On the Relation between Conservatism in Accounting Standards and Incentives for Earnings Management

QI CHEN, * THOMAS HEMMER, † AND YUN ZHANG ‡

Received 13 March 2006; accepted 13 November 2006

ABSTRACT

This paper studies the role of conservative accounting standards in alleviating rational yet dysfunctional unobservable earnings manipulation. We show that when accounting numbers serve both the valuation role (in which potential investors use accounting reports to assess a firm’s expected future payoff) and the stewardship role (in which current shareholders rely on the same reports to monitor their risk-averse manager), current firm owners have incentives to engage in earnings management. Such manipulation reduces accounting numbers’ stewardship value and leads to inferior risk sharing. We then show that risk sharing, and hence contract efficiency, can be improved under a conservative accounting standard where, absent earnings management, accounting earnings represent true economic earnings with a downward bias, compared with under an unbiased standard where, absent earnings management, accounting earnings represent true economic earnings without bias.

* Duke University; † University of Houston; ‡ Duke University. We appreciate comments from seminar participants at Duke University, 2004 Duke/UNC Fall Camp, University of Houston, University of Maryland, the 15th FEA conference at University of Southern California, an anonymous referee, and Doug Skinner (the editor). We acknowledge the financial support of the Fuqua School of Business at Duke University (for Qi Chen and Yun Zhang) and the Bauer College of Business at University of Houston (for Thomas Hemmer).
1. Introduction

In this paper, we study the role of standard-specific and thus observable accounting biases in alleviating rational yet dysfunctional unobservable biases introduced by earnings manipulation. Specifically, we study the relative performance of two alternative financial reporting regimes: one where accounting standards are designed to be bias free and one where accounting standards are designed deliberately to introduce a conservative bias in financial reports. We identify conditions under which conservatism in accounting standards is effective in reducing incentives to manage earnings upwards. In addition, we demonstrate that doing so can reduce contracting costs, thus providing insights into the source of the seemingly universal popularity of conservatism in financial accounting standards.\(^1\)

A variety of alternate perspectives on the demand for conservatism currently exist in the accounting literature. Watts [2003a] provides a number of intuitive explanations for the beneficial implications of conservative accounting, in particular from a contracting perspective. His main argument is that conservatism constrains opportunistic behavior and offsets biases introduced by self-interested parties. Our paper is somewhat related, as we also focus on the role of conservative accounting in alleviating biases in financial reports introduced by opportunistic reporting behavior. Our main point, however, differs significantly from his. By allowing for both systemic conservative accounting biases and opportunistic (liberal) reporting biases in an analytical model with endogenous demand for financial reports, we are able to show that conservatism need not offset opportunistic biases by imposing explicit constraints and reducing firm insiders’ opportunities to introduce biases. Rather, conservatism can have such an effect simply by dampening firm insiders’ incentives to manage earnings.

Our study of accounting conservatism is based on a specific model in which we seek to integrate a number of general observations about the features of standard financial reporting regimes. First, we note that financial reports are produced by jointly confronting auditable historical data and an array of subjective estimates with specific predefined accounting principles, procedures, and methods. Any biases in financial reports are therefore bound to be the net result of biases introduced by specific accounting principles, procedures, and methods, and biases introduced by estimates. While the former are largely precommitted publicly through the choice of specific accounting principles, the latter, by their subjective nature, are harder to credibly commit to and much more difficult for outsiders to identify ex post.

Second, while some biases are deterministic in the sense that the final report always over- or underestimates the truth, biases are often introduced

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\(^1\) Examples of accounting conservatism abound: Generally accepted accounting principles (GAAP) require most research and development expenditures to be expensed as incurred rather than capitalized. Similarly, many assets are valued at the “lower of the market or historical value” and recognition of appreciations in value is not permitted under most circumstances.
only in expectation. This is certainly true for biases introduced by particular accounting methods and principles, as these have to be used consistently after their adoption. Accordingly, while straight-line depreciations may be used with the intent of creating a conservative bias, in the absence of a liquid market for the asset being depreciated, the realized bias introduced into accounting income may well end up being liberal. And while there may be more degrees of freedom available when it comes to biasing estimates, the very fact that estimates are made ex ante allows for the possibility that what seems biased in one direction ex ante actually leads to realized biases in the opposite direction ex post.

Third, although biases introduced by manipulative actions such as the use of biased estimates could potentially be offset exactly “on average” by using accounting methods biased in the opposite direction, two different sources of bias are not likely to be perfect substitutes. Stated differently, two wrongs are not likely to make a right: While the different biases introduced by biased estimates and biased accounting methods may offset in a way to produce an overall unbiased report, the information content of that report is unlikely to be identical to one produced without the introduction of any bias at all. For example, “lower of cost or market” may well provide an appropriate offset for optimistic estimates used for bad debt expense. Yet the final report that incorporates such offsetting biases still differs substantively from a report where all assets are marked-to-market in an unbiased way.

