The Effects of Audit Quality Disclosure on Audit Effort and Investment Efficiency

Abstract

We examine the effects of disclosing information about audit quality on auditors’ effort provision and investors’ investment efficiency. In our setting, the usefulness of an audited financial report for investors depends both on the quality of the underlying financial reporting (e.g., as embodied by GAAP) and the quality of auditors’ reports (i.e., the likelihood audit evidence uncovers managerial misreporting). An auditor exerts an unobservable effort to influence audit quality and is motivated by liability in the event of an audit failure. We show that the effects of audit quality disclosure depend on the underlying quality of financial reporting and the argument for audit quality disclosure to motivate auditors’ effort can be supported when the underlying financial reporting quality is relatively weak. Our analyses contribute to the debate about the costs and benefits of policies aimed at improving audit transparency and offer a potential explanation for cross-country differences in auditor disclosure requirements.
1 Introduction

There is an on-going debate related to the audit quality indicator (“AQI”) project currently evaluated by the Public Company Accounting Oversight Board (PCAOB).¹ The objective of the AQI project is to “determine the feasibility of developing key indicators of audit quality and effectiveness and requiring auditing firms to publicly disclose these indicators.” (PCAOB Release No. 2015-005). Supporters of the project argue that such disclosure can assist investors’ investment decisions and improve audit quality. Others question whether the quality indicators can accurately represent audit quality, and express concerns about the unintended consequences of disclosing them, including the risks of misunderstanding and misuse by investors, as well as increased audit costs and distraction of auditors from more productive tasks which can lower audit quality and increase auditor liability.²

The discussion about the AQI project is part of a general debate about the costs and benefits of regulations aimed at improving audit transparency. This debate concerns a broad set of disclosure requirements and standards for auditors either already in place or currently under consideration. While many policies are not directly related to disclosure of audit quality as the AQI project, they are often motivated by, or have implications for, providing more information to help investors better assess audit quality. One example is the recent PCAOB requirement that audit firms disclose information about audit partners.³ One of the stated objectives of the requirement is to “provide insights into audit quality” (PCAOB Release No. 2015-008). Other examples include the proposed auditing standard by PCAOB that would require auditors to provide more disclosure in conjunction with unqualified opinions to help investors better assess the reliability of these opinions.⁴ To the extent the reliability

¹The U.S. AQI project was initiated in response to the recommendation made by the Advisory Committee on the Auditing Profession to the U.S. Department of Treasury. Similar initiatives are being evaluated by the regulatory bodies in Canada and Switzerland as well as by the International Organization of Securities Commissions (IOSCO). Audit firms in UK and Singapore are provided guidance to disclose such information on a voluntary basis, whereas audit firms in Australia, Netherlands, and New Zealand are required to disclose various audit quality indicators. For details, see https://www.accountancyeurope.eu/wp-content/uploads/1607_Update_of_Overview_of_AQIs.pdf
³Similar requirements have been adopted in many other jurisdictions, including China, EU, Japan, Russian Federation, South Africa, and Mexico.
⁴The proposal is outlined in PCAOB Release No. 2013-005, August 13, 2013. A modified version was
of these opinions reflects audit quality, a better understanding of the consequences about audit quality disclosure also has implications for evaluating the proposed standard.

The paper contributes to the debate by providing a theoretical analysis on how disclosure of audit quality affects investors’ investment efficiency and auditors’ effort, an important input to audit quality. The setting we examine incorporates several key features that reflect the various aspects of the debate. First, we focus on the auditor’s attestation role in that the auditor verifies whether the manager truthfully implements the existing financial reporting rules (which produce noisy indicators of the fundamentals). To do so, we model the usefulness of the audited financial report as jointly determined by the quality of the underlying financial reporting system that maps the firm’s fundamentals into an unobservable true accounting signal, (mis)reporting of the true signal by the firm’s manager, and the audit quality which refers to the likelihood that the auditor’s evidence uncovers the true accounting signal and reveals managerial misreporting. Second, we assume the auditor’s effort increases the audit quality stochastically. This assumption reflects the concern that audit quality indicators are unlikely to fully reveal the auditor’s effort. It also implies that information about the audit quality is useful for the investor’s investment decisions, which is a key benefit argued by proponents for audit quality disclosure. Third, to reflect the role the investor plays in monitoring and disciplining the auditor, we focus on the role of audit liability in motivating the auditor’s effort and examine the strategic interactions between the auditor and investor. To do so, we allow the investor’s decision to be based on both public information (i.e., audit opinion) and his private information. In addition to providing a realistic description of the investor’s decision process, allowing the investor to rely on private information for decision reproposed for public comment on May 11, 2016 (PCAOB Release No. 2016-003). The case of qualified opinions is less controversial as under the existing practice auditors need to provide detailed discussions on issues involved in those opinions. The assumption that expanded disclosure about critical audit matters can provide investors more information about audit quality can be illustrated by the analogy of assessing the quality of wine. Wine evaluators can assess the quality of a wine by tasting the final product (similar to investors relying on the unqualified audit opinion). Presumably, the evaluators can also gain additional insights about the wine quality if there is more disclosure about how the vineyard produces its wines: the criteria for picking the grapes, the type of containers used in fermentation, etc. Such information is akin to the disclosure of critical audit matters in that it is not directly about wine (or audit) quality, but knowledge of it can improve users’ assessment of the quality.

5The assumption that investors’ investment decision depends on both private and public information is common among theoretical models of financial markets (e.g., Grossman and Stiglitz (1980), Verrecchia
making is enables our model to reflect both the auditor’s uncertainty about the likelihood of liability due to the investor’s behaviors and the concern that audit quality disclosure can incur additional liability.

We compare two regimes: a Disclosure Regime where the investor observes the realized audit quality for both qualified and unqualified opinions; and a No Disclosure Regime where the investor can observe the audit quality only for qualified opinions.\textsuperscript{6} In both regimes, the auditor’s effort is motivated by the potential damage compensation she may pay to the investor in the case of an audit failure. An audit failure occurs when the auditor does not catch managerial misreporting when the state is bad (auditor vulnerability) and the investor invests when the state is bad (investor reliance).\textsuperscript{7} Everything else equal, the auditor prefers lower vulnerability and lower investor reliance. As shown in prior literature (Schwartz (1997)), the threat of liability due to audit failure is a double-edged sword: it provides the auditor more incentives to exert costly, unobservable effort at the expense of inefficient over-investment by the investor. The balance between these two forces is affected by whether the investor observes audit quality. In the No Disclosure regime, the auditor’s effort reduces auditor vulnerability but does not directly affect investor reliance. In the Disclosure regime, the auditor’s effort not only reduces auditor vulnerability but also affects investor reliance. The effect on investor reliance determines whether audit quality disclosure improves or reduces the auditor’s effort in equilibrium.

We obtain two main results. The first is that disclosing audit quality improves the auditor’s effort only when the financial reporting quality is relatively low. The intuition is the following. Under the audit liability rule, the investor relies on the auditor’s report for both its informativeness value and its insurance value. The informativeness value arises \textsuperscript{(1982)). A large proportion of return movements is attributed to investors trading on private information (e.g., Roll (1984, 1988)). Recent research on the feedback effect further suggests that investors’ private information can in turn affect firms’ behaviors (e.g., Chen, et al. (2007), Gao and Liang (2013), see Bond, et al. (2012) for a survey).

\textsuperscript{6}We allow investors to assess audit quality for qualified opinion to reflect the fact that auditors tend to provide more information when they issue qualified opinion. We assume that the additional information can help investors assess audit quality. Our results are robust to the alternative setting where investors cannot assess audit quality for all opinions in the No Disclosure regime.

\textsuperscript{7}We take the likelihood that the auditor will be found liable (negligent) in audit failures as exogenously determined by the legal system that can vary by jurisdictions and be altered by laws and regulations, an example of which is the Private Securities Litigation Reform Act of 1995 in the U.S.
because the investment project is risky and the investor would benefit from the information in the audited financial report about the firm’s fundamentals. The insurance value arises because the investor expects damage compensation in the case of an audit failure; therefore everything else equal, the investor would have more incentives to rely on the auditor’s report when he perceives a high likelihood of auditor vulnerability. While the informativeness value implies that the investor relies on the audit opinion more when the audit quality is high, the insurance value implies the opposite.

In the Disclosure Regime, the investor can fine-tune his use of the audit opinion according to the disclosed audit quality. When the underlying reporting quality is low, the investor primarily uses the audit opinion for its insurance value, more so when the realized audit quality is low. Since the auditor’s effort increases audit quality in expectation, the insurance-motivated reliance enhances the auditor’s incentives to exert effort. On the other hand, when the underlying reporting quality is high, the investor relies on the audit opinion primarily for its informativeness value, and therefore is less likely to invest when the realized audit quality is low. Consequently, the informativeness-motivated investor reliance reduces the auditor’s incentives to exert effort. In contrast, in the No Disclosure regime the investor cannot fine-tune his decisions based on the realized audit quality, which results in higher equilibrium auditor’s effort than the Disclosure Regime if and only if the underlying reporting quality is high.

Our second main result is related to the effect of disclosing audit quality on investment efficiency. We measure investment efficiency from the social optimal perspective, defined as the (inverse) of the expected loss from type I (forsaking a good project) and type II (undertaking a bad project) errors. We show that disclosing audit quality affects investment efficiency via three main channels. The first channel works to improve investment efficiency (from a social perspective) by allowing the investor to fine-tune his use of the audit opinion for its informativeness value. The second channel reduces investment efficiency because it also allows the investor to fine-tune his use of the audit opinion for its insurance value. The third channel affects the auditor’s effort which in turn moderates the impact of first two channels. Specifically, the benefit of the first channel is reduced by the fact that when the investor uses the audit opinion for its informativeness value, disclosing audit quality reduces the auditor’s effort. Likewise, the cost of the second channel is reduced because when this channel is in force, disclosing audit quality increases the auditor’s effort. In sum, the net
effect of disclosing audit quality on investment efficiency is a complex trade-off between these forces. Numerical examples suggest that on the net, investment efficiency is lower under the Disclosure Regime than under the No Disclosure Regime when the underlying reporting quality is relatively high.

To shed light on how the auditing regime affects the desirability of financial reporting quality, we take the auditing regime as given, and analyze the optimal financial reporting quality choice both from the perspective of a benevolent standard setter whose objective is to maximize the investment efficiency, and from the perspective of a self-interested manager whose objective is to maximize the probability of investment. We find that in both audit disclosure regimes, high financial reporting quality increases investment efficiency only when the quality of auditing technology/standards is high and when the legal regime allows investors to pose meaningful threat of audit liabilities. Numerical examples also find that when the frictions in the auditing environment are taken into account, self-interested managers would actually prefer to implement high quality reporting system, contrary to the conventional wisdom.

Our paper belongs to the broad literature on understanding how audit rules and regulations affect market participants’ behaviors (e.g., Dye (1993), Narayanan (1994), Hillegeist (1999)), and to the specific literature on evaluating their effects on audit quality and investment efficiency (e.g., Schwartz (1997), Pae and Yoo (2001), Deng, Melumad, and Shibano (2012)). While most prior studies focus on the effects of audit liability rules, we contribute to the literature by examining the effects of disclosing audit quality.

Our analyses shed light on the complex trade-offs involved in the debate about audit quality disclosure in specific and audit transparency in general. In addition, our setting also helps us gain some preliminary understanding of the interaction between financial reporting quality and the usefulness of audit opinions, an important issue that has not been analyzed in prior literature. In terms of empirical implications, we find that more audit quality related disclosure is more likely to motivate audit effort when the underlying reporting quality is relatively low. To the extent that the financial reporting quality is arguably lower in developing and emerging markets, this prediction is consistent with the casual observation

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8See also Newman, et al. (2005) and Deng, Melumad, and Shibano (2012) for reviews of related literature.

9In a related study, Dye and Sridhar (2007) analyze how disclosing information about signal quality affects the risk-sharing between different generations of owners in an overlapping generation model. There is no auditor in their setting.
that many of the rules and policies aiming at improving audit transparency were also first adopted in these markets. Our analyses suggest that audit transparency can reduce the auditor’s effort, consistent with experimental findings by Kachelmeier et al. (2014).

The rest of the paper proceeds as follows. Section 2 sets up the basic model. Section 3 analyzes the investor’s investment decisions conditional on his information set. Section 4 solves for the equilibrium auditor effort and compares how the auditor’s effort, investment efficiency, and audit fee differ between the two disclosure regimes. Section 5 examines the ex ante choice of financial reporting quality from both the standard setter’s perspective and from the manager’s perspective, and Section 6 concludes.

2 The Model

2.1 Objectives and information structure

Consider a firm managed by a manager who has access to an investment project that requires an up-front investment $K > 0$. The manager/firm does not have sufficient fund internally and must obtain $K$ from an external investor.\(^\text{10}\) If funded, the project generates a random terminal cash flow of $rK > K$ when the state of nature is good (denoted by $G$) and 0 when the state is bad (denoted by $B$). The state of nature is not known until the project concludes. The common prior for a good (bad) state is $\Pr(G) = 1 - \Pr(B) = \mu \in (0, 1)$. Without loss of generality, we assume the ex ante net present value of the project is zero, implying that $r = \frac{1}{\mu}$.

The firm is endowed with an accounting reporting system which produces a noisy signal $R \in \{R_G, R_B\}$ that can partially inform the state of nature with probability $q$ as:

$$\Pr(R_G|G) = \Pr(R_B|B) = q \in \left(\frac{1}{2}, 1\right).$$

The higher $q$ is, the more accurately the accounting signal captures the underlying state. $q$ is public information, and represents the quality of the financial reporting system determined at the beginning of the game.\(^\text{11}\)

\(^\text{10}\)The investor can be a large institutional investor, a major creditor/bank, or a group of investors with similar preferences such as hedge funds and private equity investors.

