The Effects of Asymmetric Disclosure on Price Informativeness and Firm Performance*

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Abstract

Prior literature shows that an uninformative disclosure policy may be optimal for firms to encourage investors to *acquire* private information when price performs a feedback role in that firm managers condition their investment based on information conveyed in prices (Gao and Liang (2013)). We show that a timely loss recognition policy can further encourage investors to *trade* on their private information in the presence of the feedback effect. We show that the preferences for timely loss recognition may differ between firms (whose objective is to maximize ex ante firm value) and the social planner (whose objective is to maximize firms' investment efficiency). We identify conditions under which mandatory reporting rules featuring timely loss recognition can be justified. Our analysis identifies an alternative mechanism for disclosure policies to affect price informativeness and enhance financial market's resource allocation role, and provides a new rationale for some long-standing accounting principles. It also suggests an alternative interpretation for empirical associations between timely loss recognition and firm performance.

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1 Introduction

One key feature of accounting is timely loss recognition (TLR), also known as conditional conservatism. It refers to the practice that financial accounting recognizes bad news more quickly than good news, implemented by lower verification threshold for bad news than for good news (Watts (2002); Gao (2013)). Timely loss recognition manifests in several accounting standards such as as lower-of-cost-or-market accounting for inventories, the impairment for long-lived assets, and the impairment of unrealized losses for held-to-maturity financial instruments. While few dispute that accounting reporting features conservative characteristics such as TLR, there is an ongoing debate on what purposes they serve. A commonly held view is that conservatism serves the stewardship role, demanded by investors (e.g., creditors and shareholders) to better contract with firm insiders with misaligned incentives (Kothari, Ramana and Skinner (2010)). However, Lambert (2010) notes that the stewardship view is not the consensus among standard setters and calls for more research to understand the role of conservative reporting in market setting.[1]

In this paper, we explore the role of timely loss recognition in affecting investors' incentive to trade on their private information when the feedback channel is present. The feedback channel refers to the situations when security prices contain useful information to guide firm managers' investment decisions. Prices can contain such decision-useful information because of their unique role in aggregating the diverse *private* information possessed by investors. The private information is used by investors to maximize their trading profits and is difficult to directly communicate with firm managers. Because the information reflects investors' assessments of firms' future prospects, it can be useful to guide firm decisions and improve performance.[2] A growing number of empirical studies document evidence that firm managers do condition their investment decisions on the information revealed in stock prices, consistent with the existence of feedback channel (e.g., Luo (2005), Chen, et al. (2007)). The feedback channel highlights the importance of price informativeness, i.e., price's ability to incorporate decision-useful private information from the trading process, because under the feedback channel, more informative prices not only improve how efficiently investors allocate capital across firms, but also how efficiently managers utilize capital within firms.

For price to be informative, it is important that traders have incentives both to spent resources to acquire information ex ante and to trade based on the information they observe. These incentives would be lowered, as would price informativeness, by frictions in the financial markets such as trading costs or strategic market makers who set prices guarding against informed traders. In this paper, we show that such frictions can lead

[1] On the theory side, Chen, et al. (2007) and Gao (2013) advance two explanations for conservatism, whereas Gigler, et al. (2009) find that conservatism cannot be justified on the ground of debt contracting efficiency alone.

[2] The insight that price provides useful information for guiding resource allocation decisions in the economy can be traced to Hayek (1945). See Dow and Gorton (1997) and Subrahmanyam and Titman (1999) for early theoretical studies of the feedback effects, and see Bond, Edmans, and Goldstein (2012) for a review of the related literature.

to a demand for timely loss recognition in order to improve price informativeness. In doing so, we model reporting timeliness as the probability that accounting reporting system reveals managers' forward-looking information about the state of nature before managers make investment decisions. Asymmetrically timely loss recognition means accounting recognizes managers' bad news with higher probability than it does good news.

We find that timely loss recognition changes the trading dynamics when public disclosure is absent, and mitigates the distortion on price informativeness caused by the feedback channel as identified in Edmans, Goldstein and Jiang (2015). Specifically, Edmans, et al. (2015) show that the feedback effect affects speculators' incentive to trade on good and bad news asymmetrically: it increases their incentive to trade on good news and decreases their incentive to trade on bad news. This is because when an informed trader[3] trades on her private information, the information is partially revealed through the trading process, and can assist the firm in making more efficient investment decisions. While learning from price improves firm value in expectation, it affects traders' incentives to trade on good vs. bad news asymmetrically. Specifically, conditional on observing good news, the speculator's trading profit from buying is higher with feedback than without. This is because the firm's total profit would be higher when the manager learns information from price than when the manager does not learn. On the contrary, conditional on observing bad news, the speculator' trading profit from selling is lower with feedback than without: this is because when the state is bad, the firm's profit would not be as bad with feedback as without feedback: the firm can adjust investment accordingly to offset the impact of the negative state. This implies that traders are less likely to profit from trading on negative news. Consequently, prices are less informative about downturns, reducing firms' investment efficiency and the stability of the real economy.[4]

One of the key assumptions in Edmans, et al. (2015) is that the only source of new information is from the informed trader, and the firm does not observe any private information on its own. Thus, in their model, the information asymmetry between the firm and the market is one-way: the market has strictly more information than the firm. While this assumption highlights the role of feedback on informed traders' incentive to trade on private information, it has two unrealistic implications. The first is that the market maker, conditional on observing the order flows, can predict perfectly the firm's investment decision. The second implication is that discussion about the optimal disclosure policy is a moot issue, since the firm has no private information to disclose. In our model, we allow the firm to observe the true state of the world with positive probability, but not always. In addition, the market does not observe whether the firm observes

[3]In this paper we use informed trader and speculator interchangeably.

[4]In a related study, Dow, Goldstein and Guembel (2016) show that the feedback channel generates strategic complimentarities in speculators' incentives to acquire information about weak firms. This can result in a market breakdown when speculators stop producing information about firms experiencing small negative shocks to their fundamentals, further magnifying the shocks.

useful private information or not. This allows the information asymmetry to exist both ways, and permits a role for public disclosure. At the same time, since firms do not always observe private information, learning from price is still possible. However, in the case of uninformative disclosure, it is unclear (to the market) whether the firm will actually learn from the market or not.

We show that a reporting system featuring timely loss recognition (TLR, thereafter referred to as the conservative system) reduces the asymmetry in speculator's trading incentives via two effects. The first is the State Uncertainty Effect which affects the market maker's belief about the underlying state of the world upon an uninformative disclosure by the firm. Specifically, under a TLR system, bad news is more likely to be pre-empted, and thus the market maker assesses the state is more likely to be good and sets the price higher accordingly upon uninformative disclosure by the firm. This increases the speculator's profit from selling when she observes bad news, and decreases her profit from buying when she observes good news. Thus, she is more likely to trade on bad news with TLR than without. The general uncertainty about the state enables the informed trader to profit from her private information. However, this effect would be there even in the absence of the feedback channel. The second effect is the Information Endowment Uncertainty Effect, which affects the speculator's belief that the firm will take the correct state-contingent investment action. Specifically, TLR increases the informed trader's advantage in forecasting the consequences of the firm's investment decision when she observes negative news. This is because under the TLR system, conditional on uninformative disclosure, the informed trader who observes the negative information assesses higher likelihood that the firm does not observe the true state, and therefore will not take the value-enhancing corrective investment action. This increases the informed trader's expected profit from selling on negative news (because she expects to pay a lower price to buy back the firm's share to cover her short position). Notice that the effect of this uncertainty provides an additional source of uncertainty to be exploited by the informed trader, although this opportunity exists only when the feedback channel is present.

Thus, timely loss recognition has two effects on the firm's ex ante value: on the positive side, it increases the price informativeness, by improving price's ability to reflect bad news. As a result, price provides more useful information to guide firm decisions when managers do not observe such information on their own. This effect increases the firm value, due to the increased investment efficiency. On the other hand, TLR also increases the speculator's information advantage over the market maker and the uninformed traders. Because the uninformed traders lose to informed speculators' on average due to information asymmetry, they would demand a discount when purchasing shares issued by the firm, lowering the firm's selling price.[5] Therefore, when the firm puts sufficient weight on the welfare of its liquidity traders, it will not necessarily choose the TLR reporting system. On the other hand, from a social optimal prospective, the total social

[5] Lambert (2010) has conjectured that conservative reporting may reduce market liquidity.

surplus from the firm's decision would be higher when price is more informative, whereas it is unaffected by the welfare transfer between liquidity traders and informed traders. We show that it is always optimal to implement TLR from the social perspective, but not necessarily so from the firm's perspective.

In summary, our analyses generate the following implications. First, they show that whether TLR is desirable hinges critically on the feedback channel. Without affecting the price informativeness, disclosure only affects how price reflects the firm's decisions, but does not affect the firm's decisions themselves. In this case, disclosure only affects the information asymmetry among traders, which in turn affects the stock's liquidity discount. Second, they imply firms may have different preferences over reporting properties than a benevolent social planner (say, standard setters). From the firms' perspective, only those that benefit more from learning from the market would choose a reporting system with TLR whereas firms that weight the liquidity discount more would prefer a neutral or aggressive system. However, from the social optimal perspective, timely loss recognition dominates either a neutral system or an aggressive system because it achieves higher efficiency in the real economy and because the liquidity discount is simply a wealth transfer among investors, and does not directly affect the efficiency of the real economy.

Our analysis connects accounting reporting property (TLR) to the overall informational efficiency in the financial market when the market performs resource allocation role via the feedback channel. This connection provides an alternative mechanism for timely loss recognition to be desirable other than via the stewardship channel. The stewardship view argues that the benefit of conservative reporting is to contract and discipline self-interested firm managers to take value-maximizing actions. Prior studies have mostly relied on this argument to interpret the positive associations between TLR and firm performance as supporting evidence for the stewardship role of TLR (e.g., Francis and Martin (2010)). While our analysis is silent on whether and how timely loss recognition performs the stewardship role, it does suggest that timely loss recognition can be positively associated with firm performance even when there are no agency conflicts with firm managers. In fact, in our framework, timely loss recognition motivates self-interested investors to take actions that can help firm performance. The stewardship view and the feedback channel are not mutually exclusive, although as we elaborate next, they identify different frictions and therefore have different implications for the role of financial reporting.

Our analysis identifies a unique role for financial reporting in dealing with frictions from the financial markets. Thus, it can reconcile the differences in opinion between advocates of conservatism among accounting researchers and standard setters in the U.S.. The advocates in accounting academia call for conservatism and stewardship role as a key element in FASB's conceptual framework. The FASB explicitly rejects the stewardship view but nonetheless maintain key conservative features in many standards. The FASB rejects the stewardship view of accounting standards because the key friction underlying the stewardship view is the conflict of interest between firm insiders and outside investors, which can be dealt with by various contracting agreements and governance mechanisms. It is unclear that modifying accounting standards is the most efficient and direct means to deal with such friction. In contrast, the friction identified by our analysis is the feedback channel, and its asymmetric effect on the speculator's incentives to trade on good vs. bad news. Because the feedback channel takes place precisely because firm managers want to utilize useful information in price to maximize firm value, the incentive distortion it causes on the speculators is unlikely, if possible at all, to be dealt with by contracting with the speculators. Thus, our analysis offers a theoretical foundation for FASB's practice without relying on the stewardship view.

Our analysis endogenizes price informativeness as a function of the informational properties of accounting reports. This generates a framework to interpret estimates from price-earnings regressions. Specifically, we show that even when the accounting reports are neutral (that is, neither conservative or aggressive), we can still observe an asymmetry in earnings' correlation with positive stock returns vs. with negative stock returns, as long as the feedback effect is present. This is because the feedback channel asymmetrically affects price's ability to reflect negative vs. positive news. Timely loss recognition mitigates this asymmetric effect, and therefore would generate less asymmetric correlations than under the neutral system, whereas the aggressive reporting would exacerbate the asymmetry. These factors imply that the commonly used Basu measure may not properly capture cross-sectional differences in firms' timely loss recognition: it can also differ cross-sectionally depending on the strength of the feedback effect. Since the feedback effect benefits firm performance and generates asymmetric slope estimates in earnings-stock return regressions, it can also produce positive associations between firm performance and the Basu-like measures even with neutral accounting.

Our paper contributes to the growing theoretical literature how the feedback channel affects stock market efficiency. It is closely related to Edmans, Goldstein and Jiang (2015) and Gao and Liang (2013). Edmans, et al. (2015) identify the insight that the feedback channel asymmetrically reduces price's ability to reflect negative news and therefore generates an endogenous limit to arbitrage on negative news. We build on their insight and further examine how timely loss recognition, a long-standing feature in financial reporting, may mitigate this endogenous frictions. Our analysis alleviates the concern expressed in Edmans, et al. (2015) that more developed markets may not necessarily be better at dealing with the endogenous friction generated by the feedback channel. Our analysis supports a commonly held view that more developed markets usually have more sophisticated financial reporting systems, many of feature conservative characteristics such as TLR. Gao and Liang (2013) also examine how disclosure policy can be used to motivate speculators' information acquisition when the feedback channel is present. In their model, there is no trading cost so there is no asymmetry in speculators' incentive to trade good vs. bad news. They show that partial disclosure may be desirable to motivate information acquisition. We extent their line of inquiry and focus how to structure the partial disclosure. We show fine-tuning the partial disclosure policy with asymmetric timeliness can further enhance price informativeness and facilitate firms' learning from the market.

Our paper also relates to the real effects models surveyed in Kanodia (2007) and Kanodia and Sapra (2016). Like the real effects models, our model also identifies a mechanism for financial reporting properties to affect real production efficiency. Unlike the real effects models where stock price passively reflects firms' past decisions, in our model price actively affects firms' decisions because it contains decision-useful information that the speculators acquire and trade on in their pursuits of profit seeking activities.

Lastly, our paper relates to the large literature in accounting on the role of conservatism. Whereas most existing theoretical literature formulate conservatism in the probability of observing a noisy signal conditional on true state (e.g., Chen, et al. (2007), Gigler, et al. (2009), Gao (2013)), we abstract from noise in disclosed signals (i.e., in our setting, the accounting signal is unbiased conditional on its disclosure) and focus on the timeliness of disclosure. This focus has the potential to tie our analysis the empirical studies, most of which measure conservatism in terms of *more timely* recognition of bad news relative to good news (e.g., Basu (1997), Bushman, et al. (2004)).[6] More importantly, we identify a non-governance channel for timely loss recognition to benefit firm performance: in our framework, how firms structure their disclosure policy can shape the market dynamics when there is no disclosure, and specifically, timely loss recognition can improve investors' incentive to trade on negative news. The channel highlights the intricate endogenous relation between public information and private information in the financial markets, especially when information in financial markets can affect resource allocation in the real economy.

The remaining of the paper is organized as follows. Section 2 describes the model. Section 3 derives and characterizes the possible equilibria. Section 4 focuses on how asymmetric timeliness reporting affects the likelihood of various equilibria. Section 5 derives the social welfare and the firm's net benefit under different reporting regimes and identifies conditions under which the social optimal reporting system coincides with the optimal system preferred by the firm. Section 6 discusses extensions and empirical implications, and section 7 concludes.

[6]While some empirical studies suggest that losses are less informative about firms' future prospects due to abandonment options (Hayn (1995), Lawrence, Sloan and Sun (2017)), most empirical studies do not emphasize the differential informativeness of accounting profits (good news) vs. losses (bad news). When timely loss recognition is implemented by applying a lower verifiability threshold for recognizing negative news (Kothari, et al. (2010)), accounting losses can be conditionally less informative than profits.

2 Model

2.1 Setup

The model has four dates, $T \in \{0, 1, 2, 3\}$, and four players: a firm, a speculator, a noise/liquidity trader, and a market maker. All players are assumed to be risk-neutral with zero discount rate across dates.

Investment decision and firm value The firm's terminal value, realized at $T = 3$ and denoted by $v(\theta, d)$, depends on both an underlying state and firm's investment decisions. We use $\theta \in \{H, L\}$ to denote the state. The prior probability of $\theta = H$ is $\Pr(\theta = H) = \frac{1}{2}$ and is common knowledge.[7] The firm makes an investment decision denoted by $d \in \{-1, 0, 1\}$, where $d = 1$ stands for increasing investment, $d = -1$ decreasing investment and $d = 0$ maintaining the current level of investment. Changing the investment level (i.e., $d = -1$ or 1) requires an adjustment cost $c > 0$, while $d = 0$ does not involve any cost for the firm. We assume $c < x$ so that adjustment can be profitable.

