively.

The next expected value of producing a project with mean probability $\mu = \frac{l + u}{2}$ of being viable is given by:

$$\frac{\phi - l}{u - l} \times m_W + \frac{u - \phi}{u - l} \times m_B - \frac{k}{2} (\mu - \mu)^2 - \frac{c(\phi - \phi)^2}{2(u - l)}.$$

Expressed in terms of $\mu$, $\sigma$, this equals (since $l = \mu - \sigma \sqrt{3}$ and $u = \mu + \sigma \sqrt{3}$):

$$\frac{\phi - (\mu - \sigma \sqrt{3})}{2\sigma \sqrt{3}} \times m_W + \frac{\mu + \sigma \sqrt{3} - \phi}{2\sqrt{3}} \times m_B - \frac{k}{2} (\mu - \mu)^2 - \frac{c(\phi - \phi)^2}{4\sigma \sqrt{3}}.$$

Preparers take the market values $m_W$, $m_B$ as given when they choose the distribution of projects, since investors do not get to observe directly preparers' choices. Preparers take the shadow standard $\hat{\phi}$ as given as well: if the preparers anticipate the market values $m_W$, $m_B$, optimal classifications manipulation dictates that they will engage in classifications manipulation ex post (after $\hat{\phi}$ has realized its value $\phi_i$) as long as $\phi_i \geq \phi' - \frac{m_B - m_W}{c} = \phi$. Hence, when preparers maximize with respect to the choice of $\mu$ (as required in equilibrium condition (vi)), they obtain as the first-order condition for the optimal mean $\mu^*$:

$$m_B - m_W = (\mu^* - \mu)2k\sigma \sqrt{3}.$$  \hspace{1cm} (23)

This condition determines $\mu^*$ in terms of the endogenous market prices $m_W$, $m_B$.

The market price $m_B$ is determined by the shadow standard $\hat{\phi}$ and $\mu^*$ through equilibrium condition (v). Using each of: (23), (9), $u = \mu^* + \sigma \sqrt{3}$, and $\phi = \phi' - \frac{m_B - m_W}{c}$, and simplifying, we find that $m_B$ equals:

$$m_B = \left(\hat{\beta}_i \frac{u + \phi}{2}\right)^2 = \left(\hat{\beta}_i \frac{\mu^* + \sigma \sqrt{3} + \phi}{2}\right)^2 = \left(\hat{\beta}_i \frac{\mu^* + \sigma \sqrt{3} + \phi' - \frac{m_B - m_W}{c}}{2}\right)^2$$

$$= \hat{\beta}_i^2 \left(\frac{\mu^* + \sigma \sqrt{3} + \phi' - (\mu^* - \mu) \frac{2k\sigma \sqrt{3}}{c}}{2}\right)^2 = \hat{\beta}_i^2 \left(\frac{\mu^* d + \sigma \sqrt{3} + f}{2}\right)^2.$$

(24)
where
\[ d \equiv 1 - \frac{2k\sigma\sqrt{3}}{c} \quad \text{and} \quad f \equiv \phi^s + \frac{2k\mu\sqrt{3}}{c}. \]
Likewise, using (23), (9), \( l = \mu - \sigma\sqrt{3} \), and \( \phi = \phi^s - \frac{m_B - m_W}{c} \), and simplifying, we find
\[ m_W = \left( \beta_i \left( \frac{l + \phi}{2} \right) \right)^2 = \left( \beta_i \frac{\mu - \sigma\sqrt{3} + \phi}{2} \right)^2 = \bar{\beta}_i^2 \left( \frac{\mu - \sigma\sqrt{3} + \phi^s}{2} \right)^2 = \bar{\beta}_i^2 \left( \frac{\mu^* - \sigma\sqrt{3} + f}{2} \right)^2. \]

Thus, (24) and (25) together imply:
\[ m_B - m_W = \bar{\beta}_i^2 (\mu^* d + f) \sigma\sqrt{3}. \]
Combining this last equation with (23), we conclude
\[ (\mu^* - \mu) 2k\sigma\sqrt{3} = m_B - m_W = \bar{\beta}_i^2 (\mu^* d + f) \sigma\sqrt{3}. \]
Solving this equation for \( \mu^* = \mu^*(\phi^s) \), we obtain, for arbitrary \( \phi^s \), the equilibrium Nash response function:
\[ \mu^* = \mu^*(\phi^s) = \frac{2\mu k \left( 1 + \frac{\sigma\beta_i^2\sqrt{3}}{c} \right) + \bar{\beta}_i^2 \phi^s}{2 \left( 1 + \frac{\sigma\beta_i^2\sqrt{3}}{c} \right) - \bar{\beta}_i^2}. \]
As a further step toward calculating the Stackelberg standard, we have to solve for the equilibrium shadow standard \( \phi(\phi^s) \) for each standard \( \phi^s \). To that end, we substitute \( m_B - m_W \) as specified in (26) above into the relationship \( \phi = \phi^s - \frac{m_B - m_W}{c} \), combined with the equilibrium Nash response function to obtain:
\[ \phi(\phi^s) = \phi^s - \frac{m_B - m_W}{c} = \phi^s - \frac{(\mu^*(\phi^s) - \mu) 2k\sigma\sqrt{3}}{c}. \]
We take this expression, along with the standard-dependent expressions for \( m_W \) and \( m_B \) (obtained by substituting the equilibrium Nash response function into each of (24) and (25)), and insert these expressions into (22). Then, we maximize with respect to the choice of \( \phi^s \). Omitting the details, we obtain the expression for \( \phi^{STACKELBERG} \) appearing in the statement of the lemma.