\(^\text{11}\)In later section, we analyze the optimal choices of $q$ from the standard setter’s perspective and from the manager’s perspective.
There are three risk-neutral players in the model: the investor, the manager, and the auditor. The investor decides whether to invest $K$ in the project. If the project is taken, its entire payoff directly accrues to the investor. We normalize the investor’s payoff to zero when the project is rejected. The manager is protected by limited liability, and his only payoff is a private non-pecuniary benefit $\lambda > 0$ that he extracts when the project is taken. This implies that the manager’s incentive is to maximize the probability with which the project gets funded by the investor. We assume that the manager privately observes the realized accounting signal $R \in \{R_G, R_B\}$, after which he proposes a report $\hat{R} \in \{\hat{R}_G, \hat{R}_B\}$ to the auditor and the investor. The report $\hat{R}_G$ and $\hat{R}_B$ claim that the privately observed accounting signal is $R_G$ and $R_B$, respectively. Given the manager’s incentive and the assumption that he is protected by limited liability, he strictly prefers a favorable report $\hat{R}_G$ to an unfavorable one $\hat{R}_B$, regardless of the true accounting signal.\[\text{(12)}\] While clearly a simplification, this assumption allows a role for the auditor. If it was public knowledge that managers’ reports are always truthful, auditors would not be needed in the first place.

Following the practice, we assume that the auditor gets a non-contingent fee $F$ from the firm at the beginning of their relationship. Once set, $F$ does not affect any players’ subsequent strategic behavior. We assume a competitive audit market such that the audit fee is set to equal the auditor’s cost of effort and expected liability in the event of an audit failure (to be defined more precisely next). The auditor earns her fee by providing assurance services to the manager’s financial disclosure. Specifically, after observing the manager’s report $\hat{R}$, the auditor spends resources and exerts effort to collect audit evidence $\Omega \in \{\Omega_g, \Omega_b\}$. Following prior literature (e.g., Shibano (1990), Deng, et al. (2012)), we assume that the auditing technology is imperfect such that the audit evidence reveal the underlying accounting signal with probability $\gamma \in (1/2, 1)$:

$$\Pr(\Omega_g | R_G) = \Pr(\Omega_b | R_B) = \gamma.$$ 

$\gamma$ indicates the audit quality in our model: the higher $\gamma$ is, the more likely the audit evidence

\[\text{(12)}\] Our results are qualitatively unchanged if we allow stochastic misreporting by the manager. Stochastic misreporting can be introduced in two ways. First, we can allow the manager to choose $t \in [0, \bar{t}]$ such that $\Pr(\hat{R}_G | R_B) = t$, where $\bar{t} < 1$ is an exogenous upper bound on the manager’s misreporting. It is easy to see that in this case the manager will optimally choose $\bar{t}$. Second, we can alternatively do away with the exogenous bound $\bar{t}$ and instead introduce an increasing convex cost to manager’s misreporting. Our results are robust to both modeling alternatives.
reveals the underlying accounting signal, and the more likely the audit evidence can reveal the manager’s misreporting.

To allow a role for the auditor’s effort, we assume that $\gamma$ is higher when the auditor exerts more effort. Specifically, we assume that the auditing technology is incomplete in that there are only two levels of audit quality available $\gamma \in \{\gamma_h, \gamma_l\}$ with $1 \geq \gamma_h > \gamma_l \geq \frac{1}{2}$ and that the auditor’s effort stochastically increases the chance of a high quality audit with $\Pr(\gamma = \gamma_h) = e$ and $\Pr(\gamma = \gamma_l) = 1 - e$. The auditor privately observes $e$. She also bears the cost of effort, $C(e)$, with $C'' \geq 0$, $C''(0) = 0$ and $C''(1)$ sufficiently large. For expositional ease, we impose a specific functional form $C(e) = \frac{1}{3}ce^3$ with $c$ sufficiently large and note that our results are robust with any convex cost function.

The incomplete audit technology and the stochastic effect of effort on quality can be justified on the ground that the available auditing technologies are not flexible enough to be adapted to achieve any desirable level of $\gamma$. They help capture the idea that even without information about the auditor effort, investors can still reasonably assess the minimum audit quality as long as the audit opinion states that the audit is conducted in accordance with Generally Accepted Auditing Standards (GAAS) which require that auditors plan and perform the audit to obtain reasonable assurance about the integrity of the financial statements. Accordingly, after examining these standards, investors have some information about the precision of the auditor’s signal. Modeling-wise, these assumptions serve two important roles for our analyses. The first is to reflect the concern that the realized audit quality (e.g., the audit quality index) is a noisy indicator for the auditor’s underlying effort. Second, they enable us to assess the validity of two arguments for disclosing audit quality, that is, it helps monitor and discipline auditors and assists investors’ investment decisions.\footnote{We thank Phil Stocken (the Editor) for this observation. The magnitudes of $\gamma_h$ and $\gamma_l$ are determined outside the model, and can vary cross-sectionally either because the quality of GAAS varies across jurisdictions, or because some industries are more difficult to audit due to their business models.}

\footnote{The stochastic relation between audit quality and the auditor’s effort imply that neither is a sufficient statistic for each other. This is important as otherwise, the disclosed audit quality would not be useful in the investor’s equilibrium decision. To see this, consider the alternative setup where the audit technology is complete and the auditor can adjust her effort $e$ and choose any desirable audit level $\gamma(e) \in [0, 1]$ with $\gamma'(e) > 0$. In contrast, the audit quality index (AQI) cannot perfectly reveal the true quality, in that it is only feasible to construct an index with two values, say, pass ($\gamma_h$) or fail ($\gamma_l$). The likelihood of obtaining a passing grade increases in the true underlying quality, i.e., $\Pr(\text{Pass}|\gamma) = \gamma(e)$ and $\Pr(\text{Fail}|\gamma) = 1 - \gamma(e)$. It can be shown that in this case, the equilibrium outcome is completely determined by investor’s conjecture.
We assume that the auditor may be subject to liability payment in the event of an audit failure. An audit failure occurs when the investor chooses to invest and the state turns out to be bad (B), and at the same time, the accounting signal correctly captures the state (i.e., \( R = R_B \)) but the auditor fails to detect managerial misreporting by issuing an unqualified opinion. We assume that the audit failure is a necessary condition for auditor liability, thus precluding the possibility of the auditor being liable when the investor decides not to invest. This corresponds to the case where a necessary condition for a third party (the investor in our model) to sue the auditors is to establish the existence of an actual loss (Gormley (1984)), and comports with the practice that it is typically quite difficult (if not impossible) to sue auditors for loss on investments that would have been undertaken.

To reflect the fact that an audit failure does not necessarily imply auditor negligence, which is a precondition for auditors to incur liabilities in most courts, we assume that the likelihood that the auditor is found liable in an audit failure is \( \alpha \in (0, 1) \). Thus, the auditor is expected to pay \( \alpha K \) to the investor, conditional on audit failure.\(^{15}\) Conditional on audit failure, whether the auditor is found liable depends on many factors that are outside the auditors’ control such as as jurors’ behavioral biases as shown in Kadous (2000). In this sense, we view \( \alpha \) as exogenously determined by the legal environments, which determine the likelihood that investors can successfully sue the auditor in the event of an audit failure (i.e., establish auditor negligence) as well as the severity of the auditor’s liability. An example of the legal regulation is the Private Securities Litigation Reform Act of 1995 in the U.S., which eliminated the joint and several liability under which auditors and other parties could be named to lawsuits because of “deep pockets” rather than culpability. In our model, this would represent a lower \( \alpha \).\(^{16}\)

After observing the audit evidence \( \Omega \), the auditor issues either an unqualified or a qualified opinion. Given the assumption that the audit fee is paid up front, and the auditor’s effort is about the auditor’s effort level; and disclosing Pass/Fail does not necessarily affect the investor’s investment decision. Thus, the alternative setup does not fully represent all considerations/concerns involved in the debate.

\(^{15}\)Audit failure may also induce reputation loss and loss of future business. This can be accommodated by assuming an additional lost for the auditors of \( \beta K > 0 \) that does not accrual to investors. All qualitative results carry through with this alternative setup as long as \( \alpha > 0 \). The key assumption is that investors play a role in disciplining auditors.

\(^{16}\)We thank the Editor, an anonymous referee, and Clive Lennox for helping us clarify these points.
sunk at this stage, the auditor is only concerned about her expected liability. Therefore, her incentive is always to issue qualified (unfavorable) opinion regardless of her evidence. This will lead to a trivial equilibrium where the auditor exerts no effort, always issues qualified opinions, and the investor never relies on the auditor’s opinions. Anticipating this, the firm would not pay for the auditor’s service to begin with. To circumvent this trivial case, we assume that the auditor cannot issue a qualified opinion unless her evidence supports it (i.e., she observes $\Omega_b$). This is consistent with the practice that a qualified opinion usually is accompanied with detailed discussions and hence is likely to be based on evidence collected. Together, these assumptions imply that the audit opinion will always truthfully reflect the audit evidence. As such, we refer to the audit evidence ($\Omega$) and the audit opinion interchangeably throughout the paper for notational ease.

The investor observes both the manager’s report and the audit opinion. In addition, the investor also observes a noisy signal of his own $S \in \{S_g, S_b\}$ that is informative of the underlying state with probability $p \in [\frac{1}{2}, 1]$ as

$$\Pr(S_g | G) = \Pr(S_b | B) = p.$$ 

Here $p$ reflects the quality of the investor’s signal, and is randomly drawn from a uniform distribution on $[\frac{1}{2}, 1]$. We assume both $p$ and $S$ are privately observed by the investor after the auditor issues her opinion. Except in the zero probability cases of $p = 0.5$ or $p = 1$, neither $S$ or $\Omega$ is a sufficient statistic for each other with respect to the underlying state of nature, although it is possible that the investor’s private information can be more precise than the audited financial statement. The possibility can be justified on the ground that the investor is a sophisticated financier and is able to obtain information due to his past experiences dealing with similar projects (see also footnote 5 in the Introduction). The assumption that $p$ is the investor’s private information implies that the auditor is uncertain about the extent to which the investor will rely on the audited reports in his decision. This uncertainty is necessary to reflect the concern that additional disclosure about audit quality may be “misused” by the investor and increase investor lawsuits.

17 Alternatively, Lu and Sapra (2009) assume an exogenous cost from issuing qualified opinions. The nature of audit evidence in their model differs from ours. In their model, the auditor either knows for sure whether the manager lied, or is left uncertain. In the latter case, the auditor decides whether to issue a qualified or an unqualified opinion. In our model, the audit evidence does not recover the true accounting report perfectly so the auditor cannot establish for sure whether the manager’s report is inaccurate.
Figure 1a illustrates the information structure modeled in the paper. The two right panels show how audit evidence $\Omega_j$ relates to the underlying state of nature $J \in \{G, B\}$, while the left panel corresponds to the investor’s signal $S$. Figure 1b summarizes the time line of the model.

[Insert Figure 1]

2.2 Disclosure regime and equilibrium definition

We examine both a No Disclosure regime ($ND$) and a Disclosure regime ($D$) that differ in terms of the investor’s information about the audit quality. Specifically, let $\Phi_i = \{S, p, q, \Omega, \tilde{\gamma}_i(\Omega)\}$ be the investor’s information set in regime $i \in \{D, ND\}$. In the Disclosure regime, the disclosure requirements enable the investor to observe the realized audit quality for both qualified and unqualified opinions, i.e., $\tilde{\gamma}_D(\Omega_y) = \tilde{\gamma}_D(\Omega_b) \in \{\gamma_h, \gamma_l\}$. In contrast, in the No Disclosure regime, the investor can observe $\gamma$ only when the auditor issues qualified opinions, i.e., $\tilde{\gamma}_{ND}(\Omega_b) \in \{\gamma_h, \gamma_l\}$. When the auditor issues an unqualified opinion, we assume the investor relies on his conjecture about the auditor effort $\hat{c}_{ND}$ to form a conjectured audit quality, i.e., $\tilde{\gamma}_{ND}(\Omega_y) = \tilde{\gamma}_{ND}$ according to

$$\tilde{\gamma}_{ND} = \hat{c}_{ND}\gamma_h + (1 - \hat{c}_{ND})\gamma_l. \quad (1)$$

We next define the equilibrium concept in each regime.

**Definition 1a** An equilibrium in the Disclosure regime consists of the auditor’s effort choice $c^*_D$ and the investor’s decision rule $I^*(\Phi_D)$ where $\Phi_D = \{S, p, q, \Omega, \tilde{\gamma}_D\}$ with $\tilde{\gamma}_D(\Omega_y) = \tilde{\gamma}_D(\Omega_b) \in \{\gamma_h, \gamma_l\}$ such that (i) $e^*_D = \arg \min_{e \in (0,1)} \Pr (\text{Audit Failure}|e) \alpha K + C(e)$, given $I^*(\Phi_D)$ and (ii) $I^*(\Phi_D) = \arg \max_{I \in \{0,1\}} [r \Pr (G|\Phi_D) + \alpha \Pr (\text{Audit Failure}|\Phi_D) - 1] K$, given $\Phi_D$.