When the firm chooses to maintain the status quo (i.e., $d = 0$), its value $v(\theta, 0)$ is entirely determined by its asset in place: $v(\theta, 0) = R_\theta$ with $R_H > R_L$. When the firm chooses to adjust its investment level, the firm value depends on both the asset in place R_θ and the outcome of the investment decision. When the state is H, the correct decision is to increase investment, which boosts the gross firm value by x and thus the net firm value becomes $v\left(H, 1\right) = R_H + x - c$. Decreasing investment ($d = -1$) is the wrong decision and reduces the firm value by x: $v(H, -1) = R_H - x - c$. When the state is L, reducing investment ($d = -1$) is the right decision with $v\left(L, -1\right) = R_L + x - c$, while increasing investment is the wrong decision with $v\left(L, 1\right) = R_L - x - c$.

Asymmetric timeliness in disclosure At $T = 0$, the firm's internal information system generates a signal $\delta \in \{\theta, \emptyset\}$ that reveals the true stateθ with probability f (i.e., $Pr(\delta = \theta) = f$), and is a null signal with probability $1 - f$ (i.e.,$Pr(\delta = \emptyset) = 1 - f$). One can think of f as determined by the quality of the firm's internal information system, which we take as exogenously determined by factors outside of our model. The realization of δ is the firm's private information and is not observed by anyone else in the model. Upon the realization of δ, the firm discloses to the external market a signal $r \in \{\theta, \phi\}$ according to a pre-committed disclosure policy/reporting system. We assume that the firm can only disclose a null signal ($r = \phi$) when it receives a null signal ($\delta = \emptyset$). When the firm privately observesθ, the reported signal can fully reveal θ with probabilityβ_θ, and is a null signal $r = \phi$ and uninformative about θ with probability $1 - \beta_\theta$. The assumption that the reported signal can only partially reveal the firm's internal information reflects the fact that the

[7] As in Edmans et al. (2015), we impose symmetry for all parameters other than biases in disclosure policies to ensure that our results are not driven by asymmetry in assumptions.

firm's internal information is often complex and soft in nature, and requires costly verifications to be credibly communicated to the external audience (Gao (2013)). This is particularly the case in the context of periodic performance reporting (e.g., quarterly or annual reports filed by the SEC registrants in the U.S.), where the disclosed information needs to follow accounting rules and conventions, and therefore is unlikely to convey the firm's internal information without loss.

To focus on asymmetric timeliness, we allow β_θ to depend on the nature of $\theta \in \{H, L\}$. Specifically, we use $\beta_H = \beta - \xi$ and $\beta_L = \beta + \xi$ to denote the probability of the disclosure being informative when the firm learns that the underlying state is H and when it learns the state is L, respectively. This is different from Gao and Liang (2013) who assume that disclosure informativeness is independent of the underlying state. As it is easy to see $\frac{1}{2}(\beta_H + \beta_L) = \beta$, β can be interpreted as the *overall timeliness* in disclosure, while ξ captures the degree of *asymmetric timeliness* in disclosing good versus bad news: a positive ξ implies that $\beta_L > \beta_H$ and the system is *conservative* in that bad new is disclosed in a more timely fashion than good news. In contrast, a negative ξ implies that $\beta_L < \beta_H$, and the system is *aggressive* in that good news is disclosed in a more timely fashion than bad news. In Sections 2 and 3, we treat ξ as exogenously given and study how ξ affects the equilibria that will emerge; and analyze the optimal level of ξ in Section 4. Similarly, for the time being, we assume $\beta \in (0, 1)$ is exogenously determined and mainly focus on the effects of *asymmetric timeliness.*[8]

Trading Subsequent to the firm's disclosure as described above, trading occurs at $T = 1$. A speculator (she) receives a private signal $\eta \in \{\theta, \phi\}$ which reveals θ with probability λ and is a null signal ϕ with probability $1 - \lambda$. Thus, λ reflects the speculator's information endowment. Whether the speculator is privately informed of θ is not directly observable to other players in the model. In addition to the speculator, two other players participate in the trading stage of the game: a noise/liquidity trader who trades for liquidity reasons unrelated to the realization of θ, and a risk-neutral market maker (he) who collects orders from the speculator and the noise trader, sets a price equal to the expected firm value conditional on his information, and executes the orders out of his inventory. The noise trader submits an order $z \in \{-1, 0, 1\}$, each with probability $\frac{1}{3}$. $z = 1$ (-1) means a buy (sell) order for 1 share of the firm's stock, while $z = 0$ stands for not participating in trading. Similarly, the speculator's trade is denoted by $s \in \{-1, 0, 1\}$. As with models of imperfect competition, she does not trade (i.e., $s = 0$) if she is not privately informed of θ. Trading (i.e., $s = 1$ or -1) imposes a cost $\kappa > 0$ on the speculator. The trading cost κ is commonly known and should be interpreted broadly: while direct trading costs from commissions are typically small, other indirect costs can be large, which may include borrowing costs paid to unmodeled third party financiers to finance her trades.

[8] In Section 5, we relax this assumption and allow the firm to choose both β and ξ. Our results are not qualitatively affected.

With a positive trading cost, it is also immediate to see that when θ is revealed by the firm's disclosure the speculator will not trade on her private information. To rule out any equilibrium where the speculator trades against her private information (i.e., $s = 1$ when $\eta = L$ or $s = -1$ when $\eta = H$), we require that $R_H - x > R_L + x$. That is, even if the firm makes a wrong investment decision when $\theta = H$, its terminal value is still larger than if it makes a correct investment decision when $\theta = L$.

Following Kyle (1985), market orders are submitted simultaneously and anonymously to the market maker who absorbs orders using his own inventory and sets the price equal to the expected firm's terminal value, given all available information including any information contained in the order flow. The market maker can only observe the total order flow $X = s + z$, but not its individual components s and z. Obviously, $X \in \{-2, -1, 0, 1, 2\}$ and we use $P(X, r) \equiv E[v(\theta, d) | X, r]$ to denote the pricing function set by the market maker.

Since it is possible that the firm does not observe θ but the speculator knows it, the firm can potentially use the information contained in stock prices to infer θ, a phenomenon commonly known as the feedback effect in the literature. Specifically, the firm makes its investment decision *after* trades are executed, and it can observe the order flow X, update its posterior belief regarding θ, and make investment decisions accordingly. Following the prior literature on the feedback effect (Edmans et al., 2015), we assume that the firm is able to observe the total order flow.[9] In order to highlight the feedback effect, we assume the likelihood that the speculator is privately informed of θ is sufficiently high such that $\lambda > \frac{2c}{x+c}$, which, as will be shown below, induces the firm to adjust its investment decisions in response to observed total order flow. In addition, we assume δ, $\eta$and z are all independently distributed.

Objectives, timeline and equilibrium We now lay out players' objectives, summarize the model's timeline and define the equilibrium concept used.

Objectives The speculator determines her trading strategy to maximize her expected gross trading profit net of any trading cost conditional on her information set. The firm makes investment decisions to maximize the expected firm's terminal value conditional on its information set. Finally, as mentioned earlier, the market maker sets the share price to the expected firm's terminal value conditional on his information set.

Timeline Table 1 presents the timeline of the model.

[9]Under the alternative assumption that the firm observes only P but not X, an alternative equilibrium can arise, in which the firm's investment decision is suboptimal given the information in X. The assumption of observing X is reasonable, since in practice order flow information is provided by many market microstructure databases only with a short lag.

Table 1: Timeline of the game

$T = 0$	A disclosure policy is put in place with asymmetric timeliness ξ. -The firm receives a signal $\delta = \phi$ (that reveals the state) with probability f and $\delta = \phi$ (a null signal) with probability $1 - f$. - The firm discloses $r \in \{\theta, \phi\}$ according to the disclosure policy.
$T = 1$	The speculator privately observes η, which reveals the state with probability λ. - The noise trader submits a market order $z \in \{-1, 0, 1\}$ with equal probability. - The speculator submits a market order $s \in \{-1, 0, 1\}$ and incurs the trading cost of $\|s\|\kappa$. - The market maker observes total order flow $X = s + z$, and sets price equal to the expected firm value: $P(x, r) = E(v\|X, r)$.
$T = 2$	The firm makes its investment decision (d) to maximize the expected firm value $E(v(\theta, d)\|\delta, X)$.
$T = 3$	θ, the firm value, and each player's payoff are realized and observed.

Equilibrium definition We employ the *Perfect Bayesian Equilibrium* as the solution concept for the model.

Definition. A Perfect Bayesian Equilibrium consists of the speculator's trading strategy $s(\eta, r)$, the market maker's pricing strategy $P(X, r)$ and the firm's investment strategy $d(\delta, X)$ such that:

1. Given $P(X, r)$ and $d(\delta, X)$, the speculator's trading strategy $s(\eta, r)$ maximizes her expected payoff: $s(\eta, r) \in \arg\max_s E[s(v - p) - |s|\kappa \mid \eta, r]$.

2. Given $s(\eta, r)$ and $d(\delta, X)$, the market maker sets price to the expected firm value $P(X, r) = E[v(\theta, d)|X, r]$.

3. Given $s(\eta, r)$ and $P(X, r)$, the firm makes the investment decision $d \in \{-1, 0, 1\}$ to maximize the expected firm value: $d(\delta, X) \in \arg\max_d E[v(\theta, d)|\delta, X]$.

4. All players use the Bayes' rule to update their beliefs. Beliefs on outcomes not observed on the equilibrium path have to satisfy the Cho and Kreps' (1987) Intuitive criterion.

3 Solutions

In this section, we solve the model using backward induction. We start with the firm's investment decision at $T = 2$ and then derive the speculator's optimal trading strategy at $T = 1$.

3.1 Firm's investment decision at $T = 2$

After observing internal information δ and the market order flow X, the firm updates its posterior belief about state H (which we denote as $\mu_F^{\delta,X} \equiv \Pr(\theta = H \mid \delta, X)$) and makes its investment decision. When $\delta = H$, $\mu_F^{\delta,X} = 1$ regardless of X; similarly, when $\delta = L$, $\mu_F^{\delta,X} = 0$. That is, when the firm has internal information, it is more informative than order flow X and the firm only relies on internal information. The firm uses information from order flow only if the internal signal is not available.

Let μ_F^* denote the posterior belief under which the firm is indifferent between increasing investment and maintaining the status quo:

$$\mu_F^* (R_H + x) + (1 - \mu_F^*) (R_L - x) - c = \mu_F^* R_H + (1 - \mu_F^*) R_L \Longrightarrow$$

$$\mu_F^* = \frac{1}{2} + \frac{c}{2x}.$$

Thus, the firm will increase investment if and only if $\mu_F^{\delta,X} \geq \frac{1}{2} + \frac{c}{2x}$. Similarly, let μ_F^{**} denote the firm's posterior belief under which it is indifferent between decreasing investment and maintaining the status quo, then

$$\mu_F^{**} (R_H - x) + (1 - \mu_F^{**}) (R_L + x) - c = \mu_F^{**} (R_H) + (1 - \mu_F^{**}) R_L \Rightarrow$$

$$\mu_F^{**} = \frac{1}{2} - \frac{c}{2x}.$$

Without loss of generality, we assume that when the firm is indifferent between maintaining the status quo or changing the investment level, it chooses the former ($d = 0$). The following lemma summarizes the firm's optimal investment decisions.

Lemma 1. *The firm optimally increases investment if* $\mu_F^{\delta,X} > \mu_F^*$*, decreases investment if* $\mu_F^{\delta,X} < \mu_F^{**}$*, and maintains the status quo if* $\mu_F^{**} \leq \mu_F^{\delta,X} \leq \mu_F^*$*.*

Lemma 1 implies that the firm chooses to change its investment level if and only if its posterior belief about state θ is sufficiently strong. Intuitively, any change in investment involves an adjustment cost which can be only justified from the firm's perspective if it significantly revises its belief conditional on its information.

3.2 Equilibria of trading game at $T = 1$

We now solve the equilibria for the trading game at $T = 1$. For expositional ease, we only focus on pure strategy equilibria. Note that the speculator will not trade when the firm's disclosure reveals θ. This is because the speculator have no information advantage and thus no expected trading profits, while trading incurs a positive cost κ. Thus, trading generates an expected profit for the speculator only when the firm disclose the null signal, i.e., $r = \phi$.

In this case, since $R_H - x > R_L + x$, the speculator will never buy (i.e., $s = 1$) when she learns $\eta = L$ and will never sell (i.e., $s = -1$) when she learns $\eta = H$. As such, we can categorize all possible pure strategy equilibria by how the speculator trades when the firm's disclosure does not reveal θ (i.e., $r = \phi$):

(i) NT (no-trading) equilibrium: $s(\theta, \phi) = 0$, $\forall\theta$, i.e., the speculator does not trade.

(ii) BNS (buy-not-sell) equilibrium: $s(H, \phi) = 1$ and $s(L, \phi) = 0$, i.e., the speculator buys when observing $\eta = H$ but does not sell when observing $\eta = L$.

(iii) SNB (sell-not-buy) equilibrium: $s(H, \phi) = 0$ and $s(L, \phi) = -1$, i.e., the speculator sells when observing $\eta = L$ but does not buy when observing $\eta = H$.

(iv) T (trading) equilibrium: $s(H, \phi) = 1$ and $s(L, \phi) = -1$, i.e., the speculator buys when observing $\eta = H$ and sells when observing $\eta = L$.

In what follows, we will go through each of the four cases in Propositions 1-4, characterize the firm and market maker's equilibrium play and derive conditions that sustain these equilibria. To facilitate exposition later on, we define t as the conditional probability of state $\theta = H$ conditional on the firm disclosing a null signal ($r = \phi$):

$$
\begin{aligned}
t &\equiv \Pr(\theta = H | r = \phi) \\
&= \frac{\Pr(r = \phi|\theta = H)\Pr(\theta = H)}{\Pr(r = \phi|\theta = H)\Pr(\theta = H) + \Pr(r = \phi|\theta = L)\Pr(\theta = L)} \\
&= \frac{\frac{1}{2}(1 - f\beta_H)}{\frac{1}{2}(1 - f\beta_H) + \frac{1}{2}(1 - f\beta_L)} \\
&= \frac{1}{2} + \frac{1}{2}\frac{f}{1 - f\beta}\xi;
\end{aligned}
$$

No-trading (NT) equilibrium In this equilibrium, the speculator does not trade on her private information and thus the order flow contains no information about the state on the equilibrium path ($X \in$

$\{-1,0,1\}$).[10] As such, the firm changes its investment level only when its private signal δ reveals the state: increasing (decreasing) investment when $\delta = H$ (L) and maintaining the status quo when $\delta = \phi$. While the market maker, like the firm, is not able to extract information from the order flow on the equilibrium path, he can update his belief about the state based on the fact that the firm's disclosure r is a null signal. Denote $\mu_M^{r,X}$ as the market maker's posterior belief for $\theta = H$ conditional on the firm's disclosure r and order flow X (i.e., $\mu_M^{r,X} \equiv \Pr(\theta = H \mid r, X)$). When $r = \phi$ and $X \in \{-1,0,1\}$, we have

$$\begin{aligned}
\mu_M^{\phi,X} &\equiv \Pr(\theta = H \mid r = \phi, X) \\
&= \frac{\Pr(\theta = H, r = \phi, X)}{\Pr(\theta = H, r = \phi, X) + \Pr(\theta = L, r = \phi, X)} \\
&= \frac{1}{2} + \frac{1}{2}\frac{f}{(1 - f\beta)}\xi = t \equiv \Pr(\theta = H | r = \phi),
\end{aligned}$$

as the order flow X does not convey additional information given $r = \phi$. Note, $\mu_M^{\phi,X} > \frac{1}{2}$ if and only if $\xi > 0$. Intuitively, timely loss (gain) recognition preempts (delays) the bad news and induces a more favorable (harsher) inference by the players in the market conditional on $r = \phi$.

To set price, the market maker considers not only the probability of the state but also the firm value in each state. The firm value depends on the firm's investment decision, which relies on the firm's internal information endowment. Define τ_θ as the conditional probability of the firm privately observing state θ conditional on state θ and no disclosure ($r = \phi$):

$$\begin{aligned}
\tau_\theta &\equiv Pr(\delta = \theta | \theta, r = \phi) \\
&= \frac{f(1 - \beta_\theta)}{f(1 - \beta_\theta) + (1 - f)} = \frac{f(1 - \beta_\theta)}{1 - f\beta_\theta}
\end{aligned}$$

and the expected firm value R'_θ in state θ increases with the probability that the firm is informed and takes the correct investment decision to improve firm value:

$$R'_\theta \equiv R_\theta + \tau_\theta (x - c)$$

We can see that $R'_\theta \geq R_\theta$, and the difference $\tau_\theta (x - c)$ captures the impact of the firm's information endowment. When the firm is more likely to be informed, the firm value is higher. This is considered by the market maker when setting price: $P = \mu_M^{\phi,X} R'_H + \left(1 - \mu_M^{\phi,X}\right) R'_L$.

