Information Asymmetry, Diversification, and Cost of Capital  

John Hughes, Jing Liu and Jun Liu

First Version: May 20, 2004
Current Version: April 18, 2006
Information Asymmetry, Diversification, and Cost of Capital

Abstract

Of late, there has been considerable interest among accounting researchers in the relation between asymmetric information and cost of capital. Empiricists have taken the lead with a number of studies that document associations between proxies for asymmetric information such as earnings quality and risk premiums. However, the theoretical foundation for these studies has yet to be fully established. In this study, we consider the effects of private signals that are informative of both systematic factors and idiosyncratic shocks affecting asset payoffs in a competitive noisy rational expectations setting. Taking a large economy limit, we show that i) private information about systematic factors affects risk premiums only through its effects on factor risk premiums; ii) idiosyncratic risk is fully diversifiable and affects risk premiums only as a source of noise in drawing inferences about systematic factors from private signals iii) holding total information constant, greater information asymmetry leads to higher factor risk premiums and thus higher cost of capital; and iv) controlling for betas, there is no cross-sectional effect of information asymmetries on cost of capital. These results provide guidance in interpreting the findings of existing empirical work and suggest specifications helpful for future research.
1 Introduction

It is widely held that, in theory, a firm’s cost of capital (expected return) is jointly determined by the risk premiums on systematic risk factors and firm’s exposure to those factors. Idiosyncratic risks are not priced because in large economies they can be eliminated by forming well diversified portfolios. Interesting questions arise as to how asymmetric information impacts on firm’s cost of capital. First, to what extent do firm specific information characteristics enter the determination of expected returns after controlling for firm betas (loadings on systematic risks) with asymmetric information? Second, how, if at all, are firm betas affected by asymmetric information? Third, how does asymmetric information affect factor risk premiums? These questions are important for accounting because of its central role in reducing information asymmetry in capital markets and the presumption that cost of capital is reduced as a consequence of accounting disclosure.\textsuperscript{1}

In this paper, we examine the interplay between asymmetric information and cost of capital while fully considering the forces of diversification. We conduct our analysis within the framework of a competitive noisy rational expectations economy pioneered by Grossman and Stiglitz (1980) and extended by, among others, Admati (1985), and Easley and O’Hara (2004). In order to examine the effect of diversification, we follow the standard practice in the asset pricing literature (e.g, Ross, 1976) and employ a factor structure to distinguish systematic and idiosyncratic components of asset payoffs. Private signals for each asset have components that are informative about systematic factors as well as idiosyncratic shocks of the underlying payoffs. Informed investors receive these signals and uninformed investors

\textsuperscript{1}For example, the former SEC chairman Author Levitt suggested in a speech at the Inter-American Development Bank on September 29, 1997, that high quality accounting standards “improve liquidity and reduce capital costs.”
draw inferences about the information contained in these signals from prices. Our principal focus lies with characterizing risk premiums for large economies in which the number of risky assets and related private signals go to infinity.

We show that if private signals only pertain to idiosyncratic shocks, then in large economies information leads to no resolution of uncertainty about priced risks, which implies risk premiums are unaffected.\(^2\) In the general case, we show that for large economies private information about systematic factors affects risk premiums only through factor risk premiums (firm betas are not affected) and private information about idiosyncratic shocks enters the determination of factor risk premiums as a source of noise in drawing inferences about systematic factors either directly from private signals or price, but otherwise does not affect risk premiums. We further show that holding total information constant, when information asymmetry about the systematic factors increases, the factor risk premiums increase, which causes the average cost of capital in the economy to increase. However, there is no cost of capital effect in the cross-section, because betas are not changed and idiosyncratic risks are not priced.

Our results are intuitive. Private information about idiosyncratic shocks changes expectations of future risky asset payoffs, but in large economies idiosyncratic risks are not priced. As expected, private information about systematic factors affects risk premiums by resolving uncertainty about systematic factors representing risks that are priced in large as well as small economies. *Ceteris paribus* greater disparity in posterior beliefs about systematic factors between informed and uninformed investors implies a higher cost of capital consistent with less resolution of systematic uncertainty. Since private signals are more informative than prices about

\(^2\)At the other extreme, if private signals are simply risky asset payoffs plus noise (e.g., Admati, 1985), then in the limit as the economy expands factor realizations become perfectly revealed to all investors, which implies risk premiums equal to zero.
systematic factors, then increasing the fraction of informed investors results in a
greater resolution of uncertainty about those factors. Greater sensitivity of private
signals to systematic factors, higher precision, and lower volatility of idiosyncratic
shocks imply more is learned from those signals about systematic factors, thereby
reducing risk premiums.

The above results are obtained in the space of price and dollar payoffs suitable
given the exponential utility and normal distributions structure required for ana-
lytic tractability. In the space of returns and payoffs scaled by price, we show that,
in addition to the effect of private information on factor risk premiums, firms’ costs
of capital vary cross-sectionally because the price deflated firm betas are affected
by idiosyncratic factors as well as information on idiosyncratic factors. However,
after controlling for deflator effects, firm specific information characteristics have
no influence on expected returns.

An important feature of our information structure is that private signals are
informative about systematic factors as well as idiosyncratic shocks. This feature
is necessary for information asymmetry to affect the risk premium and is consistent
with evidence from Seyhun (1992) and Lakonishok and Lee (2001) that corporate
insiders are able to time the market. Furthermore, the notion that private signals
at the firm level may contain a systematic component is supported by the observa-
tion that financial reports for which some investors may have advance knowledge
typically includes fundamentals such as revenues, earnings, and cash flows that are
plausibly affected by systematic factors as well as idiosyncratic shocks; evidence
dates back to Ball and Brown (1968).

Although private signals at the firm-level are generally understood to be far
more informative of idiosyncratic shocks than systematic factors, we will show that
an infinitesimally small amount of information on systematic factors extracted from
private signals for each firm, when aggregated in large economies, can have a finite effect on factor risk premiums.

Our model can be viewed as an extension of Easley and O’Hara (2004) to a setting where both private signal and asset payoffs follow a factor structure. Aside from the role of systematic factors, our analysis differs from theirs in that we examine both small and large economies, while they only consider a small economy where diversification is assumed to be insufficient. The factor structure extension is important for studying diversification, because in the large economy limit, if assets are i.i.d. as in Easley and O’Hara (2004), risk premiums go to zero. Like us, Admati (1985) also considers the interplay between private information and equilibrium prices in a noisy rational expectations framework. Rather than a factor structure, Admati’s principal analysis assumes asset payoffs are distributed normally and satisfy a general variance-covariance matrix. However, Admati (1982) recognizes the advantages of a factor structure in characterizing economy-wide information, but finds an explicit solution in the case of diverse information to be infeasible.

In contrast to Admati (1985), our asymmetric information structure allows us to solve for equilibrium prices and risk premiums explicitly. Having an explicit pricing solution is especially useful because it allows us to examine how changes in model parameters such as posterior information asymmetry affect factor risk premiums. Furthermore, having a closed form solution for finite economies enables us to examine the convergence properties of risk premiums as the number of assets is increased.

We note that outside of the paradigm that we employ to study asymmetric information and cost of capital, there is another stream of literature that examines

\footnote{Brennan and Cao (1997) employ a similar structure. Admati (1985) is a multi-asset generalization of Hellwig (1980).}
the effect of estimation risk on cost of capital (e.g., Barry and Brown, 1985, Handa and Linn, 1993, and Coles, Lowenstein and Suay, 1995, Clarkson, Guedes and Thompson, 1996). However, as Clarkson and Thompson (1990) observe, in theory, estimation risk is fully diversifiable in large economies.

This study has a number of important implications for the voluminous empirical literature that examines the link between information asymmetry and cost of capital. First, our analysis clarifies a commonly held misperception about the pricing of asymmetric information in a competitive market (e.g., Easley and O’Hara 2004). A large number of empirical studies [e.g., Ali, Klasa and Yeung (2005), Botosan and Plumlee (2004), Botosan, Plumlee and Xie (2004), Bushman, Piotroski and Smith (2005), Easley, Hvidkjaer and O’Hara (2002, 2004), Hope, Kang, Thomas and Vasvari (2005), Francis, LaFond, Olsson and Schipper (2004a), Gietzmann and Ireland (2005), Gore and Baber (2005), Bhattacharya, Daouk and Welker (2003)] cite the flawed intuition that information asymmetry should be priced because uninformed investors demand price protection from trading with privately informed investors, while in fact both informed and uninformed investors exploit liquidity traders whose demands are manifested in an assumption of noisy supply. Our analysis demonstrates that the pricing effect characterized in Easley and O’Hara’s (2004) Proposition 2 can be diversified away when the economy is large.