**Definition 1b** An equilibrium in the No Disclosure consists of the auditor’s effort choice $c^*_{ND}$, the investor’s conjecture of $\hat{c}_{ND}$ and $\tilde{\gamma}_{ND}$ according to (1), and the investor’s decision rule $I^*(\Phi_{ND})$ where $\Phi_{ND} = \{S, p, q, \Omega, \tilde{\gamma}_{ND}(\Omega)\}$, with $\tilde{\gamma}_{ND}(\Omega_b) \in \{\gamma_h, \gamma_l\}$ and

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18 Allowing the investor to observe audit quality for a qualified opinion under the No Disclosure regime reflects the fact that the auditor usually discloses more information about her decision when issuing a qualified opinion than when issuing an unqualified opinion. Our results are unaffected if we instead assume the investor does not observe audit quality in all cases (including with qualified opinions) in the No Disclosure regime. (Detailed analyses for this alternative setup is available upon request.)
We evaluate the two disclosure regimes using two measures: investment efficiency \((IE)\) and total surplus \((TS)\), as defined below.

**Definition 2** Let \(I = 1\) (or 0) denote the investor’s decision to invest in (or reject) the project and \(e\) denote the auditor’s effort in equilibrium. The associated investment efficiency \((IE)\) and total surplus \((TS)\) are defined as:

\[
IE = -\mu \Pr (I = 0 | G) (r - 1) K - (1 - \mu) \Pr (I = 1 | B) K.
\]

and \(TS = IE - C(e)\).

The investment efficiency is a weighted average of the expected loss from Type I errors (i.e., forgoing profitable projects) and Type II errors (i.e., investing in unprofitable projects). The dollar amount of loss conditional on the Type I and Type II error is \((r - 1) K\) and \(K\), respectively. The total surplus takes into account both the investment efficiency and the auditor’s cost of effort. In order to highlight the manager’s suboptimal reporting incentives and allow the auditor a meaningful role to improve welfare, we assume that the manager’s private benefit \(\lambda\) is negligibly small compared to either the investment efficiency or the auditor’s cost of effort, because otherwise social welfare is maximized if the project is always taken, rendering the auditor’s assurance role useless from the social welfare perspective.

### 3 Optimal investment decision rule

Because the investment decision is the last strategic play in the model, we solve the model by first deriving the investor’s optimal investment decision rule given his information set \(\Phi_i = \{S, p, q, \Omega, \tilde{\gamma}_i(\Omega)\}\). The rule is conditional on the investor’s information about audit quality, thus applying to both disclosure regimes. To facilitate later discussions, we start
with a benchmark case without any damage compensation \((\alpha = 0)\). It is easy to see that the benchmark case represents the first-best investment rule (conditional on the investor’s information) that maximizes the investment efficiency \((IE)\).

**Lemma 1** Let \(k \in \{a, b, c, d\}\) denote one of the four possible combinations of the investor’s private signal and audit evidence. Let \(\tilde{\gamma}_k\) denote the investor’s information about audit quality in scenario \(k\). The investor’s optimal investment decision is given by

\[
\begin{array}{c|c|c}
Scenario \ k & \{\Omega, S\}_k & Investment \ Decision \ I^* \\
(a) & (\Omega_b, S_b) & I^* = 0 \text{ for all } p \text{ and } \tilde{\gamma}_k \\
(b) & (\Omega_g, S_g) & I^* = 1 \text{ for all } p \text{ and } \tilde{\gamma}_k \\
(c) & (\Omega_b, S_g) & I^* = 1 \text{ if } p \geq P^*(\tilde{\gamma}_k, q); \\
(d) & (\Omega_g, S_b) & I^* = 1 \text{ if } p < P^*(\tilde{\gamma}_k, q)
\end{array}
\]  

(2)

where

\[
P^*(\tilde{\gamma}, q) = \tilde{\gamma} q + (1 - \tilde{\gamma}) (1 - q). \tag{3}
\]

Lemma 1 is proved by noting that when \(\alpha = 0\), the investor will invest if and only if \(\Pr(G|R) rK > K\) where \(\Phi\) denotes his information set. Since the project has zero expected NPV under the common prior (i.e., \(\Pr(G) r = 1\)), the investor will invest only if he has a more favorable posterior belief about the underlying state than the prior (i.e., \(\Pr(G|\Phi) > \Pr(G)\)). The posterior under scenario (a) is clearly lower than the prior since both the private signal \(S_b\) and the audit opinion \(\Omega_b\) indicate that the state is bad, therefore it is optimal not to invest. Similarly, it is optimal to invest in scenario (b) as both signals indicate the state is good. In both scenarios, the investor’s information about the audit quality \(\tilde{\gamma}\) is irrelevant.

In scenarios (c) and (d), the two signals convey conflicting messages about the state. Bayes’s rule shows that the more precise of the two signals will guide the posterior, and hence the investment decision. To see that, note the posterior under scenario (c) is given by:

\[
\Pr(G|S_g, \Omega_b, \tilde{\gamma}) = \Pr(G) \frac{p (1 - P^*(\tilde{\gamma}))}{\mu p (1 - P^*(\tilde{\gamma})) + (1 - \mu) (1 - p) P^*(\tilde{\gamma})},
\]

where \(p = \Pr(S_j|J)\) measures the informativeness of the investor’s private signal with respect to the underlying state \(J \in \{G, B\}\), and \(P^*(\tilde{\gamma}) = \Pr(\Omega_j|J)\) measures the informativeness of the audit evidence \(\Omega_j\) with respect to the state \(J\) as

\[
\Pr(\Omega_j|J) = \Pr(\Omega_j|R_j) \Pr(R_j|J) + \Pr(\Omega_j|R_{-j}) \Pr(R_{-j}|J) = \tilde{\gamma} q + (1 - \tilde{\gamma}) (1 - q) = P^*(\tilde{\gamma}, q).
\]

13
It is straightforward to show that \( \Pr (G|S_b, \Omega_g, \tilde{\gamma}) \geq \Pr (G) \) iff \( p \geq P^*(\tilde{\gamma}) \). Similarly, the posterior in scenario (d)

\[
\Pr (G|\text{scenario}(d)) = \frac{\mu (1 - p) P^*(\tilde{\gamma})}{\mu (1 - p) P^*(\tilde{\gamma}) + (1 - \mu) p (1 - P^*(\tilde{\gamma}))} > \Pr (G) \text{ iff } p < P^*(\tilde{\gamma}, q).
\]

Lemma 1 shows that the investor’s investment decision depends on both the auditor’s opinion and its quality. Because there is no audit liability, we refer to this dependence as the investor utilizing the informativeness value of the audit opinion. The informativeness use arises because the investor does not have perfect information about the underlying state and therefore can benefit from incorporating the information content in the audited financial reports about the underlying state.

When \( \alpha > 0 \), the investor can obtain damage compensation in case of an audit failure. This can distort his investment decision away from the benchmark case in scenarios (b) and (d) where an audit failure is possible, more so in scenario (d) when the investor’s private signal conflicts with the audit opinion. This intuition is confirmed in Proposition 1 below.

**Proposition 1** Define \( P^*(\tilde{\gamma}, q) \) the same as in (3). When \( \alpha > 0 \), the investor’s optimal investment decisions under scenarios (a)-(c) are the same as those given by (2). Under scenario (d), he will invest if and only if \( p < \overline{P}(\tilde{\gamma}, q, \alpha) \), where

\[
\overline{P}(\tilde{\gamma}, q, \alpha) \equiv P^*(\tilde{\gamma}, q)\rho(\tilde{\gamma}, \alpha, q) \quad \text{(4)}
\]

\[
\rho(\tilde{\gamma}, \alpha, q) \equiv \frac{1}{1 - \alpha q (1 - \tilde{\gamma})} > 1. \quad \text{(5)}
\]

To prove Proposition 1, note that \( \alpha > 0 \) does not affect the investor’s decision in scenario (b), where he would invest even if \( \alpha = 0 \). In scenario (d), he will invest if and only if

\[
r \Pr (G|S_b, \Omega_g, \tilde{\gamma}) + \alpha \Pr (B, R_B|S_b, \Omega_g, \tilde{\gamma}) - 1 > 0,
\]

where the second component, \( \alpha \Pr (B, R_B|S_b, \Omega_g, \tilde{\gamma}) \), reflects the investor’s expected damage compensation in case the investment fails. It depends on both on the legal regime \( (\alpha) \) and the investor’s belief that the auditor made a mistake in issuing an unqualified opinion conditional on \( S_b \):

\[
\Pr (B, R_B|S_b, \Omega_g, \tilde{\gamma}) = \frac{\Pr (\Omega_g|R_B, B) \Pr (S_b|B)}{\Pr (S_b, \Omega_g)} = \frac{(1 - \tilde{\gamma}) qp}{\Pr (S_b, \Omega_g)}.
\]

Note that \( \rho(\tilde{\gamma}, \alpha, q) \propto \alpha \Pr (B, R_B|S_b, \Omega_g, \tilde{\gamma}) \), which captures the extent of over-investment.
Proposition 1 shows that when $\alpha > 0$, the investor will not only use the audit opinion for its informativeness value, but also for its insurance value, as reflected by $\rho(\gamma, \alpha, q)$. The insurance value leads to overinvestment in scenario (d) relative to the benchmark case. The overinvestment occurs because the auditor’s liabilities provide the investor with a downside protection if the investment fails.

Since the investor’s informativeness use and insurance use of the audit opinion play an important role in later analyses, we summarize some simple comparative statics in Corollary 1 below.

**Corollary 1** A higher $q$ increases both the informativeness value and the insurance value of the audit opinion (i.e., $\frac{\partial P^*}{\partial q} > 0$ and $\frac{\partial \rho(\gamma, \alpha, q)}{\partial q} > 0$). A higher $\gamma$ increases the informativeness value but decreases the insurance value (i.e., $\frac{\partial P^*}{\partial \gamma} > 0$ and $\frac{\partial \rho(\gamma, \alpha, q)}{\partial \gamma} < 0$).

The proof for Corollary 1 is omitted as it follows directly from taking derivatives of (3) and (5). Not surprisingly, the informativeness of the audited report for predicting the state of nature ($P^*(q, \gamma)$) increases in both $q$ and $\gamma$. The effects of $\gamma$ and $q$ on the audit opinion’s insurance value is more subtle, and operate by affecting the investor’s beliefs about the likelihood that the auditor made a mistake in issuing an unqualified opinion. Straightforward algebra show that (6) is decreasing $\gamma$, i.e., the investor assesses a lower likelihood of auditor mistake when $\gamma$ is high. This is because a higher $\gamma$ means the audit evidence is more likely to reveal the true accounting signal and thus more likely to exonerate the auditor of audit liability. On the other hand, (6) increases in $q$. This is because a larger $q$ means $S$ and $\Omega$ are more correlated, therefore when the investor’s private signal differs from the auditor’s evidence, he is more confident that the auditor has made a mistake in issuing an unqualified opinion.

As noted earlier, the investment rule in Proposition 1 applies to both disclosure regimes except that $\overline{P}(\gamma)$ is evaluated at the actual observed $\gamma$ in the Disclosure regime; and is evaluated at the conjectured $\gamma$ (per (1)) in the No Disclosure regime. In both regimes, the investment threshold $P^*(\gamma)$ in scenario (c) is evaluated at the realized $\gamma$.

Proposition 1 shows the downside of the auditor’s liability in that it results in overinvestment by the investor, which reduces the investment efficiency from a social optimal perspective (Schwartz (1997)). However, as will be shown next, the benefit of the auditor’s liability rule is that it improves the auditor’s incentives to exert effort (above and beyond
alternative incentive mechanisms not explicitly considered here), which increases the informativeness value of the audited financial report and increases the equilibrium investment efficiency. Our focus is not to assess the desirability of the auditor’s liability regime itself; rather we are interested in how the costs and benefits of the liability change with audit disclosure regimes.

4 Comparing Equilibrium Outcomes Between Regimes

4.1 Audit effort

4.1.1 No Disclosure regime

In the No Disclosure regime, given the investor’s conjecture \( \hat{\gamma} \) in (1) and the investment rule in Proposition 1, the auditor chooses effort \( e \) to minimize

\[
[e \Pr (\text{audit failure} \mid \gamma_h, \hat{\gamma}) + (1 - e) \Pr (\text{audit failure} \mid \gamma_l, \hat{\gamma})] \alpha K + \frac{1}{3} ce^3, \quad (7)
\]

where the first term reflects the expected liability and the second term the cost of effort.

Setting the partial derivative of (7) with respect to \( e \) to zero yields the following expression for the auditor’s optimal effort choice:

\[
\alpha K \left[ l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu) \right] P^2(\hat{\gamma}) = ce^2. \quad (8)
\]

(8) shows that the auditor chooses the optimal effort to balance the marginal benefit of effort (the left-hand side (LHS) of (8)) with the marginal cost (the right-hand side (RHS) of (8)), for a given conjecture (\( \hat{e} \) and hence \( \hat{\gamma} \)).

To better understand the intuition, notice that conditional on the realized \( \gamma \) and \( \hat{\gamma} \), the auditor’s assessment of an audit failure depends on her assessment on the probability of the following two events: auditor vulnerability and investor reliance as shown in (9) below:

\[
\Pr (\text{audit failure} \mid \gamma, \hat{\gamma}) = l(q, \gamma, \mu) P^2(\hat{\gamma}) \quad (9)
\]

where \( l(q, \gamma, \mu) \equiv (1 - \mu) q (1 - \gamma). \quad (10)\)

Specifically, auditor vulnerability refers to the event the state is bad (\( \Pr (B) = 1 - \mu \)) and the underlying accounting signal is accurate (recall \( \Pr (R = R_B \mid B) = q \)), but the auditor issues an unqualified opinion (because she observes audit evidence \( \Omega_g \), which has probability
of \((1 - \gamma)\). Combining these factors, the probability of auditor vulnerability is given by 
\(l(q, \gamma, \mu)\). Easy to see that \(l(q, \gamma, \mu)\) decreases in \(\gamma\), i.e., 
\(l(q, \gamma, \mu) - l(q, \gamma_h, \mu) > 0\). This incentivizes the auditor to exert effort to achieve \(\gamma_h\) to reduce her vulnerability.

