Uniform vs. Discretionary Regimes in Reporting Information with Unverifiable Precision and a Coordination Role*

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Abstract

We examine uniform and discretionary regimes for reporting about firm performance from the perspective of a standard setter, in a setting where the precision of reported information is difficult to verify and the reported information can help coordinate decisions by users of the information. The standard setter’s task is to choose a reporting regime to maximize the expected decision value of reported information for all users. The uniform regime requires all firms to report using the same rules regardless of the precision of their information, and the discretionary regime allows firms to freely condition their reporting on the precision of their information. Our analysis identifies three main characteristics of the reported information as determinants for the choice between the two regimes: the coordination role of the reported information, the ex ante dispersion in firms’ precision, and the ex ante likelihood that the precision is high. We show that when unverifiable information precision varies across firms and users’ decisions based on reported information have strong strategic complementarities, a uniform regime can have a beneficial social effect as compared to a discretionary reporting regime.
1 INTRODUCTION

We study how much discretion firms should have in reporting information about their performance from the perspective of a benevolent social planner (a standard setter) whose objective is to maximize the collective value of reported information in assisting investors in their decisions (e.g., whether to invest in the firm’s securities) that may exhibit strategic complementarities. In our setting, the reported information is derived from transactions data and other items (“evidence” or “signals”) amassed by firms, and the precision of these signals with respect to the payoff that matters to investors is difficult to verify. The social planner/standard setter chooses between two reporting regimes: a uniform regime and a discretionary regime that differ in terms of the degree of discretion/free choice firms have in how to report their information.

We focus on the types of discretion that can affect the publicity and clarity of reported information, where publicity refers to the ease (the likelihood) with which an individual investor can access and interpret the reported information and clarity refers to the extent to which the investor can accurately assess the firm-specific precision of the reported information. In the uniform regime, firms must choose from among pre-specified reporting methods that achieve a specific level of publicity regardless of the precision of their signals. In contrast, the discretionary regime imposes no restrictions and permits firms to freely choose how to report as a function of the precision of their signals, which may result in different levels of publicity of reported information. We show that by affecting firms’ discretion over publicity, different regimes can affect clarity, and that the trade-off between publicity and clarity is a key consideration in determining the optimal regime from a social welfare perspective. We also find that the degree of strategic complementarities in investors’ decisions and the unverifiable nature of the precision of the reported information are important determinants for the uniform regime to achieve higher social welfare than the discretionary regime.

1 Investors’ decisions exhibit strategic complementarities when the marginal payoff for an individual investor’s action is increasing in the average decision of all other investors. We use the terms “investors” and “users” interchangeably to refer to current and potential equity investors and creditors, including possibly providers of trade credit.
Our focus on publicity and clarity as the distinguishing characteristics of information produced under the uniform vs. discretionary regimes is motivated by the long-standing debate over the desirability and consequences of accounting standards, viewed as a commitment to report financial information in a restricted way. On the one hand, a benefit of accounting standards is to increase publicity as they provide definitions of terms, concepts, presentation formats and classifications that make the reported information more readily accessible to a wider user group. On the other hand, reporting under standards are viewed by some as costly and even undesirable because they reduce firms’ discretion about how to report and thereby reduce the clarity of information available to users (e.g., Sunder 2010).

We contribute to this debate by formalizing the trade-off between the publicity and clarity effects of reporting under accounting standards in an analytical setting. Our primary purpose is to use the setting to identify conditions under which the trade-off favors the uniform regime, which we associate with restrictive accounting standards. In doing so, we hope to shed light on the question of what types of information are more suitably reported under accounting standards, and what types of information are more suitably reported in ways chosen freely by firms.

We model a continuum of firms; each firm is a reporting entity with an information system that provides signals/evidence, e.g., transactions data and other information items

\footnote{Since securities laws may require precommitment to apply specified financial reporting standards, for example, the US SEC’s requirement that US registrants apply US GAAP, both the commitment and the ways of reporting, the standards themselves, are sometimes viewed as little more than by-products of securities laws, which are in turn subject to their own debates. Bushman and Landsman (2010) write "the reality [is] that the regulation of corporate reporting is just one piece of a larger regulatory configuration, and that forces are at play that would subjugate accounting standards setting to other regulatory demands." The debate on whether mandatory disclosure is optimal has a long history, and remains largely unresolved. See, for example, Grossman (1981), Easterbrook and Fischel (1984), Admati and Pfleiderer (2001), and Leuz and Wysocki (2008). However, standards sometimes arise from non-regulatory sources. For example, Jamal, Maier, and Sunder (2003) document that US e-commerce firms voluntarily adopt standards on protecting customer privacy. Brockholdt (1978) describes how accounting standards were developed for the railroad industry during the Industrial Revolution. The first US association of public accountants, established in 1887, wrote accounting standards and set codes of conduct long before the passage of the Securities Acts of 1933 and 1934. For details, see: http://www.accountingfoundation.org/jsp/Foundation/Page/FAFSectionPage&cid=1351027541272.}

\footnote{We do not address the effects of uniform vs. discretionary regimes in requiring disclosure of information that otherwise would not be forthcoming from firms, due to, for example, proprietary costs or agency considerations. Rather, we consider when the way information is disclosed should follow prescribed rules such as US GAAP or IFRS versus being entirely subject to free choice.}
of interest to its investors. Each signal has an amount or realization that is verifiable (hard) and can take a continuum of values, and a precision that is soft (unverifiable), can be either high or low, and is privately observed only by the firm itself. To illustrate, a firm that reports its (estimated) warranty obligation keeps transaction records of sales of goods under warranty, including a description of the item, the warranty terms and the invoice price; these items and amounts can be verified. However, the precision of the reported warranty obligation, referring to the probability the firm will actually pay the exact amount of its reported (estimated) warranty obligation, is both soft and privately known by the firm. That is, the firm has private information about the quality of goods it sells under warranty and the probabilities and costs of warranty claims.

We assume each firm wishes to maximize the aggregate utility of its own investors whose actions upon receiving the reported information may exhibit strategic complementarities in that the marginal payoff to each investor’s action is affected by the average actions of all other investors (Morris and Shin (2002), Angeletos and Pavan (2004)). For example, investors in a secondary market for a firm’s securities will be more willing to invest if they believe other investors will likely do the same and therefore create a liquid market. While there is general incentive alignment between the firm’s interest and investors’ interest in aggregate, because each investor does not take into account the positive externality of his action for the welfare of others, investors may not sufficiently coordinate their actions, failing to recognize that the more each investor coordinates with others, the better off everyone is. This between-investor externality can cause individual preferences for coordination to diverge from the firm’s preference (Angeletos and Pavan (2004)).

Given these assumptions, we represent the standard setter as a benevolent social planner.

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4The importance of strategic complementarities in financial markets has long been recognized. Strategic complementarities can lead to coordination failure in that agents fail to coordinate their actions to achieve socially optimal outcomes. Coordination failure can have significant macro-level effects by affecting the stability of financial intermediaries specifically, and of financial markets in general (e.g., Diamond and Dybvig (1983), Goldstein and Pauzner (2005), Amador and Weill (2010), Goldstein, Ozdenoren, and Yuan (2011)). It can also have micro-, firm-level effects by affecting, for example, firms’ liquidity and financial constraints (He and Xiong (2012a, 2012b)) or stock price efficiency (e.g., Allen, Morris, and Shin (2006), Gao (2008), Chen, Huang, and Zhang (2014)).
who maximizes the aggregate utility for all investors of all firms by choosing between a uniform regime and a discretionary regime. Both regimes allow firms to directly announce their unverifiable precision, although, as we will show, such announcements may not be deemed credible by investors. The two regimes differ in terms of the degree of discretion afforded to firms. In the uniform regime, firms are required to report using pre-specified rules and methods that result in an identical level of publicity regardless of their signal precision; in the discretionary regime, firms may freely choose any rules or methods to report, which can result in different levels of publicity.\(^5\) In choosing between the regimes, the standard setter relies on three factors that are assumed to be commonly known by all players: the magnitude of the need to coordinate actions among investors (the coordination need); the \(\text{ex ante}\) dispersion of the precision of the signals obtained from firms’ information systems (i.e., the difference between the two possible levels of precision), and the \(\text{ex ante}\) likelihood of obtaining signals with greater precision.\(^6\)

We first show that when the coordination need is weak there exists an equilibrium that achieves first-best social welfare, where all firms truthfully reveal their precision and choose reporting methods that achieve full publicity. In this case, the uniform and discretionary regimes achieve the same social value as long as the uniform regime permits reporting methods that can achieve full publicity. This is because when the coordination need is low the between-investor externality is insignificant such that firms do not wish to mislead their investors with regard to their signal precision. However, when investors sufficiently care about coordinating their actions, the between-investor externality becomes so severe that investors would not view firms’ announcements of their precision as credible, and, consequently, different equilibria emerge from different regimes.

Under the uniform regime, which constrains reporting methods such that all firms offer the same level of publicity, investors cannot update their beliefs regarding firms’ precision

\(^5\)That is, we assume it is common knowledge that a mapping exists between each reporting method and the publicity of the reported information. Observing the reporting method is equivalent to observing the publicity.

\(^6\)The standard setter need not know the actual realization of the signal/evidence and its precision at each individual firm.
from firms’ publicity choices and thus can only assign the *expected* precision to all firms. In this case, the reporting lacks clarity, because investors do not fully understand the precision of the reported information. We show that, under this regime, the optimal solution for the standard setter is to require full publicity, even though investors apply the *expected* rather than *actual* precision assessment to every firm.

Under the discretionary regime, each firm can freely condition its publicity on the precision of its signal. We show there exists a separating equilibrium in which high-precision firms restrict publicity (e.g., by using presentations and methods that are difficult for some investors to understand) and low-precision firms do not. The intuition for this finding derives from the fact that firms’ incentives to truthfully reveal the precision of their signals are weakened when the coordination need is strong. This is because when investors do not fully internalize the impact of their actions on other investors’ utilities, they under-utilize the coordination value of the reported information by placing a lower weight on the information than firms would have preferred (Angeletos and Pavan (2004)). Investors’ inefficient use of the coordination value is reduced if they believe firms’ reports of their signals have higher precision than is in fact the case, so low-precision firms have incentives to pool with high-precision firms, and the incentives increase with the strength of the coordination need. To separate themselves from low-precision firms, high-precision firms are forced to reduce the publicity of their reporting, as a credible signal about their precision. The signal is costly because partial publicity reduces coordination between investors who can access and understand the reported information and those who cannot. However, the marginal cost is lower for high-precision firms than for low-precision firms, because high-precision information can move the posterior payoff assessment of investors who can access it closer to the actual fundamental value, which in turn better coordinates their actions with the actions of investors who cannot access and understand the reported information but whose private information is nonetheless correlated with the fundamental.

In the context of our model, the trade-off between publicity and clarity is the key consideration for evaluating the two regimes from the standard setter’s perspective. The uniform regime maximizes the publicity of reported information at the expense of clarity because
firms are limited in their ability to communicate the precision of the reported information. In contrast, the discretionary regime necessitates a compromise on publicity by firms with high-precision information in order to credibly communicate their actual precision. The social inefficiency from restricted publicity in the discretionary regime is traded off against the social inefficiency of reduced clarity in the uniform regime.

The trade-off between publicity and clarity also sheds light on the role played by the standard setter in our model to select a regime to optimally balance inefficiencies from two related externalities, one of which is the afore-mentioned between-investor externality. The second and more subtle between-firm externality includes two sides of the same coin: under the uniform regime, low-precision firms alleviate the between-investor externality by pooling with the high-precision firms, thus imposing a negative externality on the latter, while under the discretionary regime high-precision firms credibly separate themselves via restricted publicity, imposing a negative externality on their low-precision counterparts. The standard setter’s regime choice optimally balances the two externalities: under the uniform (discretionary) regime, the between-investor externality for the low-precision firms is alleviated (exacerbated) at the expense (benefit) of increased (reduced) between-firm externality for the high-precision firms. We compare the expected social value of information under the two regimes and show numerically that the uniform regime achieves higher social welfare when the coordination need is high, when the dispersion among firms’ information precision is high, and when the \textit{ex ante} probability of high precision is high. Otherwise, the discretionary regime is preferred.

To the extent the uniform regime resembles financial reporting standards as embodied, for example, in US GAAP and in International Financial Reporting Standards (IFRS), our analyses generate insights and implications for the evaluation of financial reporting standards. First, while criticisms of reporting standards point to their restrictive nature as a deficiency, our analysis shows that standards may be socially optimal \textit{because} they are restrictive, as long as the precision of reported information is inherently difficult to verify and the reported information plays an important coordination role. We believe that the information environment of financial reporting generally meets the first condition, especially when the
reported information requires subjective estimates by firm managers. For example, while it is easy to verify the principal amounts of loans outstanding, it is difficult to verify lenders' assessments of credit risk as embodied in their estimates of loan loss provisions. The difficulty is exacerbated if information precision varies by time, across firms, and by the type of information; for example, fair value estimates of financial assets become less precise if markets for those assets become illiquid. The second condition, referring to the importance of the coordination value of financial reports, is consistent with the IASB's and FASB's conceptual framework objective of setting standards for information that is relevant, in the sense of being capable of making a difference in the decisions of users of financial reports, including current and potential investors and creditors (e.g., SFAC 8, para. OB2; para. QC6-QC10). However, since reported information serves a coordination role only when there are strategic interactions among users' actions, our analysis points out that being decision-useful for a large number of users is not, in and of itself, sufficient to justify reporting standards; rather, the value derives from significant interactive effects among actions investors take based on the reported information.