Finally, we note that accounting numbers in financial reports generally play a multitude of roles that inevitably create diverging incentives for attempting to endow them with certain biases. For example, while from a stewardship perspective, (current) shareholders may favor a cautious approach to financial reporting, from a valuation perspective they may be more concerned about avoiding undervaluation of their shares.

Our model captures these features in the following ways. First, for specificity we concentrate on diverging reporting incentives related to the two aforementioned key uses of financial reports: “valuation,” where potential investors use accounting information to assess a firm’s expected future pay-off, and “stewardship,” where accounting information is used by the current owners to evaluate and incentivize employees. We do this by studying a setting where an innovator/entrepreneur contracts with a professional manager to help manage the firm in a way to increase the value of the firm prior to selling (all or part of) it to a second generation of shareholders, for example, through an initial public offering (IPO). Our analysis takes accounting numbers’ dual roles as given and focuses instead on their consequences.

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2 Another interpretation of our model is that it describes a venture capitalist contracting with an entrepreneur (agent) and subsequently bringing the firm to IPO. The venture capitalist is likely to assume a dominant role in the financial reporting involved in the IPO process.

3 For an in-depth discussion on accounting numbers’ multiple roles, see Watts and Zimmerman [1986]. Bushman, Engel, and Smith [2006] provide empirical evidence for the relation between the valuation role and the stewardship role of accounting reports. Also, if investors
Second, to capture the point that biases are introduced ex ante and thus typically are “in expectation,” all biases in our model are introduced by stochastically altering the representation of the underlying facts. Concretely, biases are introduced through reducing the probability that an event is represented accurately in the financial report while at the same time increasing the probability that the event is either over- or underrepresented depending on the particular bias being introduced.

This stochastic representation also allows us to capture the property that two wrongs do not make a right in a particularly parsimonious way. We do this by assuming that a liberal bias can only be introduced by actions that increase from zero (decrease from one) the probability of a favorable (unfavorable) earnings report when the underlying true earnings are unfavorable. A conservative bias, on the other hand, can only be introduced by actions that increase from zero (decrease from one) the probability of an unfavorable (favorable) earnings report when the underlying true earnings are favorable. Accordingly, a liberal bias that makes bad outcomes appear good can always be offset \textit{in expectation} by a conservative bias. The offset is not a “one for one” though, as the tools available for engineering the different biases are different themselves.

In addition to capturing the notion that biases may be offset mainly through dissimilar actions, there are several other advantages to the particular representation we adopt here. For one, the representation captures the notion that actions intended to distort financial reports are not innocuous in the sense that a user of financial statements anticipating a particular bias is able to undo the damage simply by deducting the anticipated bias from the published earnings. With the stochastic structure, the user can take out only the expected bias but cannot restore the information content of the report lost due to the introduction of biased noise. Another advantage of our representation is that the model \textit{itself} is biased against finding any benefits of conservatism in financial reporting. This is true regardless of the presence of a liberal bias since introducing conservatism here always reduces the information content of the financial report, thus making it less useful.

The analysis of our model yields a number of insights. Starting in a benchmark setting where the accounting regime is unbiased, we first establish that when there is uncertainty about the future payoff of the firm, the current owner has an incentive to manage the accounting earnings upward (in expectation) to induce more favorable (potential) investors’ belief about the firm’s prospects. Because we allow for the fact that there is a fair amount of discretion in the estimation part of preparing the financial report, this incentive leads, in equilibrium, to the introduction of a liberal bias in the financial report.

rationally rely on the accounting numbers in pricing a firm, it has to be the case that the accounting numbers are related to the true underlying output. If the true output depends on the firm manager’s productive effort, then accounting numbers, by their relation with the true output, are informative about these productive efforts. The valuation role of accounting numbers then almost surely implies their stewardship value.
The equilibrium we identify implies inefficiency, however. While the current owner in fact does choose to manipulate earnings upwards in equilibrium, he does not benefit from the manipulation for two reasons. First, although potential investors do not directly observe the earnings management, they rationally expect and price protect against it. Accordingly, they correctly incorporate the expected bias in the financial report when valuing the firm. Second, such manipulation actually reduces the current owner’s welfare relative to a (here unattainable) no-manipulation equilibrium. This is because the introduction of a nondeterministic bias also reduces the informativeness of accounting numbers for stewardship purposes, which in turn leads to inefficiencies in the incentive arrangement provided by the current owner to the manager.

Again, the central inefficiency arises in our setting essentially because of the need for making estimates in producing financial reports. Because such estimates are subjective, unverifiable, and likely unobservable, it is impossible for the current owner to credibly commit not to bias his estimates deliberately with the intent to distort accounting earnings. It is in this context that we study the potential consequences of using publicly observable accounting principles with publicly observable (expected) bias. We do so by comparing the efficiency loss (which is captured by the inefficiency in risk sharing in our model) under the two accounting regimes we contrast: an unbiased regime where, absent earnings management, accounting earnings represent the underlying true output without bias, and a conservative one where, again in the absence of earnings management, accounting earnings may underrepresent the true underlying output.