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33 It is easy to show that the second-order condition required for \( \phi^{STACKELBERG} \) to be a maximum is satisfied whenever the denominator of \( \phi^{STACKELBERG} \) is positive. It is also easy to show that if this denominator is positive, then the stability condition is also satisfied.
PROOF OF THEOREM 5. The Nash standard is the fixed point of the equilibrium Nash response function, i.e., it is the solution to
\[ \mu^*(\phi^*) = \phi^*. \]
Using the previously determined calculation of the equilibrium Nash response function in (27) yields the unique expression for the Nash standard appearing in (a).
To prove (b), note that the financial reporting environment is stable if and only if:
\[ \frac{\partial \mu^*(\phi^*)}{\partial \phi^*} < 1. \]
This last inequality can be rewritten as:
\[ \bar{\beta}_i^2 < k \left( 1 + \frac{\sigma \bar{\beta}_i^2 \sqrt{3}}{c} \right). \]
This proves (b).
To prove (c), first note that if a standard \( \phi^*_0 \) is below \( \phi^{NASH}_N \), then so are all elements of the sequence \( \phi^*_n = \mu^*(\phi^*_{n-1}) \). This follows from the monotonicity of \( \mu^*(\bullet) \):
\[ \phi^*_0 < \phi^{NASH}_N \Rightarrow \phi^*_1 = \mu^*(\phi^*_0) < \mu^*(\phi^{NASH}) = \phi^{NASH}. \]
Hence (by induction),
\[ \phi^*_n = \mu^*(\phi^*_n) < \mu^*(\phi^{NASH}) = \phi^{NASH} \quad \text{for all} \quad n \geq 1. \]
Next note that if the environment is stable, then the denominator of \( \mu^*(\bullet) \) is positive, and so (as simple algebra confirms), \( \phi^* < \mu^*(\phi^*) \) if and only if \( \phi^* < \phi^{NASH} \). As all elements of the sequence \( \phi^*_n \) have already been shown to be below \( \phi^{NASH} \), it follows that
\[ \phi^*_n < \mu^*(\phi^*_n) = \phi^*_n \]
for all \( n \). This proves (c).
(d) To show \( \phi^{NASH} < \phi^{STACKELBERG} \), write \( \phi^{STACKELBERG} \) as \( \frac{\mu x + \bar{\beta}_i^2}{x - \bar{\beta}_i^2} \) and \( \phi^{NASH} \) as \( \frac{\mu x}{x - \bar{\beta}_i^2} \), where \( x \equiv 1 + \frac{\sigma \bar{\beta}_i^2 \sqrt{3}}{c} \). So, we must show that, when the economy is stable,
\[ \frac{\mu x}{x - \bar{\beta}_i^2} < \frac{\mu \left( x + \frac{\bar{\beta}_i^2}{2k} \right)}{x - \frac{3\bar{\beta}_i^2}{2k}}. \]
Upon rearrangement, this inequality can be written as:
\[ \bar{\beta}_i^2 < 2kx = 2k \left( 1 + \frac{\sigma \bar{\beta}_i^2 \sqrt{3}}{c} \right). \]
Since the requirement for stability is \( \hat{\beta}_i^2 < k \times (1 + \frac{\hat{\beta}_i^2 \sqrt{3}}{\kappa}) \), the stability condition guarantees \( \phi^{sNASH} < \phi^{sSTACKELBERG} \). This proves (di).

(dii) follows immediately from the fact that the equilibrium Nash response function \( \mu^* (\phi^s) \) is increasing in \( \phi^s \):

\[
\phi^{sNASH} = \mu^* (\phi^{sNASH}) < \mu^* (\phi^{sSTACKELBERG}).
\]

(diii) follows immediately from the combined facts that: when the environment is stable, \( \frac{\partial \mu^* (\phi^s)}{\partial \phi^s} < 1; \phi^{sNASH} = \mu^* (\phi^{sNASH}); \) and \( \phi^{sNASH} < \phi^{sSTACKELBERG} \).

REFERENCES