Second, our analysis shows that the value of the uniform regime is maximized at full publicity for all firms, balanced against its effect on clarity. This result provides a theoretical justification for the FASB's and IASB's conceptual frameworks' focus on general purpose financial reporting to users who cannot require firms to provide information directly to them, and to "seek to provide the information set that will meet the needs of the maximum number of primary users" (SFAC 8, para OB2-OB5; para QC35-QC38; para. OB8). The conceptual frameworks describe desirable characteristics of information produced by applying standards, including comparability, consistency, understandability, faithful representation and verifiability. While our analysis does not directly speak to which regime, uniform

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7Chapter 3 of the FASB's Statement of Financial Accounting Concepts No. 8, (SFAC 8) Conceptual Framework of Financial Reporting (FASB, 2010) is converged with the relevant portions of the IASB's Framework. Both conceptual frameworks describe several enhancing qualitative characteristics of information that is relevant and faithfully represented, including comparability, consistency, understandability and verifiability. Faithful representation is a fundamental characteristic, meaning that the financial reporting depictions of economic phenomena are complete (containing all the necessary information for a user to understand the phenomenon being depicted), neutral and free from error. Freedom from error does not mean
or discretionary regime, will result in reported information with these characteristics, our analysis highlights the importance of evaluating these characteristics by their effects on the publicity and clarity of the reported information. To the extent publicity can be linked to comparability, consistency, and understandability, and clarity can be linked to faithful representation and verifiability, our analysis identifies a trade-off among these characteristics.

To the extent publicity is affected by the number of methods allowed under the uniform regime (e.g., US GAAP allows both LIFO and FIFO cost-flow assumptions for inventory whereas IFRS allows only FIFO), or the choice of how to display and present the information (e.g., presenting cash flow information following the format required by SFAS 95), our model provides a theoretical justification for rules governing these choices. Our model is silent on the content of standards, for example, whether, for specific commercial arrangements and transactions, disclosure or recognition achieves higher publicity and why, and whether, for specific accounts, a tabular reconciliation increases publicity. To the extent standards allow more than one method to calculate an estimate, our analysis is also silent as to which methods should be permitted to maximize the trade-off between publicity and clarity. Instead, our analysis suggests that standards should restrict choices that can enable firms to credibly achieve clarity but are at the same time costly to implement and instead focus on improving publicity.

Third, our analysis suggests both a reason why standard setting is sometimes left to private, non-governmental organizations, which usually make substantial efforts to address all participants’ concerns, and a reason why enforcement of the resulting standards is essential.

accurate; for example, “an estimate of an unobservable price or value cannot be determined to be accurate or inaccurate. . . . [A] representation of that estimate can be faithful if the amount is described clearly and accurately as being an estimate, the nature and limitations of the estimating process are explained, and no errors have been made in selecting and applying an appropriate process. . . . (para. QC15, Chapter 3, SFAC 8). Comparability enables users of financial information to identify and understand similarities and differences among items; consistency, which contributes to the goal of comparability, refers to using the same methods for the same items over time and across entities. Understandability refers to ways of classifying, characterizing and presenting information clearly and concisely, with the goal of making that information comprehensible. Verifiability means that different knowledgeable, independent parties would reach consensus (but perhaps not complete agreement) that a particular accounting depiction of a financial statement item faithfully represents that item. Verifiability can be direct (vouching or confirming to an external source) or indirect (rechecking the inputs to a measurement technique and recalculating the outputs). SFAC 8, para. QC26 emphasizes that both ranges of possible amounts and related probabilities can be verified.
With regard to the former, the role of the standard setter in our model is to optimally balance the two aforementioned externality problems by selecting a reporting regime to maximize aggregate welfare of firms and investors. As either regime creates winners and losers (high-precision firms would like to credibly separate under the uniform regime and low-precision firms would like to pool under the discretionary regime), it is essential for social welfare maximization that the standard setter is not captured by a self-interested party but is instead representative of all those involved. This point seems consistent with a number of institutional features of accounting rule making bodies. For example, the IASB’s constitution specifies an IFRS Advisory Council that provides advice on agenda decisions and standard setting priorities; IASB board meetings are open to the public; and the IASB seeks comment, in the form of comment letters, and consultation, in the form of hearings and roundtable discussions, on its proposals. The FASB has similar arrangements. With regard to the latter, our analysis implies that under the uniform regime high-precision firms would be better off separating via reduced publicity. Therefore, it is crucial for social welfare maximization to strictly enforce the uniform regime and prevent high-precision firms from imposing a negative between-firm externality on their low-precision counterparts. The importance of enforcement is consistent with empirical research suggesting that the beneficial effects of adopting IFRS depend on a country’s strength of enforcement, e.g., Li (2010), Landsman, et al. (2012), and Yip and Young (2012).

The rest of the paper unfolds in five sections. Section 2 reviews the related literature. Section 3 describes our basic model and section 4 provides preliminary results. Section 5 characterizes the equilibria under the uniform and discretionary regimes and explores the normative question of whether and when the uniform regime is preferred to the discretionary regime. Section 6 summarizes our findings and discuss their implications. Appendix A contains proofs of the results presented in the body of the paper.

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8The IASB constitution is available at www.ifrs.org/the-organisation/governance-and-accountability/constitution/
2 RELATED LITERATURE

Our paper is related to the literature that evaluates the costs and benefits of disclosure rules for information about firm performance in the presence of externalities in the financial markets, both among firms (Admati and Pfeiderer (2001)) and among investors (Morris and Shin (2002), Angeletos and Pavan (2004, 2007), Cornand and Heinemann (2008)). Most papers in this literature examine whether more precise public disclosure is beneficial, assuming the precision of public disclosure is publicly known by investors. We extend and contribute to this literature by examining how to disclose, specifically, by comparing uniform regime with a discretionary regime, when the precision of the disclosed information is not known by investors (i.e., unverifiable) whose actions exhibit strategic complementarities.

We show that the externalities among users’ actions lead to users’ inefficient use of public information, which, when the information precision is unverifiable, result in externalities among firms’ reporting decisions in that high-precision firms have to take costly actions to signal their types in the discretionary regime. Our analysis identifies conditions under which these social inefficiencies can be reduced by uniform reporting rules, whose social value can be assessed by the trade-off between publicity and clarity of the reported information under these rules. As such, our paper contributes to the debate about the social value of reporting under standards (Dopuch and Sunder (1980), Sunder (2010)) by providing a formal analysis that links the social value of standards to the coordination role and unverifiable precision of the reported information.

In this regard, our paper is related to Admati and Pfleiderer (2001), which to the best of our knowledge provides the first formal analytical evaluation of how different disclosure rules help reduce certain inefficiencies/externalities among firms. In their model, each firm derives a private value from providing informative public disclosure, where the precision of the disclosure is assumed to be known to all investors. The externalities arise when firms’ fundamentals are correlated and when improving precision is costly, in which case firms free-ride on others’ disclosure, resulting in socially inefficient precision levels in equilibrium. Admati and Pfleiderer (2001) show that neither regulations that require a common minimum
disclosure precision nor subsidies to the costs of disclosure can achieve strictly higher social welfare than the market equilibrium. Our paper differs from theirs in that we do not assume correlations among firms’ fundamentals and instead focus on frictions due to strategic complementarities among investors of a given firm and the unverifiable nature of the precision of firms’ information.\textsuperscript{9} Our analysis suggests that uniform reporting rules can lead to higher social welfare than the equilibrium under the discretionary regime when the coordination role of information is strong.

Our paper is also related to the accounting literature that evaluate the costs and benefits of mandatory disclosure in the presence of conflicts between different parties of a given firm (i.e., the stewardship role of financial reporting), including frictions between investors as the principal and firm managers as the agent (e.g., Gigler and Hemmer (1998, 2001)), or frictions between current shareholders and future shareholders (e.g., Kanodia and Lee (1998), Kanodia, et al. (2005)), or a combination of both (Dye and Verrecchia (1995)).\textsuperscript{10} In these studies mandatory disclosure is assumed to be verifiable, with precision publicly known by investors whose actions do not exhibit strategic complementarities. In our setup, the degree of conflict between firms and their investors is endogenized as a function of the coordination

\textsuperscript{9}We believe the assumption of unverifiable information precision captures an important aspect of the actual financial reporting environment. For example, both Boeing and Nordstrom may extend trade credit to their customers, whose probabilities of default are unlikely to be highly correlated. As far as investors are concerned, however, the two firms are alike in that the precision of their bad debt expense estimates is unknown and unverifiable. We also believe that considering interactions among current and potential investors and creditors is particularly important in evaluating disclosure rules in financial markets, as opposed to, say, in non-financial markets, for example, disclosures about food and drug safety. For example, while the collective choices of depositors to withdraw cash from a bank can directly affect an individual depositor’s welfare, the collective choices of patients upon receiving a disclosure about a drug trial does not directly affect a given patient’s chance of recovery.

\textsuperscript{10}Specifically, Gigler and Hemmer (1998, 2001) study how frequency and conservatism in mandatory reporting affect owners’ contracting costs to elicit truthful disclosure of unverifiable information privately observed by firm managers. Kanodia and Lee (1998) and Kanodia, et al. (2005) study how measurement rules in financial reporting affect the inferences investors can draw about the unverifiable information that underlies firm decisions, which in turn affect investors’ pricing of the firms and in equilibrium can affect firm decisions themselves. Dye and Verrecchia (1995) show that discretion is preferred if the only conflict is between the existing shareholders and the manager, while uniformity may be preferred if there is an additional conflict between existing and future shareholders. Fishman and Hagerty (1990) also study the optimal degree of discretion in a stewardship setting. They show that granting more discretion by allowing firms to disclose a signal from a larger permissible set may reduce disclosure efficiency because more discretion affords more opportunity to firms to cherry-pick which signal to report. In contrast, we focus on the effect of discretion on communications of unverifiable information, which is not present in their model.
role of reported information, and the conflict results in social inefficiency when the precision of the information is unverifiable.

Our result regarding how firms disclose their information under the discretionary regime also relates to the broad literature on how information is disseminated and used in the capital markets. Within this broad literature, prior research has analyzed when firms will, or will not, voluntarily disclose verifiable information, where the information can pertain to firms’ performance (e.g., Grossman (1981), Verrecchia (1983), Dye (1985), Jung and Kwon (1988)), or pertain to the precision of a performance indicator (Jorgensen and Kirschenheiter (2003) and Hughes and Pae (2004)).

Prior literature has also studied the disclosure of unverifiable information (e.g., Crawford and Sobel (1982), Gigler (1994), Stocken (2000), Fischer and Stocken (2001)), and of a hybrid form of information where a firm can manipulate its information disclosure by adding bias (e.g., Stein (1989), Fischer and Verrecchia (2000), and Dye and Sridhar (2004)). Our paper extends and contributes to this literature by studying the interaction between verifiable and unverifiable information in a setting with strategic uncertainty. Bertomeu and Marinovic (2014) also examine the interaction between verifiable and unverifiable information. In their model, firm value is the sum of two components with different degrees of verifiability. They study firms’ decision to incur costs to certify the verifiable component, in order to communicate information about the unverifiable component.

Our setting differs in that the unverifiable information pertains to the precision of the verifiable signal, and the cost of communicating the unverifiable information is endogenously determined by disclosure publicity and differs between the uniform and discretionary regimes.

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12 Specifically, in their model, firm value is the sum of two components, both privately observed by the firm. One component is always unverifiable; the firm can incur a certification cost to make the other component verifiable. They show that the firm’s equilibrium certification decision communicates the unverifiable component of its private information.
3 MODEL SET-UP

We adopt a framework similar to that in Angeletos and Pavan (2007) in which public disclosure is socially beneficial, with an important extension that the precision of public disclosure is unknown to investors and is privately observed by the firms. As will become clear soon, this extension allows a role for firms to use discretion to communicate information about the precision of their signals. This in turn allows us to compare the social welfare of two regimes: a uniform regime which imposes restrictions on firms’ disclosure publicity choices and a discretionary regime where firms are free to determine publicity.

3.1 Players’ Objective Functions and Information Structure

Consider an economy with a continuum of firms evenly distributed over a unit interval, each of which has a continuum of investors indexed by $i$ who are also assumed to be evenly distributed over a unit interval $[0, 1]$. Investors represent users of firms’ reported information, which may include current and potential shareholders or creditors. For ease of exposition, we describe our model for a representative firm whenever doing so doesn’t cause confusion.

Let $v \in \mathbb{R}$ be a random variable representing the fundamental uncertainty inherent in a specific aspect of the firm’s business activities (e.g., the dollar amount of loans that would be defaulted, or the remaining useful life of a long-lived depreciable asset). The common prior is that $v$ is uniformly distributed over the real line $\mathbb{R}$. We assume all investors are risk-neutral and have identical preferences given by

$$E_i(U_i) \equiv \max_{e_i} E \left\{ (1 - \rho) v + \rho \bar{e} \right\} e_i - \frac{1}{2} e_i^2 | \Omega_i \right\},$$

where $e_i \in \mathbb{R}$ is an action chosen by investor $i$ and $E_i$ denotes the expectation is taken conditional on investor $i$’s information set $\Omega_i$ at the time of his decision. The marginal return to action is $(1 - \rho) v + \rho \bar{e}$, with $\rho \in (0, 1/2)$ where $\bar{e} \equiv \int_0^1 e_i di$ is the "average" action.

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13Assuming a continuum of agents enables each agent to ignore the impact of his own action on other agents, affording tractability to our analysis.
of all investors.\footnote{\(\rho < 1/2\) ensures that the agent’s utility maximization problem is well behaved (i.e., concave).}

To better appreciate the economic forces embodied in (1), slightly rewrite it as

\[
U_i = (1 - \rho) \left( ve_i - \frac{e_i^2}{2} \right) + \rho \left( \bar{e}e_i - \frac{e_i^2}{2} \right),
\]

which expresses investor \(i\)'s utility as a weighted average of two objectives. The first objective, captured by \(ve_i - \frac{e_i^2}{2}\), is to choose the best "private action" to match the firm’s fundamental such that \(e_i\) is as close to \(v\) as possible. For example, if \(e_i\) is a lending decision, then the "private action" term induces \(i\) to lend according to the firm’s cash flow situation to maximize payoff safety. The second objective is to choose the best "collective action", captured by \(\bar{e}e_i - \frac{e_i^2}{2}\). Given our assumption that \(\rho > 0\), the "collective action" term is maximized by choosing \(e_i\) equal to other investors’ average action, \(\bar{e}\). That is, investors’ actions are strategic complements to each other. For example, creditors are more willing to lend if more customers and suppliers continue to do business with the firm. The magnitude of the parameter \(\rho\) measures the relative weight that investors put on the "private action" versus "collective action" term. Our formulation of (1) reflects the key idea that a firm’s investors are often concerned not only with responding to the firm’s fundamentals but also with coordinating with one another. When this is the case, public information serves two distinct roles: an informative role in helping investors predict a firm’s fundamentals and a coordination role in helping investors predict others’ behavior (Morris and Shin (2002)), with \(\rho\) reflecting the need for coordination.