We show that the degree of earnings manipulation is lower in the conservative regime than in the unbiased accounting regime. The intuition behind this result is the following. Under the conservative system, observing a low accounting earnings number does not necessarily mean that the true economic earnings are as low. This in turn reduces the benefit of earnings manipulations, as the reason for the manipulation is exactly to make the inference drawn from observed earnings numbers more favorable. Moreover, we show that conservatism increases the current owner’s marginal cost to motivate the agent, further dampening his incentive to manage earnings.

We also identify situations where the conservative accounting regime improves the contracting efficiency for the current owner. This result is somewhat counterintuitive at first: after all, conservatism introduces additional noise in the accounting report, which in and of itself reduces the stewardship value of accounting data and therefore increases the cost of motivating the manager. However, we show that under reasonable conditions, the conservative standard actually reduces earnings management so much so that the noise introduced by the conservative standard is more than offset by the reduction in earnings management. This overall net reduction in

Firms can, and are often required to, disclose the detailed estimates. But it may still be difficult for firms to convince investors that these estimates are completely bias free.
(equilibrium) noise makes the accounting reports more informative about managerial action.

In terms of empirical implications, our analysis provides a theoretical framework for interpreting existing evidence regarding both the time-series changes and the cross-country differences in the degree of accounting conservatism. Specifically, our model implies that conservative principles are more likely to arise in situations where accounting numbers play dual roles and where the self-interested parties involved in the financial reporting process (e.g., controlling shareholders, top management teams) have significant equity stakes in their firms. Casual examination of existing evidence seems to support this prediction. For example, Watts [2003b] notes that conservatism in the United States has increased after the Financial Accounting Standards Board (FASB) began managing the accounting standards, a period in which equity incentives for top management in U.S. companies have also increased significantly (Milliron [2000]).

Ball, Kothari, and Robin [2000] provide evidence that accounting measurement is more conservative in common law countries (e.g., the United States and United Kingdom) than in code law countries (e.g., France and Germany). To the extent that management in the United States and United Kingdom has higher equity stakes, and investors in the United States and United Kingdom are more diverse and rely more on accounting earnings for valuation than those in France and Germany, where ownership tends to be concentrated and direct monitoring is more prevalent (thus accounting is less likely to play dual roles), their findings are consistent with our model’s prediction.

Our paper also relates to two strands of theoretical research in accounting: earnings manipulation and accounting conservatism. Prior studies on earnings manipulation focus on identifying situations where earnings manipulations arise in equilibrium (Stein [1989] and Dye [2002]). In these papers, the cost of earnings manipulation is assumed to be exogenous. Our paper differs from theirs in that the cost of earnings manipulation is endogenized as a contract cost and is tied to the stewardship role of accounting numbers. What is significant about this difference is that conservatism here, unlike in prior studies, plays a crucial role in reducing overall earnings management.

Kwon, Newman, and Suh [2001] also study the need for conservatism in a pure agency setting. In their model, accounting numbers have only the stewardship value and conservatism arises because the agent has limited liability. There, conservatism per se enhances accounting numbers’ stewardship value by loosening the limited liability constraint. Our paper instead analyzes the implications of the dual roles of accounting numbers.

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5 Recent studies by Dye and Sridhar [2004] and Stocken and Verrecchia [2004] analyze the impact of aggregation on earnings manipulation.

6 Narayanan and Davila [1998] study a similar trade-off between the performance evaluation and belief-revision uses of accounting signals. Their focus is on delegation, while ours is on accounting conservatism.
More importantly, in our model, conservatism per se is “bad,” as it makes accounting numbers less valuable for stewardship purposes. The benefit of adopting a conservative principle stems from its effect on the current owner’s incentive to manipulate earnings. The net effect of conservatism in equilibrium is to increase both accounting numbers’ stewardship value and their usefulness in assessing firms’ future payoff. Accordingly, our paper also differs from Gigler and Hemmer [2001] and Venugopalan [2004], which take conservatism as given and focus on its implications for firms’ voluntary disclosure behavior and investment efficiency.

In what follows, we first set up the model in section 2 and solve the model under the unbiased accounting standard in section 3. We then introduce conservatism in section 4. Conclusions are offered in section 5.

2. Basic Model Setup

Our basic model commences at date 1 when a firm’s current owner hires a professional manager (the agent) to manage the operations of his firm. We assume (entirely for simplicity) that the current owner (being a venture capitalist) is risk neutral while the agent is risk and effort averse. The agent can take an (unobservable) productive action \( a \in \{a_h, a_l\} \) (the subscripts \( h \) and \( l \) represent “high” and “low,” respectively) where \( a_h \) more positively, albeit stochastically, influences the firm’s payoff than does \( a_l \). To ensure that the problem at hand is not trivial, we assume that the expected net value generated by \( a_h \) is so much larger than that generated by \( a_l \) that the current owner always finds it worthwhile to motivate the agent to work hard.