Second, we show that factor risk premiums are affected by information asymmetry. In particular, controlling for total information, high(low) information asymmetry leads to high(low) cost of capital. This result provides theoretical support for empirical studies that examine the link between aggregate information environment and market cost of capital [see, for example, Bhattacharya, Daouk and Welker (2003), Bhattacharya and Daouk (2002) and Jain (2005)]. Although quite intuitive, we are unaware of other theoretical studies that show this result.
Third, our analysis suggests that firm specific information characteristics should not enter the determination of expected returns after controlling for beta. Empirical research in this domain reports conflicting evidence. While some claim to have found a positive correlation between information asymmetry and cost of capital [e.g., Botosan (1997), Botosan and Plumlee (2002), Botosan, Plumlee and Xie (2004)], others dispute that this effect due to incomplete control of endogeneity and correlated omitted variables [e.g., Cohen (2004), Nikolaev and Van Lent (2005), Chen, Chen and Wei (2004)]. Our theory sheds useful light on this issue (see Section 4 for details).

Fourth, our analysis suggests that information asymmetry has no effect on cash flow betas, but may alter (percentage) return betas due to deflation by price. Existing empirical studies do not investigate the information effect on cash flow betas or contrast the difference between cash flow and return betas. Hence, our theory suggests that this is a promising arena for further inquiry.

Finally, an asymmetric information factor does not arise endogenously in our model. However, our model is silent on whether there exists a systematic information factor. Therefore, our model is not inconsistent with studies that assume the existence of an information factor (e.g., Aboody, Hughes and Liu 2005, Francis, Lafond, Olsson and Schipper 2004b and Easley, Hvidkjaer and O’Hara 2003). We believe both theoretical and empirical research on this issue is warranted.

The rest of the paper is organized as follows: Section 2 describes the setup for our model and studies an economy with a small number of investors and risky assets; Section 3 studies the large economy limit; Section 4 explores the empirical implications of the model; and Section 5 concludes the paper.
2 Small Economy

In this section, we consider an economy with a finite number of risky assets. We present a noisy rational expectation model in which the asset payoffs and the random supply of the assets have factor structures. We solve the equilibrium in closed form and consider special cases that serve as a benchmarks when information is symmetric.

2.1 The Setup

We assume that there is a riskless asset with return $R_f$ that has an infinitely elastic supply. We assume that payoffs of $N$ risky assets are generated by a factor structure of the form

$$\nu = \bar{\nu} + \beta F + \Sigma^{1/2} \epsilon. \quad (1)$$

The mean of asset payoffs $\bar{\nu}$ is an $N \times 1$ constant vector, the factor $F$ is a $K \times 1$ vector of mean zero normal random variables with covariance matrix $\Sigma_F$, the factor loading $\beta$ is an $N \times K$ constant matrix, the idiosyncratic risk $\epsilon$ is a vector of standard normal random variables, and $\Sigma$ is an $N \times N$ diagonal matrix.

The supply of risky assets, $x$, is a vector of $N \times 1$ random variables specified as follows

$$x = \bar{x} + \beta_x F_x + \Sigma_x^{1/2} \eta_x, \quad (2)$$

where $\bar{x}$, $\beta_x$, and $\Sigma_x$ are constant $N \times 1$ vector, $N \times 1$ vector, and $N \times N$ matrix respectively. There is a systematic component in the random supply, $F_x$, which is a mean zero normal random variable with variance $\sigma_{f_x}^2$ and an idiosyncratic component, $\eta_x$, which is a standard normal random variable.
The noisiness of the supply is necessary in our setting to prevent prices from fully revealing the informed investors’ private signal (defined below) and can be interpreted as caused by trading for liquidity reasons. The presence of a systematic component is based on the reasonable view that liquidity trading is influenced by market-wide forces that may or may not correspond to factors influencing risky asset payoffs.

If we interpret the random supply as due to a liquidity effect, then our assumption of systematic components in random supply is supported by empirical studies that find there are systematic components of liquidity; for example, Chordia, Roll, and Subrahmanyam (2000) and Huberman and Hulka (2001). IPO waves are also suggestive of systematic components. Without a systematic component in the random supply, then in the limiting case, as the number of risky assets becomes large (implying an infinite number of independent asset specific signals), prices would still be fully revealing of the informed investors’ private signals. In other words, noisy supply is necessary but not sufficient to ensure that asymmetric information is not a moot issue in large economies; there also needs to be a systematic component. We further assume for simplicity that $F_x$ is independent of the factors generating asset payoffs.\(^4\)

We assume that there are two classes of investors, informed and uninformed, with the total number denoted by $M$. In the finite economy, the number of investors can be independent of the number of assets. In the large economy limit, $M$ and $N$ must expand at the same rate to ensure that per capital wealth is neither zero nor infinite (see Section 3 for details). Investors are assumed to be price

\(^4\)Noisy rational expectation equilibrium models with many assets having a factor structure in asset payoffs, but not in the random supply of risky assets, have been considered in Caballe and Krishnan (1994); Daniel, Hirshleifer, and Subrahmanyam (2001); Kodres and Pritsker (2002); and Pasquariello (2004).
takers, though strictly speaking this is only suitable in the limiting case where the numbers of assets and investors are infinite.

The informed investors all receive private signal $s$ on asset payoffs and the uninformed can only (imperfectly) infer the signal from market prices. This specification is used by Grossman and Stiglitz (1980) and Easley and O’Hara (2004). In Admati (1985) agents receive independent signals. It can be argued that our assumption and Admati’s are two special cases of a general information structure where investors have both diverse and asymmetric information: while we emphasize asymmetry, Admati emphasizes diversity. Technically speaking, the correlation between the private signals across informed investors is perfect in our model and zero in Admati’s model. While in our analysis price will be a function of informed investors’ private information, price is a function of the realized asset payoffs in Admati’s case when the number of assets is infinite due to the elimination of signal noise through aggregation of signals across assets.

We assume all investors have the following utility

$$U = -E[\exp(-AW_1)],$$

where $A$ is the investor’s absolute risk aversion coefficient and $W_1$ is the investor’s terminal wealth. The budget constraint is:

$$W_1 = W_0 R_f + D'(\nu - R_f p),$$

where $W_0$ is the investor’s initial wealth and $D$ is a vector containing the numbers of shares invested in risky assets.

Under the normality, the utility maximization problem becomes a mean-variance problem

$$\max_D E[W_1|J] - \frac{A}{2} \text{var}[W_1|J],$$

s.t. $W_1 = W_0 R_f + D'(\nu - R_f p),$
where $J$ represents the investor’s information set. The first-order condition implies optimal demand takes the following form:

$$D^*_J = \frac{1}{A} \Sigma_{\nu,J}^{-1} \mathbb{E}[\nu - R_fp | J].$$  

(5)

When asset payoffs do not depend on systematic factors, $\beta = 0$, it is easy to show investors’ demands for securities are increasing in expected asset payoffs and the precision of information about asset payoffs, and decreasing in risk aversion. In the more general case where asset payoffs do depend on systematic factors, $\beta \neq 0$, the demand for asset $i$ depends not only on investors’ posterior precision of beliefs on payoffs for asset $i$, but also on their posterior beliefs on payoffs for other assets. The informed and the uninformed have different demands because they condition on different information sets $J$.

### 2.2 Informed Investors

The informed investors receive private signal $s$ which takes the form

$$s = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2} \eta = \Sigma^{1/2} \epsilon + bF + \Sigma_s^{1/2} \eta.$$  

(6)

The $N \times K$ constant matrix $b$ reflects the relative information content of the signal with respect to the systematic factors and $\eta$ is an $N \times 1$ standard normal random variable. To conform with the interpretation of factor models, we will assume that $F, \epsilon, \eta,$ and $\eta_x$ are are jointly normal and independent and the matrices $\Sigma_s$ and $\Sigma_s$ are diagonal.

Our specification of asset payoffs is distinct from an alternative specification where asset payoffs do not follow a factor structure, but satisfy a general variance-covariance matrix (e.g., Admati, (1985)). Though a factor structure such as (1) implies a specific variance-covariance matrix, a general variance-covariance matrix
does not imply a corresponding factor structure. Admati (1985) entertains such constructions and concludes that a factor model is the natural context in which to consider private signals on economy-wide phenomena. Under her information structure, investors receive private signals about both systematic factors and idiosyncratic shocks. However, because of mathematical complexities an explicit solution was not obtained.

The signal $s$ for each risky asset specified in the above equation is a linear combination of information about the systematic components of the asset’s payoff, information about the idiosyncratic component of that payoff, and noise. The signal $s$ can also be interpreted as a combination of two signals: a signal about the idiosyncratic component of asset payoffs, $s_1 = \Sigma^{1/2}\epsilon + (bF + \Sigma^{1/2}s\eta)$, where $(bF + \Sigma^{1/2}s\eta)$ is interpreted as noise; and a signal about the systematic component, $s_2 = bF + (\Sigma^{1/2}\epsilon + \Sigma^{1/2}s\eta)$, where $(\Sigma^{1/2}\epsilon + \Sigma^{1/2}s\eta)$ is interpreted as noise.

The assumption that informed investors receive private information not only about the idiosyncratic component, but also about the systematic components of risky asset payoffs, although uncommon in the theoretical literature, is intuitive. Informed investors such as corporate insiders are likely to know more than the general public about the firm’s fundamentals such as revenues, earnings, and cash flows. To the extent that the fundamentals are generated by a factor structure, private information is likely to contain both components. Consistent with this assumption, Seyhun (1992) and Lakonishok and Lee (2001) show that aggregated trading by corporate insiders is predictive of future market returns.