We assume each investor observes a noisy private signal \(s_i\) of \(v\):

\[
s_i = v + \varepsilon_i,
\]

where the error term \(\varepsilon_i\) is normally distributed and is independent of \(v\), with mean zero and precision \(\beta\) (i.e., the inverse of the variance, \(\beta = 1/\sigma_{\varepsilon_i}^2\)).

The firm observes a noisy signal \(z\) which may be disclosed to investors (we postpone a
detailed discussion of how \( z \) can be disclosed to section 3.2): 

\[
z = v + \eta.
\]

We interpret \( z \) as the firm’s best estimate of \( v \).\(^{15}\) The error term \( \eta \) represents the noise in the firm’s estimate, and is assumed to be normally distributed with mean zero and precision \( \lambda \beta \). \( \lambda \) reflects the relative precision of the firm’s information versus investors’ private signals and is referred to as the precision of public information throughout the paper for convenience.

We assume that \( \lambda \) can take one of two values: \( \lambda_\tau, \tau \in \{ h, l \} \), with \( \lambda_h > \lambda_l \). To capture the essential knowledge gap between investors and firms, we assume it is common knowledge that \( \lambda_h > \lambda_l > 1 \), that is, all types of signals released by firms are known to be more accurate than investors’ private signals. At date 0, all firms and investors have a common prior that \( \lambda \) equals \( \lambda_h \) with probability \( q \), i.e., \( \Pr ( \lambda = \lambda_h ) = q \) and \( \Pr ( \lambda = \lambda_l ) = 1 - q \), and understand that each firm’s \( \lambda \) is independent of others’. We assume \( \lambda_\tau \) is a characteristic of \( z \), and is privately observed by the firm when \( z \) is realized. Although our model is a one-period model, we have in mind that the reporting decision is recurring periodically whereas \( \lambda_\tau \) can change from period to period (that is, precision is not a firm-specific constant feature). That said, for the ease of presentation, from now on we will call a firm with precision \( \lambda_h \) (\( \lambda_l \)) as a type \( h \) (\( l \)) firm.

As will become clear later, investors’ optimal action choice is independent of their perception of \( \beta \) conditional on that of \( \lambda \). Therefore, it is without loss of generality that we also assume the information environments facing the investors are sufficiently complex that they do not have a prior probability assessment on the nominal level of \( \beta \).\(^{16}\) It can be justified on the grounds that market participants aren’t sufficiently well informed about how information is collected and processed to gauge its nominal level of accuracy. Investors do know that their private signal is less accurate than the firm’s signal and that the firm is asymmetrically

\(^{15}\)In the case of banks, for example, \( z \) can represent banks’ estimates of borrowers’ credit-worthiness and likelihood of defaults, or of the fair value of the derivative contracts related to banks’ hedging activities.

\(^{16}\)This technical assumption rules out the possibility that an investor can update his belief of \( \lambda \) by comparing his private signal and the disclosed public signal.
informed about the relative precision of its reported information.

After observing its signal’s relative precision, the firm chooses a disclosure policy (to be detailed next) to maximize the aggregate utility of its own investors. Formally, a type $\tau$ ($\tau = h$ or $l$) firm’s payoff $F_\tau$ is:

$$F_\tau = E \left( \int_0^1 U_i di \mid \lambda = \lambda_\tau \right).$$  \hspace{1cm} (2)

(2) implies that there is a general interest alignment between the firm and its investors (in aggregate) such that firms have no obvious desire to withhold $z$ from its investors.

Finally, a benevolent standard setter regulates firms’ reporting to their investors whose objective is to maximize the aggregate payoff of all firms, equivalent to the aggregate utility of all investors in the economy. Denoting the standard setter’s objective as $SS$. Given our assumption that $\lambda$ is independent between firms, $Pr(\lambda = \lambda_h) = q$, and firms are evenly distributed over a unit interval, it is easy to see

$$SS = q F_h + (1 - q) F_l.$$

The information structure is common knowledge among all players of the model: investors, firms, and the standard setter. We will discuss how the standard setter influences reporting by the firm to its investors in the next subsection.

### 3.2 Reporting Regimes, Timeline, Equilibrium Definition

The firm directly announces $\hat{\tau} = h$ or $l$ regarding its type (i.e. relative signal precision) and discloses $z_i$ to investor $i$, where

$$z_i = \delta_iz + (1 - \delta_i)\emptyset,$$  \hspace{1cm} (3)

where $\delta_i$ is an indicator variable taking the value 1 with probability $\bar{\delta} \in [0, 1]$ and $\delta_i$ is independent of $\delta_{i'}$, $\forall i \neq i'$. That is, an investor gains access to the firm’s signal $z$ with
probability $\bar{\delta}$ and a "null signal", $\emptyset$, with probability $1 - \bar{\delta}$. We label $\bar{\delta}$ as the firm’s disclosure publicity that determines the measure of investors who are informed of $z$. $\bar{\delta}$ ranges from 0 (disclosing to no investor) to full publicity of 1 (disclosing to every investor) and is publicly observable. Note that $\bar{\delta}$ is a measure of investors who are informed of $z$ but not necessarily of the precision of $z$ (i.e., $\lambda$). To make this distinction, from now on, we will use the term clarity to describe the extent to which investor $i$ is able to correctly infer the precision of the firm’s disclosure $z$ (if $\delta_i = 1$). A crucial distinction between $\hat{\tau}$ and $z_i$ lies in their degree of verifiability: while the former is unverifiable (i.e., the firm is free to announce $h$ or $l$ regardless of its true type), the latter is verifiable (i.e., if $\delta_i = 1$, $z$ has to be truthfully disclosed to investor $i$). For notational ease, we define $d \equiv \{\hat{\tau}, \bar{\delta}\}$ as the firm’s disclosure policy, i.e., how the firm reports information to investors.

The standard setter imposes one of two reporting regimes: a uniform regime or a discretionary regime that differs in terms of firms’ discretions in choosing how to report $z$. Specifically, the uniform regime limits firms’ discretion to the set of pre-specified reporting methods that achieve a given level of publicity regardless of the precision of their signals. In contrast, in the discretionary regime, firms can freely condition their reporting methods on the realized precision $\lambda$, and therefore can potentially communicate their private information regarding $\lambda$ via these choices.\footnote{We note that given the binary nature of $\lambda$, it is without of loss of generality to allow only one level of publicity under the uniform regime in our setting. The important assumption here is that the dimension of publicity allowed in uniform regime is smaller than the dimension of firm types. This assumption captures the key idea that the one-size-fits-all nature of uniform regime inevitably involves restricting discretion in communicating firm-specific information.} In other words, we assume a mapping exists from different reporting rules and methods to various levels of publicity, where a given level of publicity can be achieved by a finite set of rules and methods. We assume the mapping is common knowledge. As such, in the discretionary regime, while investors may not directly observe the precision of $z$, they may be able to infer it from the reporting methods firms choose. The timeline of the model is:

**Date 0**: The standard setter decides which one of the two regimes firms must follow, based on the following common knowledge about a given type of evidence/signal: the mag-
magnitude of $\rho$, the binary distribution of $\lambda$ with $\Pr(\lambda = \lambda_h) = q$ and $\Pr(\lambda = \lambda_l) = 1 - q$, and the fact that $\lambda$ is privately observed by the firm.

**Date 1:** The firm privately observes $z$, and discloses $\tilde{\tau}$ and $z_i$ according to the prevailing regime. Investors observe their private signal ($s_i$), and choose actions based on the information available to them, which differs by regime.

**Date 2:** $v$ is realized and investors’ utility materializes.

We adopt *Perfect Bayesian Equilibrium* as our solution concept. A Perfect Bayesian Equilibrium (PBE) is a strategy profile $\{(d), e_i^*(\hat{q}(d), \hat{\delta})\}$ and a set of investors’ updated probability assessments $\{\hat{q}(d)\}$ that the firm’s type is $h$ given the firms’ disclosure policy $d$, such that,

(a) *No firm wishes to deviate, given investors’ beliefs and the equilibrium strategies of the other type;*

(b) *No investor wishes to deviate, given his beliefs, the equilibrium strategies of the firm and the equilibrium strategies of other investors; and*

(c) *Whenever possible, beliefs are updated by Bayes rule from the equilibrium strategies.*

In addition, we apply Cho and Kreps’ (1987) Intuitive Criterion to impose restrictions on off-equilibrium beliefs.

**Cho and Kreps’ Intuitive Criterion:** Consider an out of equilibrium disclosure policy choice $d$. If a type $\tau$ firm ($\tau \in \{h, l\}$) cannot be strictly better off by choosing $d$ regardless of investors’ belief and a type $\tau'$ firm with $\tau' \neq \tau$ strictly benefits by choosing $d$ provided it is correctly perceived, then upon observing a firm selecting $\hat{\delta}$ investors must believe such a firm is $\tau'$ with probability 1.

The Intuitive Criterion requires that off-equilibrium disclosure policy choices be supported by "reasonable beliefs" about the type of firm that would have found it profitable to deviate from the expected equilibrium play. Application of the Intuitive Criterion helps to eliminate unreasonable equilibria and strengthen the model’s prediction.
The model outlined above is highly-stylized with both advantages and disadvantages. One advantage is that it permits technical tractability and easy comparison with prior research. More importantly, it allows us to link the costs and benefits of uniform vs. discretionary regimes to two characteristics of the evidence/signals collected by firms (i.e., whether the evidence serves a coordination role and whether the precision of the evidence is unverifiable), regardless of whether the evidence pertains to firms’ operating, investment, or financing activities. This focus ties our analysis closer to the conceptual frameworks by standard setters which are stated in terms of qualitative characteristics of reported information, as opposed to the type and nature of the underlying economic transactions (for example whether the information is pertaining to firms’ investment, operating, or financing activities). The disadvantage is that we assume the degree of the information’s coordination role ($\rho$) and the degree of unverifiability of information precision ($q$) as exogenously given. As such, our analysis is unable to answer the questions of which economic transactions are more likely to generate evidence with strong coordination role or unverifiability, and what specific uniform rules should be designed for these transactions, questions that are beyond the scope of our current study but are interesting and important to explore in future research.

4 PRELIMINARY RESULTS

To provide a building block to completely characterize the equilibrium solutions under the two regimes, we first derive investors’ equilibrium action choices conditional on their available information and shed light on firms’ incentives to reveal their information precision.

At the time investor $i$ decides his action, his information set $\Omega_i \equiv \{s_i, z_i, q, d\}$ includes his private signal $s_i$, public signal $z_i$, his prior probability assessment of a type $h$ firm ($q$), and the firm’s reporting policy $d \equiv \{\hat{r}, \hat{\delta}\}$ that consists of the firm’s announcement of its

\footnote{We conjecture, but have no way of offering formal proof, that instead of relying on the characteristics for large varieties of economic transactions firms conduct, relying on the characteristics of information provides a more succinct way to state the conceptual frameworks, which provide guidance on both the boundary/nature of standards, meaning what types of information are more suitably reported under what types of authoritative guidance (uniform vs. discretionary regimes in our analysis), and the desirable outcome of reporting according to standards, meaning what types of information should be produced by applying standards.}
type \( \hat{\tau} \) as well as its publicity choice \( \bar{\delta} \). Investor \( i \) selects \( e_i \) to solve

\[
E_i (U_i) \equiv \max_{e_i} E_i \left[ (1 - \rho) \left( v e_i - \frac{e_i^2}{2} \right) + \rho \left( \bar{e} e_i - \frac{e_i^2}{2} \right) \right],
\]

where \( E_i \) is the expectation taken conditional on investor \( i \)'s information set \( \Omega_i \).

Taking the first order condition with respect to \( e_i \) and rearranging, the solution to (4) is

\[
e_i^* = (1 - \rho) E_i [v] + \rho E_i [\bar{e}].
\]

As expected, \( e_i^* \) is increasing in the expected fundamental \( E_i [v] \) and in the expected average action \( E_i [\bar{e}] \). In principle, finding the investors’ equilibrium action from (5) is complicated by the evolution of higher-order beliefs. \( i \)'s beliefs about other investors’ actions determine \( i \)'s action; in turn, this determines other investors’ actions; in turn, this determines \( i \)'s action;...; and so forth. We are searching for a fixed point to this infinite regress that determines optimal actions. Equation (5) indicates that \( e_i^* \) is linear in the updated beliefs of \( v \) and \( \bar{e} \).

Therefore, we conjecture the equilibrium action as a linear function of the private and public signals. This conjecture turns out correct. In fact, it constitutes the unique equilibrium, as demonstrated by the following proposition.

**Proposition 1: (Investors’ Equilibrium Action Decisions)**

(a) Given publicity \( \bar{\delta} \) and investors’ perceived \( \hat{\delta} \), a unique action equilibrium exists:

\[
e_i^* (\hat{\delta}, \bar{\delta}) = \begin{cases} s_i & \text{for } \delta_i = 0 \\ w (\hat{\delta}, \bar{\delta}) z + \left[ 1 - w (\hat{\delta}, \bar{\delta}) \right] s_i & \text{for } \delta_i = 1 \end{cases}
\]

where

\[
w (\hat{\delta}, \bar{\delta}) \equiv \frac{\hat{\delta} \lambda_{h} + (1 - \hat{\delta}) \lambda_{l}}{1 - \rho \beta \left( 1 - \frac{q_{h} \lambda_{h} + (1 - q) \lambda_{l}}{1 + \lambda_{h}} \right)}.
\]

(b) Denote the expected payoff conditional on \( v \) for firm type \( \tau \) given \( \bar{\delta} \) and \( \hat{\delta} \) as
\[ F_r(\hat{q}, \delta). \] We have

\[ F_r(\hat{q}, \delta) = \frac{v^2}{2} - \frac{1}{2\beta} \left\{ \delta \left( 1 - 2\rho \delta \right) w(\hat{q}, \delta)^2 + \delta \left[ 1 - w(\hat{q}, \delta) \right]^2 + 1 - \delta \right\}. \]

(c) \( F_r(\hat{q}, \delta) \) is increasing in \( \delta \), \( \forall \tau \).