Conditional on \(B\) and \(\Omega_g\), the \(\bar{P}^2(\bar{\gamma})\) term on the LHS of (8) is the auditor’s expected likelihood for the event of investor reliance, which takes place when the auditor issues an unqualified opinion and the investor invests in the bad state. It can happen in two cases: in scenario (b) when the investor also receives an erroneous signal \(S_g\), the probability of which is 
\[\int_{1/2}^1 2(1-p)dp = \frac{1}{4}\]; and in scenario (d) when the investor ignores his correct signal \(S_b\) and instead relies on the auditor’s opinion, the probability of which is 
\[\int_{1/2}^{P^2(\bar{\gamma})} 2pdf = \bar{P}^2(\bar{\gamma}) - \frac{1}{4}\],
where \(\bar{P}\) is defined in equation (4) of Proposition 1. The sum of the two probabilities is 
\(\bar{P}^2(\bar{\gamma})\). Because \(\bar{P}^2(\bar{\gamma})\) is solely based on the investor’s conjecture, the auditor does not take into account the effect of her effort on investor reliance when choosing her optimal effort \((\partial P(\bar{\gamma})/\partial e = 0)\). As will be clear, this is no longer the case in the Disclosure regime when \(\bar{P}^2(\gamma)\) is based on the realized audit quality.

The equilibrium effort is derived by replacing the investor’s conjecture \(\hat{e}\) in (8) with the auditor’s actual effort. This ensures that the investor’s conjecture is rational in equilibrium. The auditor’s equilibrium effort \(e_{ND}^*\) is characterized by

\[
\alpha K[l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)]\bar{P}^2(e_{ND}^* \gamma_h + (1 - e_{ND}^*) \gamma l) = c e_{ND}^{2*}.
\tag{11}
\]

(11) may admit multiple solutions as both sides can be increasing in the auditor’s effort. Multiple equilibria can occur because the investor’s conjecture \(\hat{e}\) can be self-fulfilling. To sharpen our analysis, we focus on stable equilibria defined in Definition 1b. The notion of a stable equilibrium follows Stokey, Lucas, and Prescott (1989). Graphically, an equilibrium is stable if the marginal cost line crosses from below the marginal benefit line at the equilibrium. Proposition 2 and Corollary 2 characterize the stable equilibrium under the No Disclosure regime.

**Proposition 2** Under the No Disclosure regime, there exists at least one stable equilibrium where (i) the auditor’s effort \(e_{ND}^*\) is given by (11); and (ii) the investor follows the investment rule given in Proposition 1, i.e., always invests in scenario (a); never invests in scenario (b); invests iff his private signal precision \(p\) is larger than \(P^* (\gamma_j, q) = \gamma_j q + (1 - \gamma_j)(1 - q)\) upon observing \(\gamma_j \in \{\gamma_h, \gamma_l\}\) in scenario (c); and invests iff \(p < \frac{\gamma (e_{ND}) q + (1 - \gamma (e_{ND})) (1 - q)}{1 - \alpha q (1 - \gamma (e_{ND}) )}\) with \(\gamma (e_{ND}) = e_{ND}^* \gamma_h + (1 - e_{ND}^*) \gamma l\) in scenario (d).
Corollary 2 In the No Disclosure regime, \( \frac{de_N}{dq} > 0 \) for any stable equilibrium.

Corollary 2 shows that higher reporting quality \( q \) increases the equilibrium audit effort under the No Disclosure regime. The proof is straight-forward by noting that the marginal cost of effort \( c e_{N,D}^2 \) does not depend on \( q \), whereas both terms for the marginal benefit of effort, \( l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu) \) and \( \overline{P}^2(\gamma) \), unambiguously increase in \( q \). Intuitively, \( q \) increases the sensitivity of auditor vulnerability to the auditor’s effort (i.e., \( l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu) \) increases with \( q \)), therefore motivates more effort, per the familiar informativeness principle in agency theory (e.g., Holmstrom (1979), Kim (1995)). To see this, take the extreme case where \( q = 1/2 \). Then, even if the underlying state is bad, it is still likely that the underlying accounting system generates \( R_G \). This reduces the likelihood of an audit failure (recall a necessary condition for an audit failure is that the underlying accounting signal is \( R_B \) when the state is \( B \)). This in turn reduces the auditor’s effort incentive because a lower \( q \) makes the audit failure a noisier performance measure for effort. Second, a higher \( q \) also increases the expected investor reliance \( \overline{P}^2(\gamma) \) because it increases both the informativeness value and the insurance value of the auditor’s report, as shown in Corollary 1. This heightened reliance magnifies the cost of audit vulnerability, which increases the auditor’s incentives to exert effort.

4.1.2 Disclosure regime

The auditor’s effort choice under the Disclosure regime is determined similarly as that in the No Disclosure regime. The main difference is that \( \hat{\gamma} \) in the assessed probability of an audit failure is replaced by the realized \( \gamma \). As such, the auditor takes into account the effect of her effort choice on both auditor vulnerability and investor reliance. Proposition 3 characterizes the auditor’s effort choice, and Proposition 4 compares the equilibrium effort levels between regimes.

**Proposition 3** In the Disclosure regime, there is a unique equilibrium where (i) the auditor’s equilibrium effort \( e_D^* \) is determined by

\[
\max \left\{ \alpha K [l(q, \gamma_l, \mu) \overline{P}^2(\gamma_l) - l(q, \gamma_h, \mu) \overline{P}^2(\gamma_h)], 0 \right\} = ce_{D,D}^2,
\]

(12)

with \( \overline{P}(\gamma) \) and \( l(q, \gamma, \mu) \) given by (4) and (10), respectively; and (ii) the investor follows the investment rule as specified in Proposition 1, i.e., always invests in scenario (a);
never invests in scenario (b); invests iff his private signal precision \( p \) is larger than \( P^* \left( \gamma_j, q \right) = \gamma_j q + \left( 1 - \gamma_j \right) \) upon observing \( \gamma_j \in \{ \gamma_h, \gamma_l \} \) in scenario (c); and

invests iff \( p < \frac{\gamma_j q + (1-\gamma_j) (1-q)}{1-aq(1-\gamma_j)} \) upon observing \( \gamma_j \in \{ \gamma_h, \gamma_l \} \) in scenario (d).

**Proposition 4** There exists \( q^* \in (\frac{1}{2}, 1) \) such that \( e^*_D \geq e^*_ND \) if and only if \( q \leq q^* \).

Proposition 3 establishes two main differences in the auditor’s equilibrium effort under two disclosure regimes. First, the uniqueness of effort is guaranteed in the Disclosure regime but not in the No disclosure regime. Multiple equilibria do not arise in the Disclosure regime because the investor directly observes \( \gamma \) and no longer needs to use any conjectured \( \widehat{\gamma} \) in his investment decision. Second, a positive effort choice (i.e., an interior solution) is ensured in the No Disclosure regime but not in the Disclosure regime. In fact, (12) shows that a corner solution of \( e^*_D = 0 \) is possible, as the marginal benefit of effort can become negative. Since \( e^*_ND > 0 \), (12) further suggests that it is possible \( e^*_ND > e^*_D \). The possibility is formally confirmed in Proposition 4 which shows that there always exists an interior \( q^* \in (\frac{1}{2}, 1) \) such that \( e^*_ND > e^*_D \geq 0 \) if \( q > q^* \).

We can illustrate the idea behind Proposition 4 using the informativeness criterion established in Kim (1995). Specifically, Kim (1995) shows that in agency settings, the efficiency of two alternative information systems in motivating effort can be compared by the sensitivity of the performance measure to the agent’s effort under each system: the system with more sensitive measure is more efficient. In our setting, the audit failure is a performance measure for the auditor’s effort; thus the regime under which the audit failure is more sensitive to the auditor’s effort would motivate more effort. (9) shows that an audit failure depends on both auditor vulnerability and investor reliance. Therefore the effect of disclosure on effort can be analyzed by how sensitive each component is to the auditor’s effort (evaluated from the auditor’s perspective).

Since \( \gamma \) is an increasing function of \( e \) (albeit stochastically), with an abuse of notation, we express the sensitivity in the two regimes as:

\[
\frac{\partial \ln \Pr \text{ (audit failure}| \gamma, \widehat{\gamma})}{\partial \gamma} \bigg|_{ND} = \frac{\partial \ln l(q, \gamma, \mu)}{\partial \gamma} = \frac{-1}{1 - \gamma}, \quad (13)
\]

\[
\frac{\partial \ln \Pr \text{ (audit failure}| \gamma)}{\partial \gamma} \bigg|_{D} = \frac{\partial \ln l(q, \gamma, \mu)}{\partial \gamma} + 2 \frac{\partial \ln \overline{P}(\gamma)}{\partial \gamma}. \quad (14)
\]

In the No disclosure regime, the auditor takes into account the effect of her effort on vulnerability \( \frac{\partial \ln l(q, \gamma, \mu)}{\partial \gamma} < 0 \) but not on investor reliance which depends on the investor’s conjecture.
(i.e., \( \frac{\partial \ln \mathcal{P}(\gamma)}{\partial \gamma} |_{ND} = 0 \)). In contrast, in the Disclosure regime, the auditor takes into account the effect of her effort on investor reliance because the investor uses the observed \( \gamma \) in his investment decision, i.e., \( \frac{\partial \ln \mathcal{P}(\gamma)}{\partial \gamma} |_{D} \neq 0 \), and the auditor’s effort affects the likelihood of observing \( \gamma_h \) or \( \gamma_l \). Since \( \mathcal{P}(\gamma) = P^*(\gamma)\rho(\gamma) \), \( \frac{\partial \ln \mathcal{P}(\gamma)}{\partial \gamma} \) can be expressed as the sum of two sensitivities: the insurance use sensitivity \( \left( \frac{\partial \ln \rho(\gamma)}{\partial \gamma} = -\alpha q \rho(\gamma) < 0 \right) \), and the informativeness use sensitivity \( \left( \frac{\partial \ln P^*(\gamma)}{\partial \gamma} = \frac{(2q-1)}{P^*(\gamma)} > 0 \right) \). Comparing (13) and (14), it is easy to see that the Disclosure regime motivates more equilibrium effort if the insurance sensitivity is larger (in absolute magnitude) than the informativeness sensitivity.

This is because under the Disclosure regime, the insurance use motivates effort whereas the informativeness use has the opposite effect. To see this, note that when the investor primarily utilizes the auditor’s report for the insurance use, he is more likely to invest upon observing low audit quality \( \gamma = \gamma_l \) because an audit failure is more likely with \( \gamma = \gamma_l \) from his perspective (as shown in (6)). Anticipating this effect, the auditor has more incentives to exert effort to reduce the insurance-motivated investor reliance. In contrast, when the investor primarily uses the auditor’s report for its informativeness value, he is more likely to heed the advice of an unqualified audit opinion and invest when the audit quality is high \( \gamma = \gamma_h \). From the auditor’s perspective, this means higher effort increases investor reliance (because of informativeness use) and therefore expected liability. Clearly, this would reduce her effort incentive.

Figure 2 graphically represents the idea behind Proposition 4 by plotting the absolute values of the two sensitivities as functions of \( q \). As shown in the proof of Proposition 4 in the Appendix, the curve for informativeness sensitivity \( \left( \frac{\partial \ln P^*(\gamma)}{\partial \gamma} = \frac{(2q-1)}{P^*(\gamma)} \right) \) crosses the insurance sensitivity curve once from below at \( q^* \in (\frac{1}{2}, 1) \), implying that when \( q > q^* \), the informativeness effect dominates and the auditor’s equilibrium effort is lower under the Disclosure regime.

The intuition for Proposition 4 is as follows. When \( q \) is relatively large, the accounting signal is quite informative regarding the state to begin with, therefore the primary motivation for the investor to rely on the audit opinion is for its informativeness value. The marginal

\[ q^* \text{ is the interior } q \text{ such that } 2q - 1 - \alpha q^2 = 0. \text{ Notice that } \mathcal{P}(\gamma = 1/2) \text{ and } \mathcal{P}(\gamma = 1) \text{ imply that when } q > q^*, \text{ the informativeness effect dominates and the auditor’s equilibrium effort is lower under the Disclosure regime.} \]

\[ \text{[Insert Figure 2]} \]

\[ \text{The Appendix shows that } q^* \text{ is such that } 2q - 1 - \alpha q^2 = 0. \text{ Notice that } \mathcal{P}(\gamma = 1/2) = \frac{1}{2 - \alpha q} \text{ and } \mathcal{P}(\gamma = 1) = q. \text{ and } q^* \text{ is such that } \mathcal{P}(\frac{1}{2}) = \mathcal{P}(1), \text{ i.e., } \mathcal{P}'(\gamma) |_{q^*} = 0. \]
impact of a higher audit quality on the audit opinion’s informativeness value is larger than on its insurance value such that a higher effort by the auditor increases investor reliance and audit failure in expectation. Anticipating this, the auditor would rationally reduce her effort (relative to that in the No Disclosure regime). On the other hand, when \( q \) is relatively small, the investor relies more on the audit opinion for its insurance value. When this is the case, observing a smaller \( \gamma \) makes the investor more confident that the auditor has erred in her opinion. Consequently, \( \gamma_h \) reduces investor reliance, decreasing the odds of an audit failure. Anticipating this, the auditor’s incentives to exert effort are heightened in the Disclosure regime than in the No Disclosure regime.