Finally, we need to check when the firm and the market maker's beliefs/strategies are prescribed above,

[10] $X = 2$ (-2) perfectly reveals the speculator's trade and lies off the equilibrium path. As the proof to Proposition 1 will show, the only belief that survives the Intuitive Criterion induces observers of $X = 2$ (-2) to believe the speculator has privately observed $\eta = H$ (L).

the speculator finds in her best interest not to deviate and start trading on her private information. Denote π_{sell}^{NT} (π_{buy}^{NT}) as the speculator's gross trading profit from selling (buying). In order for NT to sustain as an equilibrium, it must be that $\kappa > max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\}$: when the trading cost is sufficiently high, the speculator refrains from trading regardless of her private information. Proposition 1 completely characterizes the no-trading equilibrium.

Proposition 1. *No-trading equilibrium exists if and only if $\kappa > \kappa^{NT} \equiv \max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\}$, where $\pi_{sell}^{NT} = \frac{2}{3}t\left(R_H^{'} - R_L^{'}\right)$ and $\pi_{buy}^{NT} = \frac{2}{3}(1-t)\left(R_H^{'} - R_L^{'}\right)$. In this equilibrium,*

- Informed trader's trading decision (s): $s(\eta, r) = 0$, $\forall \eta, r$;
- Firm's decision (d) is given by:$d(H, X) = 1$, $\forall X$; $d(L, X) = -1$, $\forall X$; $d(\phi, -1) = d(\phi, 1) = d(\phi, 0) = 0$; $d(\phi, 2) = 1$; $d(\phi, -2) = -1$;
- Market maker sets price (P) to be: $P(X, H) = R_H + x - c$, $\forall X$; $P(X, L) = R_L + x - c$, $\forall X$; $P(2, \phi) = R_H + x - c$; $P(-2, \phi) = R_L + x - c$; $P(1, \phi) = P(0, \phi) = P(-1, \phi) = tR_H' + (1-t)R_L'$.

Trading (T) equilibrium In this equilibrium, the speculator buys on good news ($\eta = H$) and sells on bad news ($\eta = L$) when the firm's disclosure $r$does not reveal θ. Consequently, an order flow of $X = 1$ (-1) implies that state H (L) is more likely and affects the uninformed firm's investment decision: when $\delta = \phi$, the firm's posterior beliefs are:

$$
\begin{aligned}
\mu_F^{\phi,1} &= \frac{\Pr(\theta = H, X = 1|\delta = \phi)}{\Pr(\theta = H, X = 1|\delta = \phi) + \Pr(\theta = L, X = 1|\delta = \phi)} \\
&= \frac{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}(1-\lambda)}{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}(1-\lambda) + \frac{1}{2}\frac{1}{3}(1-\lambda)} = \frac{1}{2-\lambda} \\
\mu_F^{\phi,-1} &= \frac{\Pr(\theta = H, X = -1|\delta = \phi)}{\Pr(\theta = H, X = -1|\delta = \phi) + \Pr(\theta = L, X = -1|\delta = \phi)} \\
&= \frac{\frac{1}{2}\frac{1}{3}(1-\lambda)}{\frac{1}{2}\frac{1}{3}(1-\lambda) + \frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}(1-\lambda)} = \frac{1-\lambda}{2-\lambda};
\end{aligned}
$$

Since $\lambda > \frac{2c}{x+c}$, the firm increases (decreases) investment with $X = 1$ (-1). Similarly, $X = 1$ (-1) also enables the market maker to update his belief:

$$
\begin{aligned}
\mu_M^{\phi,1} &= \frac{\Pr(\theta = H, X = 1|r = \phi)}{\Pr(\theta = H, X = 1|r = \phi) + \Pr(\theta = L, X = 1|r = \phi)} \\
&= \frac{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = 1|\theta = L, r = \phi)} \\
&= \frac{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right] + \frac{1}{3}(1-t)(1-\lambda)} = \frac{t}{t + (1-t)(1-\lambda)} > \frac{1}{2};
\end{aligned}
$$

$$\begin{aligned}
\mu_M^{\phi,-1} &= \frac{\Pr(\theta = H, X = -1 | r = \phi)}{\Pr(\theta = H, X = -1 | r = \phi) + \Pr(\theta = L, X = -1 | r = \phi)} \\
&= \frac{\Pr(\theta = H | r = \phi) \Pr(X = -1 | \theta = H, r = \phi)}{\Pr(\theta = H | r = \phi) \Pr(X = -1 | \theta = H, r = \phi) + \Pr(\theta = L | r = \phi) \Pr(X = -1 | \theta = L, r = \phi)} \\
&= \frac{t\frac{1}{3}(1-\lambda)}{t\frac{1}{3}(1-\lambda) + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = \frac{t(1-\lambda)}{t(1-\lambda) + (1-t)} < \frac{1}{2}.
\end{aligned}$$

Again, an order flow of $X = 1$ (-1) tilts the market maker's belief in favor of state H (L) and leads to a higher (lower) price set by the market maker. For the speculator to trade on her information, her expected gross trading profit from buying (selling), denoted as π_{buy}^T (π_{sell}^T) must be bigger than her trading cost: $\kappa < min\left\{\pi_{buy}^T, \pi_{sell}^T\right\}$: when the trading cost is sufficiently low, the speculator always takes advantage of her private information. Proposition 2 characterizes the trading equilibrium.

Proposition 2. *Trading equilibrium exists if and only if* $\kappa < \kappa^T \equiv min\left\{\pi_{buy}^T, \pi_{sell}^T\right\}$, *where*

$$\begin{aligned}
\pi_{buy}^T &= \frac{1}{3}(1-t)\left[R_H^{'} - R_L^{'}\right] + \frac{1}{3}\left(1 - \mu_M^{\phi,1}\right)\left[R_H - R_L + 2(1-\tau_L)x\right]; \\
\pi_{sell}^T &= \frac{1}{3}t\left[R_H^{'} - R_L^{'}\right] + \frac{1}{3}\mu_M^{\phi,-1}\left[R_H - R_L - 2(1-\tau_H)x\right].
\end{aligned}$$

In this equilibrium,

- Speculator's trading strategy: $s(H,\phi) = 1$; $s(L,\phi) = -1$; and $s = 0$, otherwise;
- Firm's investment strategy: $d(H,X) = 1$, $\forall X$; $d(L,X) = -1$, $\forall X$; $d(\phi,2) = d(\phi,1) = 1$; $d(\phi,-2) = d(\phi,-1) = -1$; $d(\phi,0) = 0$;
- The market maker's price setting strategy: $P(X,H) = R_H + x - c$, $\forall X$; $P(X,L) = R_L + x - c$, $\forall X$; $P(2,\phi) = R_H + x - c$; $P(-2,\phi) = R_L + x - c$; $P(0,\phi) = tR_H' + (1-t)R_L'$;

$$P(1,\phi) = \mu_M^{\phi,1}(R_H + x - c) + \left(1 - \mu_M^{\phi,1}\right)\left[\tau_L(R_L + x - c) + (1-\tau_L)(R_H - x - c)\right];$$

$$P(-1,\phi) = \mu_M^{\phi,-1}\left[\tau_H(R_H + x - c) + (1-\tau_H)(R_H - x - c)\right] + \left(1 - \mu_M^{\phi,-1}\right)(R_L + x - c);$$

where

$$\mu_M^{\phi,1} = \frac{t}{t + (1-t)(1-\lambda)};$$

and

$$\mu_M^{\phi,-1} = \frac{t(1-\lambda)}{t(1-\lambda) + (1-t)}.$$

Buy-not-sell (BNS) equilibrium In the BNS equilibrium, the speculator's trading strategy is one-sided: she buys on good news ($\eta = H$) but does not sell on bad news ($\eta = L$). Given this trading strategy, an order flow of $X = -1$ makes the firm and market maker believe that the speculator is likely to have observed bad news. Specifically, $X = -1$ is inconsistent with the speculator having positive information (otherwise the speculator would have bought and the order flow cannot be -1). Consequently, the firm decreases investment and market maker sets the price lower, squeezing the speculator's profit from selling on bad news and making this BNS equilibrium self fulfilling. Finally, for buy-not-sell to constitute an equilibrium strategy for the speculator, her trading cost must be lower than her trading profit from buying but higher than that from selling. Proposition 3 characterizes this equilibrium.

Proposition 3. *Buy-not-sell equilibrium exists if and only if* $\pi^T_{sell} \leq \kappa \leq \pi^{NT}_{buy}$. *In this equilibrium,*

- The speculator's trading strategy: $s(H, \phi) = 1$; $s = 0$ otherwise;

- The firm's investment strategy: $d(H, X) = 1$, $\forall X$; $d(L, X) = -1$, $\forall X$; $d(\phi, 2) = 1$; $d(\phi, -2) = d(\phi, -1) = -1$; $d(\phi, 0) = d(\phi, 1) = 0$;

- The market maker's price setting strategy: $P(X, H) = R_H + x - c$, $\forall X$; $P(X, L) = R_L + x - c$, $\forall X$; $P(2, \phi) = R_H + x - c$; $P(-2, \phi) = R_L + x - c$; $P(0, \phi) = P(1, \phi) = tR'_H + (1-t) R'_L$; $P(-1, \phi) = \mu^{\phi,-1}_M [R_H + \tau_H x - (1-\tau_H) x - c] + \left(1 - \mu^{\phi,-1}_M\right)(R_L + x - c)$; where $\mu^{\phi,-1}_M = \frac{t(1-\lambda)}{t(1-\lambda)+(1-t)}$.

Sell-not-buy (SNB) equilibrium In this equilibrium, the speculator's trading strategy is also one-sided but in a manner opposite to the BNS equilibrium described above: here, she sells on bad news ($\eta = L$) but does not buy on good news ($\eta = H$). Given this trading strategy, an order flow of $X = 1$ makes the firm and market maker believe that the speculator is likely to have observed good news. Specifically, $X = 1$ is inconsistent with the speculator having negative information (otherwise the speculator would have sold and the order flow cannot be 1). Consequently, the firm increases investment and the market maker sets the price higher. Finally, for sell-not-buy to constitute an equilibrium strategy for the speculator, her trading cost must be lower than her trading profit from selling but higher than that from buying. Proposition 4 characterizes this equilibrium.

Proposition 4. *Sell-not-buy equilibrium exists if and only if* $\pi^T_{buy} \leq \kappa \leq \pi^{NT}_{sell}$. *In this equilibrium,*

- The speculator's trading strategy: $s(L, \phi) = 1$; $s = 0$ otherwise;

- The firm's investment strategy: $d(H, X) = 1$, $\forall X$; $d(L, X) = -1$, $\forall X$; $d(\phi, -2) = -1$; $d(\phi, 2) = d(\phi, 1) = 1$; $d(\phi, 0) = d(\phi, -1) = 0$;

- The market maker's price setting strategy: $P(X,H) = R_H + x - c$, $\forall X$; $P(X,L) = R_L + x - c$, $\forall X$; $P(2,\phi) = R_H + x - c$; $P(-2,\phi) = R_L + x - c$; $P(0,\phi) = P(-1,\phi) = tR'_H + (1-t)R'_L$; $P(1,\phi) = \mu_M^{\phi,1}(R_H + x - c) + \left(1 - \mu_M^{\phi,-1}\right)[R_L + \tau_L x - (1-\tau_L)x - c]$; where $\mu_M^{\phi,1} = \frac{t}{t+(1-t)(1-\lambda)}$.

4 Effects of asymmetric timeliness in disclosure

Propositions 1 through 4 in the previous section show that the existence of the four equilibria boils down to which interval speculator's trading cost κ falls in: $\left(0, min\left\{\pi_{buy}^T, \pi_{sell}^T\right\}\right)$, $\left[\pi_{sell}^T, \pi_{buy}^{NT}\right]$, $\left[\pi_{buy}^T, \pi_{sell}^{NT}\right]$ or $\left(max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\}, \infty\right)$. Clearly, these intervals are functions of the asymmetric timeliness parameter ξ. How ξ would affect the size of these intervals is the focus of this section.

To fix idea, we start with a neutral disclosure policy, i.e., $\xi = 0$. In this case, because good news is disclosed by the firm with the same probability as bad news, conditional on the firm's null disclosure, observers are not able to update their belief regarding the state (i.e., $t = \frac{1}{2}$), nor can they update their belief about whether the firm is privately informed (i.e., $\tau_H = \tau_L = 1 - \frac{1-f}{1-f\beta}$). It is straightforward to see that with a neutral policy,

$$\pi_{sell}^T \mid_{\xi=0} < \pi_{buy}^T \mid_{\xi=0} < \pi_{buy}^{NT} \mid_{\xi=0} = \pi_{sell}^{NT} \mid_{\xi=0} . \tag{1}$$

The intuition for the above expression is the following. Under a neutral policy, because both H and L states are equally likely, the market maker cannot glean information from the firm's null disclosure in setting prices. Therefore, the speculator's expected gross trading profit (π_{buy}^{NT}or π_{sell}^{NT}), i.e., the difference between her expected firm value and the market price, is the same when she privately observes $\eta = H$ as when she observes $\eta = L$, resulting in $\pi_{buy}^{NT} \mid_{\xi=0} = \pi_{sell}^{NT} \mid_{\xi=0}$. In the trading equilibrium, the market maker will adjust prices accordingly to observed order flow because the observed order flow partially reveals the speculator's private information. This will in turn reduce the speculator's trading profit, resulting in both $\pi_{sell}^T \mid_{\xi=0} < \pi_{sell}^{NT} \mid_{\xi=0}$ and $\pi_{buy}^T \mid_{\xi=0} < \pi_{buy}^{NT} \mid_{\xi=0}$.

The comparison between $\pi_{sell}^T \mid_{\xi=0}$ and $\pi_{buy}^T \mid_{\xi=0}$ is due to the feedback effect when the firm learns from the observed order flow. Specifically, the firm increases investment when observing a positive order flow. This will increase the firm's terminal value when $\theta = H$, increasing the speculator's expected profit from a long position (buying). The opposite is true when the speculator observes $\theta = L$, however. This is because if the speculator trades, it is rational for her to take a short position (sells) on bad news. Yet, the resulting negative order flow will prompt the firm to decrease investment, which increases the firm expected terminal value, but will in turn decrease the speculator's expected profit from her short position. That the feedback effect asymmetrically influences $\pi_{sell}^T \mid_{\xi=0}$ and $\pi_{buy}^T \mid_{\xi=0}$ and makes the BNS equilibrium region strictly contains the SNB equilibrium region (i.e., $\left[\pi_{sell}^{T}, \pi_{buy}^{NT}\right] \supset \left[\pi_{buy}^{T}, \pi_{sell}^{NT}\right]$) is the central thesis in Edmans et al. (2015).

Proposition 5 demonstrates how the trading and no-trading equilibrium interval would behave if ξ marginally moves away from 0.

Proposition 5. *(Effects of asymmetric timeliness in disclosure)*

- $\frac{d\pi_{sell}^{T}}{d\xi} > 0$ and $\frac{d\pi_{sell}^{NT}}{d\xi} > 0$.
- For any $\hat{\xi} > 0$, $max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} |_{\xi=\hat{\xi}} > max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} |_{\xi=-\hat{\xi}}$, implying that a conservative disclosure policy with asymmetric timeliness parameter $\hat{\xi}$ generates a smaller no-trading equilibrium region than an aggressive disclosure policy with asymmetric timeliness parameter $-\hat{\xi}$; $max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} |_{\xi=\hat{\xi}} > max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} |_{\xi=0}$, implying that the no-trading equilibrium region shrinks as ξ increases from 0.
- $\frac{dmin\left\{\pi_{buy}^{T}, \pi_{sell}^{T}\right\}}{d\xi} |_{\xi=0} > 0$, implying that the trading equilibrium region expands (shrinks) as ξ increases (decreases) in the neighborhood around $\xi = 0$.

The key message of Proposition 5 is that a conservative system encourages the speculator to trade on her private information more so than an aggressive system. The intuition comes from two effects of a biased reporting system. The first is the State Uncertainty Effect, which affects the market maker's posterior belief of the state conditional on the disclosure of a null signal $r = \phi$. Specifically, under a conservative system ($\xi > 0$), bad news is disclosed more timely than good news, thus seeing $r = \phi$ induces the market maker to assess higher likelihood to $\theta = H$ than to $\theta = L$. This can be shown mathematically by noting that the derivative of $t \equiv Pr\left(\theta = H \mid r = \phi\right)$ with respect to ξ is strictly positive:

$$\frac{dt}{d\xi} = \frac{1}{2}\frac{f}{1 - f\beta} > 0.$$

As such, the market maker sets a higher price, ceteris paribus, enabling a speculator who receives a private signal $\eta = L$ to sell the share at a higher price and increase her trading profit from selling ($\frac{d\pi_{sell}}{d\xi} > 0$). Note that the State Uncertainty Effect also increases the speculator's trading profit from buying with an aggressive disclosure policy ($\xi < 0$). In this case, seeing $r = \phi$ induces the market maker to believe that $\theta = L$ is more likely than $\theta = H$, as an aggressive system discloses good news in a more timely fashion than bad news. As such, the market maker sets a lower price, ceteris paribus, enabling a speculator who receives a private signal $\eta = H$ to buy the share at a lower price and increase her trading profit from buying ($\frac{d\pi_{buy}}{d\xi} > 0$).