Our specification of signals differs in two respects from that of Admati (1982) in the context of her factor model: the signals in our model are perfectly correlated across informed investors while in her model investors receive diverse signals, and the “two signals” constructively received by informed investors in our model are
correlated with covariance matrix $\Sigma_s$ conditional on $\nu$ and $F$, whereas the two signals for a given investor in Admati (1982) are uncorrelated. Assuming that the signals about the idiosyncratic component of an asset’s payoff and the systematic component are uncorrelated as in Admati’s signal specification, changes the expressions in the limiting case as $N$ goes to infinity, but does not affect either the structure of the explicit solution or the qualitative results that follow from that solution.

To calculate the conditional expectations and covariance matrixes, we need to derive the joint density function of $\nu$ and $F$ conditional on information $s$.

**Remark 1** The moments of the joint distribution of $\nu$ and $F$ conditional on signal $s$ are

$$E[\nu|s, F] = \nu + \beta F + \Sigma_{\nu|s,F} \Sigma_s^{-1} (s - b F),$$
$$E[F|s] = \Sigma_F s^t (\Sigma + \Sigma_s)^{-1} s,$$
$$\Sigma_{\nu|s,F}^{-1} = \Sigma^{-1} + \Sigma_s^{-1},$$
$$\Sigma_F^{-1} = \Sigma_F^{-1} + b^t (\Sigma + \Sigma_s)^{-1} b$$
$$\hat{\Sigma}_s = \Sigma + b \Sigma_F b^t + \Sigma_s.$$

The proof is given in the Appendix. From these moments, it follows that, conditional on signal $s$, the payoff is of the form

$$\nu = \nu + \Sigma_{\nu|s,F} \Sigma_s^{-1} s + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) F + \Sigma_{\nu|s,F}^{1/2} \epsilon_{\nu|s,F},$$

(7)

where $F$ is a normal random variable with mean $E[F|s]$ and covariance matrix $\Sigma_F|s$ and conditional on $s$ and $F$, $\epsilon_{\nu|s,F}$ is a standard normal random variable. We note that from the perspective of an informed investor loadings on the systematic factors (conditional betas) are $\beta_s = \beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b$. The precision matrix of the factors has increased from $\Sigma_F^{-1}$ to $\Sigma_F^{-1} = \Sigma_F^{-1} + b^t (\Sigma + \Sigma_s)^{-1} b$. Note idiosyncratic
risks matter for the factor precision matrix in that they are a source of noise when informed investors draw inferences about systematic factors.

From equation (7), the expectation of $\nu$ conditional on $s$ is

$$E[\nu|s] = \bar{\nu} + \Sigma_{\nu|s,F} \Sigma_s^{-1} s + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1} s$$

(8)

and the variance of $\nu$ conditional on $s$ is

$$\Sigma_{\nu|s} = \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) \Sigma_{F|s} (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b)'$$

(9)

Equations (8) and (9) can be substituted into the demand function to calculate the investor’s demand $D^*_J$ for risky assets:

$$D^*_s = \frac{1}{A} \Sigma_{\nu|s}^{-1} (\bar{\nu} + \Phi_s s - R_f p),$$

(10)

where

$$\Phi_s = \Sigma_{\nu|s,F} \Sigma_s^{-1} + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1}.$$

2.3 Uninformed Investors

The uninformed investors do not observe the signal $s$, but can imperfectly infer $s$ from the equilibrium price.

We conjecture that the equilibrium prices have the following form:

$$p = C + B (s - \lambda(x - \bar{x})),$$

where $C$ is an $N \times 1$ vector and $B$ and $\lambda$ are $N \times N$ matrices. We will assume that $B$ is invertible. Therefore, observing the price $p$ is equivalent to observing $\theta$ which is defined as

$$\theta = B^{-1} (p - C) = s - \lambda(x - \bar{x}).$$
Substituting equations (2) and (6), we can write

\[ \theta = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2} \eta - \lambda \beta_x F_x - \lambda \Sigma_x^{1/2} \eta_x. \]  

(11)

Therefore, we can interpret \( \theta \) as another signal which has sensitivity \( b \) to the factor \( F \) and idiosyncratic shocks with covariance matrix \( \Sigma_\theta \), where

\[ \Sigma_\theta = \Sigma_s + \lambda(\beta_x \Sigma_{Fx} \beta_x' + \Sigma_x) \lambda'. \]

Note that signal \( \theta \) is less informative than signal \( s \), i.e., its conditional variance-covariance matrix is larger than that of \( s \), i.e., \( \Sigma_\theta = \Sigma_s + \lambda(\beta_x \Sigma_{Fx} \beta_x' + \Sigma_x) \lambda' \geq \Sigma_s \). We should remark that \( \lambda \) is in general non-diagonal; the idiosyncratic shocks \( \Sigma_s^{1/2} \eta + \lambda \Sigma_x^{1/2} \eta_x \), although independent of \( F \), are not independent of each other.

When systematic factors in the random supply are uncorrelated with systematic factors in asset payoffs, as we assumed, the signal \( s \) is a sufficient statistic for \((s, \theta)\). However, it is plausible that the two systematic factors are correlated. In this case, the signal \( s \) is no longer a sufficient statistic for \((s, \theta)\). While the uninformed will continue to condition on only \( \theta \), the informed will now condition on both \( s \) and \( \theta \), a departure from the above analysis in which the informed only conditioned on \( s \). We assume independence for tractability. Nonetheless, we are confident that our analysis can be extended to accommodate the case of correlated factors and that our results are robust with respect to the relaxation of the independence assumption. The crucial aspect for risk premiums to be affected by asymmetric information is whether the informed investors learn more about systematic factors than uninformed investors in equilibrium; this can be modeled with or without the correlation between the two classes of systematic factors.

To calculate the conditional expectations and covariance matrixes, we need to derive the moments of the joint density function of \( \nu \) and \( F \) conditional on
information $\theta$.

**Remark 2** The moments of the joint distribution of $\nu$ and $F$ conditional on the signal $\theta$ are

\[
\begin{align*}
E[\nu|\theta, F] &= \nu + \beta F + \Sigma_{\nu|\theta,F}^{-1}(\theta - bF), \\
E[F|\theta] &= \Sigma_{F|\theta}^{-1} b'(\Sigma + \Sigma_{\theta})^{-1} \theta, \\
\Sigma_{\nu|\theta,F}^{-1} &= \Sigma^{-1} + \Sigma_{\theta}^{-1}, \\
\Sigma_{F|\theta}^{-1} &= \Sigma_{F}^{-1} + b'(\Sigma + \Sigma_{\theta})^{-1} b, \\
\hat{\Sigma}_{\theta} &= \Sigma + b\Sigma_{F} b' + \Sigma_{\theta}.
\end{align*}
\]

The proof is given in the Appendix. From these moments, it follows that, conditional on $\theta$, the payoff is of the form

\[
\nu = \nu + \Sigma_{\nu|\theta,F}^{-1} \theta + (\beta - \Sigma_{\nu|\theta,F}^{-1} b) F + \Sigma_{\nu|\theta,F}^{-1/2} \epsilon_{\nu|\theta,F},
\]

where $F$ is a normal random variable with mean $E[F|\theta]$ and covariance matrix $\Sigma_{F|\theta}$ and $\epsilon_{\nu|\theta,F}$ is a standard normal random variable conditional on $\theta$ and $F$. We note that, from the perspective of an uninformed investor, loadings on the systematic factors (conditional betas) are $\beta_{\theta} = \beta - \Sigma_{\nu|\theta,F}^{-1} b$. The precision matrix of the factors has increased from $\Sigma_{F}^{-1}$ to $\Sigma_{F|\theta}^{-1} = \Sigma_{F}^{-1} + b'(\Sigma + \Sigma_{\theta})^{-1} b$.