To simplify the relation between productive effort and (expected) terminal output, we assume that there are only two possible terminal payoffs: \( x_h > x_l \), where \( x_l \) is normalized to zero for simplicity and without loss of generality. Production is represented by assuming that \( \Pr(x_h \mid a_h) = p \geq \frac{1}{2} > \Pr(x_h \mid a_l) = 0 \), and \( \Pr(x_l \mid a_h) = 1 - p < \Pr(x_l \mid a_l) = 1 \). That is, if the agent chooses \( a_l \) (i.e., shirks), the payoff is low \( (x_l) \) for sure; if he chooses \( a_h \) (i.e., works), the real output is more likely to be high. As only the marginal cost of effort is relevant, we normalize the agent’s cost of \( a_l \) to zero and let \( D > 0 \) denote the agent’s incremental disutility for taking the high rather than the low effort. His (additively separable) utility from consuming a payment of \( s \) is \( U(s) \) where \( U’ > 0 \) and, due to risk aversion, \( U” < 0 \). Without loss of generality, we normalize the agent’s reservation utility to zero.

We note that the binary production structure assumed above is without loss of generality for our purpose. This is because our goal is not to identify the optimal effort level for the principal per se; rather, it is to identify conditions under which conservatism helps reduce earnings management as well as reduce the cost of implementing any given effort level. A standard analysis of principal-agent models with multiple or continuous effort levels involves invoking assumptions to ensure only the local incentive compatible (IC) constraint is binding (relevant) in equilibrium. Typically, the monotone likelihood ratio and the convexity of the distribution function conditions are assumed, but even if left implicit (equilibrium), concavity of the agent’s
expected utility in effort is central to most, if not all, principal-agent studies (e.g., Grossman and Hart [1983]).

From a technical vantage point, this means that, to establish whether an information system is of value, one simply needs to show that the optimal effort absent the information system (regardless of what that effort level is) can be implemented over the immediate adjacent (lower) effort more cost-effectively with the information system present. We show that, between any two effort levels where, absent conservatism, the principal wants to implement the higher effort (which excludes the degenerate case where the lowest effort level is preferred), conservatism can help the principal implement the higher effort level at a lower cost. Thus, the beneficial role of conservatism is not unique and special to a binary setting. It is an insight that can be applied to general settings with multiple effort levels or continuous effort choice.

At date 2, the current owner needs to sell the firm to a second generation of shareholders (whom we refer to as investors or future shareholders) for reasons (such as liquidity needs) exogenous to our model. We assume that the second generation of shareholders bids for the firm’s shares competitively. To ensure a meaningful role for accounting information in this transaction, we assume that the sale takes place before $x$ is realized ($x$ accrues to the future shareholders) and that the accounting reports $e$ (statements of earnings) are the only publicly observable and contractible signals about $x$ available at date 2. Thus, the firm’s selling price depends on both parties’ beliefs about $x$ based on accounting earnings. To capture the idea that (some) firms last longer than their managers, we assume that the agent turns over when ownership changes hand at date 2. Accordingly, the agency problem ends at the time of transfer and the current owner contracts with the agent by specifying compensation contingent upon accounting earnings.

Accounting earnings are generated by the firm’s accounting system. How well they represent the true underlying output depends on both the prevailing accounting standards/principles adopted by the jurisdiction in which the firm operates (e.g., U.S. GAAP or international accounting standards) and the earnings manipulation chosen by the firm’s owner. Our focus here is on how conservatism in accounting standards affects earnings manipulation. As such, we confine our analysis to comparing two accounting standards that differ fundamentally with respect to conservatism.

The first accounting standard we consider is an unbiased standard that, without earnings management, simply reveals the underlying economic conditions. Accordingly, under this standard, $\Pr(e_h \mid x_h) = 1$ and $\Pr(e_l \mid x_l) = 1$. The second standard is conservative and differs from the unbiased standard in that, absent earnings management, accounting earnings are stochastically biased downward via $\Pr(e_h \mid x_h) = 1 - \Gamma$ and $\Pr(e_l \mid x_h) = \Gamma$, with $\Gamma \in (0, 1)$. As accounting standards are publicly disclosed and can be enforced by auditors and regulators, $\Gamma$ is assumed to be publicly observable. With this representation, $\Gamma$ reflects the degree of conservatism in the accounting
standard, as a higher value of $\Gamma$ implies that true high output is more likely to be reported as low earnings numbers by the accounting system.

As discussed earlier, accounting earnings may also be affected by the current owner’s preference for (opportunistic) earnings management. To parsimoniously capture the notion that earnings manipulation always reduces the informativeness of the accounting system, in our setting the current owner manages earnings by introducing some unobservable noise ($\Delta$) into the system such that $\Pr(e_l | x_l)$ is lowered from its default value of 1 to $1 - \Delta$, and $\Pr(e_h | x_l) = \Delta$. Accordingly, a higher $\Delta$ means that a true low level of payoff is more likely to be reported as high accounting earnings.\footnote{Earnings manipulation may also result in a lower value for $\Pr(e_h | x_h)$, i.e., when the current owner manages earnings downward by choosing $\Sigma$ such that $\Pr(e_h | x_h) = 1 - \Sigma$, and $\Pr(e_l | x_h) = \Sigma$. It can be easily shown that the current owner optimally sets $\Sigma$ to zero. The intuition is fairly straightforward, as the purpose of earnings management in our model is to boost earnings and selling price.}

Figures 1 and 2 provide an illustration of the information structures under the two standards and earnings management.