Proposition 1(a) reveals that we can characterize investors' action choices without regard to which regime is imposed. This is because any effect of a regime on investors' behavior operates through its effect on \( \hat{q} \) (i.e., the investors' posterior probability assessment of a type \( h \) firm conditional on the observed disclosure policy \( d \) by the firm). Recall investors have two goals: to choose an action close to the fundamental \( v \), and to match other investors' actions. As a result of the first motive, investors place greater weights on \( z \), the more accurate is \( z \) relative to their private signals. This follows because the investors choose better actions when guided by more precise public information. In addition, when investors' actions exhibit strategic complementarities (i.e., \( \rho > 0 \)), disclosure of \( z \) will have a greater impact on investors' decision: the more investors rely on \( z \), the easier it is for them to forecast and match one another's actions. In this case, public disclosure of \( z \) helps coordinate behaviors and thereby increases the firm's welfare.

Previous analyses on the value of publicity show full publicity maximizes investors' collective welfare, when the precision of the public signal is known (e.g., Cornand and Heinemann, 2008). Part 1(c) extends this finding to our setting, where investors may not directly observe the precision of the public information, by showing that full publicity maximizes the firm's payoff.

Our next task is to determine the firm's motives for revealing its type. Consider a hypothetical situation where the firm's type is publicly known. Suppose the firm could choose the weight \( w^* \) that all investors place on its public disclosure of \( z \) to maximize the firm's objective (i.e., aggregate utility of all investors). Denote \( w^{**} \) as the equilibrium weight that an individual investor would choose to maximize his utility. Then, how would \( w^* \) compare with \( w^{**} \)? The answer is provided in Lemma 1 below.
Lemma 1: Given $\delta$, investors place too little weight on disclosure as:

$$w^{**} = \frac{\lambda_r}{1 + \lambda_r - \rho \delta} < w^* = \frac{\lambda_r}{1 + \lambda_r - 2\rho \delta}.$$  

Lemma 1 shows that there exists some discord with how investors process firm’s disclosure of $z$. With strategic complementarity, investors pay too little attention to the firm’s disclosure. The firm prefers investors to coordinate their behaviors based on the information conveyed in $z$, and investors benefit by doing so. However, investors under-coordinate because they fail to account for how other investors benefit from their coordinating actions, resulting in incentive misalignment between the firm and its investors.

On the one hand, Lemma 1 suggests that a firm may have a motive for misrepresenting its type, in order to persuade investors to pay proper attention to its disclosure. This is confirmed by the following lemma:

Lemma 2: Denote $\frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}}|_{\hat{q}=1}$ as the marginal change in type $h$ firm’s expected payoff with respect to $\hat{q}$ when investors believe the firm to be type $h$ with probability 1. Similarly, denote $\frac{\partial F_l(\hat{q}, \hat{\delta})}{\partial \hat{q}}|_{\hat{q}=0}$ as the marginal change in type $l$ firm’s expected payoff with respect to $\hat{q}$ when investors believe the firm to be type $h$ with probability 0. $\frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}}|_{\hat{q}=1} > 0$ and $\frac{\partial F_l(\hat{q}, \hat{\delta})}{\partial \hat{q}}|_{\hat{q}=0} > 0$.

Lemma 2 states that a type $l$ firm, when correctly perceived by its investors (i.e., at $\hat{q} = 0$), strictly benefits if it can induce a higher perceived $\hat{q}$ on the margin. This implies that a type $l$ may have incentives to pretend to be a type $h$ to induce investors to put a higher weight on its public disclosure. In contrast, a type $h$ firm, when it is correctly perceived (i.e. at $\hat{q} = 1$) and thus unable to further increase investors’ perceived $\hat{q}$, is concerned with being "contaminated" by a mimicking type $l$ and thus receiving a lower perceived $\hat{q}$ on the margin. A type $h$ firm has incentives to credibly convey its type to investors in order to avoid this contamination. On the other hand, Lemma 1 suggests that the degree of incentive misalignment between the firm and its investors varies with $\rho$ (the need for coordinating investor actions). Specifically, the difference between $w^{**}$ and $w^*$ vanishes as $\rho$ approaches
0, implying that when \( \rho \) is small firms may not wish to be perceived as the other type. This reasoning is confirmed by Proposition 2.

**Proposition 2: (Costless Signaling Equilibrium)** Under either regime, there exists a \( \bar{\rho} \) such that with \( \rho \in [0, \bar{\rho}] \) firms truthfully announcing their types directly and selecting full publicity \((\bar{\delta} = 1)\) constitute an equilibrium that satisfies the Intuitive Criterion. That is, in such a costless signaling equilibrium, a type \( h \) firm chooses \( d_h = \{h, \bar{\delta}_h = 1\} \), a type \( l \) firm chooses \( d_l = \{l, \bar{\delta}_l = 1\} \) and investors updated posteriors are \( \hat{q}(d_h) = 1 \) and \( \hat{q}(d_l) = 0 \).

Proposition 2 pertains to settings where conflicts between the firm and its investors are insignificant. The source of disagreement stems from different preferences for coordinating actions, as reflected by the size of \( \rho \). When \( \rho \) is small, in spite of the fact that a type \( l \) firm wishes to increase its investors’ perceived \( \hat{q} \) on the margin, going all the way and pretending to a type \( h \) firm comes at too large a cost of deceiving investors to pay more attention to the firm’s disclosure of \( z \) than is warranted. As a result, in this case, investors’ reaction to the disclosure is aligned with the firm’s preferences; the firm has no incentive to distort information by mis-reporting its type.

We characterize equilibrium solutions under the two regimes for the case where \( \rho > \bar{\rho} \) in the next section.

## 5 MODEL SOLUTIONS

### 5.1 Equilibrium Under Uniform Regime

When \( \rho > \bar{\rho} \), Lemmas 1 and 2 imply that *ceteris paribus* investors will view any direct announcement of \( \hat{\tau} = h \) with suspicion, as type \( l \) has a strict preference to be viewed by investors as type \( h \). As such, the direct disclosure channel \( \hat{\tau} \) becomes cheap talk and would not be believed by investors just by itself. That is, \( \hat{q}(d) \), investors’ perception is independent of the firm’s announcement \( \hat{\tau} \) in equilibrium. Furthermore, because under the uniform regime
the standard setter requires all firms to use the same publicity level, investors are not able to update their beliefs about firm type based on the observed publicity either. Consequently, the uniform regime induces investors to apply their prior probability assessments in determining their equilibrium action choices.

**Corollary 1:** when $\rho > \bar{\rho}$, under the uniform regime, there exists a unique equilibrium that satisfies the Intuitive Criterion:

(a) For a given $\tilde{\delta}$ imposed by the standard setter, investor $i$ chooses action $e_i^* (\tilde{q} = q, \tilde{\delta})$, where $e_i^* (\ast, \ast)$ is defined in Proposition 1.

(b) Standard setter’s objective is increasing in $\tilde{\delta}$, thus maximized at $\tilde{\delta} = 1$. Denoting standard setter’s payoff under the uniform regime with full publicity ($\tilde{\delta} = 1$) as $SS_{UD}$, we have

$$SS_{UD} = qF_h (q, 1) + (1 - q) F_i (q, 1).$$

Corollary 1(a) is a direct consequence of Proposition 1(a) with $\tilde{q} = q$ due to the reasoning preceding the corollary. Part 1(b) shows that the standard setter under this regime will optimally set publicity to 1, thus requiring all firms to disclose their signal $z$’s to all investors. This result directly follows Proposition 1(b), coupled with the fact that the standard setter maximizes the expected payoff of both firm types. We note that the optimality of full publicity ($\tilde{\delta} = 1$) under the uniform regime is consistent with the FASB’s objective to focus on general purpose financial reporting to users who cannot require firms to provide information directly to them, to issue standards that “seek to provide the information set that will meet the needs of the maximum number of primary users” (SFAC 8, para OB2-OB5; para QC35-QC38; para. OB8).

Corollary 1 describes an equilibrium outcome that all investors access and interpret all firms’ reports in the same way, even though that interpretation, represented by investors’ perceived precision, is inaccurate when applied to the actual report by an individual firm.\(^{19}\)

\(^{19}\)It is the rational interpretation from investors’ perspective given their information set.
This result corresponds to a common intuition for what financial reporting standards do: they provide coherent reporting guidance, including definitions, recognition and measurement rules, and rules for classification and presentation that users can apply to interpret reports of the same event or transaction occurring at different firms. At the same time, this result also highlights (endogenizes) a commonly-held view about a cost of reporting under the uniform regime: investors cannot use all the information that is potentially available from firms because the regime limits firms’ discretion to convey all available information, specifically, information about differing precision. Specifically, when the conflict between firms and their investors is significant ($\rho > \bar{\rho}$), the requirement that all firm types adopt the same publicity level renders firms unable to communicate their unverifiable precision to investors. Consequently, investors have to rely on their prior probability assessment of firm types and view every firm as an "average" firm.\textsuperscript{20}

### 5.2 Equilibrium Under Discretionary Regime

When $\rho > \bar{\rho}$, regardless of the disclosure regime, investors do not believe any direct communication of the firm’s unverifiable precision via $\hat{\tau}$ just by itself. However, unlike in the previous section, the firm under the discretionary regime is free to choose any publicity level. While Proposition 1(c) shows that a firm’s payoff is reduced by a decrease in publicity ($\theta$), we note that for the same marginal change in publicity, the magnitude of the corresponding change in the firm’s payoff depends on the firm’s type, thus making publicity choice a possible communication channel for the precision of its disclosure $z$.

To elaborate, with strategic complementarities in investors’ actions, both types of firms benefit from making investors’ actions close to each other. Reduced publicity is costly as

\textsuperscript{20}Recall that firms are still allowed to freely report $\hat{\tau}$, corresponding to the observation that accounting standards do not prohibit firms from providing alternative financial reports outside the uniform regime, for example, pro forma earnings, although the credibility of the alternative reporting is not ensured. What matters in the uniform regime is that standards do not permit choices that can result in a different level of publicity for the reported information, corresponding to the fact that in the US alternative measures must be described and reconciled to the amounts reported under the uniform regime of US GAAP. Refer to the US SEC’s rule, Conditions for the Use of Non-GAAP Financial Measures, Release No. 33-8176; 34-47226; FR-65, effective March 2003.
it adds heterogeneity to investors’ information sets which in turn induces less-coordinated behaviors. This occurs because investors who observe only the null public signal (i.e., \( \delta_i = 0 \)) rely exclusively on their private information \( s_i \) to determine their actions; investors who observe the firm’s signal (i.e. \( \delta_i = 1 \)) choose actions based on both their private information and the firm’s signal \( z \). The cost of heterogeneity is lower when the correlation is greater between the information sets of the two groups of investors. Investors’ information sets are correlated via their correlation to the firm’s fundamental. The correlation is higher when \( z \) is more precise. This in turn implies that reduced publicity is less costly for a type \( h \) firm than for a type \( l \) firm, enabling the former to use publicity as a credible tool to reveal its type. Formally, we have

**Lemma 3:** Let \( MRS_\tau (\hat{q}, \hat{\delta}) = -\frac{\partial F_\tau (\hat{q}, \hat{\delta})}{\partial \hat{\delta}} \) denote the marginal rate of substitution between publicity \( \hat{\delta} \) and investors’ perceived \( \hat{q} \) for a firm of type \( \tau \).

(a) \( \frac{dMRS_\tau (\hat{q}, \hat{\delta})}{d\lambda_\tau} < 0 \).

(b) Consider perception-publicity combinations \((\hat{q}_1, \hat{\delta}_1)\) and \((\hat{q}_2, \hat{\delta}_2)\), with \( \hat{q}_2 > \hat{q}_1 \). If type \( l \) is indifferent between choosing \((\hat{q}_1, \hat{\delta}_1)\) and \((\hat{q}_2, \hat{\delta}_2)\), then type \( h \) strictly prefers \((\hat{q}_2, \hat{\delta}_2)\) to \((\hat{q}_1, \hat{\delta}_1)\).

Lemma 3 formalizes the above intuition and demonstrates that the type \( h \) firm is willing to substitute publicity for clarity at a lower rate than the type \( l \) firm. It documents the usual "single-crossing property" by showing that the marginal substitution of publicity for perceived \( \hat{q} \) is a decreasing function of the precision of \( z \). Specifically, Lemma 3(b) implies that type \( h \) is more eager to credibly communicate its type than type \( l \). Figure 1 illustrates the rationale behind Lemma 3(b). It shows in a publicity-perception space that type \( l \)’s indifference curve (denoted by \( I^l \)) intersects type \( h \)’s indifference curve (denoted by \( I^h \)) once from above at point \( B \) (where \( \hat{q} = 0 \) and \( \hat{\delta} = 1 \)). This implies type \( h \) has a smaller marginal rate of substitution of publicity for \( \hat{q} \) than type \( l \). Consequently, as the figure confirms, if type \( l \) is indifferent to points \( A \) and \( B \) that lie on its indifference curve \( I^l \), then type \( h \) strictly
prefers point $A$ to $B$, since $A$ lies in type $h$'s "preferred-to" set. Notice, point $A$ consists of lower publicity and higher perceived $\hat{q}$ than $B$, implying that type $h$ is more eager to credibly communicate its type by reducing publicity than type $l$.

The aforementioned preference ordering in Lemma 3 are all we require to characterize "reasonable equilibria" for our game, by which we mean equilibria supported by Cho and Kreps’ (1987) Intuitive Criterion.