From equation (12), the expectation of $\nu$ conditional on $\theta$ is

\[
E[\nu|\theta] = \nu + \Sigma_{\nu|\theta,F}^{-1} \theta + (\beta - \Sigma_{\nu|\theta,F}^{-1} b) \Sigma_{F|\theta}^{-1} b'(\Sigma + \Sigma_{\theta})^{-1} \theta
\]

and the variance of $\nu$ conditional on $\theta$ is

\[
\Sigma_{\nu|\theta} = \Sigma_{\nu|\theta,F} + (\beta - \Sigma_{\nu|\theta,F}^{-1} b) \Sigma_{F|\theta}^{-1} \beta - (\beta - \Sigma_{\nu|\theta,F}^{-1} b)'.
\]

Equations (13) and (14) can be substituted into the demand function to calculate the uninformed investor’s demand $D^*_J$ for risky assets:

\[
D^*_a = \frac{1}{A} \Sigma_{\nu|\theta}^{-1} (\nu + \Phi_{\theta} - R_f p),
\]
where
\[
\Phi_\theta = \Sigma_{\nu|\theta,F} \Sigma_{\theta}^{-1} + (\beta - \Sigma_{\nu|\theta,F} \Sigma_{\theta}^{-1} b) \Sigma_{F|\theta} b' (\Sigma + \Sigma_{\theta})^{-1}.
\]

2.4 Equilibrium

Imposing the market clearing condition that the total demand from the informed and the uninformed investors equals the supply, we obtain the following equation:
\[
x = M \left( \frac{\mu}{A} \Sigma_{\nu|s}^{-1} (\bar{\nu} + \Phi_s s - R_f p) + \frac{1 - \mu}{A} \Sigma_{\nu|\theta}^{-1} (\bar{\nu} + \Phi_\theta \theta - R_f p) \right),
\]
where \( \mu \) is the proportion of informed investors. Defining
\[
\bar{\Sigma}_\nu = \left( \mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1},
\]
we derive the following expression for the prices of risky assets:
\[
p = \frac{1}{R_f} \left( \bar{\nu} + \bar{\Sigma}_\nu \left( \mu \Sigma_{\nu|s}^{-1} \Phi_s s + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \theta - \frac{A}{M} x \right) \right) = \frac{1}{R_f} \left( \bar{\nu} - \bar{\Sigma}_\nu \frac{A}{M} \bar{x} \right) + \frac{1}{R_f} \bar{\Sigma}_\nu \mu \Sigma_{\nu|s}^{-1} \Phi_s \left( s - \left( \mu \Sigma_{\nu|s}^{-1} \Phi_s \right)^{-1} \frac{A}{M} (x - \bar{x}) \right) + \frac{1}{R_f} \bar{\Sigma}_\nu (1 - \mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta (s - \lambda (x - \bar{x})). \quad (16)
\]
Comparing the above expression to the conjectured form of the price \( p \), it must be true that
\[
\lambda = (\mu \Sigma_{\nu|s}^{-1} \Phi_s)^{-1} \frac{A}{M}. \quad (17)
\]
Note that \( \lambda \) is solved in terms of the parameters of the model. The matrices \( \Sigma_{\nu|\theta} \), \( \Phi_\theta \), and \( \bar{\Sigma}_\nu \) are expressed in terms of \( \lambda \) as well as the parameters of the model; they are solved once \( \lambda \) is solved.

**Theorem 1** Given that informed investors receive a private signal, \( s \), that is informative about both idiosyncratic and systematic components of asset payoffs, a
partially revealing noisy rational expectations equilibrium exists, and prices of risky assets satisfy

\[
p = \frac{1}{R_f} \tilde{\nu} - \frac{1}{R_f} \hat{\Sigma} \nu A \bar{x} + \frac{1}{R_f} \tilde{\Sigma} \nu \left( \mu \Sigma_{\nu|s}^{-1} \Phi_s + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \right) (s - \lambda(x - \bar{x})). \tag{18}
\]

This equation confirms the conjectured form of the price

\[
p = C + B(s - \lambda(x - \bar{x})),
\]

where \( C = \frac{1}{R_f} (\tilde{\nu} - \hat{\Sigma} \nu A \bar{x}) \) and \( B = \frac{1}{R_f} \tilde{\Sigma} \nu \left( \mu \Sigma_{\nu|s}^{-1} \Phi_s + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \right) \). The risk premium of assets satisfies

\[
E[\nu - R_fp] = \frac{A}{M} \hat{\Sigma} \nu \bar{x} = \frac{A}{M} \left( \mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1} \bar{x}. \tag{19}
\]

Proof: The price \( p \) and the expressions for \( B \) and \( C \) are derived by combining the equations (16) and (17). The equation for the risk premium follows immediately. Note that the posterior precisions \( \Sigma_{\nu|s}^{-1} \) and \( \Sigma_{\nu|\theta}^{-1} \) do not depend on realizations of signals \( s \) and \( \theta \), respectively.

The first term in the price \( p \) is the expected payoff without information discounted by the risk-free return. This is the price if investors are risk-neutral \((A = 0)\) and there are no signals in the economy. The second term is the discount in price associated with risk, thus the risk premium. The third term is the correction to the expected payoff associated with signals and noisy supply.

The risk premium is determined by the geometric average of the covariance matrices of asset payoffs conditional on \( s \) and \( \theta \), \( \Sigma_{\nu|s} \) and \( \Sigma_{\nu|\theta} \). That is, the risk premium compensates the average of the risks conditional on \( s \) and \( \theta \). Two properties of the risk premium follow. First, from equation (9), \( \Sigma_{\nu|s} = \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F} \Sigma_{s,F}^{-1} b) \Sigma_{F|s}(\beta - \Sigma_{\nu|s,F} \Sigma_{s,F}^{-1} b)' \) and similarly for \( \Sigma_{\nu|\theta} \), the average risk includes idiosyncratic risk \( \Sigma_{\nu|s,F} \) and \( \Sigma_{\nu|\theta,F} \). Therefore, idiosyncratic risks are priced. Second, the average covariance matrix, \( \Sigma_{\nu} \) depends on \( \beta \) nonlinearly, thus the risk premium depends on \( \beta \) nonlinearly.
2.5 Symmetric Information

When all investors are informed, \( \mu = 1 \), Theorem 1 implies that the risk premium is

\[
E[\nu - R_f p] = \frac{\Sigma_{\nu|s} A}{M} \bar{x} = \left( \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) \Sigma_{F|s}(\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b)' \right) \frac{A}{M} \bar{x}.
\]

In such an economy, the factor risk premiums are decreased by information because \( \Sigma_{F|s} \) is smaller than \( \Sigma_F \). In addition, an econometrician who observes the return but not the signal will conclude that the risk premium depends on \( \beta \) as well as some firm-specific characteristics, \( \Sigma_{\nu|s,F} \Sigma_s^{-1} b \). Thus, firms with the same \( \beta \) but different \( \Sigma_{\nu|s,F} \Sigma_s^{-1} b \) may have different expected returns. This economy seems potentially to provide a theory for the empirical findings of Daniel and Titman (1998).

At the other extreme, when all investors are uninformed, \( \mu = 0 \), \( \lambda \to \infty \); i.e., the inferred signal \( \theta \) is infinitely more noisy than \( s \) and thus is not informative at all. It follows immediately that the covariance matrix conditional on \( \theta \), \( \Sigma_{\nu|\theta} \), is the same as \( \Sigma \) and the factor covariance matrix conditional on \( \theta \), \( \Sigma_{F|\theta} \), is the same as \( \Sigma_F \). Furthermore, factor loadings conditional on \( \theta \) are the same as unconditional factor loadings, i.e., \( \beta_{\theta} = \beta \). From Theorem 1, the risk premium is

\[
E[\nu - R_f p] = \frac{\Sigma_{\nu|\theta} A}{M} \bar{x} = \left( \frac{\Sigma + \beta \Sigma_F \beta'}{M} \right) \frac{A}{M} \bar{x}.
\]

\(^5\)In a recent paper, Lambert, Leuz, and Verrecchia (2005) also consider effects of information on cost of capital in a general variance-covariance matrix framework. They show that if all pairwise correlations of asset payoffs are positive, then information can reduce the average cost of capital. Our factor structure does not require a restriction to positive pair-wise correlations and allows us to distinguish effects of information (split into systematic and idiosyncratic components) on asset betas and factor risk premiums. Moreover, the principal focus of our study is on the effects of asymmetric information and price discovery, issues that are absent in Lambert, Leuz, and Verrecchia (2005).
The above can be described as the risk premium in a small economy with homogeneous beliefs. In this case, there is no updating of beliefs, idiosyncratic risk is priced, and $\beta$ appears linearly in the risk premium. The idea of diversification is clearly embedded in the above expression because the risk premiums on systematic risks $\Sigma^{A}_M$ are larger than the risk premium on idiosyncratic risks $(\beta \Sigma^{F}_{A} \beta')^{A}_M$ by a factor of $N$. When the economy ($N$ and $M$) is small, the risk premium on idiosyncratic risk can not be ignored; when the size of the economy increases, idiosyncratic risk premium becomes less and less important relative to the systematic risk premiums. As we show in the next section, in the large economy limit, idiosyncratic risk premium goes to zero.

We conclude this section by observing that in an economy with a finite number of assets idiosyncratic as well as systematic risk is priced, information on idiosyncratic shocks reduces idiosyncratic risk and hence the risk premium, information can increase or decrease factor loadings, the risk premium depends on beta non-linearly, and information on the systematic factor reduces systematic risk and hence the factor risk premium. As we will demonstrate in Section 3, only the last property survives in the limit as the number of risky assets goes to infinity.