While not crucial to our results, we assume that there is some upper bound for $\Delta$, that is, $\Delta \in [0, \Delta_{\text{max}}]$, with $\Delta_{\text{max}} < 1$. The purpose of this assumption is simply to eliminate the potential for a “pathetic” equilibrium where $\Delta$ is set equal to one and the current owner therefore is unable to motivate
the agent to take \( a_h \). Imposing an upper bound on \( \Delta \) also makes intuitive sense as accounting standards and auditors constrain the current owner’s earnings manipulation behavior.

We assume that the earnings manipulation, \( \Delta \), is observed (only) by the current owner and the agent, but not by the future shareholders/investors. However, investors are not naive about the reported accounting earnings. Although they do not observe \( \Delta \) directly, they take into account the current owner’s incentives to manipulate earnings and use their (rational) conjecture of \( \Delta \) (denoted \( \hat{\Delta} \)) when pricing the firm. The current owner, in choosing his optimal \( \Delta \), takes \( \hat{\Delta} \) as given. In equilibrium, investors’ conjecture is correct. A formal definition for an equilibrium is given below:

**Definition.** An equilibrium in this model is characterized by \((\Delta, \hat{\Delta}, P_{e_k}, s_k)\), where \( k = l \) or \( h \), such that

1) investors’ conjecture, \( \hat{\Delta} \), is correct in equilibrium, i.e., \( \hat{\Delta} = \Delta \);
2) given the compensation contract \( s_k \) and \( \Delta \), the agent chooses \( a_h \);
3) given \( \hat{\Delta} \), \( \Delta \) and \( s_k \) maximize the current owner’s expected payoff from selling the firm to investors, net of the expected employment compensation for the agent;
4) given \( \hat{\Delta} \), the market price, \( P_{e_k} \), equals the investors’ expected future payoff from purchasing the firm.

Figure 3 summarizes the timeline and main events of the model.

### 3. Unbiased Accounting Standard

In this section, we solve the model under the unbiased accounting standard following the standard approach of backward induction. Accordingly, we first determine how much investors are willing to pay for the firm at date 2 as a function of both the realized earnings and investors’ conjecture about \( \Delta \). We then use this pricing function to identify the current owner’s choice of earnings manipulation, \( \Delta \), and compensation contract for the agent, \( s \), at date 1.

Investors purchase the firm for the expected output \( x \). Given their conjecture of the current owner’s earnings manipulation, investors’ expected benefit when the realized earnings is \( e_h \) and \( e_l \) can be respectively written
as:
\[
\hat{P}_{eh} = \frac{p}{p + (1 - p)\Delta} x_h > 0, \\
\hat{P}_{el} = x_l = 0.
\]

That is, investors pay a higher price upon observing \(e_h\) than upon observing \(e_l\). Ceteris paribus, this implies that the current owner would like to make \(e_h\) appear more frequently, which can be achieved by increasing \(\Delta\).

In addition to the earnings management \(\Delta\), at date 1, the current owner also needs to choose the compensation contract for the agent. Specifically, the contract will pay the agent \(s_h\) upon observing the high earnings realization \(e_h\) (or equivalently upon obtaining the high price of \(\hat{P}_{eh}\)) and \(s_l\) upon observing \(e_l\) (or equivalently upon obtaining the low price of \(\hat{P}_{el}\)).

The triplet \((s_h, s_l, \Delta)\) is jointly chosen to maximize the current owner’s expected payoff as follows:

\[
\max_{s_h, s_l, \Delta} \left( p + (1 - p)\Delta \right) \left( \hat{P}_{eh} - s_h \right) + \left( 1 - (p + (1 - p)\Delta) \right) \left( \hat{P}_{el} - s_l \right)
\]

\[
\text{s.t.} \quad (p + (1 - p)\Delta) U(s_h) + (1 - (p + (1 - p)\Delta)) U(s_l) \geq D \geq 0. 
\]

To solve this maximization problem, we can simplify the IC constraint to

\[
U(s_h) - U(s_l) \geq \frac{D}{p(1 - \Delta)}.
\]

The solution has both the IR and IC constraints binding. Thus, the optimal \(s_h^*\) and \(s_l^*\) can be obtained directly from these constraints as:

\[
U(s_h^*) = \frac{D}{p}, \\
U(s_l^*) = \frac{D}{p} \left( 1 - \frac{1}{1 - \Delta} \right).
\]