Since \( e^*_N \) is always increasing in \( q \) (as shown in Proposition 2), the finding in Proposition 4 that \( e^*_D > e^*_N \) only for \( q < q^* \) suggests that a higher \( q \) can decrease the equilibrium effort in the Disclosure regime. Corollary 3 below identifies a set of sufficient conditions for this to be the case.

**Corollary 3** Under the Disclosure regime, \( \frac{de^*_D}{dq} \leq 0 \) when \( \gamma_h \) and \( \alpha \) are relatively small, and \( \frac{de^*_D}{dq} \geq 0 \) when \( \gamma_f \) and \( \alpha \) are relatively large.

To see the intuition for Corollary 3, note that \( q \) affects the auditor’s incentives via three channels. The first is to increase the marginal benefit of effort in reducing auditor vulnerability (via the \( l(q, \gamma, \mu) = (1 - \mu)q(1 - \gamma) \) term), similar to that in the No Disclosure regime. Second, it also promotes more insurance use by the investor (as shown in Corollary 1). These two channels increase the auditor’s effort incentive. At the same time, higher \( q \) also improves the informativeness value of the audited financial report (via \( P^* \)), which reduces the auditor’s effort incentives. The magnitude of \( \alpha \) and \( \gamma \) determines the relative trade-off between these opposing forces. For example, when \( \alpha \) is small, the investor mostly uses the auditor’s report for its informativeness value, heightening the negative informativeness effect from increasing \( q \), resulting in \( \frac{de^*_p}{dq} \leq 0 \). On the other hand, when \( \gamma_f \) is sufficiently big (which implies \( \gamma_h \) is close to 1), obtaining the more accurate audit evidence almost for sure exonerates the auditor from liability, thus improving the auditor’s effort incentives, which implies \( \frac{de^*_p}{dq} > 0 \).

### 4.2 Investment efficiency

We next examine the effect of disclosure on investment efficiency (IE) as defined in Section 2. The optimal investment rule to maximize IE is laid out in Lemma 1. Proposition 1
shows that the investor deviates from this rule due to the insurance use of the audit opinion. Conditional on the auditor’s effort, the insurance use results in a zero-sum transfer between the investor and the auditor, lowering the investment efficiency from the social optimal perspective. While Proposition 4 shows that the No Disclosure regime results in higher audit effort than the Disclosure regime when $q$ is relatively high, the same ranking cannot be ensured for investment efficiency.

Specifically, disclosing audit quality affects investment efficiency via three channels. The first two channels operate by allowing the investor to fine-tune his use of both the informativeness and the insurance values of the auditor’s report, holding the auditor’s effort constant. Since the informativeness use follows the social optimal investment rule, *ceteris paribus*, the flexibility to adjust the optimal investment decision ($P^*$) as a function of $\gamma$ improves the *ex ante* investment efficiency. We term this channel the social Blackwell channel, and note this channel tilts the efficiency comparison in favor of the Disclosure regime. On the other hand, because the insurance use of the auditor’s report leads to suboptimal investment decisions, allowing the investor more flexibility in adjusting the insurance use decreases the overall investment efficiency. We term this channel the private Blackwell channel, which tilts against the Disclosure regime.

The third channel operates by affecting the auditor’s effort, which works to moderate the full effects of the previous two channels. Specifically, the benefit from the social Blackwell channel is reduced by the fact the audit effort is lowered by audit quality disclosure when the investor mainly uses the audit opinion for its informativeness value (as shown in Proposition 4). On the other hand, the cost of the private Blackwell channel is partially offset by the higher auditor’s effort induced as a result of the investor exploiting the insurance value of the audit report. Taken together, the third channel tilts the comparison in favor of the No Disclosure regime for large enough $q$. Overall, the efficiency comparison of the two regimes is determined by a fairly complex trade-off among these forces, which does not lend itself to a complete analytical solution. To fix idea, Claim 1 below provides a partial trade-off between the social and private Blackwell effect.

**Claim 1** Holding the auditor’s effort constant at the same level for the two regimes, $IE_D > IE_{ND}$ if and only if $q > q^*$ where $q^*$ is defined in Proposition 4.

Claim 1 shows that, in a hypothetical situation without considering the differential effort
effects between the two regimes, the social Blackwell effect dominates the private Blackwell effect if and only if \( q > q^* \). It is not surprising to see that the threshold is the same as that for Proposition 4. This is because at \( q^* \) the marginal impact of the audit effort on the informativeness use and on the insurance use cancels out (recall \( \frac{dT(y)}{dy} \bigg|_{y=q^*} = 0 \)), and the social and private Blackwell effects exactly offset each other, making \( IE_D = IE_{ND} \).

When the incentive effect is present, the picture becomes more complicated. To gain some insight, we rely on numerical solutions and plot in Figure 3 the comparison of investment efficiency between the two regimes under all combinations of \( \alpha \in [0, 1] \) and \( q \in (0.5, 1) \).\(^{20}\) Specifically, the two vertical solid curves identify the combinations of \( \alpha \) and \( q \) under which the investment efficiency is the same under both regimes. Figure 3 also shows that \( IE_D < IE_{ND} \) when \( \alpha \) is relatively large and \( q \) is relatively small (the northwest corner of the graph) and when \( q \) is relatively large.

To illustrate the driving forces, we also plot the equilibrium effort levels as a function of \( q \) under the two regimes for \( \alpha = 0.5 \) in the two horizontal curves in Figure 3, with the solid and the dashed curves representing the efforts under the No Disclosure and Disclosure regime, respectively. Consistent with Proposition 4, the \( e_N^*(q, \alpha) \) curve rises above the \( e_D^*(q, \alpha) \) curve after \( q^*(\alpha) \).\(^{21}\) Also consistent with Claim 1, the investment efficiency is identical at \( q^* \). When \( q < q^* \), the investor mainly uses the auditor’s opinion for its insurance value which distorts the investment decision from the social optimal level, resulting in lower investment efficiency despite the positive incentive effect (i.e., \( e_D^* > e_N^* \)). When \( q \) is slightly larger than \( q^* \) such that the negative incentive effect (i.e., \( e_D^* < e_N^* \)) is not too large, the social Blackwell effect dominates the private effect, resulting in \( IE_D > IE_{ND} \). As \( q \) becomes sufficiently large, the negative incentive effect dominates, resulting in \( IE_D \) lower than \( IE_{ND} \) again.

[Insert Figure 3]

Similarly, Figure 4 plots the comparison of the two regimes based on total surplus, defined as the investment efficiency net of the auditor’s cost of effort. It shows similar patterns as those for the investment efficiency: \( TS_{ND} > TS_D \) when \( q \) is either very small or very large.

\(^{20}\)We use \( \gamma_h = 0.7 \) and \( \gamma_l = 0.5 \) in the plot, and verify that the qualitative conclusions are robust to alternative values (results available upon request).

\(^{21}\)The proof for Proposition 4 in the Appendix shows that the threshold \( q^* \) is determined by the unique solution of \( q \in [0.5, 1] \) such that \( 2q - 1 - \alpha q^2 = 0 \), which implies \( q^*(0.5) = 2 - \sqrt{2} \approx 0.586 \).
For illustration, we keep the two curves for the investment efficiency comparison as the dashed south-north curves in Figure 4. It shows that when \( q \) is sufficiently large, the region where \( TS_{ND} > TS_D \) is smaller than the region where \( IE_{ND} > IE_D \). This is because \( TS \) also considers the auditor’s cost of effort which is higher under the No Disclosure regime when \( q \) is large.

[Insert Figure 4]

4.3 Audit fee

We next compare the audit fee \( F_i \) between the two regimes \( i \in \{D, ND\} \). The assumption of a competitive audit market implies that \( F \) equals the sum of the expected auditor’s liability and the cost of effort evaluated at the equilibrium effort and investment level as

\[
F_i = \alpha K \cdot \Pr(\text{audit failure} | e^*_{i}) + \frac{1}{3} e^3_{i}, i \in \{D, ND\}.
\]

It is clear that the auditor will demand a higher fee in the regime with higher effort and higher expected liability in equilibrium. Proposition 4 shows that when \( q \geq q^*, e^*_{ND} \geq e^*_{D} \), suggesting a larger fee for cost of effort in the No Disclosure regime. The expected liability is proportional to the likelihood of an audit failure, which depends on investor reliance, as represented by \( \overline{P}^2(\gamma) \), and auditor vulnerability, as represented by \( (1 - \mu) q (1 - \gamma) \). Define \( M(\gamma) = (1 - \gamma) \overline{P}^2(\gamma) \). It is easy to see that the probability of an audit failure in each regime can be written in terms of \( M(\gamma) \). Specifically,

\[
\Pr(\text{audit failure} | ND) = (1 - \mu) q[1 - (e^*_{ND} \gamma_h + (1 - e^*_{ND}) \gamma_l)] \overline{P}^2(\hat{\gamma}(e^*_{ND})) \quad (15)
\]

\[
= (1 - \mu) q M(\hat{\gamma}(e^*_{ND}))
\]

where \( \hat{\gamma}(e^*_{ND}) \equiv e^*_{ND} \gamma_h + (1 - e^*_{ND}) \gamma_l \) is the investor’s conjectured audit quality given \( e^*_{ND} \). Similarly,

\[
\Pr(\text{audit failure} | D) = (1 - \mu) q[e^*_{D}(1 - \gamma_h) \overline{P}^2(\gamma_h) + (1 - e^*_{D})(1 - \gamma_l) \overline{P}^2(\gamma_l)] \quad (16)
\]

\[
= (1 - \mu) q [e^*_{D} M(\gamma_h) + (1 - e^*_{D}) M(\gamma_l)].
\]

It is clear from (15) and (16) that holding effort constant between the two regimes, the expected liability would be higher (lower) under the No Disclosure regime if \( M(\gamma) \) is concave (convex) in \( \gamma \). The proof in the Appendix shows that \( M(\gamma) \) is concave when \( q \geq q^* \).
implying that the investor’s reliance is also higher under the No Disclosure regime when $q \geq q^*$. Together the two forces yield a higher fee for the No Disclosure regime, as shown in Proposition 5 below.

**Proposition 5** Let $F_i$ denote the audit fee under disclosure regime $i \in \{D, ND\}$ and $q^*$ be defined as in Proposition 4. Then $F_{ND} \geq F_D$ if $q \geq q^*$ where the equality sign holds when $q = q^*$.

The comparison is not as clear-cut for the case of $q < q^*$ as two countervailing forces are at play. First, holding investor reliance constant, $e^*_D > e^*_{ND}$ suggests a higher audit fee under the Disclosure regime. On the other hand, higher effort $e^*_D$ reduces auditor vulnerability and therefore reduces the expected liability. The net effect is therefore unclear, and does not yield a closed-form analytical comparison. We rely on numerical examples to gain some insight and plot the result in Figure 5. It shows that the audit fee is higher under the Disclosure regime when $q < q^*$, suggesting the latter effect is dominated by the former.

[Insert Figure 5]

5 Additional analysis: endogenous $q$

So far, we have taken the financial reporting quality as exogenously determined and focused instead on the effects of audit quality disclosure. However, financial reporting quality can be affected, either by standard setters or by firms’ management. Since financial reporting and auditing environments are closely related, it is also of interest to examine the effects of varying $q$ both from an efficiency maximizing standard setter’s perspective and from the manager’s perspective. In both cases, we assume without loss of generality that the decision-maker (the standard setter or the manager) can choose any $q \in [\underbar{q}, \overbar{q}] \subseteq [1/2, 1]$ without additional cost at the beginning of the game and the choice is publicly known.\(^{22}\) In addition, he takes into account the strategic interactions that will take place in the subsequent games.

\(^{22}\)This ex ante perspective is common in the literature, and has been adopted in recent studies by Chen, et al. (2017), and Gao (2015).
5.1 From the standard setter’s perspective

We assume that the standard setter chooses \( q \) to maximize the investment efficiency \( (IE) \) as defined earlier.\(^{23}\) Thus, insights about the optimal \( q \) can be obtained by examining the comparative statics of expected \( IE \) with respect to \( q \) evaluated at the equilibrium effort and investment level. We summarize our results in Proposition 6 below.

**Proposition 6** (a) In the No Disclosure regime, when \( \gamma_h \) (\( \gamma_i \)) is relatively small (large), the investment efficiency is strictly decreasing (increasing) in \( q \). (b) In the Disclosure regime, when both \( \alpha \) and \( \gamma_h \) (\( \gamma_i \)) are relatively small (large), the investment efficiency is decreasing (increasing) in \( q \).

Recall that Corollary 2 shows \( \frac{d \omega_N}{dq} > 0 \) in the No Disclosure regime. The same cannot be said for investment efficiency. In general, a higher \( q \) increases the investment efficiency in the absence of the insurance effect (i.e., \( \rho = 1 \)) as the investor is better able to forecast the underlying state with high quality financial reporting. However, in the presence of the insurance effect, a higher \( q \) also distorts the investor’s investment decision away from the social optimum (as shown in Corollary 1 \( \frac{d \omega}{dq} > 0 \)). Proposition 6(a) shows that this latter negative effect can dominate the former when \( \gamma_h \) is sufficiently small. Intuitively, when the auditing technology is poor (\( \gamma_h \) is low), the audit opinion is not that informative of the underlying state, higher \( q \) primarily increases the investor’s insurance-driven investment, resulting in lower investment efficiency.

In the Disclosure regime, increasing \( q \) on the margin affects the investment efficiency via two channels. The first is similar to the forces in the No Disclosure regime, where \( q \) increases the insurance value of the audit opinion and can reduce investment efficiency. Furthermore, under the Disclosure regime, a higher \( q \) has an additional impact on investment efficiency via its adverse impact on the auditor’s effort provision, as shown in Corollary 3.