The second effect of a biased reporting system is the Information Endowment Uncertainty Effect, which affects the speculator's belief about whether the firm observes the true state conditional on the disclosure of a null signal. Specifically, under a conservative disclosure policy, conditional on the speculator observing $\eta = L$, observing $r = \phi$ increases the speculator's belief that the firm is more likely to be uninformed of the state (i.e., $\delta = \emptyset$). Mathematically, the derivative of $\tau_L \equiv Pr(\delta = L \mid L, r = \phi)$ with respect to ξ is strictly negative:

$$\frac{d\tau_L}{d\xi} = -\frac{f(1-f)}{(1-f\beta_L)^2} < 0.$$

When the speculator believes that the firm has not observed the true state, she is also more confident that the firm will not decrease investment in the bad state. When the state is bad and the firm does not take corrective action, the firm's terminal value is lower, enabling the speculator to buy back share at a lower price at $T = 3$ to cover her short position at $T = 1$. This in turn increases her trading profit from selling ($\frac{d\pi_{sell}}{d\xi} > 0$).

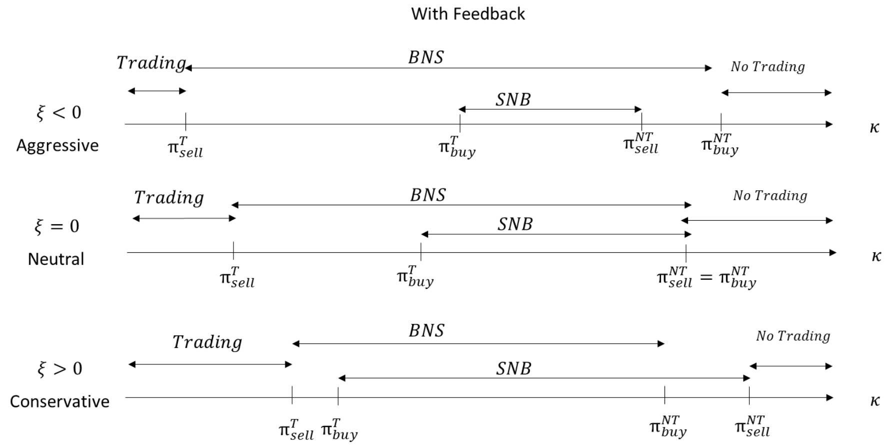
Unlike the State Uncertainty Effect, the Information Endowment Uncertainty Effect reduces the speculator's trading profit from buying with an aggressive disclosure policy. To see this, note that under an aggressive policy $\xi < 0$, observing $r = \phi$ makes the speculator (who has privately observed $\eta = H$) believe that the firm is more likely to be uninformed of the state (i.e., $\delta = \phi$) and hence not increase investment, which leads to a lower terminal price and reduces the speculator's trading profit from buying.

At $\xi = 0$, 1 shows $\pi^{NT}_{buy} = \pi^{NT}_{sell}$. Both the State Uncertainty Effect and the Information Endowment Uncertainty Effect enhance π^{NT}_{sell} when ξ increases from zero, while the two effects counteract with each other when ξ decreases from zero. As such, the no-trading region shrinks under a conservative system with $\hat{\xi} > 0$ compared to either a neutral system or an aggressive system with $-\hat{\xi}$.

At $\xi = 0$, 1 has $\pi^{T}_{buy} > \pi^{T}_{sell}$ due to the presence of the feedback effect, implying that the trading region is entirely determined by π^{T}_{sell} in a small neighborhood around $\xi = 0$. Consequently, because both the State Uncertainty Effect and the Information Endowment Uncertainty Effect increase (decrease) π^{NT}_{sell} when ξ increases (decreases) from zero, the trading equilibrium region expands (shrinks) when the disclosure policy becomes more conservative (aggressive). Figure 1 illustrates the equilibrium intervals under the aggressive, neutral, and conservative system, respectively.

Proposition 5 offers a key insight of our paper: timely loss recognition provides more incentive for the speculator to trade on her private information, which increases the information role of market prices to help the firm make more efficient investment decisions.

Figure 1: Trading and no-trading equilibrium under conservative, neutral, and aggressive system



5 Disclosure policy choices at $T = 0$

Up to this point, we have treated the disclosure policy parameter ξ as given and mainly focused on characterizing the resulting equilibria. In this section, we will endogenies the choice of ξ at $T = 0$. Specifically, we now consider ξ's choice from two perspectives. The first is from the perspective of a social planner who can dictate ξ via disclosure regulations. The social planner's objective is to maximize the firm's terminal value v, which is a measure of social welfare in the model because the speculator's trading gain and cost are simply transfers from the noise trader and transfers to any third party financiers, respectively. Thus, the social planner's problem can be characterized as

$$\max_{\xi} E\left\{v\left[\theta, d\left(\delta, X\right)\right]\right\}$$
$$s.t. 0 \leq \beta + \xi \leq 1$$
$$0 \leq \beta - \xi \leq 1.$$

The second is from the perspective of the firm whose objective is to maximize its terminal value v net of a liquidity discount. Specifically, the firm's problem is

$$\max_{\xi} E\left\{v\left[\theta, d\left(\delta, X\right)\right]\right\} - \alpha E\left(\pi\right)$$
$$s.t. 0 \leq \beta + \xi \leq 1$$
$$0 \leq \beta - \xi \leq 1,$$

where π is the speculator's trading profit and $\alpha \in [0, 1]$ is a commonly known parameter. Such an objective function can arise in a situation where the firm chooses ξ anticipating with probability α that it needs to issue shares to liquidity traders to raise capital and these traders price protect themselves against future losses to the speculator during the trading game at $T = 1$.

Proposition 6. *For any level of trading cost κ, there exists a conservative system that is weakly preferred by the social planner.*

Note that the firm's expected terminal value is maximized when informed trading is maximized and the firm is able to glean the most amount of speculator's private information from the order flow to make correct investment decisions. The key in proving Proposition 6 is to establish the social welfare ranking under the four equilibria. Specifically, in terms of the firm's expected terminal value, the trading equilibrium dominates the BNS/SNB equilibrium; and the BNS/SNB equilibrium dominates the no-trading equilibrium. Intuitively, the trading equilibrium brings both good news and bad news into prices via the speculator's trading; the SNB/BNS equilibrium brings only one-sided news into prices (given that state $\theta = H$ and $\theta = L$ are equally likely ex ante, these two equilibria generate the same expected firm value); and prices don't reflect any of the speculator's information in the no-trading equilibrium. Proposition 6 is thus immediate because the trading region expands and no trading equilibrium region shrinks under a conservative system with $\hat{\xi} > 0$ compared to an aggressive system with $-\hat{\xi}$ as shown by Proposition 5.

In contrast, Proposition 7 suggests that from the firm's perspective, the firm may prefer an aggressive disclosure policy.

Proposition 7. *Fix any $\hat{\xi} > 0$. Consider a neutral policy ($\xi = 0$), a conservative policy with $\beta_H = \beta - \hat{\xi}$ and $\beta_L = \beta + \hat{\xi}$, and an aggressive policy with $\beta_H = \beta + \hat{\xi}$ and $\beta_L = \beta - \hat{\xi}$. Suppose the trading equilibrium is sustained under all three policies. Then, the firm strictly prefers the aggressive disclosure policy.*

When the trading cost is sufficiently low, the trading equilibrium prevails under all disclosure regimes. In this case, the speculator always trades and the firm always learns from prices, and the expected firm value is not affected by any asymmetry in timeliness, i.e. $E[v(\theta, d)]$ is the same for all three regimes. Compared to the aggressive regime, a conservative regime gives the speculator a higher expected trading profit, thus lowers the firm's objective by increasing the liquidity discount due to adverse selection. The next proposition gives sufficient conditions under which the social planner and firm's preferences coincide or diverge.

Proposition 8. *Suppose $\kappa > \kappa^T |_{\xi=0}$ and $\kappa - \kappa^T |_{\xi=0}$ is sufficiently small.*

1) The social planner strictly prefers a conservative system.

2) The firm prefers a conservative system if either of the following holds:

(a) α (the weight the firm puts on liquidity discount) is sufficiently small;

(b) f is sufficiently small, and x is sufficiently big relative to $R_H - R_L$;

(c) f is sufficiently small and λ is sufficiently big.

3) The firm prefers an aggressive system if α is sufficiently big, and any of the following holds:

(b) f is sufficiently big, and x is sufficiently small relative to $R_H - R_L$;

(c) f is sufficiently big and λ sufficiently small.

When the trading cost κlies just slightly above $\kappa_{sell}^{T}\mid_{\xi=0}$, the BNS equilibrium prevails with a neutral disclosure policy or with an aggressive policy for which ξ is not too negative, as implied by Proposition 5. When ξ increases from zero and the disclosure policy becomes more conservative, there will be a discrete jump in the expected firm's terminal value because it can learn more information from prices when the trading equilibrium prevails. In contrast, as far as the firm's objective is concerned, a switch to the trading equilibrium also entails a discontinuous jump in the speculator's trading profit and thus a larger price discount. Whether the firm prefers a conservative policy depends on its tradeoff between benefits of learning from prices and the cost of adverse selection discount in share offerings. Proposition 8 shows that conservatism is strictly preferred by the firm when it doesn't care too much about the price discount in its objective. That is, when $r$becomes small, the firm's objective converges to that of the social planner who strictly prefers a conservative system. The conservative policy dominates also when the firm doesn't have much private information and its investment decision's impact is big relative to its asset in place. As f becomes smaller, the firm needs to increasingly rely on information gleaned from prices to make its investment decision, which has a higher economic impact relative to the asset in place when x is large compared to $R_H - R_L$. Finally, conservatism also prevails when f is small but λ is big, because a big λ relative to f implies the speculator's information advantage over the firm is substantial and the benefits for the firm to learn from prices are big. In contrast, when the firm already has a lot of private information (f is big), or when the speculator is not that well informed (λ is small), or when the firm's investment decision doesn't have much of an economic impact relative to its asset in place ($x$is small relative to $R_H - R_L$), the liberal policy that reduces the price discount dominates. By identifying conditions under which the social planner's preferences diverge from the firm's, our paper sheds light on the important normative question when it is socially beneficial for financial reporting regulators such FASB or IASB to impose conservative disclosure requirements on firms.

6 Extensions and empirical implications

6.1 Extension: Endogenize both overall and asymmetric timeliness

In the main analysis, we assume that the firm only chooses the degree of asymmetric timeliness, while the overall timeliness (β) is fixed. In this subsection, we allow the firm to endogenize both β and ξ. Next we show that, when learning from feedback channel is important, even if the firm can choose both β and ξ, the firm's optimal reporting system is conservative.

Proposition 9. *Suppose the firm can choose both β and ξ to maximize $E\left(v\left(\theta,d\right)\right)-\alpha E\left(\pi\right)$, subject to ensuring that the trading equilibrium is played in trading stage. Denote $\underline{\kappa}=\left(\frac{1}{6}+\frac{1}{3}\left(\frac{1-\lambda}{2-\lambda}\right)\right)\left(R_H-R_L\right)-\frac{2}{3}\left(\frac{1-\lambda}{2-\lambda}\right)x$. When $\kappa>\underline{\kappa}$ and $\kappa-\underline{\kappa}$ is sufficiently small, the optimal reporting system has $\beta<1$ and $\xi>0$.*

Proposition 9 considers the endogenous choice of both overall timeliness and asymmetric timeliness (β and ξ). In this case, the firm's problem is to choose β and ξ such that:

$$
\begin{aligned}
&\max_{\beta,\xi} E^T\left(v\left(\theta,d\right)\right)-\alpha E^T\left(\pi\right)\\
&s.t. d\in\arg\max_{\{-1,0,1\}} E\left(v\left(\theta,d\right)|\delta,X\right)\\
&\quad s\in\arg\max_{\{-1,0,1\}} sE\left(v-P\left(X\right)|\eta\right)\\
&\quad P\left(X\right)=E\left(v\left(\theta,d\left(\delta,X\right)\right)|r,X\right)\\
&\quad \pi^T_{buy}\geq\kappa\\
&\quad \pi^T_{sell}\geq\kappa
\end{aligned}
$$

When learning from feedback is valuable, there exists a range of trading cost under which the optimal reporting system is conservative. To understand this result, recall that conservatism improves firm value through enabling trading equilibrium. That is, it increases the speculator's profit form selling on bad news and thus ensures such information is incorporated into price and can guide firm investment. Intuitively, when κ is sufficiently small so that the trading equilibrium always prevails, thus a conservative system only leads to larger liquidity discount and is suboptimal. When κ is in this intermediate range, conservatism reporting enables the speculator to always trade on her private information and improves firm value. Under the optimal system, the speculator who observes $\theta=L$ just breaks even while the speculator who observes $\theta=H$ makes a positive profit. Thus timely loss recognition ($\xi>0$) allows the firm to maintain investment efficiency but reduce overall timeliness β to minimize the liquidity discount, and thus is optimal.

6.2 Empirical implications

Feedback effect and timely loss recognition Our analysis shows that timely loss recognition improves firm value via facilitating feedback effect, that is, through affecting price's ability to incorporate trader's private information. In the absence of feedback effect, a conservative system is not preferred compared to an aggressive system, since conservatism leads to higher information asymmetry between informed and uninformed investors, and thus to a higher liquidity discount. Our analysis shows that other things equal, conservatism is more beneficial, and thus more likely to be adopted by a firm, when (1) the firm value depends more on the subsequent decision taken and thus can potentially benefit from feedback effect; (2) when it is less costly to adjust the firm's investment, (3) when the firm's internal information is of lower quality, or when the speculator's information is of higher quality.

Also, from a social welfare perspective, the price discount from the firm to noise trader, or form noise traders to speculators, is merely a wealth transfer and does not affect the overall welfare. If a social planner wants to design the optimal reporting standards to maxis social welfare, then the socially optimal system would maximize the firm's investment efficiency and thus would be a conservative system. Furthermore, our analysis shows that there is an optimal amount of conservatism: more conservatism is beneficial only to the extent that it increases price informativeness.

Short-selling constraint In practice, the cost of selling on bad news is often higher than the cost of buying on good news, due to short selling constraints. When $\kappa_{sell} > \kappa_{buy}$, this *amplifies* the asymmetry in trading behavior caused by the feedback effect. As a result, speculator's private information about bad news is further less likely to get incorporated stock price. In this case, timely loss recognition would be more useful at increasing the profit from selling on bad news, which increases overall price informativeness and ultimately improves real investment efficiency.

Implication on return-earning relation Our analysis shows that in the presence of feedback effect, even when the accounting reports are neutral, we can still observe an asymmetry in earnings' correlation with positive stock returns vs. with negative stock returns. This is because of the asymmetric effect the feedback channel has on price's ability to reflect negative vs. positive news. Thus, the commonly used Basu measure may not properly capture cross-sectional differences in firms' timely loss recognition: it can also differ cross-sectionally depending on the differences in returns, i.e., the strength of the feedback effect. Since the feedback effect benefits firm performance and generates asymmetric slope estimates in earnings-stock return regressions, it can also produce positive associations between firm performance and the Basu-like measures even with neutral accounting. This provides a potential explanation for prior studies that finds supposedly neutral measures also exhibit asymmetric correlation with positive versus negative returns. For example, Collins et al. (2014) find asymmetry in cash flow's correlation with positive vs. with negative stock returns, and Patatoukas and Thomas (2011) show lagged earnings also exhibits similar asymmetry. We also show that timely loss recognition mitigates the feedback channel's asymmetric effect on price informativeness. This implies that timely loss recognition would generate less asymmetric correlations than under the neutral system, whereas the aggressive reporting would exacerbate the asymmetry.