Because of the dual uses of accounting information, the contract with the agent also depends on the earnings reports and is thus, in turn, affected by the current owner’s \(\Delta\) choice. Recall when \(1 - \Delta\) approaches its maximum of one, the resulting accounting reports are the most informative. Thus, as \(1 - \Delta\) increases, the risk imposed on the agent, and hence the difference between \(s_h^*\) and \(s_l^*\), should become smaller. This is easily verified by noting that

\[
\frac{\partial (s_h^* - s_l^*)}{\partial (1 - \Delta)} = \frac{\partial s_h^*}{\partial (1 - \Delta)} - \frac{\partial s_l^*}{\partial (1 - \Delta)} = -\frac{D}{p} \frac{1}{U'(s_l)(1 - \Delta)^2} < 0.
\]

\(^8\)Conditioning the agent’s payments on earnings is equivalent to conditioning on price because price here is uniquely determined by a given earnings realization.
Stated differently, earnings management is costly to the current owner because a higher $\Delta$ introduces more noise into the system and leads to inferior risk sharing between the current owner and the agent. As a result, earnings management increases the current owner’s expected compensation to the agent, as shown formally by the following derivative:

$$\frac{\partial E(s^*)}{\partial \Delta} = -(1 - p) \left[ (s^*_h - s^*_l) - \frac{U(s^*_h) - U(s^*_l)}{U'(s^*_l)} \right] > 0$$  \hspace{1cm} (1)$$

The last inequality follows as $U(\cdot)$ is a strictly concave function.

To solve for the optimal $\Delta$, substitute in the optimal $s^*_h$ and $s^*_l$ and simplify the current owner’s problem to:

$$\max_{\Delta} (p + (1 - p)\Delta)(\tilde{P}_{e_h} - s^*_h) + (1 - (p + (1 - p)\Delta))(\tilde{P}_{e_l} - s^*_l).$$

Here, we only consider interior solutions. For convenience, to obtain the first-order condition, we take the derivative with respect to $(1 - \Delta)$ instead of $\Delta$:

$$\text{FOC} = -(1 - p)(\tilde{P}_{e_h} - \tilde{P}_{e_l}) - \frac{\partial E(s^*)}{\partial (1 - \Delta)}.$$  

Substituting in $\tilde{P}_{e_h} - \tilde{P}_{e_l}$ yields$^9$

$$\frac{-\partial E(s^*)}{\partial (1 - \Delta)} = \frac{p(1 - p)}{1 - (1 - p)(1 - \Delta)} x_h.$$  \hspace{1cm} (2)$$

The equilibrium $\Delta^*$ is obtained by setting $\tilde{\Delta} = \Delta$ in the above equation. As plotted in figure 4, the left-hand side of equation (2) is decreasing in $1 - \Delta$, while the right-hand side is concavely increasing in $1 - \Delta$, starting from the y axis at $p(1 - p)x_h$ with the two curves intersecting at most once.

The following proposition summarizes our results so far.

**PROPOSITION 1.** Suppose the parameter values are such that an interior solution to equation (2) exists. Then, there exists an equilibrium where the current owner

$^9$ Notice that the objective function is everywhere concave in $1 - \Delta$, as $-\frac{\partial^2 E(s^*)}{\partial (1 - \Delta)^2} < 0$ and hence any point satisfying equation (2) is the maximum. To see this,

$$-\frac{\partial^2 E(s^*)}{\partial (1 - \Delta)^2} = (1 - p) \left\{ \frac{U''(s^*_l) \frac{\partial s^*_l}{\partial (1 - \Delta)} (U(s^*_h) - U(s^*_l))}{(U'(s^*_l))^2} \right\} - \left\{ \frac{\frac{\partial (U(s^*_h) - U(s^*_l))}{\partial (1 - \Delta)} 1}{U'(s^*_l)} - \frac{\partial (s^*_h - s^*_l)}{\partial (1 - \Delta)} \right\}.$$ 

Note that $\frac{\partial (U(s^*_h) - U(s^*_l))}{\partial (1 - \Delta)} \frac{1}{U'(s^*_l)} = \frac{\partial U(s^*_l)}{\partial (1 - \Delta)} \frac{1}{U'(s^*_l)} = -\frac{\partial s^*_l}{\partial (1 - \Delta)}$ and $\frac{\partial (s^*_h - s^*_l)}{\partial (1 - \Delta)} = -\frac{\partial s^*_l}{\partial (1 - \Delta)}$. Thus, the two terms in the square brackets cancel each other out and the first term is strictly negative.
introduces upward earnings manipulation to the accounting information system, i.e., $\Delta^* > 0$.

While Proposition 1 establishes that the current owner may optimally choose to manipulate earnings, in equilibrium outside investors are not fooled by the earnings manipulation. Investors price protect themselves by taking $\Delta$ into account when bidding for the firm. However, with the manipulation, the current owner ends up paying more to motivate the agent (recall $E(s)$ is an increasing function of $\Delta$). Thus, the manipulation is inefficient for the current owner. This inefficiency arises from the current owner’s inability to commit not to manipulate earnings and the fact that accounting numbers are used in both external and internal reporting. In the next section, we show that under the conservative standard the equilibrium manipulation can be reduced, and the current owner can achieve better risk sharing with the agent and lower his expected payment to the agent.