Figure 6 presents several numerical plots of \( IE \) (dashed line) and \( TS \) (solid line) as functions of \( q \), evaluated at the equilibrium effort and investment levels.\(^{24}\) For comparison, each panel also plots the \( IE \) and \( TS \) lines evaluated at the equilibrium effort level and the

---

\(^{23}\)We are not able to obtain analytical results on the effects of \( q \) on total surplus \((TS)\), although numerical results (in Figure 6 below) show qualitative conclusions.

\(^{24}\)The proof for Claim 1 shows \( IE = (1 - \mu) K (\Pi - 1) < 0 \) where \( \Pi = \Pr (I = 1 | G) - \Pr (I = 1 | B) > 0 \). For convenience, Figure 6 plots \( \Pi \) for \( IE \) and \( \Pi - c(e) \) for \( TS \).
socially optimal investment rule as specified in Lemma 1. For obvious reasons, they lie above their equilibrium counterparts. The top 3 panels plot the No Disclosure regime and confirm the results in Proposition 6. Both $IE$ and $TS$ decrease in $q$ when $\alpha$ and $\gamma_k$ are relatively small (the left panel); they increase in $q$ as $\alpha$ and $\gamma_i$ become relatively large (the right panel). The bottom three panels plot the Disclosure regime and show similar patterns.

Together, these results suggest a complementary relation between the optimal quality of financial accounting standards ($q$) and the quality of the audit environment as captured by the degree of audit liability ($\alpha$) and audit technology ($\gamma$). Specifically, the need to improve the reporting quality is made even more urgent in environments where the auditing standards are reasonably advanced (high $\gamma$) and that the legal system imposes credible threat for penalizing auditor negligence based on investor behaviors ($\alpha$ is high).

5.2 From the manager’s perspective

To the extent that financial reporting standards often allow and entail managerial judgement and discretion, it is possible that managers may take actions to influence $q$. Consistent with the assumption that the manager’s utility depends only on the private benefits from implementing the project, we assume that the manager chooses $q$ to maximize the probability of the investor investing in the project.

Intuitively, the manager would prefer a higher $q$. This is because when $q = \frac{1}{2}$, the investor would rarely rely on the audited financial report for his investment decision, rendering the manager no control over the likelihood of investment. Corollary 1 shows that increasing $q$ increases both the informativeness value and the insurance value of the auditor’s report.\footnote{A caveat is in place. In the absence of insurance and effort effects, the optimal $q$ to maximize the ex ante unconditional probability of investment depends on the common prior. In this case, one can show that there exists $\mu^* \in (0, 1)$ such that the manager prefers a noisy (perfect) accounting report, i.e., set $q^*$ to lower (upper) bound $q$ (7) if $\mu < \mu^*$ ($\mu > \mu^*$). The intuition is that a higher $q$ makes the audited financial report closer to the true state of nature, increasing the investor’s reliance on the report rather than his private signal. When $\mu$ is small, therefore the investor is a priori unlikely to invest, a very precise accounting report is for sure likely to be low, and therefore further reduces the likelihood of investment. A noisy accounting signal on the other hand would increase the chances of investment as the investor relies more on his private signal (which is independent from the accounting signal, conditional on the state). The algebra to prove}
In the No Disclosure regime, improving $q$ also has added benefit of increasing the auditor’s equilibrium effort, which further improves investor reliance. That said, the effort effect is not as significant in the Disclosure regime, as Corollary 3 shows that the equilibrium effort $e_D$ may decrease with $q$.

However, proving this intuition analytically turns out to be challenging as a closed-form expression for how $q$ affects the likelihood of investment is difficult to obtain in both regimes. We rely on numerical examples to verify the intuition. In Figure 7, we present the 3-D plots for $\Pr(\text{Project taken})$ for all combinations of $\alpha$ and $q$ under three values of $\mu$, for both the No Disclosure regime (in the top row) and the Disclosure regime (in the bottom row). All plots show that for a given $\alpha$, the probability of investment increases in $q$, consistent with our conjecture.

The analyses in Section 5.1 and 5.2 reveal an interesting divergence of preferences for the financial reporting quality between a benevolent standard setter and a self-interested manager when $\alpha$ and $\gamma_h$ are relatively small. The conventional wisdom is that standard setters prefer higher quality whereas self-interested managers prefer obfuscation. Our analyses suggest that the preference divergence may be reversed when we take into account the frictions in the audit market.

[Insert Figure 7]

6 Conclusion

We examine the effects of disclosing audit quality on auditors’ effort provision and investors’ investment efficiency in a setting where audit quality is affected by the auditor’s effort, which is in turn motivated by her liability in the event of an audit failure. We show that while disclosing audit quality enhances the information decision usefulness of financial reports for the investor, it can also adversely affect the auditor’s incentives and consequently lower the expected audit quality and investment efficiency in equilibrium. We show that the underlying quality of financial reporting is an important determinant for this trade-off, and the argument for using audit quality disclosure to motivate auditor effort can be supported only when the underlying financial reporting quality is relatively low.

\[\text{this intuition is fairly complex and is available upon request. Since numerical examples show that this force appears to be dominated by the insurance effect, we do not include the proof for space constraint.}\]
References


Figure 1a: Information Structure

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Date 2</th>
<th>Date 3</th>
<th>Date 4</th>
<th>Date 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reporting quality $q$ given.</td>
<td>Accounting report $R \in {R_D, R_B}$ generated by accounting system and privately observed by the manager.</td>
<td>The auditor exerts costly private effort $e$ that determine audit quality $Pr(\gamma_R) = e$ and $Pr(\gamma_R) = 1 - e$.</td>
<td>The investor observes $\Phi_i = {(S, p, q, \Omega, \hat{\lambda}_i(\Omega)}$ and makes investment decision.</td>
<td>State of nature revealed.</td>
</tr>
<tr>
<td>Audit disclosure regime $i \in {ND, B}$ set.</td>
<td>The manager reports $R_D$.</td>
<td>Observes and discloses $\Omega_D$ or $\Omega_B$.</td>
<td>In Disclosure regime, $\hat{\lambda}_D(\Omega) \in {\gamma, \gamma_1}$.</td>
<td>Project payoff realized and distributed.</td>
</tr>
<tr>
<td>Non-contingent audit fee paid.</td>
<td></td>
<td></td>
<td>In No Disclosure regime, $\hat{\lambda}<em>{ND}(\Omega_B) \in {\gamma, \gamma_1}$ and $\hat{\lambda}</em>{ND}(\Omega_B) = \gamma_{ND} = \gamma_{ND} = (1 - e)\gamma_1$.</td>
<td>Auditor liability assessed.</td>
</tr>
</tbody>
</table>
Figure 2: Threshold $q$ for Effort Comparison in Two Regimes
Figure 3: Compare Investment Efficiency Between Regimes
Figure 4: Compare Total Surplus Between Regimes
Figure 5: Compare Audit Fees Between Regimes
Figure 6a: Effects of $q$ on $IE$ and $TS$ in the No Disclosure Regime

Figure 6b: Effects of $q$ on $IE$ and $TS$ in the Disclosure Regime

\{\alpha, \gamma_0, \gamma_1\} = \{0.5, 0.7, 0.5\}

\{\alpha, \gamma_0, \gamma_1\} = \{0.8, 0.9, 0.5\}

\{\alpha, \gamma_0, \gamma_1\} = \{0.99, 0.99, 0.88\}

\{\alpha, \gamma_0, \gamma_1\} = \{0.99, 0.99, 0.88\}
Figure 7a: Effect of $q$ on $\Pr \left(\text{Investment}\right)$ in the No Disclosure Regime

Figure 7b: Effect of $q$ on $\Pr \left(\text{Investment}\right)$ in the Disclosure Regime
7 Appendix

**Proof of Lemma 1**  The proof is directly discussed in the main text and therefore omitted here.

**Proof of Proposition 1**

The proof for scenarios (a) to (c) is the same as that for Lemma 1. For scenario (d) when \( \alpha > 0 \), the investor’s expected payoff from undertaking the project net of the initial investment is

\[
\Pr (G|\Omega_g, S_b, \gamma) rK + \Pr (B, R_B|\Omega_g, S_b, \gamma) \alpha K - K = \\
\frac{\mu (1 - p) \rho * (\gamma, q)}{\mu (1 - p) \rho * (\gamma, q) + (1 - \mu) p [1 - \rho * (\gamma, q)]} \frac{K}{\mu} + \\
\frac{(1 - \mu) pq (1 - \gamma)}{\mu (1 - p) \rho * (\gamma, q) + (1 - \mu) p [1 - \rho * (\gamma, q)]} \alpha K - K
\]

where \( \rho * (\gamma, q) = \Pr (\Omega_j|J) \) is given in (3). It is easy to see that the investment payoff is non-negative if and only if \( p \leq \frac{\rho * (\gamma, q)}{1 - \alpha q (1 - \gamma)} = \rho * (\gamma, q) \rho (\gamma, \alpha, q) = \overline{P} (\gamma, \alpha, q) \). Q.E.D.

**Proof of Proposition 2**

We prove for the existence of a stable equilibrium. Note that both sides of (8) are continuous in \( e \). The RHS of (8) \( C'(0) = 0 \) and \( C'(1) \) is sufficiently large, while the LHS is bounded and strictly positive for all \( e \). Thus, there must exist at least one solution to equation (11) and all solutions to equation (11) must lie strictly between 0 and 1. Further, at \( e = 0 \), \( \text{LHS} > \text{RHS} = 0 \) of equation (11). Also, at each \( e^* \), \( \text{LHS} = \text{RHS} \). Suppose there does not exist any stable equilibrium, i.e., whenever \( \frac{\partial [\text{LHS of (11)}]}{\partial e} |_{e=e^*} \geq 2ce^* \). Then, it must be that \( \text{LHS} \geq \text{RHS} \), \( \forall e \). But this contradicts the fact that \( C'(1) \) is sufficiently large and LHS is bounded. Q.E.D.

**Proof for Corollary 2**

Taking the total derivative on (11) with respect to \( q \) and rearranging terms yields \( \frac{dc_{ND}}{dq} = \frac{\partial [\text{LHS of (11)}]}{\partial q} = \frac{2(1+\alpha(1-\gamma)^2)}{[1-\alpha q (1-\gamma)]^2} > 0 \) and

\[
\frac{\partial (\text{LHS of (11)})}{\partial q} = \alpha K (1 - \mu) (\gamma_h - \gamma_i) \overline{P}^2 + \alpha K (1 - \mu) q (\gamma_h - \gamma_i) 2 \overline{P} \frac{\partial \overline{P}}{\partial q}
\]
where $\overline{P}$ is defined in (4). In addition, the denominator for $\frac{de_{\text{ND}}}{dq}$ is positive because by definition in a stable equilibrium $\frac{\partial\text{LHS of (11)}}{\partial e}\big|_{e=e_{\text{ND}}^*} < 2ce_{\text{ND}}^*$. Thus $\frac{de_{\text{ND}}}{dq} > 0$. Q.E.D.

**Proof of Proposition 3**

In the Disclosure regime, the auditor chooses $e$ to maximize

$$\max_e \left[ l(q, \gamma_h, \mu)\overline{P}^2(\gamma_h) - l(q, \gamma_l, \mu)\overline{P}^2(\gamma_l) \right] \alpha K + l(q, \gamma_l, \mu)\overline{P}^2(\gamma_l) + \frac{1}{3}ce^3.$$

(17)

Taking a first-order derivative on (17) with respect to $e$ and setting it to zero, we obtain equation (12). Since (17) is strictly convex in $e$, the solution to (12) is indeed a global minimizer for the auditor. Note that (12) is free of investors’ conjecture $\hat{e}$ and $\hat{\gamma}$. This is because observing the realization of $\gamma$ is a sufficient statistic for investors’ investment decision. As such, (12) is also the equilibrium condition for the auditor’s effort under the Disclosure regime. Finally, since only the RHS of (12) is a function of $e$ with $C'(0) = 0$ and $C'(1)$ sufficiently large and the LFS is a non-negative constant independent of $e$, there is only one solution to (12). Q.E.D.

**Proof of Proposition 4**

Our strategy to prove the proposition is to compare the equilibrium conditions under the two regimes: (11) versus (12). Note that

$$\overline{P}(\gamma_h)^2 > \overline{P}(\gamma_l)^2 \iff \overline{P}(\gamma_h) > \overline{P}(\gamma_l) \iff \overline{P}'(\gamma) > 0.$$

Since $\overline{P}'(\gamma) = \frac{2q-1-\alpha q^2}{(1-\gamma)(1-\gamma q)}$, $\overline{P}'(\gamma) > 0$ if and only if $2q-1-\alpha q^2 > 0$. Because $2q-1-\alpha q^2 = 0$ only admits one solution between 1/2 and 1, there exists a $q^*$ such that $\overline{P}'(\gamma) > 0$ if and only if $q > q^*$, where $q^*$ is the unique solution to

$$2q - 1 - \alpha q^2 = 0 \text{ s.t. } q \in [1/2, 1].$$

Consider the first case of $q \in [1/2, q^*)$ which implies $\overline{P}'(\gamma) < 0$. As Proposition 2 has established $e_{\text{ND}}^* \in (0, 1)$, we have

$$\overline{P}(\gamma_h) < \overline{P}(e_{\text{ND}}^* \gamma_h + (1 - e_{\text{ND}}^*) \gamma_l) < \overline{P}(\gamma_l).$$

Recall $l(q, \gamma, \mu) \equiv (1 - \mu)(1 - \gamma)q$ is decreasing in $\gamma$, we have

$$l(q, \gamma_l, \mu)\overline{P}(e_{\text{ND}}^* \gamma_h + (1 - e_{\text{ND}}^*) \gamma_l)^2 - l(q, \gamma_h, \mu)\overline{P}(e_{\text{ND}}^* \gamma_h + (1 - e_{\text{ND}}^*) \gamma_l)^2 <$$

$$l(q, \gamma_l, \mu)\overline{P}(\gamma_l)^2 - l(q, \gamma_h, \mu)\overline{P}(\gamma_l)^2 \implies$$

$$\text{LHS of (11)} < l(q, \gamma_l, \mu)\overline{P}(\gamma_h)^2 - l(q, \gamma_h, \mu)\overline{P}(\gamma_l)^2$$
Note LHS of (11) > 0. Thus, LHS of (11) < max \{ \alpha K[l(q, \gamma_l, \mu)\bar{p}(\gamma_h)^2 - l(q, \gamma_h, \mu)\bar{p}(\gamma_l)^2], 0 \} = LHS of (12), hence \( e_{ND}^* < e_D^* \).