7 Conclusion

This paper analyzes how timely loss recognition affects firm performance via the feedback channel of financial markets. To do so we first adopt a new modeling approach that directly captures the *asymmetric timeliness* in loss recognition. Our analysis reveals a novel channel through which conservatism affects firm's real decisions and firm value. By preempting more bad news, timely loss recognition changes the market dynamics when public disclosure is absent, and helps mitigate the distortion the feedback channel has on price informativeness identified in Edmans, Goldstein and Jiang (2015). Specifically, Edmans, et al. (2015) show that feedback effect affects speculators' incentive to trade on good and bad news *asymmetrically*: it increases (reduces) their incentive to trade on positive (negative) news. Thus bad news faces an endogenous limit to arbitrage and is less likely to be reflected in price than good news, which reduces firms' investment efficiency and add to the instability of the real economy. We show that timely loss recognition in accounting reporting can mitigate the distortions the feedback channel has on speculators' incentives to trade on negative news. This is because under a conservative system, the asymmetric withholding of firm' internal information enlarges the gap in firm value between two states, which increases the speculator's trading profit (corrective action effect). Furthermore, since bad news is more likely to have already been pre-empted, no disclosure implies the firm is more likely to be in a good state. Thus the market sets price higher, which further increases the profit of selling on bad news and counteracts the asymmetry in trading behavior by feedback effects.

We show that the desirability of conservatism critically hinges on its impact on feedback effect. With feedback effect, timely loss recognition improves the firm value through enhancing price informativeness and enabling the firm to make better investment decisions. Without feedback effect, however, timely loss recognition leads to higher information asymmetry among traders and thus a larger liquidity discount.

Appendix: Proof

Proof of Proposition 1

In the no-trading equilibrium, the speculator does not trade on her private information (on the equilibrium path) and thus the order flow is not informative about state (i.e., no feedback effect). As such, the firm increases (decreases) investment when it is privately informed of $\delta = H$ ($\delta = L$) with a resulting firm value $v(H,1) = R_H + x - c$ ($v(L,-1) = R_L + x - c$). When the firm receives a null signal ($\delta = \phi$), $\mu_F^{\phi,X} = \frac{1}{2}$, $\forall X$. Consequently, by Lemma 1 it keeps the status quo. Lastly, when the firm observes $\delta = \phi$ and $X = -2$ or 2 (which is an off equilibrium play), the only belief that satisfies the Intuitive Criterion is $\left\{\mu_F^{\phi,2} = 1, \mu_F^{\phi,-2} = 0\right\}$, which results in the firm increasing (decreasing) investment with $X = 2$ (-2). To see this, suppose that a speculator who privately observes $\eta = L$ and submits a buying order and that the total order flow is $X = 2$. In this case, the most favorable belief that benefits such a speculator is that everyone else thinks she has observed $\eta = L$, because otherwise the purchasing price would be even higher. But even with this most favorable belief, the speculator is still not able to generate a positive net trading profit due to the existence of the trading cost.

As to the market maker, let's use $\mu_M^{r,X}$ to denote his posterior belief for $\theta = H$ conditional on the firm's disclosure r and order flow X (i.e., $\mu_M^{r,X} \equiv Pr(\theta = H \mid r, X)$). When the firm discloses θ, obviously the market maker sets the price $P(X,\theta) = R_\theta + x - c$, $\forall X$. When the firm discloses a null signal ($r = \phi$) and the market maker observes $X = -2$ or 2, the Intuitive Criterion requires $\left\{\mu_M^{\phi,2} = 1, \mu_M^{\phi,-2} = 0\right\}$ (as argued above), which leads to $P(2,\phi) = R_H + x - c$ and $P(-2,\phi) = R_L + x - c$. In contrast, when $r = \phi$ and $X \in \{-1, 0, 1\}$, we have

$$\begin{aligned}
\mu_M^{\phi,X} &= Pr(\theta = H \mid r = \phi, X) \\
&= \frac{\Pr(\theta = H, r = \phi, X)}{\Pr(\theta = H, r = \phi, X) + \Pr(\theta = L, r = \phi, X)} \\
&= \frac{1}{2} + \frac{1}{2}\frac{f}{(1 - f\beta)}\xi = t.
\end{aligned}$$

Note that any order flow in the set $\{-1, 0, 1\}$ generate the same posterior belief for the firm when its disclosure does not reveal θ. This is because the order flow in this equilibrium doesn't provide any information about the state. Thus, the market maker sets the price as follows:

$$\begin{aligned}
P(1,\phi) &= Pr(\theta = H \mid r=\phi, X=1)\left[Pr(\delta = H|\theta = H, r=\phi)\, v(H,1) + (1 - Pr(\delta = H|\theta = H, r=\phi))\, v(H,0)\right] + \\
&\quad \left[1 - Pr(\theta = H \mid r=\phi, X=1)\right]\left[Pr(\delta = L|\theta = L, r=\phi)\, v(L,-1) + (1 - Pr(\delta = L|\theta = L, r=\phi))\, v(L,0)\right] \\
&= \mu_M^{\phi,1}\left[\tau_H (R_H + x - c) + (1-\tau_H) R_H\right] + \left(1 - \mu_M^{\phi,1}\right)\left[\tau_L (R_L + x - c) + (1-\tau_L) R_L\right] \\
&= t\left[R_H + \tau_H (x-c)\right] + (1-t)\left[R_H + \tau_H (x-c)\right] \\
&= tR_H^{'} + (1-t) R_L^{'}.
\end{aligned}$$

To understand the first equality, note when $\theta = H$ (L) the firm value depends on $\delta's$ realization: if $\delta = \theta$ (ϕ), $v(\theta,1) = R_\theta + x - c$ $(v(\theta,0) = R_\theta)$. This is because in this equilibrium the firm changes its investment if and only if it is private informed of θ and maintains the status quo when it is unformed. Similarly, we have $P(-1,\phi) = P(0,\phi) = tR_H^{'} + (1-t) R_L^{'}$.

Finally, we need to verify that given the the firm's investment decision and the market maker's pricing strategy as prescribed in the proposition the speculator indeed doesn't want to trade on her private information. Let's denote the speculator's gross trading profit (i.e., without taking the trading cost κinto account) from buying (selling) one share of the firm's stock in the no–trading equilibrium as π_{buy}^{NT} (π_{sell}^{NT}). If the speculator observes $\eta = H$ and deviates by buying, with equal probability of $\frac{1}{3}$, the order flow would be 2, 1, or 0. Thus, we have

$$\begin{aligned}
\pi_{buy}^{NT} &= \frac{1}{3}\left[R_H' - P(0,\phi)\right] + \frac{1}{3}\left[R_H' - P(1,\phi)\right] + \frac{1}{3}\left[R_H' - P(2,\phi)\right] \\
&= \frac{2}{3}(1-t)\left(R_H^{'} - R_L^{'}\right).
\end{aligned}$$

In contrast, if the speculator observes $\eta = L$ and deviates by selling, with equal probability of $\frac{1}{3}$, the order flow would be -2, -1, or 0. We have

$$\begin{aligned}
\pi_{sell}^{NT} &= \frac{1}{3}\left[P(-1,\phi) - R_L'\right] + \frac{1}{3}\left[P(0,\phi) - R_L'\right] + \frac{1}{3}\left[R_L' - P(-2,\phi)\right] \\
&= \frac{2}{3}t\left(R_H^{'} - R_L^{'}\right).
\end{aligned}$$

As such, no trading by the speculator constitutes an equilibrium if and only if the cost of trading exceeds the gross trading profits, i.e., $\kappa > \max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} = \frac{2}{3}(R_H - R_L)\max\{t,(1-t)\}$. Q.E.D.

Proof of Proposition 2

In this equilibrium, the speculator buys on good news and sells on bad news when the firm's disclosure doesn't reveal θ. For a firm that is not privately informed of θ, its posterior belief upon observing X is as follows:

$$\begin{aligned}
\mu_F^{\phi,1} &= \frac{Pr(\theta=H,z=0,\eta=H)+Pr(\theta=H,z=1,\eta=\phi)}{Pr(\theta=H,z=0,\eta=H)+Pr(\theta=H,z=1,\eta=\phi)+Pr(\theta=L,z=1,\eta=\phi)}\\
&= \frac{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}(1-\lambda)}{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}(1-\lambda)+\frac{1}{2}\frac{1}{3}(1-\lambda)}=\frac{1}{2-\lambda};\\
\mu_F^{\phi,0} &= \frac{Pr(\theta=H,z=-1,\eta=H)+Pr(\theta=H,z=0,\eta=\phi)}{Pr(\theta=H,z=-1,\eta=H)+Pr(\theta=H,z=0,\eta=\phi)+Pr(\theta=L,z=1,\eta=L)+Pr(\theta=L,z=0,\eta=\phi)}\\
&= \frac{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}(1-\lambda)}{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}(1-\lambda)+\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}(1-\lambda)}=\frac{1}{2};\\
\mu_F^{\phi,-1} &= \frac{Pr(\theta=H,z=-1,\eta=\phi)}{Pr(\theta=H,z=-1,\eta=\phi)+Pr(\theta=L,z=0,\eta=L)+Pr(\theta=L,z=-1,\eta=\phi)}\\
&= \frac{\frac{1}{2}\frac{1}{3}(1-\lambda)}{\frac{1}{2}\frac{1}{3}(1-\lambda)+\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}(1-\lambda)}=\frac{1-\lambda}{2-\lambda};\\
\mu_F^{\phi,2} &= 1;\\
\mu_F^{\phi,-2} &= 0.
\end{aligned}$$

When the order flow equals either -2 or 2, the speculator's private information is perfectly revealed and thus the firm's posterior uncertainty disappears (i.e., $\mu_F^{\phi,2}$ and $\mu_F^{\phi,-2}$ take extreme values), which results in the firm optimally increasing (decreasing) investment with $X=2$ $(X=-2)$. In contrast, the firm is not able to update its belief when observing a order flow of 0 and thus optimally maintains the status quo. Finally, when $X=1$ (-1), though the firm is not able to perfectly back out the speculator's information, such an event is likely to come from the speculator buying (selling) on $\eta=H$ (L). In other words, seeing $X=1$ (-1) tilts the firm's posterior belief in favor of state H (L). As $\lambda>\frac{2c}{x+c}$ implies $\mu_F^{\phi,1}>\mu_F^{*}$ and $\mu_F^{\phi,-1}<\mu_F^{**}$, $X=1$ (-1) constitutes a strong enough indicator to induce the firm to increase (decrease) investment.

Next, let's move to study the market maker's strategy. When the market maker observes $r=\phi$ and $X=1$, his posterior belief for $\theta=H$ is

$$\begin{aligned}
\mu_M^{\phi,1} &= \frac{\Pr(\theta=H,X=1|r=\phi)}{\Pr(\theta=H,X=1|r=\phi)+\Pr(\theta=L,X=1|r=\phi)}\\
&= \frac{\Pr(\theta=H|r=\phi)\Pr(X=1|\theta=H,r=\phi)}{\Pr(\theta=H|r=\phi)\Pr(X=1|\theta=H,r=\phi)+\Pr(\theta=L|r=\phi)\Pr(X=1|\theta=L,r=\phi)}\\
&= \frac{t\left[\frac{1}{3}\lambda+\frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda+\frac{1}{3}(1-\lambda)\right]+\frac{1}{3}(1-t)(1-\lambda)}\\
&= \frac{t}{t+(1-t)(1-\lambda)}.
\end{aligned}$$

Consequently, the market maker sets the price as

$$\begin{aligned}
P(1,\phi) &= Pr(\theta = H \mid r = \phi, X = 1)\left[Pr(\delta = H|\theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H|\theta = H, r = \phi))\, v(H,1)\right] + \\
&\quad \left[1 - Pr(\theta = H \mid r = \phi, X = 1)\right]\left[Pr(\delta = L|\theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L|\theta = L, r = \phi))\, v(L,1)\right] \\
&= \mu_M^{\phi,1}(R_H + x - c) + \left(1 - \mu_M^{\phi,1}\right)\left[\tau_L(R_L + x - c) + (1 - \tau_L)(R_L - x - c)\right] \\
&= \mu_M^{\phi,1}(R_H + x - c) + \left(1 - \mu_M^{\phi,1}\right)\left[R_L + \tau_L x - (1 - \tau_L)x - c\right].
\end{aligned}$$

To understand the first equality, note when $\theta = H$ the firm value is $v(H,1) = R_H + x - c$, since the firm increases investment whether $\delta = H$ or ϕ. When $\theta = L$, however, the firm value depends on $\delta' s$ realization. An privately informed firm decreases investment and generates a value of $v(L,-1) = R_L + x - c$ while an uninformed firm relies on order flow and increase investment, which reduces the firm value to $v(L,1) = R_L - x - c$. Similarly, we can calculate the market maker's posterior beliefs and prices for all other realizations of X:

$$\begin{aligned}
\mu_M^{\phi,0} &= \frac{\Pr(\theta = H, X = 0|r = \phi)}{\Pr(\theta = H, X = 0|r = \phi) + \Pr(\theta = L, X = 0|r = \phi)} \\
&= \frac{\Pr(\theta = H|r = \phi)\Pr(X = 0|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = 0|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = 0|\theta = L, r = \phi)} \\
&= \frac{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right] + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = t;
\end{aligned}$$

$$\begin{aligned}
P(0,\phi) &= Pr(\theta = H \mid r = \phi, X = 0)\left[Pr(\delta = H|\theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H|\theta = H, r = \phi))\, v(H,0)\right] + \\
&\quad \left[1 - Pr(\theta = H \mid r = \phi, X = 0)\right]\left[Pr(\delta = L|\theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L|\theta = L, r = \phi))\, v(L,0)\right] \\
&= t\left[R_H + \tau_H(x - c)\right] + (1-t)\left[R_L + \tau_L(x - c)\right] \\
&= tR'_H + (1-t)R'_L;
\end{aligned}$$

$$\begin{aligned}
\mu_M^{\phi,-1} &= \frac{\Pr(\theta = H, X = -1|r = \phi)}{\Pr(\theta = H, X = -1|r = \phi) + \Pr(\theta = L, X = -1|r = \phi)} \\
&= \frac{\Pr(\theta = H|r = \phi)\Pr(X = -1|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = -1|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = -1|\theta = L, r = \phi)} \\
&= \frac{t\frac{1}{3}(1-\lambda)}{t\frac{1}{3}(1-\lambda) + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = \frac{t(1-\lambda)}{t(1-\lambda) + (1-t)};
\end{aligned}$$

$$
\begin{aligned}
P(-1,\phi) &= Pr(\theta = H \mid r=\phi, X=-1)\left[Pr(\delta=H|\theta=H, r=\phi)\, v(H,1) + (1-Pr(\delta=H|\theta=H,r=\phi))\, v(H,-1)\right] + \\
&\quad [1-Pr(\theta=H \mid r=\phi, X=-1)]\left[Pr(\delta=L|\theta=L,r=\phi)\, v(L,-1) + (1-Pr(\delta=L|\theta=L,r=\phi))\, v(L,-1)\right] \\
&= \mu_M^{\phi,-1}\left[\tau_H(R_H+x-c) + (1-\tau_H)(R_H-x-c)\right] + \left(1-\mu_M^{\phi,-1}\right)(R_L+x-c);
\end{aligned}
$$

$$
\mu_M^{\phi,2} = 1;
$$

$$
P(2,\phi) = R_H + x - c;
$$

$$
\mu_M^{\phi,-2} = 0;
$$

and

$$
P(-2,\phi) = R_L + x - c.
$$

Finally, we need to verify that given the the firm's investment decision and the market maker's pricing strategy as prescribed in the proposition the speculator indeed want to trade on her private information. Let's denote the speculator's gross trading profit (i.e., without taking the trading cost κinto account) from buying (selling) one share of the firm's stock in the trading equilibrium as π_{buy}^T (π_{sell}^T). If the speculator observes $\eta = H$ and buys a share, with equal probability of $\frac{1}{3}$, the order flow would be 2, 1, or 0. Thus, we have

$$
\begin{aligned}
\pi_{buy}^T &= \frac{1}{3}\left[R_H' - P(0,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(1,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(2,\phi)\right] \\
&= \frac{1}{3}(1-t)(R_H' - R_L') + \frac{1}{3}\left(1-\mu_M^{\phi,1}\right)\left[R_H' + (1-\tau_H)(x-c) - R_L' + (1-\tau_L)(x+c)\right] \\
&= \frac{1}{3}(1-t)(R_H' - R_L') + \frac{1}{3}\left(1-\mu_M^{\phi,1}\right)\left[R_H - R_L + 2(1-\tau_L)x\right].
\end{aligned}
$$

In contrast, if the speculator observes $\eta = L$ and sells one share, with equal probability of $\frac{1}{3}$, the order flow would be -2, -1, or 0. We have

$$
\begin{aligned}
\pi_{sell}^T &= \frac{1}{3}\left[P(0,\phi) - R_L'\right] + \frac{1}{3}\left[P(-1,\phi) - (R_L + x - c)\right] + \frac{1}{3}\left[P(-2,\phi) - (R_L + x - c)\right] \\
&= \frac{1}{3}t(R_H' - R_L') + \frac{1}{3}\mu_M^{\phi,-1}\left[R_H - R_L - 2(1-\tau_H)x\right]
\end{aligned}
$$

As such, no trading by the speculator constitutes an equilibrium if and only if the cost of trading is lower the gross trading profits, i.e., $\kappa < min\left\{\pi^T_{buy}, \pi^T_{sell}\right\}$. Q.E.D.