Before moving on to section 4, we detour for a moment to look at the impact of a liberal accounting standard on the equilibrium. A liberal accounting system is characterized by $\Pr(e_h | x_h) = 1$, $\Pr(e_h | x_l) = q$, and $\Pr(e_l | x_l) = 1 - q$. $q$ here measures the degree of overestimation of true
output by the accounting system. Similar derivation yields the following condition for the equilibrium earnings manipulation $\Delta^*$:

$$-\frac{\partial E(s^*)}{\partial (q - \Delta)} = \frac{p(1 - p)}{1 - (1 - p)(q - \Delta)} x_h.$$  

Notice that the above expression implies $\frac{d\Delta^*}{dq} = 1$, thus $\frac{d(q - \Delta^*)}{dq} = 0$, because both sides are independent of $q$ when holding $q - \Delta$ constant. This implies that a better equilibrium risk sharing is not achieved by increasing $q$, that is, any attempt to make the accounting principles more liberal is perfectly offset by the current owner’s manipulation in equilibrium.

4. Conservative Accounting Standard

We now turn to solve the model under the conservative accounting standard. Let $\Delta^\Gamma$ represent the amount of earnings manipulation introduced by the current owner at date 1 under the conservative accounting standard.\(^{10}\) For notational ease and for reasons that will be clear soon, define

$$Z = 1 - \Delta^\Gamma - \Gamma.$$  

Intuitively, since both $\Delta^\Gamma$ and $\Gamma$ are biases, $Z$ roughly measures the informativeness of the accounting system that eventually generates the earnings number.

Let $\Delta^\Gamma$ be investors’ conjecture of $\Delta^\Gamma$ and $\tilde{P}_{e_h}^\Gamma (\tilde{P}_{e_l}^\Gamma)$ be their bidding price for the firm when the earnings report is $e_h (e_l)$. Then

$$\tilde{P}_{e_h}^\Gamma \equiv \Pr (x_h | e_h, \tilde{\Delta}^\Gamma) x_h + \Pr (x_l | e_h, \tilde{\Delta}^\Gamma) x_l = \frac{p(1 - \Gamma) x_h}{\tilde{\eta}},$$

$$\tilde{P}_{e_l}^\Gamma \equiv \Pr (x_h | e_l, \tilde{\Delta}^\Gamma) x_h + \Pr (x_l | e_l, \tilde{\Delta}^\Gamma) x_l = \frac{p\Gamma x_h}{1 - \tilde{\eta}},$$

where

$$\eta \equiv \Pr (e_h | \Delta^\Gamma, \Gamma) = p(1 - \Gamma) + (1 - p)\Delta^\Gamma$$

$$= 1 - \Gamma - (1 - p)Z$$

is the probability of observing $e_h$ under the conservative standard when the high effort is taken and $\tilde{\eta}$ is investors’ conjecture of $\eta$ under $\Delta^\Gamma$.

Since

$$\tilde{P}_{e_h}^\Gamma - \tilde{P}_{e_l}^\Gamma = \frac{p(1 - p) \tilde{\eta} x_h}{\tilde{\eta}(1 - \tilde{\eta})} > 0,$$

the current owner has an incentive to make $e_h$ appear more frequently by choosing a positive $\Delta^\Gamma$. However, $\Delta^\Gamma$ is now a function of $\Gamma$, the degree of conservatism in the accounting system. Next, we identify conditions under

\(^{10}\)An implicit assumption in this section is that the current owner cannot reduce $\Gamma$. This can be justified because accounting standards are enforced by regulators and monitored by auditors. It is therefore unlikely that firms can completely undo the effect of these standards.
which a positive degree of conservatism is desirable in that it reduces $\Delta^\Gamma$ and improves contracting efficiency.

The current owner’s problem at date 1 is very similar to that in section 3. Let $s_h^\Gamma$ and $s_l^\Gamma$ be the agent’s compensation if $e = e_h$ and $e = e_l$, respectively. The current owner chooses $s_h^\Gamma$, $s_l^\Gamma$, and $\Delta^\Gamma$ to

$$
\max_{s_h^\Gamma, s_l^\Gamma, \Delta^\Gamma} \eta (\hat{P}_{e_h} - s_h^\Gamma) + (1 - \eta) (\hat{P}_{e_l} - s_l^\Gamma)
$$

s.t. $\eta U(s_h^\Gamma) + (1 - \eta) U(s_l^\Gamma) - D \geq \Delta^\Gamma U(s_h^\Gamma) + (1 - \Delta^\Gamma) U(s_l^\Gamma),$

$$
\eta U(s_h^\Gamma) + (1 - \eta) U(s_l^\Gamma) - D \geq 0.
$$

Since the solution binds at both constraints, the optimal $s_h^\Gamma^*$ and $s_l^\Gamma^*$ solve the following equations:

$$
U(s_h^\Gamma^*) = \left(1 + \frac{\Gamma}{Z}\right) \frac{D}{p},
$$

(3)

$$
U(s_l^\Gamma^*) = \left(1 + \frac{\Gamma}{Z} - \frac{1}{Z}\right) \frac{D}{p}.
$$

(4)

Let $E(s^\Gamma)$ denote the expected payment to the agent. The following lemma establishes that $E(s^\Gamma)$ is decreasing in $Z$. All remaining proofs are relegated to the appendix.