**Proposition 3: (Costly Signaling Equilibrium)** For $\rho > \hat{\rho}$, there exists a separating equilibrium that satisfies the Intuitive Criterion where type $h$ selects publicity $\delta^h < 1$ and investors update beliefs $\hat{q}(\delta^h) = 1$, and type $l$ selects publicity $\delta^l = 1$ and investors update beliefs to $\hat{q}(1) = 0$. Type $l$ is indifferent between selecting publicities equal to $\delta^h$ and to 1. Denote the standard setter’s payoff in this equilibrium as $SS^{DD}$. We have

$$SS^{DD} = qF_h\left(1, \delta^h\right) + (1 - q) F_l(0, 1).$$

Furthermore, for any given $\rho$ there exists a $\hat{\lambda}$ such that $\forall \lambda_h < \hat{\lambda}$, the equilibrium uniquely satisfies the Intuitive Criterion.$^{21}$

The rationale for the separation pattern is clear. The type $h$ firm is more eager to credibly reveal that the precision of its signal $z$ is high when it is concerned with being imitated by the type $l$ firm. More important, though, is the reason why the firm restricts its disclosure publicity. In contrast to Morris and Shin (2002), public information is rationed to clarify its precision rather than to prevent investors from (mis)using it.

While our analysis so far views publicity as a key feature of public reporting, it does not address the issue of how type $h$ firm achieves partial publicity. We assume that a given level of publicity is achievable by a finite set of reporting rules and methods. We do not address the contents of these rules, which we believe are interesting and important questions in their own right but are beyond the scope of our paper. Our analysis is also silent on

$^{21}$For $\forall \lambda_h \geq \hat{\lambda}$, this equilibrium generate the highest payoff for firm type $h$ among all separating equilibria that satisfy the Intuitive Criterion. Our subsequent numerical analyses in Section 5.3 are not qualitatively changed if we use any other equilibrium when $\lambda \geq \hat{\lambda}$.  

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how alternative rules outside the pre-specified set achieve different levels of publicity.\footnote{See the discussions in Hirshleifer and Teoh (2003), Morris and Shin (2007), Cornand and Heinemann (2008) for further elaboration on how partial disclosures of information can be implemented. See also Mayew (2008) and Bushee and Miller (2012) for related empirical evidence on firms’ voluntary disclosure policies.} We conjecture that one way to affect publicity is by the way public information is communicated and explained, for example, by using highly technical vs. plain and simple language, or by applying sophisticated estimation methods.\footnote{A disclosure of derivative-based hedging activities can be made close to incomprehensible to all but a few who are well versed in both the economics of derivatives and the complexity of hedge accounting. Recent empirical research also suggests that while all investors can in principle access firms’ Form 10-K, the degree to which they incorporate the information in 10K is affected by the readability of 10K (e.g., Li (2008)).} To the extent that different publicity level is associated with different voluntary disclosure practices and strategies, including, for example, conference calls, webcasts, or press releases, or the linguistic complexity of the disclosure, our results from the discretionary regime can provide a theoretical justification for these practices.

5.3 Comparing Regimes

We now compare the standard setter’s payoffs between the two regimes ($SS^U$ versus $SS^D$) in the parameter region where there is an acute conflict between investors and firms (i.e., $\rho > \bar{\rho}$). The uniform regime requires firms of all types to choose the same rules that achieve full publicity and bars them from adopting rules that result different publicity as a function of their privately observed precision. In this case, while all investors receive the firm’s disclosure of $z$, they do not know its precision because they are not able to update their prior beliefs through the firm’s publicity choices. Here, full publicity is achieved at the cost of reduced clarity. The alternative is the discretionary regime which permits the firm to tailor its disclosure policy to communicate $z'$s precision, the equilibrium (characterized by Proposition 3) of which shows that the type $h$ firm restricts its publicity. In a separating equilibrium, investors are able to perfectly infer the firm’s type by its choice of publicity. Here, perfect clarity is achieved at the cost of restricted publicity. The trade-off between publicity and clarity is the key to understanding the benefits and costs of the two regimes.
The trade-off between clarity and publicity reflects a key role of the standard setter in our paper, whose objective is to select a regime to optimally balance social inefficiencies from two types of externalities. The first type of externalities arises when an individual investor chooses his action ignoring the impact of his action on other investors' utility. In equilibrium, individual investors under-coordinate their actions, giving rise to a between-investor externality problem. In response, type \( l \) firms would like to correct this between-investor externality problem by pooling with type \( h \) firms in order to induce investors to place a higher weight on their disclosure \( z \). However, because type \( h \) firms only care about the aggregate utility of their own investors, they would credibly separate themselves in the discretionary regime via restricted publicity, creating a between-firm externality problem that renders it impossible for type \( l \) firms to pool with type \( h \) firms.

When selecting the reporting regime, the standard setter faces a trade-off between these two types of externalities: the uniform regime prevents type \( h \) firms from credibly separating, alleviating the between-firm externality at the cost of exacerbating the between-investor externality problem for type \( h \) firms (as their investors will under-coordinate their actions to an even greater extent). In contrast, the discretionary regime prevents type \( l \) firms from pooling, exacerbating between-firm externality problem but alleviating type \( h \) firms’ between-investor externality problem (so that these investors do not further under-coordinate their actions).

Unfortunately, \( SS^{DD} \) and \( SS^{UD} \) are complicated functions of \( \rho, q \) and \( \lambda' \)s that do not lend themselves to a simple analytical comparison. We can, however, provide some indications of their respective superiority by calculating and comparing \( SS^{DD} \) and \( SS^{UD} \) for a representative set of numerical examples, which are illustrated in Figure 2. This figure depicts the \((\rho, q)\) plane for three sets of precision \((\lambda_h, \lambda_l)\), each with a different threshold value \( \bar{\rho} \) (represented in Figure 2 by the straight vertical line for each parameter combination \((\lambda_h, \lambda_l)\)) above which costless signaling is not possible.\(^{24}\) Regions \( DD \) (\( UD \)) are parameter regions where \( SS^{DD} \) is larger (smaller) than \( SS^{UD} \) and hence the discretionary (uniform)

\(^{24}\) All three sets set \( \lambda_l = 2 \) and vary \( \lambda_h \). The plots are qualitatively similar (and hence omitted) under different \( \lambda_l \)s.
regime is optimal. Summarizing, Figure 2 provides us with the following predictions.

[Insert Figure 2.]

- *Ceteris paribus, the discretionary (uniform) regime is preferred when* \( \rho \) *the need for coordination* or *\( q \) the prior probability that the firm is of type* \( h \) *is relatively small (large).*

Figure 2 suggests that the uniform regime is preferred when \( \rho \) or \( q \) is sufficiently large. This may seem surprising. After all, clarity is most important when there are strong collective action externalities among investors and discretion seems beneficial when more firms are eager to demonstrate their types. However, in these settings, to credibly communicate reliability, type \( h \) firms separate from type \( l \) firms by reducing their publicity; the larger \( \rho \) is the greater the reduction in publicity and the larger \( q \) is the larger proportion of firms is called on to prove their types. As a result, from the standard setter’s perspective, the cost of reduced clarity exceeds its benefit from a social welfare perspective. The standard setter is thus better off by enforcing restrictive uniform rules, thereby depriving type \( h \) firms of their discretion and duty to demonstrate their types.

- *Ceteris paribus, the discretionary (uniform) regime is preferred when the difference in precision between types* \( \lambda_h - \lambda_l \) *is sufficient large (small).*

Proposition 2 shows that while on the margin type \( l \) firms are better off from slightly increasing investors’ perception of their precision, their incentive to pretend to be type \( h \) diminishes as the difference in \( \lambda \)'s increases. This is because investors will attach a much higher weight on disclosure \( z \) than what type \( l \) firms ideally prefer (i.e., \( w^* \) in Lemma 1), reducing their benefits from inducing investors to perceive them as type \( h \) firms. Thus, when \( \lambda_h - \lambda_l \) is sufficient large, only moderate reduction in disclosure publicity is required for type \( h \) firms to credibly communicate their precision. From the standard setter’s perspective, this implies that the benefits from achieving clarity
outweigh the costs of separation via restricted publicity. In this case, the discretionary regime yields a higher expected payoff for the standard setter than the uniform regime.

6 CONCLUSIONS

6.1 Summary

We examine the optimal degree of discretion for reporting information about financial performance from the perspective of a standard setter whose task is to choose a reporting regime that decreases the costs (increases the efficiency) of information transmission from firms to their current and potential investors, meaning, broadly, suppliers of debt and equity capital. In our setting, reported information has unverifiable precision and the information is used by multiple investors, so it has a potential coordination role, to reduce investors’ uncertainty about other investors’ actions when the marginal payoff to an investor’s action depends on what other investors do. The standard setter wishes to maximize the aggregate utility of all investors at all firms, taking into account the incentives and information structures for firms and their investors.

The key feature that distinguishes the two reporting regimes we consider is the level of firms’ discretion over publicity, referring to investors’ ease of understanding of reported information. We show that the level of publicity can affect clarity, the extent to which users of the reported information correctly assess its (firm-specific) precision. In our model, if everyone could observe firm-specific precision, firms would prefer full publicity, meaning the maximum feasible level of information accessibility and ease of use. However, when firms privately observe unverifiable precision, they have incentives to restrict publicity in order to communicate information about precision. In the discretionary reporting regime, which allows firms free choice about publicity so as to signal their information precision, firms may find it optimal to incur the cost of sacrificing publicity. In contrast, we find that the uniform regime, which specifies rules that result in the same reporting publicity for all firms regardless of information precision, sacrifices clarity (i.e., investors assign the
same precision to all firms), but provides a benefit in that firms no longer sacrifice publicity. When information precision is unverifiable, the uniform reporting regime achieves higher ex ante aggregate utility when reported information plays a strong coordination role.

While one view of standards faults them for their restrictive nature, in our setting it is precisely this restrictiveness that makes uniform rules socially valuable, as long as information precision is unverifiable and reported information plays a coordination role. As an empirical matter, we believe that the environment of financial reporting is characterized by unverifiable information precision, ranging from relatively simple and pervasive estimates of fixed asset service lives and salvage values to fair value estimates in the absence of observable market inputs. As discussed in more detail in the next subsection, we also believe that, empirically, the strength of the coordination role of reported information varies, possibly across types of entities.

6.2 Implications

In this subsection we discuss several implications of our analysis for financial reporting, based on the three factors a standard setter would consider in choosing between the uniform and discretionary reporting regimes: the ex ante dispersion of the precision of firms’ information signals, the ex ante likelihood of obtaining higher precision signals, and the strength of the coordinating role of reported information. We acknowledge that some implications can be viewed as positive or explanatory, and some as normative or prescriptive, without focusing on this distinction.

First, our analysis suggests that effective reporting standards will aim to maximize publicity and will be strictly enforced. With regard to publicity, an effective standard setter will take care to promulgate rules whose application results in reporting that aids investors in understanding and processing reported information by, for example, specifying classification rules and presentation formats. With regard to enforcement, firms must follow the uniform reporting regime even though their privately-observed information precision varies, meaning that effective enforcement is a key contributor to the social welfare of the uniform reporting
Second, our analysis sheds light on the two-part question: should there be only one set of financial reporting standards, applicable to all entities and if not, what should be the basis for differences? The answer is relevant to understanding both the movement toward a single set of global accounting standards applicable regardless of where firms are domiciled and the development of differences in standards based on ownership structure or size or some combination of the two. In the context of our model, the question can be analyzed by looking to variation in the strength of the coordination role of reported information.

With regard to both the convergence of global financial reporting standards and the possibility of differences in reporting rules for differing types of reporting entities, our model shows that using the same standards (the uniform reporting regime) is more important when the standards govern the reporting of information that serves a stronger coordinating role, referring to the existence of complementarities among user actions. To the extent that the increasing flow of capital, goods and services across borders create more opportunities for interactions to take place among different user groups dispersed across multiple jurisdictions, our analysis suggests that a single set of global standards matters more for firms that operate globally, for example, banking and insurance, or whose securities are traded globally, and less so for smaller, localized entities such as not-for-profits, smaller unlisted firms and firms with few or no non-domestic financial statement users.

Our model is silent on how a standard setter may discern the presence or absence of complementarities. Casual empiricism suggests that certain relatively stable firm features can be indicative of the degree of interaction among user payoffs. One such indicator, used by both the FASB and the IASB, is ownership structure, for example, a not-for-profit entity with no owners versus a profit seeking entity with no traded securities versus a profit seeking entity with traded securities. Both standard setters interpret differences in ownership structure as indicating differences, across entities, in both the types of financial report users and the types of decisions they make. These differences in user types and user decisions form the

\(^{25}\)Laux and Stocken (2014) also analyze the importance of enforcement in a different setting.
basis for standard setting decisions that exempt certain entities from otherwise-applicable requirements. For example, the IASB has created IFRS for SMEs, a special simplified and condensed set of reporting standards that may be applied by specified unlisted firms (but excluding financial institutions). The FASB sometimes exempts private (unlisted) firms from certain reporting requirements, for example, the requirement to disclosure fair values of financial instruments.

Third, our focus on publicity, referring to the ease with which users of financial reports access and interpret the reported information, and clarity, referring to users’ understanding of the precision of the reported numbers, may speak to aggregation and disaggregation in financial reporting. On the one hand, grouping like items in a single category reduces the number of categories of financial statement items to be analyzed and thereby makes the information more accessible (that is, publicity). On the other hand, some reporting standards require explicit disaggregations specifically linked to providing information about information precision, for example, the requirement to disclose three levels of inputs to fair value measures, in decreasing order of verifiability.

Fourth, and holding constant the strength of the coordination role of reported information, we find that uniform reporting is preferred when ex ante information precision is more dispersed and when ex ante information precision is higher. We believe that the latter condition can be interpreted as consistent with the coexistence of uniform reporting regimes with mandatory audits of financial reports that apply specified auditing rules and procedures, because information signals that do not reach a certain level of precision could not be audited. The former condition captures the idea that a standard setter would not focus much attention on rules for information that is both relatively precise and of low precision variation, for example, cash, while devoting substantial effort to creating uniform rules for items such as asset impairments and derivatives without active markets.

To summarize, our model shows that consideration of interactions among users of financial reporting information is fundamental to understanding how standards should function (a normative question) and how they do function (a positive question). This perspective enriches and extends the conceptual frameworks of the FASB and IASB, which focus on
improving the decision usefulness of financial reports without explicit consideration and analysis of how users of the reports make decisions. Our model highlights that users of financial reports often consider what other users may do in reaching their own decisions, and this characteristic has implications for financial reporting standard setting, especially when the precision of the reported information is unverifiable.