Proof of Proposition 3

In this equilibrium, when $r = \phi$ (i.e., θ is not revealed by the firm), the speculator buys when she privately observes $\eta = H$ but does not trade when she observes $\eta = L$. For a firm that is not privately informed of θ, its posterior belief upon observing X is as follows:

$$
\begin{aligned}
\mu_F^{\phi,1} &= \frac{Pr\left(\theta = H, z = 0, \eta = H\right) + Pr\left(\theta = H, z = 1, \eta = \phi\right)}{Pr\left(\theta = H, z = 0, \eta = H\right) + Pr\left(\theta = H, z = 1, \eta = \phi\right) + Pr\left(\theta = L, z = 1, \eta = \phi\right) + Pr\left(\theta = L, z = 1, \eta = L\right)} \\
&= \frac{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}\left(1-\lambda\right) + \frac{1}{2}\frac{1}{3}\left(1-\lambda\right) + \frac{1}{2}\frac{1}{3}\lambda} = \frac{1}{2}; \\
\mu_F^{\phi,0} &= \frac{Pr\left(\theta = H, z = -1, \eta = H\right) + Pr\left(\theta = H, z = 0, \eta = \phi\right)}{Pr\left(\theta = H, z = -1, \eta = H\right) + Pr\left(\theta = H, z = 0, \eta = \phi\right) + Pr\left(\theta = L, z = 0, \eta = L\right) + Pr\left(\theta = L, z = 0, \eta = \phi\right)} \\
&= \frac{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}\left(1-\lambda\right) + \frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}\left(1-\lambda\right)} = \frac{1}{2}; \\
\mu_F^{\phi,-1} &= \frac{Pr\left(\theta = H, z = -1, \eta = \phi\right)}{Pr\left(\theta = H, z = -1, \eta = \phi\right) + Pr\left(\theta = L, z = -1, \eta = L\right) + Pr\left(\theta = L, z = -1, \eta = \phi\right)} \\
&= \frac{\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}{\frac{1}{2}\frac{1}{3}\left(1-\lambda\right) + \frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}\left(1-\lambda\right)} = \frac{1-\lambda}{2-\lambda}; \\
\mu_F^{\phi,2} &= 1; \\
\mu_F^{\phi,-2} &= 0.
\end{aligned}
$$

When the order flow equals 2, the speculator's private information is perfectly revealed and thus the firm's posterior uncertainty disappears (i.e., $\mu_F^{\phi,2} = 1$), which results in the firm optimally increasing investment with $X = 2$. Because the speculator doesn't sell on bad news when $r = \phi$, $X = -2$ is off the equilibrium path. Using the same argument as in the no-trading equilibrium, the only belief that survives Intuitive Criterion is that the speculator is privately informed of $\theta = L$. When $X = 0$ and $X = 1$, the firm is not able to update its belief and thus optimally maintains the status quo. Finally, when $X = -1$, though the firm is not able to perfectly back out the speculator's information, such an event is likely to come from a speculator who has privately observed $\eta = L$ but doesn't participate in trading. In other words, seeing $X = -1$ tilts the firm's posterior belief in favor of state L. As $\lambda > \frac{2c}{x+c}$ implies $\mu_F^{\phi,-1} < \mu_F^{**}$, $X = -1$ constitutes a strong enough indicator to induce the firm to decrease investment.

Next, let's move to study the market maker's strategy. When the market maker observes $r = \phi$ and

$X = 1$, his posterior belief for $\theta = H$ is

$$\begin{aligned}
\mu_M^{\phi,1} &= \frac{\Pr(\theta = H, X = 1|r = \phi)}{\Pr(\theta = H, X = 1|r = \phi) + \Pr(\theta = L, X = 1|r = \phi)} \\
&= \frac{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = 1|\theta = L, r = \phi)} \\
&= \frac{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right] + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = t.
\end{aligned}$$

Consequently, the market maker sets the price as

$$\begin{aligned}
P(1,\phi) &= Pr(\theta = H \mid r = \phi, X = 1)\left[Pr(\delta = H|\theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H|\theta = H, r = \phi))\, v(H,0)\right] + \\
&\quad \left[1 - Pr(\theta = H \mid r = \phi, X = 1)\right]\left[Pr(\delta = L|\theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L|\theta = L, r = \phi))\, v(L,0)\right] \\
&= \mu_M^{\phi,1}\left[\tau_H(R_H + x - c) + (1-\tau_H)R_H\right] + \left(1 - \mu_M^{\phi,1}\right)\left[\tau_L(R_L + x - c) + (1-\tau_L)R_L\right] \\
&= t\left[R_H + \tau_H(x - c)\right] + (1-t)\left[R_L + \tau_L(x - c)\right].
\end{aligned}$$

To understand the first equality, when $\theta = H$ (L) and the firm's private signal δ reveals the state the firm value is $v(H,1) = R_H + x - c$ $(v(L,1) = R_L + x - c)$, since the firm increases (decreases) investment. In contrast, when $\theta = H$ (L) and the firm's private signal δ is a null signal, the firm chooses to maintain the status quo with a firm value $v(H,0) = R_H$ $(v(L,0) = R_L)$. Similarly, we can calculate the market maker's posterior beliefs and prices for all other realizations of X:

$$\begin{aligned}
\mu_M^{\phi,0} &= \frac{\Pr(\theta = H, X = 0|r = \phi)}{\Pr(\theta = H, X = 0|r = \phi) + \Pr(\theta = L, X = 0|r = \phi)} \\
&= \frac{\Pr(\theta = H|r = \phi)\Pr(X = 0|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = 0|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = 0|\theta = L, r = \phi)} \\
&= \frac{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right] + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = t;
\end{aligned}$$

$$\begin{aligned}
P(0,\phi) &= Pr(\theta = H \mid r = \phi, X = 0)\left[Pr(\delta = H|\theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H|\theta = H, r = \phi))\, v(H,0)\right] + \\
&\quad \left[1 - Pr(\theta = H \mid r = \phi, X = 0)\right]\left[Pr(\delta = L|\theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L|\theta = L, r = \phi))\, v(L,0)\right] \\
&= t\left[R_H + \tau_H(x - c)\right] + (1-t)\left[R_L + \tau_L(x - c)\right] \\
&= tR'_H + (1-t)R'_L;
\end{aligned}$$

$$
\begin{aligned}
\mu_M^{\phi,-1} &= \frac{\Pr(\theta = H, X = -1 | r = \phi)}{\Pr(\theta = H, X = -1 | r = \phi) + \Pr(\theta = L, X = -1 | r = \phi)} \\
&= \frac{\Pr(\theta = H | r = \phi)\Pr(X = -1 | \theta = H, r = \phi)}{\Pr(\theta = H | r = \phi)\Pr(X = -1 | \theta = H, r = \phi) + \Pr(\theta = L | r = \phi)\Pr(X = -1 | \theta = L, r = \phi)} \\
&= \frac{t\frac{1}{3}(1-\lambda)}{t\frac{1}{3}(1-\lambda) + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = \frac{t(1-\lambda)}{t(1-\lambda)+(1-t)};
\end{aligned}
$$

$$
\begin{aligned}
P(-1,\phi) &= Pr(\theta = H \mid r = \phi, X = -1)\left[Pr(\delta = H | \theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H | \theta = H, r = \phi))\, v(H,-1)\right] + \\
&\quad [1 - Pr(\theta = H \mid r = \phi, X = -1)]\left[Pr(\delta = L | \theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L | \theta = L, r = \phi))\, v(L,-1)\right. \\
&= \mu_M^{\phi,-1}\left[\tau_H(R_H + x - c) + (1-\tau_H)(R_H - x - c)\right] + \left(1 - \mu_M^{\phi,-1}\right)(R_L + x - c);
\end{aligned}
$$

$$\mu_M^{\phi,2} = 1;$$

$$P(2,\phi) = R_H + x - c;$$

$$\mu_M^{\phi,-2} = 0;$$

and

$$P(-2,\phi) = R_L + x - c.$$

This buy-not-sell equilibrium is sustained if the trading cost is lower than the profit from buying on good news, but higher than the profit from selling on bad news. The speculator's expected gross trading profit from buying on good news is:

$$
\begin{aligned}
\pi_{buy}^{BNS} &= \frac{1}{3}\left[R'_H - P(0,\phi)\right] + \frac{1}{3}\left[R'_H - P(1,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(2,\phi)\right] \\
&= \frac{2}{3}(1-t)(R'_H - R'_L) = \pi_{buy}^{NT}
\end{aligned}
$$

If the speculator privately observes $\eta = L$ and deviates to sell one share, with equal probability of $\frac{1}{3}$, the order flow would be -2, -1, or 0. We have

$$\begin{aligned}\pi_{sell}^{BNS} &= \frac{1}{3}\left[P\left(0,\phi\right)-R_L'\right]+\frac{1}{3}\left[P\left(-1,\phi\right)-\left(R_L+x-c\right)\right]+\frac{1}{3}\left[P\left(-2,\phi\right)-\left(R_L+x-c\right)\right] \\ &= \frac{1}{3}t\left(R_H'-R_L'\right)+\frac{1}{3}\mu_M^{\phi,-1}\left[R_H-R_L-2\left(1-\tau_H\right)x\right]=\pi_{sell}^{T}\end{aligned}$$

Thus, the BNS equilibrium exists if and only if $\pi_{buy}^{NT} > \kappa > \pi_{sell}^{T}$. Q.E.D.

Proof of Proposition 4

In this equilibrium, when $r = \phi$ (i.e., θ is not revealed by the firm), the speculator sells when she privately observes $\eta = L$ but does not trade when she observes $\eta = H$. For a firm that is not privately informed of θ, its posterior belief upon observing X is as follows:

$$\begin{aligned}\mu_F^{\phi,1} &= \frac{Pr\left(\theta=H,z=1,\eta=H\right)+Pr\left(\theta=H,z=1,\eta=\phi\right)}{Pr\left(\theta=H,z=1,\eta=H\right)+Pr\left(\theta=H,z=1,\eta=\phi\right)+Pr\left(\theta=L,z=1,\eta=\phi\right)} \\ &= \frac{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)+\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}=\frac{1}{2-\lambda}; \\ \mu_F^{\phi,0} &= \frac{Pr\left(\theta=H,z=0,\eta=H\right)+Pr\left(\theta=H,z=0,\eta=\phi\right)}{Pr\left(\theta=H,z=0,\eta=H\right)+Pr\left(\theta=H,z=0,\eta=\phi\right)+Pr\left(\theta=L,z=1,\eta=L\right)+Pr\left(\theta=L,z=0,\eta=\phi\right)} \\ &= \frac{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}{\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)+\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}=\frac{1}{2}; \\ \mu_F^{\phi,-1} &= \frac{Pr\left(\theta=H,z=-1,\eta=\phi\right)+Pr\left(\theta=H,z=0,\eta=L\right)}{Pr\left(\theta=H,z=-1,\eta=\phi\right)+Pr\left(\theta=H,z=0,\eta=L\right)+Pr\left(\theta=L,z=0,\eta=L\right)+Pr\left(\theta=L,z=-1,\eta=\phi\right)} \\ &= \frac{\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)+\frac{1}{2}\frac{1}{3}\lambda}{\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)++\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}\lambda+\frac{1}{2}\frac{1}{3}\left(1-\lambda\right)}=\frac{1}{2}; \\ \mu_F^{\phi,2} &= 1; \\ \mu_F^{\phi,-2} &= 0.\end{aligned}$$

When the order flow equals -2, the speculator's private information is perfectly revealed and thus the firm's posterior uncertainty disappears (i.e., $\mu_F^{\phi,-2} = 0$), which results in the firm optimally decreasing investment with $X = -2$. Because the speculator doesn't buy on good news when $r = \phi$, $X = 2$ is off the equilibrium path. As before, the only belief that survives Intuitive Criterion is that the speculator is privately informed of $\theta = H$. When $X = 0$ and $X = -1$, the firm is not able to update its belief and thus optimally maintains the status quo. Finally, when $X = 1$, though the firm is not able to perfectly back out the speculator's information, such an event is likely to come from a speculator who has privately observed $\eta = H$ but doesn't participate in trading. In other words, seeing $X = 1$ tilts the firm's posterior belief in favor of state H. As $\lambda > \frac{2c}{x+c}$ implies $\mu_F^{\phi,1} > \mu_F^{*}$, $X = 1$ constitutes a strong enough indicator to induce the firm to increase investment.

Next, let's move to study the market maker's strategy. When the market maker observes $r = \phi$ and $X = 1$, his posterior belief for $\theta = H$ is

$$\begin{aligned} \mu_M^{\phi,1} &= \frac{\Pr(\theta = H, X = 1|r = \phi)}{\Pr(\theta = H, X = 1|r = \phi) + \Pr(\theta = L, X = 1|r = \phi)} \\ &= \frac{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = 1|\theta = L, r = \phi)} \\ &= \frac{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right] + (1-t)\frac{1}{3}(1-\lambda)} = \frac{t}{t + (1-t)(1-\lambda)}. \end{aligned}$$

Consequently, the market maker sets the price as

$$\begin{aligned} P(1,\phi) &= Pr(\theta = H \mid r = \phi, X = 1)\left[Pr(\delta = H|\theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H|\theta = H, r = \phi))\, v(H,1)\right] + \\ &\quad \left[1 - Pr(\theta = H \mid r = \phi, X = 1)\right]\left[Pr(\delta = L|\theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L|\theta = L, r = \phi))\, v(L,1)\right] \\ &= \mu_M^{\phi,1}(R_H + x - c) + \left(1 - \mu_M^{\phi,1}\right)\left[\tau_L(R_L + x - c) + (1 - \tau_L)(R_L - x - c)\right]. \end{aligned}$$

To understand the first equality, when $\theta = H$ (L) and the firm's private signal δ reveals the state, the firm value is $v(H,1) = R_H + x - c$ $(v(L,1) = R_L + x - c)$, since the firm increases (decreases) investment. In contrast, when $\theta = H$ (L) and the firm's private signal δ is a null signal, the firm chooses to increase investment with a firm value $v(H,1) = R_H + x - c$ $(v(L,1) = R_L - x - c)$. Similarly, we can calculate the market maker's posterior beliefs and prices for all other realizations of X:

$$\begin{aligned} \mu_M^{\phi,0} &= \frac{\Pr(\theta = H, X = 0|r = \phi)}{\Pr(\theta = H, X = 0|r = \phi) + \Pr(\theta = L, X = 0|r = \phi)} \\ &= \frac{\Pr(\theta = H|r = \phi)\Pr(X = 0|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = 0|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = 0|\theta = L, r = \phi)} \\ &= \frac{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right] + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = t; \end{aligned}$$

$$\begin{aligned} P(0,\phi) &= Pr(\theta = H \mid r = \phi, X = 0)\left[Pr(\delta = H|\theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H|\theta = H, r = \phi))\, v(H,0)\right] + \\ &\quad \left[1 - Pr(\theta = H \mid r = \phi, X = 0)\right]\left[Pr(\delta = L|\theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L|\theta = L, r = \phi))\, v(L,0)\right] \\ &= t\left[R_H + \tau_H(x - c)\right] + (1-t)\left[R_L + \tau_L(x - c)\right] \\ &= tR_H' + (1-t)R_L'; \end{aligned}$$

$$
\begin{aligned}
\mu_M^{\phi,-1} &= \frac{\Pr(\theta = H, X = -1 | r = \phi)}{\Pr(\theta = H, X = -1 | r = \phi) + \Pr(\theta = L, X = -1 | r = \phi)} \\
&= \frac{\Pr(\theta = H | r = \phi)\Pr(X = -1 | \theta = H, r = \phi)}{\Pr(\theta = H | r = \phi)\Pr(X = -1 | \theta = H, r = \phi) + \Pr(\theta = L | r = \phi)\Pr(X = -1 | \theta = L, r = \phi)} \\
&= \frac{t\left[\frac{1}{3}(1-\lambda) + \frac{1}{3}\lambda\right]}{t\left[\frac{1}{3}(1-\lambda) + \frac{1}{3}\lambda\right] + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = t;
\end{aligned}
$$

$$
\begin{aligned}
P(-1,\phi) &= Pr(\theta = H \mid r = \phi, X = -1)\left[Pr(\delta = H | \theta = H, r = \phi)\, v(H,1) + (1 - Pr(\delta = H | \theta = H, r = \phi))\, v(H,0)\right] + \\
&\quad \left[1 - Pr(\theta = H \mid r = \phi, X = -1)\right]\left[Pr(\delta = L | \theta = L, r = \phi)\, v(L,-1) + (1 - Pr(\delta = L | \theta = L, r = \phi))\, v(L,0)\right] \\
&= tR'_H + (1-t)\, R'_L;
\end{aligned}
$$

$$
\mu_M^{\phi,2} = 1;
$$

$$
P(2,\phi) = R_H + x - c;
$$

$$
\mu_M^{\phi,-2} = 0;
$$

and

$$
P(-2,\phi) = R_L + x - c.
$$

This sell-not-buy equilibrium is sustained if the trading cost is lower than the profit from selling on bad news, but higher than the profit from buying on good news. If the speculator observes $\eta = H$ and deviates to buy one share, with equal probability of $\frac{1}{3}$, the order flow X would be 2, 1, or 0. Her expected gross trading profit is:

$$
\begin{aligned}
\pi_{buy}^{SNB} &= \frac{1}{3}\left[R'_H - P(0,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(1,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(2,\phi)\right] \\
&= \frac{1}{3}(1-t)(R'_H - R'_L) + \frac{1}{3}\left(1 - \mu_M^{\phi,1}\right)(R'_H - R'_L) + \frac{1}{3}\left(1 - \mu_M^{\phi,1}\right)\left[(1-\tau_H)(x-c) + (1-\tau_L)(x+c)\right] \\
&= \frac{1}{3}(1-t)(R'_H - R'_L) + \frac{1}{3}\left(1 - \mu_M^{\phi,1}\right)\left[R_H - R_L + 2(1-\tau_L)x\right] = \pi_{buy}^{T}
\end{aligned}
$$