3 Large Economy Limit

In this section, we study the effects of private signals on risk premiums when the economy is large in the sense that the number of assets and the number of investors go to infinity. The idea of diversification requires that number of assets be sufficiently large for idiosyncratic shocks to cancel each other out thereby reducing aggregate portfolio uncertainty to the point where it reflects only systematic risks. If we only expand the number of assets and fix the number of investors, however, the
large economy becomes unrealistic since each individual would have infinite wealth in the limit. In addition, because each individual is assumed to have constant absolute risk aversion, infinite wealth implies infinite relative risk aversion and, hence, infinite risk premiums. Even in this extreme case, one can show that the risk premiums on the systematic factors are bigger than the risk premiums on idiosyncratic factors by an order of $N$. In other words, compared with the factor risk premiums, idiosyncratic risk can be ignored when the number of assets is large.\textsuperscript{6}

To rule out such an unrealistic scenario, we require that the number of investors expand at the same rate as the number of asset so that in the limit $\frac{N}{M}$ approaches a constant. Without loss of generality, we assume the constant is unity. In analysis that follows, when we talk about taking the “large economy limit” or “large $N$ limit”, we are implicitly referring to the case where $M$ and $N$ are expanding at the same rate. Similar restrictions have been adopted in studies that concern large economies (e.g., Leuz and Verrecchia, 2005; Ou-Yang, 2004). In his formal derivation of that relation in an economy without private signals, Ross (1976) directly imposes the restriction that relative aggregate risk aversion be uniformly bounded as the number of assets and, hence, wealth increases. Our assumption satisfies his restriction. To be precise, the risk premium from the previous section for a large economy in which the numbers of both assets and investors go to

\textsuperscript{6}We have assumed that the net supply of each individual asset is bounded below at a level above zero – stocks and bonds are such securities. If we allow for the possibility that the net supply of each individual asset to converge to zero as one expands the number of asset, e.g., increase the number of derivative securities which have net zero supply, wealth and risk premium will also be finite without the expansion of the number of investors. However, in this case diversification will not be complete, even in the case of symmetric information. We rule out this case because we need diversification to work at least under symmetric information in order to study the joint effect of diversification and information asymmetry.
infinity at the same rate when there are no private signals (i.e., homogeneous beliefs) becomes:

\[ E[\nu - R_f] = \beta A \Sigma_F \beta' \bar{x} / N, \]  

(21)

where \( A \Sigma_F \beta' \bar{x} / N \) is a vector of factor premiums. Note that the magnitude of \( \beta' \bar{x} \) is of order \( N \) and hence the magnitude of \( \beta' \bar{x} / N \) is of order 1 when \( N \to \infty \). Thus, we have finite risk premiums. In equilibrium, the risk premium on idiosyncratic risk goes to zero as \( N \to \infty \); firm’s risk premiums increase in betas and factor risk premiums, where the factor risk premiums in turn increase in risk aversion, aggregate uncertainty in the economy and supply of risky assets.

We begin our analysis of the effects of private signals on risk premiums in the large economy limit with two special cases that have appeared in the literature; private information only on idiosyncratic shocks and private information on total asset payoffs. We then consider the case where private information on systematic factors has finite aggregate precision.

3.1 Special Cases

3.1.1 Private Information Only on Idiosyncratic Components of Asset Payoffs

Suppose informed investors receive private signals on just the idiosyncratic components of risky asset payoffs. In this case, \( b = 0 \) and the signals can be written as

\[ s = \nu - \bar{v} - \beta F + \Sigma_s^{1/2} \eta = \Sigma_s^{1/2} \epsilon + \Sigma_s^{1/2} \eta. \]  

(22)

Note that when \( \beta \neq 0 \), the asset payoffs are correlated. In the special case where all asset payoffs are uncorrelated, i.e., \( \beta = 0 \), this structure reduces to the setting
considered by Easley and O’Hara (2004). It is easy to see that, for finite \( N \), information solely about idiosyncratic shocks reduces uncertainty about priced risks; \( \Sigma_{\nu|J,F} \). However, as we show below, in the limit as \( N \to \infty \), elimination of idiosyncratic risks through diversification implies that private signals containing only idiosyncratic components have no effects on risk premiums:

**Proposition 1** Given that informed investors receive private signals only about the idiosyncratic components of asset payoffs, in the limit as \( N \to \infty \), the risk premium satisfies

\[
E[\nu - R_f p] = \beta A \Sigma_F \beta' \bar{x}/N.
\]  

(23)

The proof is given in the Appendix. Notably, the risk premium in this case is the same as the risk premium without information, \( \mu = 0, \beta A \Sigma_F \beta' \bar{x}/N \), implying that this is the risk premium for all \( \mu \). In other words, there is no resolution of uncertainty about systematic factors from private signals that do not contain a systematic component; investors remain with their prior (homogeneous) beliefs.\(^7\) It is also clear that, in the setting studied by Easley and O’Hara (2004), where \( \beta = 0 \), the risk premium is reduced to zero, i.e., \( E[\nu - R_f p] = 0 \).

More generally, we expect that the same results will hold as long as \( \beta'(\Sigma + \Sigma_s)^{-1}b \to 0 \) and \( \beta'(\Sigma + \Sigma_s)^{-1}b \to 0 \) when \( N \to \infty \). Intuitively, diversification works at the power of \( 1/N \), implying that if the systematic component of the signal has a power less than \( 1/N \), then it will be eliminated by diversification.

\(^7\)Note further, that the proposition holds notwithstanding systematic components in the random supply, \( \beta_s \sigma_{fs} \neq 0 \). This result is quite intuitive: Even if all the agents are informed, \( \mu = 1 \), there is no resolution of uncertainty about the factors that affect asset payoffs, implying that the random supply of assets is irrelevant for asset pricing when it is independent of systematic factors.
Note that although private information on idiosyncratic shocks does not affect risk premiums in this case, it does affect asset prices and portfolio holdings of informed and uninformed investors and, hence, their expected utilities.

### 3.1.2 Private Information on Total Risky Asset Payoffs

Suppose now that informed investors receive private signals about total asset payoffs. In this case, \( b = \beta \) and the signals can be written as

\[
s = \nu - \bar{\nu} + \Sigma_s^{1/2}\eta. \tag{24}
\]

This is a special case of Admati (1985) where the covariance matrix of the assets has the form of a factor structure and the signals for different assets are uncorrelated.

In this case, \( \Sigma^{-1}_{F|s} = \Sigma^{-1}_F + \beta'(\Sigma + \Sigma_s)^{-1}\beta \), which goes to infinity as \( N \to \infty \). Therefore, we have

\[
\Sigma_{F|s} = 0.
\]

Similarly, \( \Sigma^{-1}_{F|\theta} = \Sigma^{-1}_F + (\beta + \beta_x)'(\Sigma + \Sigma_\theta)(\beta + \beta_x) \), which also goes to infinity as long as \( \beta + \beta_x \) goes to a constant as \( N \to \infty \); thus we also have

\[
\Sigma_{F|\theta} = 0.
\]

It is easy to show that the above two equations imply that the risk premium is zero.

The intuition here is also clear. Infinitely many private signals about asset payoffs implies that informed investors learn the systematic factor perfectly and set their demands such that prices fully reveal the systematic factor \( F \) and, thus, eliminate the risk associated with that factor.
3.2 General Case

We have considered the special cases where \((b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_d)^{-1}b) \rightarrow 0\) and \((b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_d)^{-1}b) \rightarrow \infty\). The more interesting case is where the limit of \((b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_d)^{-1}b)\) is a non-zero finite constant; what we call finite aggregate precision. This happens, for instance, if \(\sqrt{Nb}\) converges to a non-zero constant vector when \(N \rightarrow \infty\). In effect, under this structure, as the economy expands the informativeness of the private signal for a given asset about the factor is decreasing. Thus, even though signal noise and idiosyncratic risk are becoming diversified away as the number of assets increases, informed investors’ aggregate information about the factor in the limit is imperfect.

The model in our paper is a metaphor to capture the idea that there is residual aggregate uncertainty in the economy. For example, each firm’s earnings are influenced by the general health of the economy. However, even if we learn a little bit from each company about the aggregate economy, we would not be able remove the aggregate uncertainty completely. For this reason, the market usually reacts strongly when interest rates are raised or lowered or when government releases statistics about aggregate measurements such as unemployment, GDP etc. The intuition is that as one expands the number of assets, at the same time as one gets more signals about the aggregate economy, the strength of each signal is also weaker because a firm in a larger economy is relatively less important than a firm in a smaller economy. Our assumption exactly captures this idea - as the economy becomes bigger, what one can learn independently from a single firm about the market is also smaller.\(^8\)

\(^8\)As mentioned earlier, an alternative information structure that would preclude learning the factor realization perfectly and preserve our qualitative results is to assume informed investors receive two uncorrelated signals; one about idiosyncratic shocks and the other about the sys-
The risk premium in this case is given by the following proposition.

**Proposition 2** Given that informed investors receive private signals informative about both idiosyncratic and systematic components of asset payoffs with finite aggregate precision, in the limit as \( N \to \infty \), the risk premium is

\[
E[\nu - R_f p] = \beta A \left( \mu \Sigma^{-1}_{F|s} + (1 - \mu)\Sigma^{-1}_{F|\theta} \right)^{-1} \frac{\beta' \bar{x} N}{N}
\]

and the factor risk premium is

\[
\lambda = A \left( \mu \Sigma^{-1}_{F|s} + (1 - \mu)\Sigma^{-1}_{F|\theta} \right)^{-1} \frac{\beta' \bar{x} N}{N}.
\]

The proof is given in the Appendix.