Finally, our approach highlights both a special characteristic of public information that contrasts with private information observed only by a countable number of agents, and the implication of strategic uncertainty, meaning that users of reported information are uncertain about what other users will do. In addition to having practical implications for the task of standard setting, we speculate that this perspective, which explicitly considers the coordination role of public information and its implication for optimal standards, may also provide a direction for future research that extends recent work by, for example, Morris and Shin (2007), Plantin, et al. (2008), and Gigler, et al. (2014).

References


7 APPENDIX

Let $E_i(*)$ and $e_i^*$ denote, respectively, investor $i$’s expectation of $*$ and his optimal action to maximize the expected utility, conditional on his information set $\Omega_i$. Define $\bar{E}(*) \equiv \int_0^1 E_i(*) \, di$ and $\bar{E}^k(*) \equiv \overline{E^k\ldots E(*)}$. Let $I$ denote the set of investors who observe signal $z$ (i.e. $\delta_i = 1$) and $J$ denote the set of investors who observe the null signal $\emptyset$ (i.e. $\delta_j = 0$).

Lemma A1

\[
\bar{E}^k(v) = \left[1 - \tilde{\delta}^k A(k)\right] v + \tilde{\gamma} A(k) z
\]  

(A1)

where

\[
A(k) \equiv \frac{1 - \tilde{\delta}^k (1 - \tilde{\gamma})}{1 - \delta (1 - \tilde{\gamma})}, k = 1, 2, ..., \\
\tilde{\gamma} \equiv \tilde{\varrho} \frac{\lambda_h}{\lambda_h + 1} + (1 - \tilde{\varrho}) \frac{\lambda_l}{\lambda_l + 1};
\]
Further,

\[ E_j (\tilde{E}^k (v)) = s_j \]  \hspace{1cm} (A2)

and \[ E_i (\tilde{E}^k (v)) = [1 - \gamma A (k + 1)] s_i + A (k + 1) \gamma z. \]

**Proof for Lemma A1:** For \( \forall k \), given (A1), (A2) are obtained by the standard normal updating, as shown below:

\[ E_j (\tilde{E}^k (v)) = [1 - \delta \gamma A (k)] E_j (v) + \delta \gamma A (k) E_j (z) = s_j \]

and

\[ E_i (\tilde{E}^k (v)) = [1 - \delta \gamma A (k)] [(1 - \gamma) s_i + \gamma z] + \delta \gamma A (k) z \\
= [1 - \delta \gamma A (k)] (1 - \gamma) s_i + \left[ 1 - \delta \gamma A (k) + \delta A (k) \right] \gamma z \\
= \{ 1 - \gamma \left[ 1 + \delta (1 - \gamma) A (k) \right] \} s_i + \left[ 1 + \delta A (k) (1 - \gamma) \right] \gamma z \\
= [1 - \gamma A (k + 1)] s_i + A (k + 1) \gamma z. \]

The last equality was obtained by noting that

\[ 1 + \delta (1 - \gamma) A (k) = \frac{1 - \delta (1 - \gamma) + \delta (1 - \gamma) \left[ 1 - \delta \gamma A (k) \right]^k}{1 - \delta (1 - \gamma)} \\
= \frac{1 - \delta (1 - \gamma) + \delta (1 - \gamma) - \delta (1 - \gamma) \delta \gamma A (k)^k}{1 - \delta (1 - \gamma)} = A (k + 1). \]

To prove (A1), we use the method of induction. First for \( k = 1 \), normal updating and a diffuse prior on \( v \) imply

\[ E_j (v) = s_j; \]
\[ E_i (v) = \left[ \hat{q} \frac{1}{\lambda_h + 1} + (1 - \hat{q}) \frac{1}{\lambda_l + 1} \right] s_i + \left[ \hat{q} \frac{\lambda_h}{\lambda_h + 1} + (1 - \hat{q}) \frac{\lambda_l}{\lambda_l + 1} \right] z; \]
\[ = (1 - \gamma) s_i + \gamma z \]
Thus,
\[
\bar{E}(v) \equiv \int_i E_i(v) \, di + \int_j E_j(v) \, dj
\]
\[
= \int_i [(1 - \bar{\gamma}) s_i + \bar{\gamma} \bar{z}] \, di + \int_j s_j \, dj
= \delta (1 - \bar{\gamma}) v + \bar{\gamma} \bar{z} + (1 - \delta) v
= (1 - \delta \bar{\gamma}) v + \bar{\gamma} \bar{z}.
\]

Note that $A(1) \equiv 1$, thus $(A1)$ is true for $k = 1$.

Suppose $(A1)$ hold for $k = n$. Then substitute in $(A2)$, we have
\[
E^{n+1}(v) \equiv \int_i E_i(\bar{E}^k(v)) \, di + \int_j E_j(\bar{E}^k(v)) \, dj
= \delta [1 - \bar{\gamma} A(k + 1)] v + \bar{\gamma} A(k + 1) \bar{\gamma} z + (1 - \delta) v
= [1 - \delta \bar{\gamma} A(k + 1)] v + \bar{\gamma} A(k + 1) z.
\]

\[\square\]

**Proof for Proposition 1(a)**

Equation (5) shows the optimal action for investors receiving public signal is
\[
e_i^* = (1 - \rho) E_i(v) + r E_i(\bar{e}) = (1 - \rho) E_i(v) + (1 - \rho) E_i(\bar{E}(v)) + \rho^2 (1 - \rho) E_i(\bar{E}^2(v)) + \ldots
= (1 - \rho) \sum_{k=0}^{\infty} \rho^k E_i(\bar{E}^k(v)).
\]

Substitute $E_i(\bar{E}^k(v))$ from Lemma A1 into $(A3)$ yields the weight on the public signal
\[ w \left( \tilde{q}, \tilde{\delta} \right) = (1 - \rho) \tilde{\gamma} \sum_{k=0}^{\infty} \rho^k A(k + 1) \]
\[ = \frac{\tilde{\gamma} (1 - \rho)}{1 - \delta (1 - \tilde{\gamma})} \left( \sum_{k=0}^{\infty} \rho^k - \sum_{k=0}^{\infty} \rho^k \tilde{\delta}^{k+1} (1 - \tilde{\gamma})^{k+1} \right) \]
\[ = \frac{\tilde{\gamma}}{1 - \delta (1 - \tilde{\gamma})} \left( 1 - \tilde{\delta} (1 - \tilde{\gamma}) (1 - \rho) \right) \]
\[ = \frac{\tilde{\gamma}}{1 - \delta \rho (1 - \tilde{\gamma})}, \]

and the weight on private signal \( s_i \) as
\[ (1 - \rho) \sum_{k=0}^{\infty} \rho^k - (1 - \rho) \tilde{\gamma} \sum_{k=0}^{\infty} \rho^k A(k + 1) \]
\[ = 1 - w \left( \tilde{q}, \tilde{\delta} \right), \]

where \( \tilde{\gamma} \) is defined in Lemma A1. Similarly, the optimal action for investor \( A_j \) in \( J \) (receiving no public signal) is
\[ e^*_j = (1 - \rho) \sum_{k=0}^{\infty} \rho^k E_j \left( E^k(v) \right) = (1 - \rho) \sum_{k=0}^{\infty} \rho^k s_j = s_j. \]

\[ \Box \]

Proof of Proposition 1(b)

Substitute \( U_i \) into \( F_r \left( \tilde{q}, \tilde{\delta} \right) \equiv E \left[ \int_0^1 U_i d_i \mid \lambda = \lambda_r \right], \) we have
\[ F_r \left( \tilde{q}, \tilde{\delta} \right) = (1 - \rho) E \left( \int_0^1 \left( \bar{e}^*_i \frac{e^*_i}{2} \right) d_i \mid \lambda = \lambda_r \right) + \rho E \left( \int_0^1 \left( \bar{e}^*_i \frac{e^*_i}{2} \right) d_i \mid \lambda = \lambda_r \right). \]

Substitute \( e^*_i \) and \( e^*_j \) from Proposition 1(a), and straightforward algebra leads to the expression for \( F_r \left( \tilde{q}, \tilde{\delta} \right). \)

\[ \Box \]

Proof of Proposition 1(c)
To simplify notation, we drop the argument \((\tilde{q}, \tilde{\delta})\) in \(w(\tilde{q}, \tilde{\delta})\) when it doesn’t cause confusion. Proposition 1(a) implies

\[
\frac{\partial w}{\partial \delta} = \frac{\rho (1 - \gamma) \gamma}{[1 - \rho \tilde{\delta} (1 - \gamma)]^2} = \frac{\rho (1 - \gamma) w}{1 - \rho \tilde{\delta} (1 - \gamma)}
\]

where \(\gamma\) is defined in Lemma A1. The partial derivative of \(F_{\tau}(\tilde{q}, \tilde{\delta})\) with respect to \(\tilde{\delta}\) can be simplified to

\[
\frac{\partial F_{\tau}(\tilde{q}, \tilde{\delta})}{\partial \tilde{\delta}} = \frac{\partial}{\partial \delta} \left\{ \frac{v^2}{2} - \frac{1}{2\beta} \left[ \frac{\delta (1-2\rho \tilde{\delta}) w^2}{\lambda_r} + \tilde{\delta} (1 - w)^2 + 1 - \tilde{\delta} \right] \right\} = w \left\{ \frac{\lambda_r (2 - w) - w}{\beta \lambda_r} + w (1 + \gamma) + (1 - w) (1 - \gamma) \lambda_r \right\} \frac{\rho \tilde{\delta}}{1 - \rho \tilde{\delta} (1 - \gamma)}.
\]

\(\rho > 0, w, \gamma \in [0, 1]\) and \(\lambda_r > 1\) imply that \(w (1 + \gamma) + (1 - w) (1 - \gamma) \lambda_r > 0\) and \(\lambda_r (2 - w) - w > 0\). Hence, \(\frac{\partial F_{\tau}(\tilde{q}, \tilde{\delta})}{\partial \tilde{\delta}} > 0\).

**Proof of Lemma 1**

Taking derivative on \(F_{\tau}(\tilde{q}, \tilde{\delta})\) with respect to \(w\) and setting it to zero yields \(w^*\).

\[
\frac{\partial F_{\tau}(\tilde{q}, \tilde{\delta})}{\partial w} = \frac{\partial}{\partial w} \left\{ \frac{v^2}{2} - \frac{1}{2\beta} \left[ \frac{\delta (1-2\rho \tilde{\delta}) w^2}{\lambda_r} + \tilde{\delta} (1 - w)^2 + 1 - \tilde{\delta} \right] \right\} = -\frac{1}{\beta} \left[ \frac{\tilde{\delta} (1 - 2\rho \tilde{\delta}) w}{\lambda_r} - \tilde{\delta} (1 - w) \right] = 0
\]

\[
\Rightarrow w^* = \frac{\lambda_r}{1 + \lambda_r - 2\rho \tilde{\delta}}.
\]

The first order condition is both necessary and sufficient for a unique maximum as \(F_{\tau}(\tilde{q}, \tilde{\delta})\) is strictly concave in \(w\). To see this, note \(\frac{\partial^2 F_{\tau}(\tilde{q}, \tilde{\delta})}{\partial w^2} = -\frac{\tilde{\delta}}{\beta} \left( \frac{1 - 2\rho \tilde{\delta}}{\lambda_r} + 1 \right)\). Clearly, when \(\rho \in [0, 1/2]\) and \(\tilde{\delta} \in [0, 1]\), \(\frac{\partial^2 F_{\tau}(\tilde{q}, \tilde{\delta})}{\partial w^2} < 0\). \(w^{**} = \frac{\lambda_r}{1 + \lambda_r - 2\rho \tilde{\delta}}\) is obtained by setting \(\tilde{q}\) to either 0 or 1 in the \(w\)'s expression in Proposition 1(a). Since \(w^* > w^{**}\) if and only if \(\rho > 0\), the rest of Lemma 1 is then immediate.
Proof of Lemma 2

The derivative on \( F_r(\hat{q}, \hat{\delta}) \) with respect to \( \hat{q} \),

\[
\frac{\partial F_r(\hat{q}, \hat{\delta})}{\partial \hat{q}} = \frac{\partial F_r(\hat{q}, \hat{\delta})}{\partial w} \frac{\partial w}{\partial \hat{q}},
\]

where

\[
\frac{\partial w}{\partial \hat{q}} = \frac{\left( \frac{\lambda_h}{1+\lambda_h} - \frac{\lambda_l}{1+\lambda_l} \right) (1 - \rho \hat{\delta})}{\left( 1 - \rho \hat{\delta} \left( \frac{\lambda_h}{1+\lambda_h} + \frac{(1-\hat{q})\lambda_l}{1+\lambda_l} \right) \right)^2} > 0.
\]

From Lemma 1, when the firm’s type is correctly perceived, investors place the weight \( w^{**} = \frac{\lambda_r}{1+\lambda_r - \rho \hat{\delta}} \) to the disclosure of \( z \). Evaluate \( \frac{\partial F_r(\hat{q}, \hat{\delta})}{\partial w} \) at \( w^{**} \),

\[
\left. \frac{\partial F_r(\hat{q}, \hat{\delta})}{\partial w} \right|_{w = w^{**}} = -\frac{1}{\beta} \left[ \lambda_r \left( 1 - 2 \rho \hat{\delta} \right) w^{**} - \hat{\delta} (1 - w^{**}) \middle/ \lambda_r \right] = \frac{\rho \hat{\delta}}{\beta (1 + \lambda_r - \rho \hat{\delta})}.
\]

The lemma is thus obtained by noting that \( \left. \frac{\partial F_r(\hat{q}, \hat{\delta})}{\partial w} \right|_{w = w^{**}} > 0 \) if and only if \( \rho > 0 \). \( \blacksquare \)

Proof of Proposition 2

Lemma 2 establishes that type \( h \) does not wish to be perceived as type \( l \). This implies that the costless signaling equilibrium is guaranteed if we can obtain a condition under which type \( l \) does not find in its own interest to be perceived as type \( h \) at full publicity (\( \hat{\delta} = 1 \)).