If the speculator privately observes $\eta = L$ and sells one share, with equal probability of $\frac{1}{3}$, the order flow would be -2, -1, or 0. We have

$$\begin{aligned}\pi_{sell}^{SNB} &= \frac{1}{3}\left[P\left(-1,\phi\right)-R_L'\right]+\frac{1}{3}\left[P\left(0,\phi\right)-R_L'\right]+\frac{1}{3}\left[P\left(-2,\phi\right)-\left(R_L+x-c\right)\right] \\ &= \frac{2}{3}t\left(R_H'-R_L'\right)=\pi_{sell}^{NT}\end{aligned}$$

Thus, the SNB equilibrium exists if and only if $\pi_{buy}^{T}<\kappa<\pi_{sell}^{NT}$. Q.E.D.

Proof of Proposition 5

- $\frac{d\pi_{sell}^{T}}{d\xi}>0$ and $\frac{d\pi_{sell}^{NT}}{d\xi}>0$ can be easily established by taking a derivative on π_{sell}^{T} and π_{sell}^{NT} with respective to ξ.

- The no-trading equilibrium region is determined by $\kappa>\kappa_{NT}\equiv\max\left\{\pi_{buy}^{NT},\pi_{sell}^{NT}\right\}$. Note $t>\frac{1}{2}$ and $\tau_H>\tau_L$ if and only if $\xi>0$. Thus, given our assumption that $R_H-R_L>2x$ and $x>c$, we have

$$\pi_{sell}^{NT}-\pi_{buy}^{NT} = \frac{2}{3}\left(2t-1\right)\left[\left(R_H-R_L+\left(\tau_H-\tau_L\right)\left(x-c\right)\right)\right]>0$$

  if and only if $\xi>0$. Consequently, for any $\hat{\xi}>0$.

$$\begin{aligned}\kappa_{NT}\mid_{\xi=\hat{\xi}} &= \pi_{sell}^{NT}=\frac{2}{3}t\left[R_H-R_L+\left(\tau_H-\tau_L\right)\left(x-c\right)\right] \\ &> \frac{2}{3}t\left(R_H-R_L\right) \\ &= \frac{2}{3}\left(\frac{1}{2}+\frac{1}{2}\frac{f}{1-f\beta}\hat{\xi}\right)\left(R_H-R_L\right) \\ &> \frac{1}{3}\left(R_H-R_L\right); \\ \kappa_{NT}\mid_{\xi=-\hat{\xi}} &= \pi_{buy}^{NT}=\frac{2}{3}\left(1-t\right)\left[R_H-R_L+\left(\tau_H-\tau_L\right)\left(x-c\right)\right] \\ &< \frac{2}{3}\left(1-t\right)\left(R_H-R_L\right) \\ &= \frac{2}{3}\left[1-\left(\frac{1}{2}-\frac{1}{2}\frac{f}{1-f\beta}\hat{\xi}\right)\right]\left(R_H-R_L\right) \\ &= \frac{2}{3}\left(\frac{1}{2}+\frac{1}{2}\frac{f}{1-f\beta}\hat{\xi}\right)\left(R_H-R_L\right).\end{aligned}$$

  Clearly, $\kappa_{NT}\mid_{\xi=\hat{\xi}}>\frac{2}{3}\left(\frac{1}{2}+\frac{1}{2}\frac{f}{1-f\beta}\xi^*\right)\left(R_H-R_L\right)>\kappa_{NT}\mid_{\xi=-\hat{\xi}}$, and $\kappa_{NT}\mid_{\xi=\hat{\xi}}>\kappa_{NT}\mid_{\xi=0}=\frac{1}{3}\left(R_H-R_L\right)$.

- With a neutral disclosure policy $\xi=0$, we have $\pi_{sell}^{T}\mid_{\xi=0}<\pi_{buy}^{T}\mid_{\xi=0}$. As such, the trading equilibrium region is determined by $\kappa<\kappa^{T}\equiv min\left\{\pi_{buy}^{T}\mid_{\xi=0},\pi_{sell}^{T}\mid_{\xi=0}\right\}=\pi_{sell}^{T}\mid_{\xi=0}$. Thus, $\frac{d\kappa}{d\xi}\mid_{\xi=0}>0$ because $\frac{d\pi_{sell}^{T}}{d\xi}>0$. Q.E.D.

Proof of Proposition 6

The firm's terminal value consists of two parts: asset in place (R_H in state H and R_L in state L) and value created by firm's investment decision. With probability f, the firm is privately informed of θ and makes the right investment decision, which improves firm value by $(x-c)$. With probability $1-f$, the firm receives a null signal and may utilize information gleaned from the observed order flow.

Under the no-trading equilibrium, the firm does not learn from the market and changes investment level only when it is privately informed of the state. In this case, the expected firm value is:

$$E^{NT}\left[v\left(\theta,d\right)\right]=\frac{1}{2}\left(R_H+R_L\right)+f\left(x-c\right).$$

Under the trading equilibrium, the speculator buys on good news and sells on bad news. When the firm receives a null signal, if the state is H, with probability λ the speculator is present, and submits an buying order of 1. Thus, the order flow can be 2, 1 or 0, each with probability $\frac{1}{3}$. The firm invests when $X=2$ and 1, which improves firm value by $x-c$. With probability $1-\lambda$, the speculator is not present, and the order flow can be 1, 0 or -1. The firm increases investment when $X=1$, which increases the firm value by $x-c$, and decreases investment when $X=-1$, which reduces the firm value by $x+c$. Thus, for an uninformed firm, the net effect of learning from market prices when $\theta=H$ is: $\lambda\left[\frac{2}{3}\left(x-c\right)\right]+\left(1-\lambda\right)\left[\frac{1}{3}\left(x-c\right)+\frac{1}{3}\left(-x-c\right)\right]=\frac{2}{3}\left(\lambda x-c\right)$. Similarly, when $\theta=L$, with probability λ the speculator is present and submits a selling order of -1. Thus, the order flow can be -2, -1 or 0, each with probability $\frac{1}{3}$. The firm decreases investment when $X=-2$ and -1, which improves the firm value by $x-c$. With probability $1-\lambda$, the speculator is not present and the order flow can be 1, 0 or -1. The firm increases investment when $X=1$, which decreases the firm value by $x+c$, and decreases investment when $X=-1$, which increases the firm value by $x-c$. Thus, for an uninformed firm, the net effect of learning from market prices when $\theta=L$ is: $\lambda\left[\frac{2}{3}\left(x-c\right)\right]+\left(1-\lambda\right)\left[\frac{1}{3}\left(x-c\right)+\frac{1}{3}\left(-x-c\right)\right]=\frac{2}{3}\left(\lambda x-c\right)$. Since state H and L are equally likely, the expected benefit from learning under the trading equilibrium is $\frac{2}{3}\left(\lambda x-c\right)$. Hence, we have

$$E^{T}\left[v\left(\theta,d\right)\right]=\frac{1}{2}\left(R_H+R_L\right)+f\left(x-c\right)+\left(1-f\right)\frac{2}{3}\left(\lambda x-c\right).$$

Under the buy-no-sell equilibrium, the speculator buys on good news but does not trade on bad news. When the firm receives the null signal, with probability λ, the speculator is present. When the state is H, she submits a buying order of 1. Thus, the order flow can be 2, 1 or 0, each with probability $\frac{1}{3}$. The firm increases investment only when $X=2$, which improves firm value by $x-c$. When the state is L, the speculator does not trade, and the order flow can be -1, 0 or 1. With probability $\frac{1}{3}$, $X=-1$ and the firm decreases investment, which also improve firm value by $x-c$. However, with probability $1-\lambda$, the speculator is not present, and the order flow can be 1, 0 or -1. The firm decreases investment when $X=-1$, which affects firm value by $\frac{1}{2}(x-c)+\frac{1}{2}(-x-c)=-c$. Thus, for an uninformed firm, the net effect of learning from order flow is $\frac{1}{6}\lambda(x-c)+\frac{1}{6}\lambda(x-c)+(1-\lambda)(-c)=\frac{1}{3}(\lambda x-c)$. So the ex-ante expected firm value is:

$$E^{BNS}[v(\theta,d)]=\frac{1}{2}(R_H+R_L)+f(x-c)+(1-f)\frac{1}{3}(\lambda x-c).$$

Under the sell-no-buy equilibrium, the speculator sells on bad news but does not trade on good news. When the firm receives the null signal, with probability λ, the speculator is present. When the state is L, she submits a selling order of -1. Thus, the order flow can be 0, -1 or -2, each with probability $\frac{1}{3}$. The firm decreases investment only when $X=-2$, which improves firm value by $x-c$. When the state is H, the speculator does not trade, and the order flow can be 1, 0 or -1. With probability $\frac{1}{3}$, $X=1$ and the firm increases investment, which also improve the firm value by $x-c$. However, with probability $1-\lambda$, the speculator is not present, and the order flow can be 1, 0 or -1. The firm increases investment when $X=1$, which affects firm value by $\frac{1}{2}(x-c)+\frac{1}{2}(-x-c)=-c$. Thus, for an uninformed firm, the net effect of learning from order flow is $\frac{1}{6}\lambda(x-c)+\frac{1}{6}\lambda(x-c)+(1-\lambda)(-c)=\frac{1}{3}(\lambda x-c)$. So the ex-ante expected firm value is:

$$E^{SNB}[v(\theta,d)]=\frac{1}{2}(R_H+R_L)+f(x-c)+(1-f)\frac{1}{3}(\lambda x-c).$$

Given our assumption that $\lambda>\frac{2c}{x+c}$, $\lambda x-c>0$. As such,

$$E^{T}[v(\theta,d)]>E^{BNS}[v(\theta,d)]=E^{SNB}[v(\theta,d)]>E^{NT}[v(\theta,d)].$$

Note that the four expected firm value is independent of ξ. This implies that ξ only influences the expected firm value only through which of the four equilibrium emerges. Proposition 5 shows the no trading equilibrium region shrinks and trading equilibrium region expands with a conservative policy compared to an aggressive or neutral policy. Consequently, the social planner weakly prefers a conservative policy. Q.E.D.

Proof of Proposition 7

When the trading equilibrium prevails, the proof of Proposition 6 shows that the firm value is maximized and does not depend on ξ. Thus, the firm's preference is entirely determined by $E(\pi)$.

$$\begin{aligned}
E(\pi) &= \frac{1}{2}(1-f\beta_H)\lambda\pi_{buy}^T + \frac{1}{2}(1-f\beta_L)\lambda\pi_{sell}^T \\
&= \lambda(1-f\beta)\left[t\pi_{buy}^T + (1-t)\pi_{sell}^T\right] \\
&= \lambda(1-f\beta)\{t\left[\frac{1}{3}(1-t)(R'_H-R'_L) + \frac{1}{3}\left(1-\mu_M^{\phi,1}\right)[R_H-R_L+2(1-\tau_L)(x-c)]\right] \\
&+ (1-t)\left[\frac{1}{3}t(R'_H-R'_L) + \frac{1}{3}\mu_M^{\phi,-1}[R_H-R_L-2(1-\tau_H)(x-c)]\right]\} \\
&= \lambda(1-f\beta)\{\frac{2}{3}t(1-t)(R'_H-R'_L) + \frac{1}{3}\left[t\left(1-\mu_M^{\phi,1}\right)+(1-t)\mu_M^{\phi,-1}\right](R_H-R_L) \\
&+ 2(x-c)\left[\frac{1}{3}\left[t\left(1-\mu_M^{\phi,1}\right)(1-\tau_L)-(1-t)\mu_M^{\phi,-1}(1-\tau_H)\right]\right]\}
\end{aligned}$$

For any conservative policy $\hat{\xi}>0$, we now show that there exists an aggressive system with $-\hat{\xi}$ that can generate a lower $E(\pi)$. As $\tau_H>\tau_L$ when if and only if $\xi>0$, we have

$$R'_H-R'_L = R_H-R_L+(\tau_H-\tau_L)(x-c) \Longrightarrow$$

$$(R'_H-R'_L)\mid_{\xi=\hat{\xi}} > (R'_H-R'_L)\mid_{\xi=0} > (R'_H-R'_L)\mid_{\xi=-\hat{\xi}}.$$

Furthermore, $t\mid_{\xi=-\hat{\xi}} = 1-t\mid_{\xi=\hat{\xi}}$, and $\left(1-\mu_M^{\phi,1}\right)\mid_{\xi=-\hat{\xi}} = \mu_M^{\phi,-1}\mid_{\xi=\hat{\xi}}$. Thus, the first term in the curly bracket for $E(\pi)$ is larger under a conservative system than under an aggressive system, the second stays the same under both, while the third term is positive when the system is conservative, 0 when it is neutral, and negative while it is aggressive. Conservatism increases the speculator's trading profit.

Finally, because both $t(1-t)$ and $t\left(1-\mu_M^{\phi,1}\right)+(1-t)\mu_M^{\phi,-1}$ are maximized at $t=\frac{1}{2}$, the trading profit is lower under an aggressive system than under a neutral system. Q.E.D.

Proof of Proposition 8

1. When $\kappa>\kappa^T\mid_{\xi=0}$ and $\kappa-\kappa^T\mid_{\xi=0}$ is sufficiently small, the BNS equilibrium emerges with a neutral disclosure policy, with

$$E^{BNS}[v(\theta,d)] = \frac{1}{2}(R_H+R_L)+f(x-c)+(1-f)\frac{1}{3}(\lambda x-c).$$

With a small $\hat{\xi} > 0$, $\pi_{sell}^{T} \geq \kappa$ and the trading equilibrium can be sustained. We have

$$E^{T}\left[v\left(\theta, d\right)\right] = \frac{1}{2}\left(R_H + R_L\right) + f\left(x - c\right) + \left(1 - f\right)\frac{2}{3}\left(\lambda x - c\right).$$

Clearly, $E^{T}\left[v\left(\theta, d\right)\right] > E^{BNS}\left[v\left(\theta, d\right)\right]$.