Compared with the no private information case (equation 21), it is immediately evident that equilibrium risk premiums only differ with respect to the posterior aggregate uncertainty in the economy; the prior factor precision is transformed into the posterior factor precision \( \left( \mu \Sigma^{-1}_{F|s} + (1 - \mu)\Sigma^{-1}_{F|\theta} \right)^{-1} \). Idiosyncratic risks are not priced in the sense that two assets with same beta but different idiosyncratic risks will have the same risk premium. Although information about idiosyncratic shocks affects factor risk premiums, such information does not generate cross-sectional effects on risk premiums because betas are not affected.

To isolate the effect of information asymmetry while controlling for the total information revealed by prices, we can rewrite the posterior factor precision as follows:

\[
\left( \Sigma^{-1}_{F|s} - (1 - \mu) \left( \Sigma^{-1}_{F|s} - \Sigma^{-1}_{F|\theta} \right) \right)^{-1}
\]

Idiosyncratic factor. This is similar to the information structure assumed by Admati (1982) in the context of her factor model, the difference being that in our model all informed investors receive the same signals while in Admati they receive diverse signals.
Interpreting the above expression, when all investors are informed, $\mu = 1$, maximum resolution of uncertainty is achieved and the posterior factor precision is the first term in the outer parentheses, $\Sigma_{F|s}^{-1}$. However, when some investors are not informed, $\mu < 1$, the resolution of uncertainty is diminished by the degree of information asymmetry, measured as $\Sigma_{F|s}^{-1} - \Sigma_{F|\theta}^{-1}$. It follows immediately that, other things being equal, factor risk premiums increase as information asymmetry $\Sigma_{F|s}^{-1} - \Sigma_{F|\theta}^{-1}$ increases. Factor risk premiums also increase in risk aversion and supply of risky assets, a feature preserved from the symmetric information case.

We note that even though idiosyncratic risks do not matter in the cross-section for cost of capital, they affect (systematic) factor risk premiums because they act as a source of noise when investors draw inferences about the factors from signals and prices.

### 3.2.1 Risk Premiums and the Size of the Economy

To gauge the speed at which the finite economy approaches the limiting case where $N$ goes to infinity, we explicitly calculate the risk premiums for a case where we assume identical distribution for risky asset payoffs with one factor and related signals: i.e., $\Sigma = \sigma^2 I_N$ ($I_N$ is the $N$-dimensional identity matrix), $\Sigma_s = \sigma_s^2 I_N$, $\Sigma_F = \sigma_f^2$, $\beta = \beta_1 1_{N \times 1} (1_{N \times 1}$ is a $N \times 1$ vector with all elements being 1), $\bar{x} = \bar{x} 1_{N \times 1}$ and $b = \frac{k}{\sqrt{N}} 1_{N \times 1}$, where $\sigma$, $\sigma_s$, $\sigma_f$, $\beta_1$, $\bar{x}$, and $k$ are all constant. Thus, all the covariance matrices are proportional to the identity matrix; the betas of all risky asset payoffs are equal; and the sensitivities of the signals to (for convenience) a single factor are equal. Note that since there is a factor, the distributions of asset payoffs are not independent although they are identical.

Under a set of plausible parameter values where $A = 3$, $\sigma = 30\%$, $\beta = 1$, $\sigma_f = 20\%$, $\sigma_s = 25\%$, $\sigma_{fx} = 30\%$, $\beta_x = 1$, $\sigma_x = 30\%$, and $k = -1$, Figure 1
plots the risk premium against the fraction of the informed investors for various numbers of risky assets. The risk premium decreases with $N$ as we would expect. In particular, we observe that there is substantial convergence to the risk premium in the limiting case as the number of assets reaches the hundreds. This suggests that the risk premium in the limit as the number of assets goes to infinity may be a reasonable approximation to the risk premium in a finite economy where the number of assets measures in the thousands.

### 3.2.2 Deflation by Price

We note that above results are expressed in terms of dollar risk premiums derived under the standard assumptions of negative exponential utility and normal distributions. To derive results in terms of returns, one can simply deflate both sides of dollar risk premiums by the stock price (e.g., Ou-Yang, 2004, and Leuz and Verrecchia, 2005). An effect of this approach is that the percentage expected return conditional on the current price will be dependent on idiosyncratic information apart from its effects on factor risk premiums because the price depends on idiosyncratic information. Equivalently, conditional risk premiums in percentage returns can be expressed in terms of return betas (i.e., payoff betas scaled by price) and factor risk premiums as depicted in our analysis. Therefore, unlike the previous results based on dollar risk premiums, asymmetric information generates a cross-sectional effect on return risk premiums because return betas are altered by idiosyncratic information.

---

9 The parameter values for $A$ and $\beta$ are standard. Typical stock volatility is 30% to 50%, thus $\sigma = 30\%$. The values of $\sigma_s$, $\sigma_{fx}$, and and $\sigma_x$ are chosen in the range of $\sigma$. The value of $\beta_x$ is chosen to be the value of $\beta$. The values $\sigma_s$, $\sigma_{fx}$, $\beta_x$, and $\sigma_x$ need to be comparable to the parameter values that describe the underlying returns, otherwise the information effect will be either negligible or overwhelming.
We caution that characterizing unconditional risk premiums in terms of percentage returns is problematic given division by a random variable that could take on zero as a value. Moreover, it can be argued that in the space of negative exponential utility functions risk premiums are more meaningful when expressed in the same dollar terms as asset payoffs. The assumptions of negative exponential utility and normal distributions are adopted in rational expectations models not because of their realism but because of their tractability. Since the absolute risk aversion is a constant for negative exponential utility, the demand for risky securities is independent of investors’ wealth. As a result, the risk premium enters the dollar return function in an additive fashion. Division by beginning of the period price introduces idiosyncratic factors since expectations of future cash flows depends on systematic as well as idiosyncratic factors. This is true even in the absence of information asymmetry. Consistent with this argument, most rational expectations models in the literature do not draw implications from deflated prices; risk premiums in this literature are usually understood to be in dollar terms.

Even with the above mentioned caveats, suppose one nevertheless still wants to derive a return specification by deflating the dollar risk premiums using the current stock prices, all cross-sectional effect of information is contained in return betas. After controlling for the return betas, firm specific information characteristics are again irrelevant for the determination of the cross-section of expected returns.

4 Empirical Implications

Several important implications for empirical inquiries unfold from our results. First, given the substantial impact of Easley and O’Hara (2004) in accounting, it is useful to clarify the intuition for the risk premium characterized by their Propo-
sition 2. Our results imply that the common interpretation of the risk premium characterized by Easley and O’Hara in their Proposition 2 as a consequence of uninformed investors seeking compensation for the risk that they may be trading against informed traders is flawed. Both informed and uninformed investors exploit liquidity traders whose demands are manifested in an assumption of noisy supply. Uninformed investors have an information advantage over liquidity traders based on what they can infer from price. Liquidity traders absorb expected losses from both classes of investors. The only risks for which both classes of investors receive compensation are risks that cannot be diversified away, whether because of a restriction to a finite set of assets as in Easley and O’Hara or because risks are systematic as in our limiting economy. The equilibrium risk premium reflects the average posterior uncertainty.

Many empirical papers, mainly in accounting, cite Easley and O’Hara’s results as an explanation for an asymmetric information risk premium without recognizing the role played by under diversification due to the restriction to a finite set of assets[e.g., Ali, Klasa and Yeung (2005), Botosan and Plumlee (2004), Botosan, Plumlee and Xie (2004), Bushman, Piotroski and Smith (2005), Easley, Hvidkjaer and O’Hara (2002, 2004), Francis, LaFond, Olsson and Schipper (2004a), Gietzmann and Ireland (2005), Gore and Baber (2005), Bhattacharya, Daouk and Welker (2003)]. Some authors even mistakenly conclude that Easley and O’Hara (2004) have proven ”asymmetric information risk” is not diversifiable[e.g., Hope, Kang, Thomas and Vasvari (2005)].

Second, no matter whether we consider price space or return space, risk premiums on systematic factors are affected by information asymmetry. This implication applies in both cross-section and in time series. For example, Bhattacharya, Daouk and Welker (2003), Bhattacharya and Daouk (2002) and Jain (2005) find cross-
sectional variation in average cost of capital across countries where the markets are segmented by regulation and capital control. Although all these paper cite Easley and O’Hara (2004) for theoretical motivation, we believe our model provides a more direct theoretical explanation for such empirical findings. Our analysis suggests that similar market level analysis can be done in time series, though power of the test is an issue since there may not be sufficient number of significant disclosure regime changes in any one particular country.