Note that \( F_l(\hat{\lambda}, \hat{\delta}) \) is a quadratic function in \( w \) (investors’ weight on the disclosure of \( z \)) and that Lemma 1 shows type \( l \)’s expected payoff achieves its maximum at \( w^* = \frac{\lambda_l}{1+\lambda_l - 2\rho} \).

Thus, type \( l \) does not wish to mimic type \( h \) if and only if

\[
\frac{\lambda_l}{1 + \lambda_l - 2\rho} - \frac{\lambda_l}{1 + \lambda_l - \rho} < \frac{\lambda_h}{1 + \lambda_h - \rho} - \frac{\lambda_l}{1 + \lambda_l - 2\rho}, \tag{8}
\]

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which is equivalent to

\[ -2\lambda_h \rho^2 + (3\lambda_h - \lambda_l + 3\lambda_h \lambda_l - \lambda_l^2) \rho + \lambda_l^2 + \lambda_l - \lambda_h - \lambda_h \lambda_l < 0. \] (9)

It is easy to see that the LHS of the above expression is a quadratic function in \( \rho \) and peaks at

\[ \rho = \frac{3\lambda_h - \lambda_l + 3\lambda_h \lambda_l - \lambda_l^2}{4\lambda_h} > 1. \]

As such, the solution to (8) that can possibly be smaller than 1/2 is

\[ \tilde{\rho} (\lambda_h) = \min \left\{ \frac{3\lambda_h - \lambda_l + 3\lambda_h \lambda_l - \lambda_l^2 - \sqrt{1 + \frac{(3\lambda_h - \lambda_l + 3\lambda_h \lambda_l - \lambda_l^2)^2 + 8\lambda_h (\lambda_l^2 + \lambda_l - \lambda_h - \lambda_h \lambda_l)}}{4\lambda_h}, \frac{1}{2} \right\}. \]

\[ \blacksquare \]

**Proof of Corollary 1** The proof is obtained by replacing \( \hat{q} \) with \( q \) in Proposition 1. \[ \blacksquare \]

**Proof of Lemma 3**
• Lemma 3(a): we use =, to represent "equal in sign":

\[
\frac{d}{d\lambda} \left[ -\frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}} \right] = s \frac{d}{d\lambda} \left[ \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{\delta}} \right] = \frac{d}{d\lambda} \left\{ \frac{\partial w}{\partial \hat{\delta}} + \frac{\lambda_r - \lambda_r (1 - w)^2 + (4\rho \hat{\delta} - 1) w^2}{2 [(2\rho \hat{\delta} - 1) w + \lambda_r (1 - w)]} \frac{\partial w}{\partial \hat{q}} \right\} = s [1 - (1 - w)^2] \left[ (2\rho \hat{\delta} - 1) w + \lambda_r (1 - w) \right] - (1 - w) \left[ \lambda_r - \lambda_r (1 - w)^2 + (4\rho \hat{\delta} - 1) w^2 \right] = s 2\rho \hat{\delta} w - 1 < 0.
\]

• Lemma 3(b): Since \((\hat{q}_1, \hat{\delta}_1)\) and \((\hat{q}_2, \hat{\delta}_2)\) are on type \(l\)'s indifference curve, we can define \(\tilde{\delta}(\hat{q})\) as an implicit function of \(\hat{q}\) as we move from \((\hat{q}_1, \hat{\delta}_1)\) to \((\hat{q}_2, \hat{\delta}_2)\) along type \(l\)'s indifference curve, with \(\frac{d\delta(\hat{q})}{d\hat{q}} = -\frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}}\). Thus, for the type \(h\) firm,

\[
F_h(\hat{q}_2, \hat{\delta}_2) - F_h(\hat{q}_1, \hat{\delta}_1)
= \int_{\hat{q}_1}^{\hat{q}_2} \frac{dF_h(\hat{q}, \hat{\delta})}{d\hat{q}} d\hat{q} = \int_{\hat{q}_1}^{\hat{q}_2} \left[ \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{\delta}} + \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}} \frac{d\hat{\delta}(\hat{q})}{d\hat{q}} \right] d\hat{q}
= \int_{\hat{q}_1}^{\hat{q}_2} \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{\delta}} \left[ \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}} \frac{d\hat{\delta}(\hat{q})}{d\hat{q}} + \frac{d\hat{\delta}(\hat{q})}{d\hat{q}} \right] d\hat{q}
= \int_{\hat{q}_1}^{\hat{q}_2} \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{\delta}} \left[ \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}} \frac{d\hat{\delta}(\hat{q})}{d\hat{q}} - \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{q}} \right] d\hat{q} > 0.
\]

The last inequality obtains because \(\frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{\delta}} > 0\) from Proposition 1(c) and \(\frac{d}{dx} \left[ \frac{\partial F_h(\hat{q}, \hat{\delta})}{\partial \hat{\delta}} \right] < 0\) from Lemma 3(a). \(\blacksquare\)

A key component in proving Proposition 3 is to characterize the shape of the firm's indifference curve in the \(\hat{q} - \hat{\delta}\) space. To streamline the presentation of the results, we first establish a set of lemmas that shows the indifference curve is monotonic only when \(\lambda_h\) and
\( \lambda_i \) are not too far apart. When the indifference curve is non-monotone, multiple equilibria exist that survive the Intuitive Criterion.

**Lemma A2** Define \( \hat{\lambda} \) as

\[
\frac{\hat{\lambda}}{1 + \hat{\lambda}} = \frac{\hat{q} \lambda_h}{1 + \lambda_h} + \frac{(1 - \hat{q}) \lambda_i}{1 + \lambda_i}.
\]

\[
\frac{\partial F_N(\hat{q}, \delta)}{\partial \hat{q}} > 0 \text{ if and only if } \frac{\lambda_i}{\lambda} > \frac{1 - 2\rho \delta}{1 - \rho \delta}; \text{ and } \frac{\partial F_N(\hat{q}, \delta)}{\partial \hat{q}} > 0.
\]

**Proof of Lemma A2**

- Since \( \hat{q} \) influences \( F_N(\hat{q}, \delta) \) only through the term \( \frac{\hat{q} \lambda_h}{1 + \lambda_h} + \frac{(1 - \hat{q}) \lambda_i}{1 + \lambda_i} \), by a change of variable from \( \hat{q} \) to \( \hat{\lambda} \), we can rewrite \( F_N(\hat{q}, \delta) \) as

\[
F_N(\hat{\lambda}, \delta) = \frac{v^2}{2} - \frac{1}{2\beta} \left\{ \frac{\delta (1 - 2\rho \delta) w(\hat{\lambda})^2}{\lambda} + \delta \left[ 1 - w(\hat{\lambda}) \right]^2 + 1 - \delta \right\},
\]

where

\[
w(\hat{\lambda}) = \frac{\hat{\lambda}}{1 + \hat{\lambda}} = \frac{\hat{\lambda}}{1 + \lambda - \rho \delta}.
\]

Intuitively, this change of variable implies that the firm’s expected payoff is the same whether investors believe it is type \( h \) with probability \( \hat{q} \) or investors believe it has a precision of \( \hat{\lambda} \). As such, like \( \hat{q}, \hat{\lambda} \) can also be equivalently viewed as investors’ perception.

- By the Chain Rule,

\[
\frac{\partial F_N(\hat{q}, \delta)}{\partial \hat{q}} = \frac{\partial F_N(\hat{\lambda}, \delta)}{\partial \hat{\lambda}} \frac{\partial \hat{\lambda}}{\partial \hat{q}}.
\]

By the definition of \( \hat{\lambda}, \hat{\lambda} \) monotonically increases with \( \hat{q} \), i.e., \( \frac{\partial \hat{\lambda}}{\partial \hat{q}} > 0 \). Thus, \( \frac{\partial F_N(\hat{q}, \delta)}{\partial \hat{q}} \) has the same sign as \( \frac{\partial F_N(\hat{\lambda}, \delta)}{\partial \hat{\lambda}} \).

- Note that \( F_N(\hat{\lambda}, \delta) \) is a quadratic function in \( w \), investors’ weight on \( z \). Lemma 1 shows that type \( l \)’s expected payoff achieves its maximum at \( w^* = \frac{\lambda_i}{1 + \lambda_i - 2\rho \delta} \). Thus,
\[
\frac{\partial F_l(\lambda, \delta)}{\partial \lambda} > 0 \text{ if and only if } \frac{\lambda}{1+\lambda - \rho \delta} < w^* = \frac{\lambda_l}{1+\lambda_l - 2\rho \delta} \text{ which in turn is equivalent to } \frac{\lambda_l}{\lambda} \geq \frac{1-2\rho \delta}{1-\rho \delta}. \text{ Similarly, } \frac{\partial F_h(\hat{q}, \delta)}{\partial \hat{q}} > 0. \]

**Lemma 3** There exists a \( \hat{\lambda} \) such that the type \( l \)'s indifference curve that passes perception-publicity combination \((\hat{q}, \delta) = (0, 1)\) is downward sloping if and only if \( \lambda_h < \hat{\lambda} \).

**Proof of Lemma A3**

- As discussed in the first two bullet points in the proof for Lemma A2, \( F_l(\hat{q}, \delta) \) can be equivalently expressed in terms of \( F_l(\hat{\lambda}, \delta) \). In the \( \hat{\lambda} - \delta \) space with the vertical axis \( \hat{\lambda} \in [\lambda_l, \lambda_h] \) and horizontal axis \( \delta \in [0, 1] \), firm type \( l \)'s indifference curve that passes through \((\hat{\lambda}, \delta) = (\lambda_l, 1)\) can be described by \( I_l \), such that

\[ I_l = \left\{ \left( \hat{\lambda}, \delta \right) | F_l(\lambda_l, 1) = F_l(\hat{\lambda}, \delta) \right\}. \tag{A4} \]

- Let the perception-publicity combination \((\tilde{\lambda}, \tilde{\delta})\) be the solution to the following equations:

\[ F_l(\lambda_l, 1) = \frac{v^2}{2} - \frac{1}{2\beta} \left[ \frac{\delta^* (1-2\rho \delta^*) \left( \frac{\lambda_l}{1+\lambda_l - 2\rho \delta^*} \right)^2}{\lambda_l} + \tilde{\delta}^* \left( 1 - \frac{\lambda_l}{1+\lambda_l - 2\rho \delta^*} \right)^2 + 1 - \tilde{\delta}^* \right] \tag{A5a} \]

\[ \frac{\tilde{\lambda}}{1 + \lambda_l - \rho \delta^*} = \frac{\lambda_l}{1 + \lambda_l - 2\rho \delta^*}. \tag{A5b} \]

Note that \((\tilde{\lambda}, \tilde{\delta}^*)\) is unique determined. To see this, note that the right-hand-side (RHS) of \((A5a)\) is type \( l \)'s expected payoff when it is correctly perceived and investors apply the socially optimal weight \( \frac{\lambda_l}{1+\lambda_l - 2\rho \delta^*} \) (as opposed to the equilibrium weight, \( \frac{\lambda_l}{1+\lambda_l - 2\rho \delta^*} \)) on the public signal. Thus, by design, the RHS of \((A5a)\) evaluated at \( \tilde{\delta}^* = 1 > F_l(\lambda_l, 1) \) and the RHS of \((A5a)\) evaluated at \( \tilde{\delta} = 0 \) is equal to \( F_l(\lambda_l, \tilde{\delta} = 0) < F_l(\lambda_l, 1) \). Further, it is easy to show that the RHS of \((A5a)\) is increasing in \( \tilde{\delta}^* \). Therefore, \( \tilde{\delta}^* \in (0, 1) \) and is unique. \((A5b)\) shows that \( \tilde{\lambda} \) is also uniquely determined at \( \tilde{\lambda} = \frac{1-\rho \delta^*}{1-2\rho \delta^*} \lambda_l > \lambda_l \) for a given \( \delta^* \).
• Replacing \( \frac{\lambda_l}{1 + \lambda_l - 2\rho^*} \) in \( (A5a) \) with \( \frac{\lambda}{1 + \lambda - \rho^*} \), the RHS of \( (A5a) \) can be written as \( F_l \left( \bar{\lambda}, \bar{\delta}^* \right) \): type \( l' \)'s expected utility when it chooses \( \bar{\delta}^* \) and investors perceive it to be \( \bar{\lambda} \) and apply the corresponding equilibrium weight \( \frac{\lambda}{1 + \lambda - \rho^*} \). \( (A5a) \) implies \( F_l \left( \bar{\lambda}, \bar{\delta}^* \right) = F_l (\lambda_l, 1) \), therefore \( \left( \bar{\lambda}, \bar{\delta}^* \right) \in I^l \).

• Consider any perception-publicity combination \( \left( \lambda', \delta' \right) \in I^l \) with \( \lambda' > \bar{\lambda} \). We claim that \( \delta' > \delta^* \). To see this, observe that \( \left( \lambda', \delta^* \right) \notin I^l \) as \( F_l \left( \lambda', \delta^* \right) < F_l \left( \bar{\lambda}, \delta^* \right) = F_l (\lambda_l, 1) \). This follows because type \( l \)'s expected payoff is quadratic in \( w \) and maximizes at \( w^* = \frac{\lambda}{1 + \lambda - \rho^*} \). However, in equilibrium investors apply a higher weight \( w = \frac{\lambda'}{1 + \lambda' - \rho^*} > w^* \). Since \( F_l \left( \lambda, \delta \right) \) is increasing in \( \delta \) (Proposition 1(c)), in order for \( F_l \left( \lambda', \delta' \right) \) to equal \( F_l (\lambda_l, 1) \), \( \delta' \) needs to be larger than \( \delta^* \).