Similarly for the case \( q \in (q^*, 1) \) when \( \bar{P}(\gamma) > 0 \). Given \( e_{ND}^* \in (0, 1) \), we have \( \bar{P}(\gamma_h) > \bar{P}(\gamma_l) \). Thus,

\[
\begin{align*}
&l(q, \gamma_l, \mu)\bar{P}(e_{ND}^*\gamma_h) + (1 - e_{ND}^*)\gamma_l)^2 - l(q, \gamma_h, \mu)\bar{P}(e_{ND}^*\gamma_h) + (1 - e_{ND}^*)\gamma_l)^2 > \\
&l(q, \gamma_l, \mu)\bar{P}(\gamma_l)^2 - l(q, \gamma_h, \mu)\bar{P}(\gamma_h)^2 \implies \\
&LHS \text{ of (11)} > l(q, \gamma_l, \mu)\bar{p}(\gamma_h)^2 - l(q, \gamma_h, \mu)\bar{p}(\gamma_l)^2.
\end{align*}
\]

Note LHS of (11) > 0. Thus, LHS of (11) > max \{ \alpha K[l(q, \gamma_l, \mu)\bar{p}(\gamma_h)^2 - l(q, \gamma_h, \mu)\bar{p}(\gamma_l)^2], 0 \} = LHS of (12), which implies \( e_{ND}^* > e_D^* \).

Lastly, consider the case \( q = q^* \), then \( \bar{P}(\gamma) = 0 \) which implies \( \bar{P}(\gamma_h) = \bar{P}(e_{ND}^*\gamma_h) + (1 - e_{ND}^*)\gamma_l) = \bar{P}(\gamma_l) \). It then follows that \( e_{ND}^* = e_D^* \) as

\[
\begin{align*}
&l(q, \gamma_t, \mu)\bar{P}(e_{ND}^*\gamma_h) + (1 - e_{ND}^*)\gamma_l)^2 - l(q, \gamma_h, \mu)\bar{P}(e_{ND}^*\gamma_h) + (1 - e_{ND}^*)\gamma_l)^2 = \\
&l(q, \gamma_t, \mu)\bar{P}(\gamma_l)^2 - l(q, \gamma_h, \mu)\bar{P}(\gamma_h)^2.
\end{align*}
\]

Q.E.D.

**Proof for Corollary 3**

We first show that \( e_D^* \) decreases with \( \gamma_h \) and \( \alpha \) are sufficiently small. To prove this part, we first establish the following claim: if there exists a \( \bar{q} \in (1/2, 1) \) such that \( e_D^* (\bar{q}) = 0 \), then \( e_D^* (q) = 0 \) for all \( q > \bar{q} \). To prove this claim, note that from the first order condition (12), \( e_D^* = 0 \) if and only if \( (1 - \gamma_t)\bar{P}(\gamma_l)^2 - (1 - \gamma_h)\bar{P}(\gamma_h)^2 \leq 0 \), or equivalently, \( \left( \frac{\bar{P}(\gamma_l)}{\bar{P}(\gamma_h)} \right)^2 \leq \frac{1 - \gamma_h}{1 - \gamma_t} \).

If we can show that \( \frac{\bar{P}(\gamma_l)}{\bar{P}(\gamma_h)} \) is weakly decreasing in \( q \) then the claim is proved. Note that

\[
\frac{d}{dq} \left( \frac{\bar{P}(\gamma_l)}{\bar{P}(\gamma_h)} \right) = -\frac{(\gamma_h - \gamma_l)V(q)}{[1 - \alpha(1 - \gamma_l)q^2][1 - q + \gamma_h(2q - 1)]^2}
\]

where

\[
V(q) = 1 - \alpha\{(1 - \gamma_h)(1 - \gamma_l) + 2[\gamma_l - \gamma_h(2\gamma_l - 1)]\}q \\
+ [\alpha(1 - \gamma_h)(1 - \gamma_l) - (2\gamma_h - 1)(2\gamma_l - 1)]q^2.
\]

\( V(q) \) is a quadratic function of \( q \) and cumbersome algebra shows that its minimum is always non-negative, thus we have \( V(q) \geq 0 \) and \( \frac{d}{dq} \left( \frac{\bar{P}(\gamma_l)}{\bar{P}(\gamma_h)} \right) \leq 0 \).
We now provide sufficient conditions for $e_D^*$ to decrease with $q$. Proposition 3 indicates that a sufficient condition for $e_D^*$ to decrease with $q$ is for the LHS of (12) to decreases with $q$ whenever it is positive. Note that $\partial_{LHS}$ of (12) \[\frac{\partial g}{\partial q} = \alpha K (1 - \mu) [g (\gamma_l, q, \alpha) - g (\gamma_h, q, \alpha)]\] where
\[
g (\gamma, q, \alpha) = \frac{\partial}{\partial q} [q (1 - \gamma) \overline{P} (\gamma)^2] = (1 - \gamma) \overline{P} (\gamma)^2 + 2q (1 - \gamma) \overline{P} (\gamma) \frac{\partial \overline{P} (\gamma)}{\partial q}.
\] (18)

Thus a sufficient condition for the LHS of (12) to decrease in $q$ is $\frac{\partial g (\gamma, q, \alpha)}{\partial \gamma} \geq 0$. Cumbersome but manageable algebra shows $\text{sign} \left( \frac{\partial g (\gamma, q, \alpha)}{\partial \gamma} \right) = \text{sign} [h (\gamma, q, \alpha)]$ where
\[
h (\gamma, q, \alpha) \equiv -3 (1 - \gamma)^2 + 4 (1 - \gamma) [4 - \alpha (1 - \gamma)^2 - 6 \gamma] q
\[
- [15 + 4 \alpha (1 - \gamma)^2 - \alpha^2 (1 - \gamma)^4 - 48 \gamma + 36 \gamma^2] q^2
\[
- 4 \alpha (1 - \gamma) (3 - 2 \gamma) (2 \gamma - 1) q^3 - \alpha^2 (1 - \gamma)^2 [3 - 4 (2 - \gamma) \gamma] q^4.
\]

When $\gamma = \frac{1}{2}$, $h (\frac{1}{2}, q, \alpha) = \frac{1}{16} \{ -12 + 32 q + \alpha q [ -8 - (16 - \alpha) q] \}$. Note that $h (\frac{1}{2}, q, \alpha)$ is a quadratic function of $q$ which achieves its minimum of $\frac{1}{16} \{ \alpha^2 - 32 \alpha + 16 \} \equiv W (\alpha)$ when $q = \frac{1}{2}$. Further $W' (\alpha) < 0$ with $W (0) > 0$ and $W (1) < 0$. It is easy to verify that $W (\alpha) < 0$ as long as $\alpha < 0.5$. Then by continuity, $\frac{\partial g (\gamma, q, \alpha)}{\partial \gamma} \geq 0$, $g (\gamma_l, q, \alpha) - g (\gamma_h, q, \alpha) \leq 0$ and $\frac{\partial e_D^*}{\partial q} \leq 0$ when $\alpha$ and $\gamma_h$ is sufficiently small.

We now show that $e_D^*$ increases with $q$ when $\gamma_l$ and $\alpha$ are sufficiently large. Clearly, a necessary condition for $e_D^*$ to strictly increase with $q$ is that $e_D^* > 0$. $e_D^*$ is strictly positive, iff,
\[
(1 - \gamma_l) \overline{P} (\gamma_l)^2 - (1 - \gamma_h) \overline{P} (\gamma_h)^2 > 0.
\]

Thus, a sufficient condition for $e_D^* > 0$ is for $\frac{\partial (1 - \gamma_l) \overline{P} (\gamma_l)^2}{\partial \gamma} < 0$. Simple algebra shows that $\text{sign} \left( \frac{\partial (1 - \gamma_l) \overline{P} (\gamma_l)^2}{\partial \gamma} \right) = \text{sign} (v (\gamma, q, \alpha))$, where
\[
v (\gamma, q, \alpha) = -3 (1 - \gamma) + (5 - 6 \gamma) q - \alpha q (1 - \gamma) [ (1 - \gamma) (2 q - 1) + q ] < \left( 2 - \frac{\gamma}{1 - \gamma} \right) q.
\]

Thus, a sufficient condition for $\frac{\partial (1 - \gamma_l) \overline{P} (\gamma_l)^2}{\partial \gamma} < 0$ is $\gamma_l \geq \frac{2}{3}$.

Conditional on $e_D^* > 0$, we now prove that when $\alpha$ and $\gamma_l$ are sufficiently large, $\frac{de_D^*}{dq} > 0$ by proving $q [(1 - \gamma_l) \overline{P} (\gamma_l)^2 - (1 - \gamma_h) \overline{P} (\gamma_h)^2]$ increases in $q$ with sufficiently large $\alpha$ and $\gamma_l$. Similar to earlier proof, this can be shown by establishing conditions for $\frac{\partial g (\gamma, q, \alpha)}{\partial \gamma} < 0$ with $g (\gamma, q, \alpha)$ defined in (18); or equivalently the conditions for $h (\gamma, q, \alpha) < 0$ with $h (\gamma, q, \alpha)$ defined in (19). Cumbersome algebra can show that $h (\gamma, q, \alpha = 1)$ decreases in $q$ when $\gamma = 1$; and $h (1, q, 0) = -3 q^2 < 0$. This implies that by continuity, $q [(1 - \gamma_l) \overline{P} (\gamma_l)^2 - (1 - \gamma_h) \overline{P} (\gamma_h)^2]
increases in $q$ and thus $e_D^*$ strictly increases with respect to $q$ when $\alpha$ and $\gamma_l$ is sufficiently large. Q.E.D.

**Proof of Claim 1**

Recall that

$$IE \equiv -\mu \Pr (I = 0|G) (rK - K) - (1 - \mu) \Pr (I = 1|B) K$$

$$= -\mu [1 - \Pr (I = 1|G)] \left( \frac{K}{\mu} - K \right) - (1 - \mu) \Pr (I = 1|B) K$$

$$= (1 - \mu) K \left[ \Pr (I = 1|G) - \Pr (I = 1|B) - 1 \right],$$

where the second equality obtains because $r = \frac{1}{\mu}$ and $\Pr (I = 0|G) = 1 - \Pr (I = 1|G)$.

Define

$$\Pi \equiv \Pr (I = 1|G) - \Pr (I = 1|B).$$

Clearly, our comparative static analysis on $IE$ with respect to $q$ can be equivalently performed on $\Pi$.

Based on Proposition 1, investment takes place in three scenarios. We detail the probabilities of these scenarios conditional on the auditor’s effort $e_{ND}^*$ and the true state being $G$. The case for true state being $B$ can be derived analogously and therefore details omitted (available upon request).

The first scenario is when the investor observes $S_g$ and the auditor issues an unqualified opinion $\Omega_g$. In this case, the investor always invests regardless of the precision of his private signal, i.e., the ex ante probability of investing is

$$\Pr (I = 1|S_g, \Omega_g) \Pr (S_g, \Omega_g|G, e_{ND}^*) = \Pr (\Omega_g|G, e_{ND}^*) \int_{1/2}^1 2pdp$$

with

$$\Pr (\Omega_g|G, e_{ND}^*) = \Pr (\Omega_g|R_G) \Pr (R_G|G, e_{ND}^*) + \Pr (\Omega_g|R_B) \Pr (R_B|G, e_{ND}^*)$$

$$= \Pr (\Omega_g|G, \gamma_h) \Pr (\gamma_h|e_{ND}^*) + \Pr (\Omega_g|G, \gamma_l) \Pr (\gamma_l|e_{ND}^*)$$

$$= e_{ND}^* \left[ q\gamma_h + (1 - q)(1 - \gamma_h) \right] + (1 - e_{ND}^*) \left[ q\gamma_l + (1 - q)(1 - \gamma_l) \right]$$

The second scenario is when the investor observes $S_b$ and the auditor issues unqualified opinion $\Omega_g$. In this case the investment will occur when $p \leq \bar{p}$, i.e. when $S_b$ is not very informative. The ex ante probability of the project undertaken is given by

$$\Pr (I = 1|S_b, \Omega_g) \Pr (S_b, \Omega_g|G, e_{ND}^*) = \Pr (\Omega_g|G, e_{ND}^*) \int_{1/2}^{\bar{p}} 2(1 - p)dp$$

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where \( \Pr (\Omega_g|G, e_{ND}^*) \) is given in (20).