Third, in both price space and return space, our theory suggests that firm specific (idiosyncratic) characteristics should not enter the determination of expected returns after controlling for beta. Empirical inquiries that correlate expected returns with such firm characteristics include Botosan (1997), Botosan and Plumlee (2002), Botosan, Plumlee and Xie (2004), Easley, Hvidkjaer and O’Hara (2002), Gietzmann and Ireland (2005), Gore and Baber (2005), Mansi, Maxwell and Miller (2005). The significant findings in these papers could have a number of explanations. It could be that the investors are not fully diversified and hence idiosyncratic risk is priced. In turn, because the available asset space is large, incomplete diversification on the part of investors suggests behavioral biases or institutional frictions. Given these prospects, it is difficult to reach an unambiguous conclusion that (information) risk is driving the observed results. Alternatively, it could be some risk factors are omitted and the information characteristics are correlated with the firm’s exposure to the omitted factors (Fama and French, 1993). But if this is the case, we then need to modify the research design to directly measure such risk factors similar to the Fama-French’s three factor model. Last, it could simply be spurious correlation that has nothing to do with "information risk". Recent papers by Cohen (2004), Nikolaev and Van Lent (2005), Chen, Chen and Wei (2004) address this concern and find the correlation between cost of capital
and information asymmetry becomes insignificant when endogeneity and correlated omitted variables are considered. In sum, no matter which perspective one adopts to judge the empirical studies, our theory is likely to shed light on both empirical design and the interpretation of significant findings.

Fourth, our theory suggests that in price space there should be no cross-sectional effect of asymmetric information while in the return space betas could be affected cross-sectionally due entirely to the effect on price as the deflator. The existing empirical papers do not investigate the information effect in price space. Hence, our theory suggests that this is a promising arena for further inquiry.

Finally, our model is silent on whether there exists a systematic information factor, implying it is not inconsistent with studies that assume the existence of an information factor (e.g., Aboody, Hughes and Liu 2005, Francis, Lafond, Olsson and Schipper 2002 and Easley, Hvidkjaer and O’Hara 2003). For empirical studies that assume a systematic information factor, existing theoretical studies on the cost of capital effect of information, including ours, do not support the research design. One can of course draw inspiration from Merton’s (1973) Intertemporal Capital Asset Pricing Model and assume that the aggregate information structure of the economy is a state variable that helps to predict future investment opportunities. But, as in the case of other empirically determined systematic risk factors, such as the price to book factor and the size factor, the theoretical ground for drawing conclusions is less firm.

5 Conclusion

Our objective in this study is to contribute to the development of a theoretical foundation for empirical inquiries about the relation between asymmetric informa-
tion and cost of capital. We assume a factor structure for both asset payoffs and private signals in the context of a competitive noisy rational expectations model of market behavior. The advantage of this structure is that it allows us to consider the effects of private information on systematic factors as well as on idiosyncratic shocks.

Taking the large economy limit, we show that equilibrium risk premiums are entirely determined by the product of betas and factor risk premiums. This is true whether risk premiums are measured in dollars, as is the convention when assuming normal distributions and exponential utility, or percentage returns after scaling by price. The risk associated with idiosyncratic shocks affects risk premiums only as a source of noise in drawing inferences about systematic factors from private signals; idiosyncratic risk per se is eliminated through diversification. Greater information asymmetry about systematic factors leads to less resolution of uncertainty as manifested by a larger aggregate posterior factor covariance matrix and, hence, higher factor risk premiums.

Of particular note to empiricists is that controlling for return betas, there is no cross-sectional effect of information asymmetries on cost of capital. Accordingly, the more promising avenues for investigating the effects of asymmetric information would appear to be transnational studies where institutions governing disclosure policies may vary or inter-temporal studies where institutions and related disclosure policies have changed in a significant across the board manner such as with the Sarbanes-Oxley legislation.

A limitation of our pure exchange model is that it is silent on the existence of a systematic factor related to asymmetric information; a specification that has been used in several empirical studies. Investigating conditions under which such a factor might emerge in equilibrium is a topic we commend to future research.
References


Appendix

In the Appendix, we will use the following identity extensively:

$$ (\Sigma + \beta \Omega \beta')^{-1} = \Sigma^{-1} - \Sigma^{-1} \beta (\Omega^{-1} + \beta' \Sigma^{-1} \beta) \beta' \Sigma^{-1}. $$

The Proof of Remark 1.

We solve for the filtering rule, given signal \( s \). Our assumptions have specified the distribution functions \( f(\nu|F,s) \), \( f(\nu|F) \), and \( f(F) \). Therefore,

$$ f(\nu, F, s) = f(s|\nu, F) f(\nu|F) f(F). $$

We can rewrite the above as

$$ f(\nu, F, s) = f(\nu|s, F) f(F|s) f(s). $$

Focusing on the exponential terms of the joint normal distribution densities, we obtain

$$ - \ln f(\nu, F, s) = - \ln f(s|\nu, F) - \ln f(\nu|F) - \ln f(F) $$

$$ + \frac{1}{2} (s - (\nu - \varphi - \beta F) - bF)' \Sigma_{s}^{-1} (s - (\nu - \varphi - \beta F) - bF) $$

$$ + \frac{1}{2} (\nu - \varphi - \beta F)' \Sigma^{-1} (\nu - \varphi - \beta F) + \frac{1}{2} F' \Sigma_{F}^{-1} F $$

$$ = \frac{1}{2} (\nu - \varphi - \beta F)' \left( (\Sigma^{-1} + \Sigma_{s}^{-1} ) (\nu - \varphi - \beta F) - (\nu - \varphi - \beta F)' \Sigma_{s}^{-1} (s - bF) \right. $$

$$ + \frac{1}{2} (s - bF)' \Sigma_{s}^{-1} (s - bF) + \frac{1}{2} F' \Sigma_{F}^{-1} F $$

$$ = \frac{1}{2} (\nu - \mathbb{E}[\nu|s, F]) \Sigma_{\nu|s,F}^{-1} (\nu - \mathbb{E}[\nu|s, F]) $$

$$ + \frac{1}{2} (s - bF)' (\Sigma + \Sigma_{s})^{-1} (s - bF) + \frac{1}{2} F' \Sigma_{F}^{-1} F $$

$$ = \frac{1}{2} (\nu - \mathbb{E}[\nu|s, F]) \Sigma_{\nu|s,F}^{-1} (\nu - \mathbb{E}[\nu|s, F]) + \frac{1}{2} s' (\Sigma + \Sigma_{s})^{-1} s $$

37
\[
\begin{align*}
&+\frac{1}{2}(bF)'(\Sigma + \Sigma_s)^{-1}bF - s'(\Sigma + \Sigma_s)^{-1}bF + \frac{1}{2}F'\Sigma_F^{-1}F \\
&= \frac{1}{2}(\nu - E[\nu|s,F])\Sigma_{\nu|s,F}^{-1}(\nu - E[\nu|s,F]) \\
&+ \frac{1}{2}(F - E[F|s])\Sigma_{F|s}^{-1}(F - E[F|s]) + \frac{1}{2}s'\Sigma_s^{-1}s \\
&= -\ln f(\nu|s,F) - \ln f(F|s) - \ln f(s),
\end{align*}
\]

The distribution functions \(f(\nu|s,F), f(F|s), \text{ and } f(s)\) can then identified from the above equation, with

\[
\begin{align*}
E[\nu|s,F] &= \nu + \beta F + \Sigma_{\nu|s,F}\Sigma_{\nu}^{-1}(s - bF), \\
E[F|s] &= \Sigma_{F|s}b'(\Sigma + \Sigma_s)^{-1}s, \\
\Sigma_{\nu|s,F}^{-1} &= \Sigma^{-1} + \Sigma_{s}^{-1}, \\
\Sigma_{F|s}^{-1} &= \Sigma_{F}^{-1} + b'(\Sigma + \Sigma_s)^{-1}b, \\
\hat{\Sigma}s &= \Sigma + b'\Sigma_F b + \Sigma_s.
\end{align*}
\]

**The Proof of Remark 2.**

The structure of the filtering rule, given signal \(\theta\), is the same as that for \(s\). The proof proceeds in exactly the same fashion.