• Next, consider another perception-publicity combination \( \left( \lambda'', \delta'' \right) \in I^l \) with \( \lambda'' > \lambda' \). We claim that \( \delta'' > \delta' \). To see this, first observe that \( \left( \lambda'', \delta' \right) \notin I^l \). This is because

\[
\frac{\lambda''}{1 + \lambda'' - \rho \delta'} > \frac{\lambda'}{1 + \lambda' - \rho \delta'} > \frac{\lambda'}{1 + \lambda' - \rho \delta^*} > \frac{\lambda}{1 + \lambda - \rho \delta^*}.
\]

The first and last inequality obtains as \( \frac{\lambda}{1 + \lambda - \rho \delta^*} \) is increasing in \( \bar{\lambda} \), while the second inequality obtains as \( \frac{\lambda}{1 + \lambda - \rho \delta^*} \) is increasing in \( \bar{\delta} \). Therefore, \( F_l \left( \lambda'', \delta'' \right) < F_l \left( \lambda', \delta' \right) \). Since \( F_l \left( \lambda'', \delta' \right) \) is increasing in \( \delta' \) (by Proposition 1(c)), for \( \left( \lambda'', \delta'' \right) \) to be on the indifference curve, \( \delta'' \) needs to be larger than \( \delta' \). We thus have established that, for any \( \lambda'' \) and \( \lambda' \) such that \( \lambda'' > \lambda' > \bar{\lambda} \), \( \delta'' > \delta' > \bar{\delta} \). This implies that type \( l' \)'s indifference curve that passes perception-publicity combination \( (\lambda_l, 1) \) is positively sloped at any \( \lambda > \bar{\lambda} \).

• We now consider an perception-publicity combination \( \left( \lambda', \delta' \right) \in I^l \) with \( \lambda' < \bar{\lambda} \). We claim that \( \delta' > \delta^* \). To see this, observe that \( F_l \left( \lambda', \delta^* \right) < F_l \left( \bar{\lambda}, \delta^* \right) = F_l (\lambda_l, 1) \), as

\[
\frac{\lambda'}{1 + \lambda' - \rho \delta^*} < \frac{\bar{\lambda}}{1 + \lambda - \rho \delta^*}.
\]
Since $F^t\left(\lambda', \delta\right)$ is increasing in $\delta$, for $F^t_h\left(\lambda', \delta\right)$ and $F^t_h\left(\lambda, \delta^*\right) = F^t_h(\lambda, 1)$, $\delta^*$ needs to be larger than $\delta^*$.

To see this, note $\frac{\lambda_1}{\lambda} > 1 - 2\rho \delta^*$ and $\frac{\lambda_1}{\lambda} = 1 - 2\rho \delta^*$. Further, Lemma A2 shows $\frac{\partial F}{\partial \lambda} > 0$ if and only if $\frac{\lambda_1}{\lambda} > \frac{1 - 2\rho \delta^*}{1 - \rho \delta^*}$. Observe that $\frac{\lambda_1}{\lambda}$ is strictly decreasing in $\lambda$ and $1 - 2\rho \delta^*$ strictly decreasing in $\delta$. Hence, we have

\[ \frac{\lambda_1}{\lambda} > \frac{1 - 2\rho \delta^*}{1 - \rho \delta^*} > \frac{1 - 2\rho \delta'}{1 - \rho \delta'}. \]

Lemma A2 then implies $\frac{\partial F}{\partial \lambda} > 0$, $\forall \lambda < \tilde{\lambda}$. Further, Proposition 1(c) implies that $\frac{\partial F}{\partial \delta^*} > 0$. By the Implicit Function Theorem, the slope for $\tau^r$'s indifference curve in the $\lambda - \delta$ space is $-\frac{\partial F(\lambda, \delta^*)}{\partial \delta^*}$. Thus, the slope of the indifference curve is negative $\forall \lambda < \tilde{\lambda}$.

- Lemma A3 is thus immediate by noticing that $\lambda$ monotonically increases with $\tilde{q}$. And $\tilde{\lambda}$ described in the lemma can be then established as the unique value satisfying $\frac{\lambda}{1 + \lambda} \equiv \frac{\tilde{q} \lambda}{1 + \lambda} + \frac{(1 - \tilde{q}) \lambda}{1 + \lambda}$. ■

**Lemma A4** Let $I^t$ be the indifference curve in the $\tilde{q} - \delta$ space such that

\[ I^t = \{(\tilde{q}, \delta) | F^t(0, 1) = F^t(\tilde{q}, \delta)\}. \quad (A6) \]

When $\lambda_h < \tilde{\lambda}$, $\frac{\partial F^t(\tilde{q}, \delta)}{\partial \tilde{q}} > 0$ for any perception-publicity combination $(\tilde{q}, \delta')$ that lies to the right of $I^t$ (i.e. $\exists (\tilde{q}', \delta'') \in I^t$ s.t. $\delta'' < \delta'$).

**Proof for Lemma A4**

Figure 1 plots $I^t$ for the case of $\lambda_h < \tilde{\lambda}$. For any combination $(\tilde{q}', \delta')$ that lies to the right of $I^t$, consider a unique combination $(\tilde{q}', \delta'')$ with the same perception $\tilde{q}'$ but lies on $I^t$ with $\delta'' < \delta'$. Since $(\tilde{q}', \delta'') \in I^t$ and $\lambda_h < \tilde{\lambda}$, by Lemma A3, $\frac{\partial F^t(\tilde{q}', \delta'')}{\partial \tilde{q}} > 0$. By the "only

\[ 26 \text{To see this, note } \frac{\partial F^t(\tilde{q}, \delta)}{\partial \delta} > 0 \text{ and the slope for type } \tau^r \text{'s indifference curve in the } \tilde{q} - \delta \text{ space is } -\frac{\partial F^t(\lambda, \delta^*)}{\partial \delta^*}. \]
if" part of Lemma A2, this suggests that \( \frac{\lambda_1}{\lambda} > \frac{1-2\rho^3}{1-\rho^3} \), where \( \lambda = \frac{\hat{q}'\lambda_h}{1+\lambda_1} + \frac{(1-\hat{q}')\lambda_l}{1+\lambda_1} \). Since \( \frac{1-2\rho^3}{1-\rho^3} \) is strictly decreasing in \( \tilde{d} \) and \( \tilde{d}' \), we must have \( \frac{\lambda_1}{\lambda} > \frac{1-2\rho^3}{1-\rho^3} > \frac{1-2\rho^3}{1-\rho^3} \), which, by the "if" part of Lemma A2, implies that \( \frac{\partial F_1(\hat{q}, \tilde{d})}{\partial \tilde{q}} > 0 \). ∎

**Proof of Proposition 3**

- Lemma A3 implies that when costless signaling is not feasible type \( l \)'s indifference curve takes only two shapes as depicted in Figure 1 and Figure A1, respectively. Specifically, let \( \hat{\lambda} \) be as identified in Lemma A3 such that type \( l \)'s indifference curve as defined by \( I_l \) in (A6) is downward sloping if and only if \( \lambda_h < \hat{\lambda} \). Figure 1 plots the \( I_l \) curve for \( \lambda_h < \hat{\lambda} \) and Figure A1 plots the \( I_l \) curve for \( \lambda_h > \hat{\lambda} \). First consider the case when \( \lambda_h < \hat{\lambda} \).

  - Claim: any separating equilibrium must have type \( l \) choosing full publicity. Otherwise, it would be strictly better off by deviating to full publicity. This is because \( \frac{\partial F_1(\hat{q}, \tilde{d})}{\partial \tilde{d}} > 0 \) for any \( \hat{q} \). If such a move leads to an increase in \( \hat{q} \) from 0, type \( l \) would benefit even more. This is because the downward sloping indifference curve implies that \( (\hat{q}, 1) \) is to the right of \( I_l \) \( \forall \hat{q} > 0 \), which in turns implies \( \frac{\partial F_1(\hat{q}, 1)}{\partial \hat{q}} > 0, \forall \hat{q} \).

  - From Figure 1, it is easy to obtain that in any separating equilibrium type \( h \) selects a combination \( (\hat{q} = 1, \tilde{d} \in [\tilde{\delta}^{A'}, \tilde{\delta}^A] < 1) \) (i.e. any point between \( A \) and \( A' \) on Figure 1) and type \( l \) selects \( (\hat{q} = 0, \tilde{d} = 1) \) (i.e. point \( B \) on Figure 1). However, in the next three subpoints, we show that the equilibria with \( \tilde{d} < \tilde{\delta}^A \) do not survive the Intuitive Criterion as firm type \( h \) would have incentive to deviate to \( \tilde{\delta}^A \).

    - Claim: when type \( l \) deviates from full to partial publicity at \( \tilde{\delta}^A \), the most favorable perception for it is \( \hat{q} = 1 \). To see this, note Lemma A3 implies \( \frac{\partial F_1(\hat{q}, \tilde{d})}{\partial \hat{q}} > 0 \) at \( (1, \tilde{\delta}^A) \) as \( (1, \tilde{\delta}^A) \) lies on \( I_l \). By the "only if" part of Lemma A2, it must be

which has the opposite sign to \( \frac{\partial F_1(\hat{q}, \tilde{d})}{\partial \tilde{q}} \). Lemma A3 shows for \( \lambda_h < \hat{\lambda} \), \( I_l \) is downward sloping. Therefore, \( \frac{\partial F_1(\hat{q}, \tilde{d})}{\partial \hat{q}} > 0, \forall (\hat{q}, \tilde{d}) \) on \( I_l \).
\[ \frac{\lambda_l}{\lambda(\hat{q} = 1)} > \frac{1 - 2\beta^A}{1 - \beta^A}. \] Since by definition \( \lambda \) strictly increases with \( \hat{q} \), \( \frac{\lambda_l}{\lambda(\hat{q} < 1)} > \frac{\lambda_l}{\lambda(\hat{q} = 1)} > \frac{1 - 2\beta^A}{1 - \beta^A}. \) Thus by the "if" part of Lemma A2, it must be that \( \frac{\partial F_l(\hat{q}, \delta)}{\partial \delta} > 0 \) at \( (\hat{q}, \delta^A) \) for all \( \hat{q} < 1 \). This implies that at publicity \( \delta^A \), the most favorable perception that generates the highest payoff for type \( l \) is \( \hat{q} = 1 \).

- Since \( (1, \delta^A) \) lies on type \( l \)'s the indifference curve of \( I^l \), the previous bullet point implies that in any separating equilibrium type \( l \) cannot strictly benefit from deviating to \( \delta^A \) regardless of what investors' beliefs. By the Intuitive Criterion, investors must believe it is type \( h \) who chooses an off-equilibrium publicity \( \delta^A \).

- Given the above belief, consider any separating equilibrium where type \( h \) selects a combination \( (\hat{q} = 1, \delta \in [\delta^A, \delta^H]) \). Clearly, it doesn’t survive the Intuitive Criterion, as type \( h \) would strictly be better off by deviating to \( \delta^A \) as \( \frac{\partial F_h(1, \delta)}{\partial \delta} > 0 \) for any \( \delta \). This implies that the only remaining separating equilibrium with \( (\hat{q} = 1, \delta = \delta^A) \) for type \( h \) and \( (\hat{q} = 0, \delta = 1) \) for type \( l \) survives the Intuitive Criterion.

- Next, consider an arbitrary pooling (or partial pooling) equilibrium at an perception-publicity combination \( (\hat{q}^{pool}, \delta^{pool}) \). Clearly, such a combination must lie on or to the right of \( I^l \). Otherwise, type \( l \) would be strictly better off by deviating to full publicity. Further, let \( I^{l, pool} \) denote type \( l \)'s indifference curve that passes through \( (\hat{q}^{pool}, \delta^{pool}) \). By definition, \( I^{l, pool} \) must also lie to the right of \( I^l \). As a result, by Lemma A4, at any point on this indifference curve, we must have \( \frac{\partial F_l}{\partial q} > 0 \).

- Let \( (1, \delta^') \) be the intersection between \( I^{l, pool} \) and \( \hat{q} = 1 \). Following the same logic as in bullet point \#3, the Intuitive Criterion requires that investors believe a firm who chooses an off-equilibrium publicity \( \delta^' \) must be type \( h \) for sure, i.e. \( \hat{q} = 1 \). Under this belief, by Lemma 3(b), type \( h \) would strictly benefit from deviating to \( \delta^' \), thus guaranteeing the perception \( \hat{q} = 1 \) and breaking the pooling equilibrium. Hence, any pooling (or partial pooling) equilibrium does not survive the Intuitive Criterion.

- Next we show that multiple equilibria survive the Intuitive Criterion when \( \lambda_h > \lambda \).
$I''$ in Figure A1 represents type $l$’s indifference curve that passes perception-publicity combination $(\tilde{q}, \tilde{\delta}) = (0, 1)$ when $\lambda_h > \hat{\lambda}$. Consider any separating equilibrium where type $l$ chooses full publicity. The Intuitive Criterion is not able to pin down a unique belief for an off-equilibrium publicity $\tilde{\delta}^{A'}$ in Figure A1. This is because type $l$ is strictly better off by deviating to $\tilde{\delta}^{A'}$ if investors believe such a deviation is made by type $h$ with any probability $\tilde{q} \in (\tilde{q}', 1)$ as the vertical segment $A' - C$ lies to the right of $I''$ (shown in Figure A1). Applying the same logic, the Intuitive Criterion cannot pin down a unique off-equilibrium belief for any publicity between $\tilde{\delta}^{A'}$ and $\tilde{\delta}^{A''}$. Consequently, any publicity between $\tilde{\delta}^{A'}$ and $\tilde{\delta}^{A''}$ could constitute a separating equilibrium for type $h$. Particularly, type $h$ selecting a combination $(\tilde{q} = 1, \tilde{\delta} = \tilde{\delta}^{A'})$ and type $l$ selecting $(\tilde{q} = 0, \tilde{\delta} = 1)$ is a separating equilibrium that survives the Intuitive Criterion. Clearly, in this equilibrium, type $l$ is indifferent between selecting full publicity (hence revealing it type) and choosing $\tilde{\delta}^{A'}$ (hence mimicking type $h$). From an inspection of Figure A1, it is obvious that this equilibrium has the highest publicity level and thus generates the highest payoff for type $h$ firm among all separating equilibria that survive the Intuitive Criterion.

- The expression for $SS^{DD}$ is immediate. ■
Figure 1

Indifference Curve for High versus Low Type when $\lambda_0 \leq \hat{\lambda}$.
Figure 2

Social Welfare under Discretionary and Uniform Regimes
Figure A1

Indifference Curve for High versus Low Type when $\lambda_s > \hat{\lambda}$. 