The third scenario is when the investor observes \( S_g \) and the auditor issues qualified opinion. In this case \( \gamma \) will be disclosed so the investment will occur when \( p \geq P^*(\gamma) \), i.e. when \( S_g \) is relatively informative. The auditor issues qualified opinion when she observes \( \Omega_b \) regardless of whether the true accounting earnings is \( R_G \) or \( R_B \). The probability of the project undertaken is then

\[
\Pr (I = 1|S_g, \Omega_b) \Pr (S_g, \Omega_b|G, e_{ND}^*) = \sum_{\gamma=\{\gamma_l, \gamma_h\}} \left\{ \left( \int_{P^*(\gamma)}^1 2pdf \right) \Pr (\Omega_b|G, \gamma) \Pr (\gamma|e_{ND}^*) \right\}
\]

where

\[
\Pr (\Omega_b|G, \gamma_h) \Pr (\gamma_h|e_{ND}^*) = e_{ND}^* \left[ 1 - P^*(\gamma_h) \right]
\]

and

\[
\Pr (\Omega_b|G, \gamma_l) \Pr (\gamma_l|e_{ND}^*) = (1 - e_{ND}^*) \left[ 1 - P^*(\gamma_l) \right]
\]

with \( P^*(\gamma) = q\gamma + (1 - q)(1 - \gamma) \).

The probability of the project undertaken conditional on the underlying state being \( B \) can be analogously derived. Combining terms and after fair intense algebra, we can express \( \Pi_D \) and \( \Pi_{ND} \) as

\[
\Pi_D = e_D^*f(\gamma_h) + (1 - e_D^*)f(\gamma_l) + e_D^*[1 - P^*(\gamma_h)]^2 + (1 - e_D^*)[1 - P^*(\gamma_l)]^2 \tag{21}
\]

\[
\Pi_{ND} = f(e_{ND}^*\gamma_h + (1 - e_{ND}^*)\gamma_l) + e_D^*[1 - P^*(\gamma_h)]^2 + (1 - e_D^*)[1 - P^*(\gamma_l)]^2 \tag{22}
\]

where \( f(\gamma) \) is defined as

\[
f(\gamma) = -[P(\gamma) - P^*(\gamma)]^2 + P^{2*}(\gamma) = [1 - (\rho(\gamma) - 1)^2] P^{2*}(\gamma).
\]

Thus when \( e_D^* = e_{ND}^* = e^* \),

\[
\Pi_D - \Pi_{ND} = [e^*f(\gamma_h) + (1 - e^*)f(\gamma_l)] - f(e^*\gamma_h + (1 - e^*)\gamma_l).
\]

Taking the second order derivative on \( f \), we have

\[
f''(\gamma) = \frac{2(1 - 2q + \alpha q^2)}{[1 - \alpha q (1 - \gamma)]^4} \Psi(\gamma)
\]

with \( \Psi(\gamma) = (1 - 2q) + \alpha q^2 (7 - 8\gamma) + 2\alpha q (\alpha q^2 + 2) (\gamma - 1) \).

We now claim that \( \Psi < 0 \) when \( \gamma \geq \frac{1}{2} \). To see this, note that it is obvious when \( \gamma \geq \frac{7}{8} \) as the entire term is negative. Next consider the case \( \frac{1}{2} \leq \gamma < \frac{7}{8} \). Since the expression is linear
with respect to $\gamma$, if we can show that the expression is negative at both $\frac{1}{2}$ and $\frac{7}{8}$, then we are done. When $\gamma = \frac{1}{2}$, the expression is $1 - 2q - 2\alpha q + 3\alpha q^2 - \alpha^2 q^3$. Standard maximization techniques can show that this function achieves its maximum of $-\frac{1}{8}\alpha(2 + \alpha) < 0$ when $\alpha > 0$. When $\gamma = \frac{7}{8}$, the expression is obviously negative as all terms are either strictly negative or zero. Therefore, when $q < q^*$, $1 - 2q + \alpha q^2 > 0$ and $f''(\gamma) < 0$, which implies that $f(\gamma)$ is concave and the insurance effect dominates the Blackwell effect. Similarly, when $q > q^*$, $1 - 2q + \alpha q^2 < 0$ and $f''(\gamma) > 0$. Thus, $f(\gamma)$ is convex, implying that Blackwell effect dominates the insurance effect. When $q = q^*$, $f''(\gamma) = 0$, implying $IE_D = IE_{ND}$. Q.E.D.

Proof of Proposition 5

At $q = q^*$, $\overline{P}(\gamma_h) = \overline{P}(\gamma_l) = \overline{P}(\gamma_{ND})$ and $e^*_ND = e^*_D$ at $q = q^*$, implying that the audit fee is the same across the two regimes when $q = q^*$.

When $q > q^*$, with a slight abuse of notation, define $M(\gamma) = (1 - \gamma)\overline{P}^2(\gamma)$. Taking the second order derivative of $M(\gamma)$ with respect to $\gamma$ results in

$$\frac{\partial^2 M(\gamma)}{\partial \gamma^2} = \frac{2(2q - 1 - \alpha q^2)[-3(1 - \gamma) + q(4 - 6\gamma) - \alpha(1 - \gamma)q^2]}{[1 - \alpha(1 - \gamma)q]^4}.$$ 

When $q > q^*$, $2q - 1 - \alpha q^2 > 0$. Furthermore, $-3(1 - \gamma) + q(4 - 6\gamma) - \alpha(1 - \gamma)q^2 \leq -3(1 - \gamma) + q(4 - 6\gamma)$. Note that $q(4 - 6\gamma)$ is either increasing or decreasing in $q$. When $q = 1$, $-3(1 - \gamma) + q(4 - 6\gamma) = 1 - 3\gamma < 0$ as $\gamma \geq \frac{1}{2}$. When $q = \frac{1}{2}$, $-3(1 - \gamma) + q(4 - 6\gamma) = -1 < 0$. Therefore $-3(1 - \gamma) + q(4 - 6\gamma) - \alpha(1 - \gamma)q^2 \leq -3(1 - \gamma) + q(4 - 6\gamma) < 0$. This implies that $M(\gamma)$ is strictly concave when $q > q^*$.

$$\begin{align*}
\alpha K(1 - \mu)q[e^*_D(1 - \gamma_h)\overline{P}^2(\gamma_h) + (1 - e^*_D)(1 - \gamma_l)\overline{P}^2(\gamma_l)] &+ \frac{1}{3}ce^3_D \\
&< \alpha K(1 - \mu)q[e^*_ND(1 - \gamma_h)\overline{P}^2(\gamma_h) + (1 - e^*_ND)(1 - \gamma_l)\overline{P}^2(\gamma_l)] + \frac{1}{3}ce^3_{ND} \\
&= \alpha K(1 - \mu)q[e^*_NDM(\gamma_h) + (1 - e^*_ND)M(\gamma_l)] + \frac{1}{3}ce^3_{ND} \\
&< \alpha K(1 - \mu)qM(e^*_ND\gamma_h + (1 - e^*_ND)\gamma_l) + \frac{1}{3}ce^3_{ND} \\
&= \alpha K(1 - \mu)q[(1 - e^*_ND\gamma_h - (1 - e^*_ND)\gamma_l)\overline{P}^2(\gamma_{ND}) + \frac{1}{3}ce^3_{ND} \\
&= \alpha K(1 - \mu)q[e^*_ND(1 - \gamma)\overline{P}^2(\gamma_{ND}) + (1 - e^*_ND)(1 - \gamma)\overline{P}^2(\gamma_{ND}) + \frac{1}{3}ce^3_{ND}.
\end{align*}$$

The first inequality obtains because $e^*_D$ is the unique minimizing solution to $\alpha K(1 - \mu)q[e(1 - \gamma_h)\overline{P}^2(\gamma_h) + (1 - e)(1 - \gamma_l)\overline{P}^2(\gamma_l)] + \frac{1}{3}ce^3$ and $e^*_ND \neq e^*_D$ at $q \neq q^*$. The second inequality is because of the concavity of $M$. We therefore showed that the audit fee is greater in the No Disclosure regime when $q > q^*$. Q.E.D.
Proof of Proposition 6

For the No Disclosure regime: as shown in the proof of claim 1, our comparative static analysis on $IE$ with respect to $q$ can be equivalently performed on $\Pi = \Pr(I = 1|G) - \Pr(I = 1|B)$. Note that

$$d\Pi_{ND} \over dq = \partial \Pi_{ND} \over \partial q + \partial \Pi_{ND} \over \partial e_{ND}^* \over dq.$$

Based on (22), the first term is

$$\partial \Pi_{ND} \over \partial q = e_{ND}^* [(q_1 - 2)P + (q_1 - 2)\gamma_h + 2 - 2q - 2P \partial P \over \partial q + (q_1 - 2) [q_1 + (1 - q)(1 - \gamma_h) - 1] ]
+ (q_1 - 2)P + (q_1 - 2) \gamma_l + 2 - 2q - 2P \partial P \over \partial q + (q_1 - 2) [q_1 + (1 - q)(1 - \gamma_l) - 1].$$

At $\gamma_h = \gamma_l = 1/2$,

$$\partial \Pi_{ND} \over \partial q = (1 - 2P) \partial P \over \partial q = -\frac{1}{2} \frac{\alpha}{2} \frac{(2 - \alpha)\alpha}{2} < 0.$$

Earlier proof shows

$$d e_{ND} \over dq = \frac{\partial (LHS of (11)) \over dq}{\partial e_{ND}^* \over dq} = \frac{\gamma_h - \gamma_l}{2} (1 - \mu) \alpha K \left[P + 2q (\gamma_h - \gamma_l) \partial P \over \partial q\right].$$

Note that when $\gamma_h = \gamma_l = 1/2$, the numerator equals zero while the denominator is strictly positive due to the assumption that $C''(0) > 0$. Hence, $\partial \Pi_{ND} \over \partial q < 0$ when $\gamma_h = 1/2$. By continuity, there must exist a $\gamma_0 > 0$ such that $\forall \gamma_h < \gamma_0$, $\partial \Pi_{ND} \over \partial q < 0$.

At $\gamma_h = \gamma_l = 1$, $\partial \Pi_{ND} \over \partial q = 4q - 2 > 0$. Also recall $d e_{ND} \over dq > 0$. After some tedious algebra, we have

$$\partial \Pi_{ND} \over \partial e_{ND}^* = [P^*(\gamma_h) - P^*(\gamma_l)] [P^*(\gamma_h) + P^*(\gamma_l) + 2 \over \partial P \over \partial q - 2]$$

When $\gamma_h = \gamma_l = 1$, $P^*(\gamma_h) + P^*(\gamma_l) + 2 \over \partial P \over \partial q - 2 = 4q - 2 > 0$. Hence, $\partial \Pi_{ND} \over \partial q > 0$ when $\gamma_h = \gamma_l = 1$. By continuity, there must exist a $\gamma_0 < 1$ such that $\forall \gamma_l > \gamma_0$, $\partial \Pi_{ND} \over \partial q > 0$ and thus $\partial \Pi_{ND} \over \partial e_{ND}^* > 0$.

Similarly for the Disclosure regime, we show that $IE$ decreases (increases) with respect to $q$ when $\alpha$ and $\gamma_h$ ($\gamma_l$) are sufficiently small (large). We first check the three components in $d \Pi_{dis} \over dq$, expressed as:

$$d \Pi_{dis} \over dq = \partial \Pi_{dis} \over \partial q + \partial \Pi_{dis} \over \partial e_{dis}^* \over dq.$$

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Corollary 3 shows that $\frac{d\varepsilon_D}{dq} \leq 0$ when $\alpha$ and $\gamma_h$ are relatively small. Further $\frac{\partial \Pi_D}{\partial \varepsilon_D}$ evaluated at $\alpha = 0$ is positive, as

$$\frac{\partial \Pi_D}{\partial \varepsilon_D}|_{\alpha=0} = 2[P^*(\gamma_h) - P^*(\gamma_l)][P^*(\gamma_h) + P^*(\gamma_l) - 1] > 0,$$

which implies $\frac{\partial \Pi_D}{\partial \varepsilon_D} > 0$ when $\alpha$ is sufficiently small. Finally, based on (21), it is easy to show that at $\gamma_h = \gamma_l = 1/2$, $\frac{\partial \Pi_D}{\partial q} = -\frac{\frac{1}{2}aq}{\frac{1}{2}aq} < 0$, which implies that $\frac{\partial \Pi_D}{\partial q} < 0$ when $\gamma_h$ is relatively small. Thus, $\frac{\partial \Pi_D}{\partial q} < 0$ under the condition specified in the proposition.

Consider the function $z(\gamma) = P(\gamma)[2P^*(\gamma) - P(\gamma)]$. Straight-forward algebra shows that when evaluated at $\alpha = 1$,

$$\frac{dz(\gamma)}{d\gamma} = 2 \left[ -1 + (5 - 3\gamma)q - (1 - \gamma)(7 - \gamma)q^2 + (3 - \gamma)(2 - \gamma)q^3 \right] \frac{1 - q + \gamma(2q - 1)}{1 - (1 - \gamma)q}$$

When $\gamma = 1$, $\frac{dz(\gamma)}{d\gamma} = q(2q - 1 + 2q^3) > 0$. Thus, by continuity $\frac{dz(\gamma)}{d\gamma} > 0$ and $\frac{\partial \Pi_D}{\partial \varepsilon_D} > 0$ when $\gamma$ and $\alpha$ is sufficiently large. Finally, at $\gamma_h = \gamma_l = 1$, one can show similarly to that in the No Disclosure regime that $\frac{\partial \Pi_D}{\partial q} = 4q - 2 > 0$, which implies that $\frac{\partial \Pi_D}{\partial q} > 0$ when $\gamma_l$ is sufficiently large. Q.E.D.