**Proof of Proposition 1.**

Because \(b = 0\), we have

\[
\begin{align*}
\Sigma_{F|s} &= \Sigma_{F}; \\
\Sigma_{\nu|s} &= \Sigma_{\nu|s,F} + \beta\Sigma_{F}\beta'; \\
\Sigma_{F|\theta} &= \Sigma_{F}; \\
\Sigma_{\nu|\theta} &= \Sigma_{\nu|\theta,F} + \beta\Sigma_{F}\beta'.
\end{align*}
\]
Intuitively, the matrices $\Sigma_{\nu|s}$ and $\Sigma_{\nu|\theta}$ differ only in the idiosyncratic matrices $\Sigma_{\nu|s,F}$ and $\Sigma_{\nu|\theta,F}$ which do not matter for the risk premium and thus should produce the risk premium $A\beta \Sigma_F \beta' \bar{x}$. The formal proof is as follows. From

$$\Sigma_{\theta} = \Sigma_s + \lambda (\beta_s \Sigma_F \beta_s' + \Sigma_x) \lambda' \geq \Sigma_s,$$

we know that

$$\Sigma \geq (\Sigma^{-1} + \Sigma_{\theta}^{-1})^{-1} = \Sigma_{\nu|\theta,F} \geq (\Sigma^{-1} + \Sigma_{\theta}^{-1})^{-1} = \Sigma_{\nu|s,F}. $$

It follows that

$$\Sigma + \beta \Sigma_F \beta' = (\mu (\Sigma + \beta \Sigma_F \beta')^{-1} + (1 - \mu) (\Sigma + \beta \Sigma_F \beta')^{-1})^{-1} \geq (\mu (\Sigma_{\nu|s,F} + \beta \Sigma_F \beta')^{-1} + (1 - \mu) (\Sigma_{\nu|\theta,F} + \beta \Sigma_F \beta')^{-1})^{-1} = \Sigma_{\nu}$

$$ \geq (\mu (\Sigma_{\nu|s,F} + \beta \Sigma_F \beta')^{-1} + (1 - \mu) (\Sigma_{\nu|s,F} + \beta \Sigma_F \beta')^{-1})^{-1} = \Sigma_{\nu|s,F} + \beta \Sigma_F \beta'. $$

Hence, we find the upper and lower bounds for $\lim_{N \to \infty} \frac{1}{N} \Sigma_{\nu}$

$$\lim_{N \to \infty} \frac{1}{N} (\Sigma + \beta \Sigma_F \beta') = \lim_{N \to \infty} \frac{1}{N} \beta \Sigma_F \beta' \geq \frac{1}{N} \bar{\Sigma}_{\nu} \geq \frac{1}{N} \Sigma_{\nu|s,F} + \beta \Sigma_F \beta' = \lim_{N \to \infty} \frac{1}{N} \beta \Sigma_F \beta. $$

Therefore, the average risk premium is

$$E[\nu - R_f p] = A \frac{1}{N} \bar{\Sigma}_{\nu} \bar{x} \to A \frac{1}{N} \left ( \Sigma_{\nu|s,F} + \beta \Sigma_F \beta' \right) \bar{x} \to A \frac{1}{N} \beta \Sigma_F \beta' \bar{x}. $$

**Proof of Proposition 2.**

For the case of non-identically distributed risky asset payoffs, the leading order terms in the large $N$ limit are

$$\Sigma_{\nu|s,F} = (\Sigma^{-1} + \Sigma_{s}^{-1})^{-1},$$

$$\Sigma_{F|s} = \left ( \Sigma_{F}^{-1} + \frac{1}{N} k' (\Sigma + \Sigma_s)^{-1} k \right)^{-1}. $$
The variance of \( \nu \) conditional on \( s \)

\[
\Sigma_{\nu|s} = \Sigma_{\nu|s,F} + \beta \Sigma_{F|s} \beta' + O(N^{-1/2}),
\]

\[
\Phi_s = \Sigma_{\nu|s,F}^{-1} + \frac{1}{\sqrt{N}} \beta \Sigma_{F|s} k'(\Sigma + \Sigma_s)^{-1} + O(N^{-1}).
\]

Both first terms in the above equations are diagonal matrices. The second terms are due to factors. We use \( O(N^\alpha) \) to denote matrices with all of their elements generally non-zero and of order \( N^\alpha \). In the case of identical assets, \( O(N^\alpha) \propto N^\alpha 1_{N \times N} \). These terms will be negligible, in the large \( N \) limit, as far as the risk premium is concerned. The \( \Phi_s^{-1} \) matrix is

\[
\Phi_s^{-1} = \Sigma_s \left( I_N + \frac{1}{\sqrt{N}} \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \right)^{-1} \Sigma_{\nu|s,F}^{-1}
\]

and

\[
\Phi_s^{-1} \Sigma_{\nu|s} = \Sigma_s \left( I_N - \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} (\sqrt{N} I_K + k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F} \beta \Sigma_{F|s})^{-1} k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \right) \Sigma_s^{-1}
\]

\[
\times \left( I_N + \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} \beta' \right)
\]

\[
= \Sigma_s \left( I_N - \Sigma_{\nu|s,F}^{-1} \beta \left( \sqrt{N} \Sigma_{F|s}^{-1} + k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F} \beta \right)^{-1} k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \right)
\]

\[
- \Sigma_{\nu|s,F}^{-1} \beta \left( \sqrt{N} \Sigma_{F|s}^{-1} + k' (\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F} \beta \right)^{-1} \sqrt{N} \beta'
\]

\[
\rightarrow \Sigma_s \left( I_N + \frac{1}{\sqrt{N}} \Sigma_{\nu|s,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma_s^{-1} \beta \right)^{-1} \beta' \right).
\]

Therefore,

\[
\lambda = \frac{A}{\mu N} \Phi_s^{-1} \Sigma_{\nu|s} = \frac{1}{\sqrt{N^3}} A \mu^{-1} \Sigma_s \Sigma_{\nu|s,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma_s^{-1} \beta \right)^{-1} \beta'.
\]

The signal \( \theta \) is now

\[
\theta = s - \frac{1}{\sqrt{N}} A \mu^{-1} \Sigma_s \Sigma_{\nu|s,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma_s^{-1} \beta \right)^{-1} \beta' \beta_x \frac{F_x}{N} \equiv s - \frac{1}{\sqrt{N}} \Lambda F_x,
\]
with \( \Lambda = \frac{1}{N} \mu^{-1} \Sigma_{\nu,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma^{-1} \beta \right)^{-1} \frac{\beta' \beta}{N} \). The idiosyncratic component of the random supply disappears; it is diversified away. The covariance matrix of the payoffs, conditional on \( \theta \), is

\[
\Sigma_\theta = \Sigma_s + \frac{1}{N} \Lambda \beta_x \Sigma_{F,F} \beta' \Lambda'.
\]

Note that \( \Sigma_s \) is a diagonal matrix while \( \Lambda \beta_x \Sigma_{F,F} \beta' \Lambda' \) is a matrix with all of its matrix elements being of order 1. Therefore, when \( \Sigma_\theta \) is multiplied by a vector of 1’s from the right, the second term has the same order of magnitude as the first term. We can show that

\[
\Sigma_{\nu|\theta,F} = \Sigma_{\nu|s,F} + O \left( N^{-1} \right).
\]

As will be shown later, the contribution of such terms to the risk premium goes to zero in the limit as \( N \to \infty \). The factor covariance matrix, conditional on \( \theta \), is

\[
\Sigma_{F|\theta}^{-1} = \Sigma_{F|s}^{-1} + \frac{1}{N} k' \left( \Sigma + \Sigma_s + \frac{1}{N} \Lambda \beta_x \Sigma_{F,F} \beta' \Lambda' \right)^{-1} k.
\]

Note that, when multiplied by vectors of 1’s from left and from right, the term \( \frac{1}{N} \Lambda \beta_x \Sigma_{F,F} \beta' \Lambda' \) produces a \( K \times K \) matrix with elements of order \( N \), the same as matrix \( \Sigma + \Sigma_s \).

The variance of \( \nu \), conditional on \( \theta \),

\[
\Sigma_{\nu|\theta} = \Sigma_{\nu|s,F} + \beta \Sigma_{F|\theta} \beta'.
\]

The matrix \( \Sigma_{\nu|s,F} \) is diagonal, while all the elements of the matrix \( \beta \Sigma_{F|\theta} \beta' \) are of order 1. The terms neglected earlier produce matrices with all elements of order \( N^{-1} \).

From the identity,

\[
\mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1}
= \Sigma_{\nu|s,F}^{-1} - \Sigma_{\nu|s,F}^{-1} \beta' \left( \mu \left( \Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|\theta,F}^{-1} \beta \right)^{-1} \right) \beta' \Sigma_{\nu|s,F}^{-1}
\]

41
we can write

\[
\left( \mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta} \right)^{-1} = \Sigma_{\nu|s,F} + \beta M^{-1}\beta',
\]

where

\[
M = \left( \mu \left( \Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \right)^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \right)^{-1} \right)^{-1} - \beta' \Sigma_{\nu|s,F} \beta
\]

\[
= \left( \mu \left( \Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \right)^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \right)^{-1} \right)^{-1}
\times \left( \mu \left( \Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \right)^{-1} \Sigma_{F|s}^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \right)^{-1} \Sigma_{F|\theta}^{-1} \right).
\]

In the large \( N \) limit, \( \beta' \Sigma_{\nu|s,F}^{-1} \beta \) is of order \( N \), therefore, \( \Sigma_{F|s}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \to \beta' \Sigma_{\nu|s,F}^{-1} \beta \).

Similarly, \( \Sigma_{F|\theta}^{-1} + \beta' \Sigma_{\nu|s,F}^{-1} \beta \to \beta' \Sigma_{\nu|s,F}^{-1} \beta \), so

\[
M \to \beta' \Sigma_{\nu|s,F}^{-1} \beta \left( \mu \left( \beta' \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} \Sigma_{F|s}^{-1} + (1 - \mu) \left( \beta' \Sigma_{\nu|\theta,F}^{-1} \beta \right)^{-1} \Sigma_{F|\theta}^{-1} \right)^{-1} \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1}.
\]

The risk premium is given by

\[
A\beta \left( \mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta' \bar{x}}{N}
\]

and the factor risk premium is given by

\[
A \left( \mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta' \bar{x}}{N}.
\]