2. When $\kappa > \kappa^{T} \mid_{\xi=0}$ and $\kappa - \kappa^{T} \mid_{\xi=0}$ is sufficiently small, the BNS equilibrium emerges with a neutral disclosure policy, with

$$\begin{aligned} E^{BNS}\left(\pi\right) \mid_{\xi=0} &= \frac{1}{2}\lambda\left(1 - f\beta\right)\frac{2}{3}\left(1 - \frac{1}{2}\right)\left(R_H - R_L\right) \\ &= \frac{1}{6}\lambda\left(1 - f\beta\right)\left(R_H - R_L\right). \end{aligned}$$

With a small $\hat{\xi} > 0$, $\pi_{sell}^{T} \geq \kappa$ and the trading equilibrium is sustained. At $\xi = 0$, change in ξ has no first order effect on $E_c\left(\pi\right)$:

$$\begin{aligned} &\frac{\partial E_c\left(\pi\right)}{\partial \xi}\Big|_{\xi*=0} \\ &= \frac{\partial E_c\left(\pi\right)}{\partial t}\frac{\partial t}{\partial \xi}\Big|_{\xi*=0} \\ &= \frac{\partial E_c\left(\pi\right)}{\partial t}\Big|_{t*=0} \times \frac{1}{2}\frac{f}{1 - f\beta} \\ &= \frac{1}{3}\left(1 - f\beta\right)\left(R_H - R_L\right)\frac{1}{2}\frac{f}{1 - f\beta} \\ &\quad \times \left\{\left(1 - 2t\right)\left(2 + \frac{\left(1 - \lambda\right)}{1 - \lambda\left(1 - t\right)} + \frac{\left(1 - \lambda\right)}{1 - \lambda t}\right) + t\left(1 - t\right)\left(\frac{\left(1 - \lambda\right)\lambda}{\left(1 - \lambda\left(1 - t\right)\right)^2} - \frac{\left(1 - \lambda\right)\lambda}{\left(1 - \lambda t\right)^2}\right)\right\}\Big|_{t^*=\frac{1}{2}} \\ &= 0 \end{aligned}$$

When ξ is small, the speculator's profit is approaching

$$\lim_{\xi \to 0} E^{T}\left(\pi\right) = \frac{1}{3}\lambda\left(1 - f\beta\right)\left(\frac{1}{2} + \frac{1 - \lambda}{2 - \lambda}\right)\left(R_H - R_L\right)$$

Thus, the firm prefers a conservative system to a neutral system when $E^{T}\left[v\left(\theta, d\right)\right] - E^{BNS}\left[v\left(\theta, d\right)\right] \geq \alpha\left[\underset{\xi \to 0}{lim} E^{T}\left(\pi\right) - E^{BNS}\left(\pi\right)\right]$, i.e.,

$$\begin{aligned} \frac{1}{3}\left(1 - f\right)\left(\lambda x - c\right) &\geq \frac{1}{3}\lambda\left(1 - f\beta\right)\frac{1 - \lambda}{2 - \lambda}\left(R_H - R_L\right)\alpha \\ x &\geq \frac{c}{\lambda} + \frac{\left(1 - f\beta\right)}{\left(1 - f\right)}\frac{1 - \lambda}{2 - \lambda}\left(R_H - R_L\right)\alpha. \end{aligned} \tag{2}$$

When λ, x are sufficiently large, or $R_H - R_L$, α are sufficiently small, the firm prefers a conservative system to a neutral system. The lower f, the higher the β, the more the firm benefits from learning and the more likely (depending on other parameters) that it prefers a conservative system to a neutral system. Under an aggressive disclosure policy, the equilibrium is BNS:

$$\begin{aligned} E^{BNS}(\pi) &= \frac{1}{2}\lambda(1 - f\beta_H)\frac{2}{3}(1-t)[R_H - R_L + (\tau_H - \tau_L)](x-c) \\ &= \frac{2}{3}\lambda t(1-t)(1-f\beta)\underbrace{[R_H - R_L + (\tau_H - \tau_L)(x-c)]}_{\downarrow as\ system\ gets\ aggressive}. \end{aligned}$$

Easy to see that when $\alpha \to 0$, the firm prefers a conservative system: $E^T[v(\theta,d)] - E^{BNS}[v(\theta,d)] \geq \alpha\left(E_c(\pi) - E^{BNS}(\pi)\right)$.

Note that $E^{BNS}(\pi)$ decreases as the absolute degree of bias $|\xi|$ increases, e.g., it decreases as the disclosure policy gets more aggressive. Thus, the firm prefers a conservative system to the aggressive system when $E^T[v(\theta,d)] - E^{BNS}[v(\theta,d)] \geq \alpha\left(E_c(\pi) - E^{BNS}(\pi)\right)$, i.e.,

$$\frac{1}{3}(1-f)(\lambda x - c) \geq \frac{1}{3}\alpha\lambda(1-f\beta)\left\{\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)(R_H - R_L) - 2t(1-t)[R_H - R_L + (\tau_H - \tau_L)(x-c)]\right\}. \tag{3}$$

Take ξ to the most aggressive value, i.e., $\xi = \beta - 1$, implying that $\beta_H = 1$, $\beta_L = 2\beta - 1$, $\tau_H = 0$, $\tau_L = \frac{2f(1-\beta)}{1-f\beta_L} = \frac{2f(1-\beta)}{1-f(2\beta-1)}$, and $t = \frac{1-f}{2(1-f\beta)}$. In this case, the BNS equilibrium is still sustained given our assumption $(R_H - R_L) > 2x$,[11] giving the highest payoff to the firm among all aggressive systems

[11]To see this, the no trading equilibrium sustains if the following holds:

$$\begin{aligned} \kappa > \kappa^{NT} &= \frac{2}{3}\left(R'_H - R'_L\right)\max[1-t,t] \\ &= \frac{2}{3}\underbrace{[R_H - R_L + (\tau_H - \tau_L)(x-c)]}_{spread}\underbrace{\left(\frac{1}{2} + \frac{1}{2}\frac{f}{1-f\beta}|\xi|\right)}_{} \end{aligned}$$

and as $\xi < 0$ gets more aggressive, $\kappa^{NT}|_{\xi<0} > \kappa^{NT}|_{\xi=0}$ if and only if

$$\begin{aligned} [R_H - R_L + (\tau_H - \tau_L)(x-c)]\left(1 + \frac{f}{1-f\beta}|\xi|\right) &> (R_H - R_L) \\ [R_H - R_L + (\tau_H - \tau_L)(x-c)] + [R_H - R_L + (\tau_H - \tau_L)(x-c)]\frac{f}{1-f\beta}|\xi| &> (R_H - R_L) \\ (\tau_H - \tau_L)(x-c) + [R_H - R_L + (\tau_H - \tau_L)(x-c)]\frac{f}{1-f\beta}|\xi| &> 0 \\ (R_H - R_L)\frac{f}{1-f\beta}|\xi| &> -(\tau_H - \tau_L)(x-c)\left(1 + \frac{f}{1-f\beta}|\xi|\right) \\ (R_H - R_L)\frac{1}{1-f\beta} &> 2\frac{(1-f)}{(1-f\beta)^2 - f^2|\xi|^2}(x-c)\left(\frac{1-f\beta+f|\xi|}{1-f\beta}\right) \\ (R_H - R_L) &> \frac{2(1-f)}{(1-f\beta) - f|\xi|}(x-c) \end{aligned}$$

due to $\frac{dE^{BNS}(\pi)}{d|\xi|} < 0$. 3 becomes

$$(1-f)(\lambda x - c) \geq \alpha\lambda(1-f\beta)\left\{\left(\frac{1}{2}+\frac{1-\lambda}{2-\lambda}\right)(R_H - R_L) - 2\frac{1-f}{2(1-f\beta)}\left[1-\frac{1-f}{2(1-f\beta)}\right]\left[R_H - R_L - \frac{2f(1-\beta)}{1-f(2\beta-1)}\right.\right.$$

$$\frac{1-f}{1-f\beta}(\lambda x - c) \geq \alpha\lambda\left\{\left(\frac{1}{2}+\frac{1-\lambda}{2-\lambda}\right)(R_H - R_L) - \frac{1-f}{1-f\beta}\left[1-\frac{1-f}{2(1-f\beta)}\right]\left(R_H - R_L - \frac{2f(1-\beta)}{1-f(2\beta-1)}(x-c)\right)\right\}.$$

When $f \to 0$ or $\beta \to 1$, the above becomes

$$\begin{aligned}\lambda x - c &\geq \alpha\lambda\frac{1-\lambda}{2-\lambda}(R_H - R_L)\\ x &\geq \frac{c}{\lambda} + \alpha\frac{1-\lambda}{2-\lambda}(R_H - R_L). \qquad (4)\end{aligned}$$

It is obvious that each of Part 2a - 2c is sufficient for 2 and 4.

3. Part 3 can be derived similarly to Part 2, hence omitted. Q.E.D.

Proof of Proposition 9

Proof. Note that due to the feedback effect, when the reporting is neutral, $\pi^T_{buy} > \pi^T_{sell}$ regardless of β. Since $\tau_\theta = \frac{f(1-\beta_\theta)}{1-f\beta_\theta}$ and decreases with β_θ, the feedback effect increases with β_θ. This makes sense as β increases, the more likely that the firm does not know and learns from stock market, which makes feedback effect more significant. With neutral reporting,

$$\begin{aligned}\pi^T_{buy} &= \frac{1}{3}\left(\frac{1}{2}+\frac{1-\lambda}{2-\lambda}\right)(R_H - R_L) + \frac{2}{3}\frac{1-f}{1-f\beta}\frac{1-\lambda}{2-\lambda}x\\ \pi^T_{sell} &= \frac{1}{3}\left(\frac{1}{2}+\frac{1-\lambda}{2-\lambda}\right)(R_H - R_L) - \frac{2}{3}\frac{1-f}{1-f\beta}\frac{1-\lambda}{2-\lambda}x\end{aligned}$$

Easy to see that $\pi^T_{buy} > \pi^T_{sell}$, and the gap between π^T_{buy} and π^T_{sell} is due to feedback effect and is larger when β is larger, and even if $\beta = 0$ the gap still exists. That is, withholding disclosure can alleviate, but never eliminate the feedback effect. To prove that the optimal system must has $\xi > 0$, we proceed in three steps.

Step 1. At most one of the two constraints about trading profits ($\pi^T_{buy} \geq \kappa$ and $\pi^T_{sell} \geq \kappa$) is binding.

Since $\frac{(1-f)}{(1-f\beta)-f|\xi|} \in (0,1)$, sufficient condition for $\kappa^{NT}|_{\xi<0} > \kappa^{NT}|_{\xi=0}$ is:

$$(R_H - R_L) > 2(x-c),$$

which is implied by our assumption that

$$(R_H - R_L) > 2x.$$

Otherwise, the manager can increase β by an infinitesimal amount if $\beta < \bar{\beta}$, or decrease ξ by an infinitesimal amount if $\beta = \bar{\beta}$, which raise net firm value, without violating the constraints.

Step 2. The constraint on selling must be binding. To see this, suppose it is not binding under the optimal solution (β^*, ξ^*) sustains trading equilibrium and has $\pi^T_{buy} = \kappa$ and $\pi^T_{sell} > \kappa$. Fix β^*. Since $\pi^T_{buy} < \pi^T_{sell}$, it must be the case that $\xi > 0$. Then the firm can slightly lower ξ by an infinitesimal amount, and as a result, $\pi^T_{buy} > \kappa$ and $\pi^T_{sell} > \kappa$, the constraints are satisfied, and the expected loss to informed traders is lower, and thus the firm is better off. So the constraint on selling is binding.

Step 3. Now we have

$$\kappa = \pi^T_{sell} = \frac{1}{3}t\left(R'_H - R'_L\right) + \frac{1}{3}\frac{(1-\lambda)\,t}{1-\lambda t}\left(R_H - R_L - 2\frac{1-f}{1-f\left(\beta-\xi\right)}x\right) \tag{5}$$

$$\pi^T_{buy} = \frac{1}{3}\left(1-t\right)\left(R'_H - R'_L\right) + \frac{1}{3}\frac{(1-\lambda)\,(1-t)}{1-\lambda\,(1-t)}\left(R_H - R_L + 2\frac{1-f}{1-f\left(\beta+\xi\right)}x\right) \tag{6}$$

Note $\mu^{\phi,-1}_M = \frac{t(1-\lambda)}{t(1-\lambda)+(1-t)}$ and $\mu^{\phi,1}_M = \frac{t}{t+(1-\lambda)(1-t)}$, and we can write

$$\left(R'_H - R'_L\right) = \left(\frac{3\kappa}{t} - \frac{(1-\lambda)}{1-\lambda t}\left(R_H - R_L\right) + \frac{(1-\lambda)}{1-\lambda t}\frac{1-f}{1-f\beta_H}2x\right)$$

We can rewrite the firm's expected loss to informed trader as:

$$\begin{aligned}
E_{\beta,\xi}\left(\pi\right) &= \frac{1}{2}\lambda\left(1-f\beta_H\right)\pi^T_{buy} + \frac{1}{2}\lambda\left(1-f\beta_L\right)\pi^T_{sell} \\
&= \frac{1}{2}\lambda\left(1-f\beta_H\right)\left(\frac{1}{3}\left(1-t\right)\left(R'_H - R'_L\right) + \frac{1}{3}\frac{(1-\lambda)\,(1-t)}{1-\lambda\,(1-t)}\left(R_H - R_L + 2\frac{1-f}{1-f\beta_L}x\right)\right) \\
&+ \frac{1}{2}\lambda\left(1-f\beta_L\right)\left(\frac{1}{3}t\left(R'_H - R'_L\right) + \frac{1}{3}\frac{t\,(1-\lambda)}{1-t\lambda}\left(R_H - R_L - 2\frac{1-f}{1-f\beta_H}x\right)\right) \\
&= \frac{1}{3}\left(1-f\beta\right)\lambda t\left(1-t\right)\left(R'_H - R'_L + \frac{(1-\lambda)}{1-\lambda\,(1-t)}\left(R_H - R_L + 2\frac{1-f}{1-f\beta_L}x\right) + \left(R'_H - R'_L\right) + \frac{(1-\lambda)}{1-\lambda t}\left(R_H - R_L - 2\frac{1-f}{1-f\beta_H}\right.\right. \\
&= \frac{1}{3}\left(1-f\beta\right)\lambda t\left(1-t\right)\left[2\frac{3\kappa}{t} - \frac{(1-\lambda)}{1-\lambda t}\left(R_H - R_L\right) + \frac{1}{t}\frac{(1-\lambda)}{1-\lambda t}\frac{1-f}{1-f\beta}x + \frac{(1-\lambda)}{1-\lambda\,(1-t)}\left(\left(R_H - R_L\right) + \frac{1-f}{1-f\beta}\frac{1}{1-t}x\right)\right] \\
&= \underbrace{\left(1-f\beta\right)\lambda\left(1-t\right)2\kappa}_{H_1} + \underbrace{\frac{1}{3}\lambda\left(1-\lambda\right)\left(1-f\right)\left(\frac{(1-t)}{1-\lambda t} + \frac{t}{1-\lambda\,(1-t)}\right)x}_{H_2} \\
&\quad \underbrace{+\frac{1}{3}\left(1-f\beta\right)\lambda t\left(1-t\right)\left(1-\lambda\right)\left(R_H - R_L\right)\left[\frac{1}{1-\lambda\,(1-t)} - \frac{1}{1-\lambda t}\right]}_{H_3}
\end{aligned}$$

Compare two cases: $\xi_0 = 0$ and $\xi_c > 0$. Then from the binding constraint 5 we have $\beta_0 < \beta_c$; intuitively, if the firm does not use asymmetric reporting, it has to withhold more information to make trading profitable and thus to elicit feedback. Thus, easy to see $H_1\left(\xi_c\right) < H_1\left(\xi_0\right)$, and $H_3\left(\xi_c\right) < 0 = H_1\left(\xi_0\right)$ and take FOC of

H_2:

$$
\begin{aligned}
H_2 &= Constant \times \left(\frac{1-t}{1-\lambda t} + \frac{t}{1-\lambda(1-t)} \right) \\
&= C \times \frac{(1-t)(1-\lambda(1-t)) + t(1-\lambda t)}{(1-\lambda t)(1-\lambda(1-t))} \\
&= C \times \frac{1-\lambda(1-t)^2 - \lambda t^2}{(1-\lambda t)(1-\lambda(1-t))} \\
\frac{\partial H_2}{\partial \xi} &= \frac{\partial H_2}{\partial t}\frac{\partial t}{\partial \xi} \\
&= C \times \frac{-2\lambda(2t-1)(1-\lambda t)(1-\lambda(1-t)) - \lambda^2(1-2t)\left(1-\lambda(1-t)^2-\lambda t^2\right)}{(1-\lambda t)^2(1-\lambda(1-t))^2} \\
&= C \times \frac{(2t-1)\lambda}{(1-\lambda t)^2(1-\lambda(1-t))^2}\left(-2(1-\lambda t)(1-\lambda(1-t)) + \lambda\left(1-\lambda(1-t)^2-\lambda t^2\right)\right) \\
&= C \times \frac{(2t-1)\lambda}{(1-\lambda t)^2(1-\lambda(1-t))^2}(-2+\lambda)(1-\lambda) \\
&< 0
\end{aligned}
$$

So $H_2(\xi_c) < H_2(\xi_0)$. So the loss to trading profit to informed traders is lowest under a conservative regime, followed by a neutral system, and the highest under an aggressive system. So in this case a conservative system is optimal. □

More papers to be added to the reference list.

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