**Lemma.** $\frac{\partial E(s^\Gamma)}{\partial Z} < 0$.

Intuitively, since $Z \equiv 1 - \Delta^\Gamma - \Gamma$ measures the informativeness of accounting earnings, a higher $Z$ reduces the agency cost.

Substituting $s_h^\Gamma^*$ and $s_l^\Gamma^*$ into the current owner’s problem yields:

$$
\max_{\Delta^\Gamma} \eta (\hat{P}_{e_h} - s_h^\Gamma^*) + (1 - \eta) (\hat{P}_{e_l} - s_l^\Gamma^*).
$$

Using a similar approach to that in section 3, it can be shown that this objective function is concave in $Z$. Since the objective function does not depend on $\Delta^\Gamma$ once $Z$ is held constant, we can reparameterize the problem using $Z$ instead of $\Delta^\Gamma$. Again, assuming an interior solution, we have the following first-order condition with respect to $Z$:

$$
(1 - p) (\hat{P}_{e_h} - \hat{P}_{e_l}^\Gamma) = -\frac{\partial E(s^\Gamma)}{\partial Z}.
$$

(5)

The optimal $Z^*$ is obtained by setting $\hat{Z} = Z$ in the above equation.

Our main focus is to analyze the effect of conservatism on the current owner’s earnings manipulation choice, $\frac{d\Delta^\Gamma}{dt}$, which is equivalent to analyzing $\frac{dZ}{dt}$ with $Z^*$ defined by equation (5). The left-hand side of equation (5), $(1 - p) (\hat{P}_{e_h}^\Gamma - \hat{P}_{e_l}^\Gamma)$, represents the current owner’s marginal benefit from earnings manipulation. The marginal effect of conservatism on this term is

$$
\left. \frac{\partial (\hat{P}_{e_h}^\Gamma - \hat{P}_{e_l}^\Gamma)}{\partial \Gamma} \right|_{\Gamma=0} = \frac{(1 - 2\eta) p (1 - p)^2 Z}{\eta^2 (1 - \eta)^2} x_h.
$$
Since \( p \geq \frac{1}{2} \), \( \eta|_{\Gamma=0} = p + (1 - p) \Delta^\Gamma > \frac{1}{2} \), the above term is strictly negative, implying that a higher \( \Gamma \) decreases the marginal benefit of manipulating earnings.

The right-hand side of equation (5) represents the current owner’s marginal cost of earnings manipulation in terms of the agent’s incentive compensation. The impact of \( \Gamma \) on this term is complicated and depends on the agent’s utility function. The following proposition, which is a key result of our paper, identifies a sufficient condition under which conservatism helps reduce the equilibrium level of earnings manipulation by the current owner.

**Proposition 2.** When \( \Gamma \) is relatively small, a sufficient condition for \( Z^* \) to be increasing in \( \Gamma \) is that

\[
-\frac{U_l'}{U_l} \frac{U_h}{U_h - U_l} \leq \frac{1}{1 - p}.
\]

Proposition 2 outlines a sufficient condition for \( \frac{dZ^*}{d\Gamma} > 0 \). Since \( Z = 1 - \Delta^\Gamma - \Gamma, \frac{dZ^*}{d\Gamma} > 0 \) implies that \( \frac{d\Delta^*}{d\Gamma} < -1 \), that is, one unit of conservative noise reduces the noise introduced by earnings manipulation by more than one unit. The sufficient condition in Proposition 2 guarantees that

\[
-\frac{\partial^2 E(s^*)}{\partial Z^2} \bigg|_{\Gamma=0} > 0,
\]

which ensures that accounting conservatism increases the marginal cost of earnings management (captured by \( -\frac{\partial E(s^*)}{\partial Z} \)). Because we are not confining our attention to a specific class of preferences/utility functions, this condition appears somewhat technical. The appearance is deceiving since the condition is satisfied for a wide range of standard utility functions, such as the negative exponential, the logarithmic, the quadratic, and the power class with relative risk aversion less than or equal to one-half. In the case of higher levels of relative risk aversion, the condition remains satisfied as long as the difference in the agent’s payments is not too high.\(^{11}\)

Thus, Proposition 2 shows that conservatism can reduce the equilibrium level of earnings manipulation under fairly general conditions.

The intuition behind Proposition 2 is the following. Observable conservative noise reduces the current owner’s desire to manipulate earnings for two reasons. First, given investors’ conjecture \( \Delta^\Gamma \), such noise makes good news less good and bad news less bad. That is, adding some known (stochastic) conservative bias reduces the impact of news on share prices and consequently reduces the benefit of earnings management, given potential investors’ belief. Second, at the same time, conservative noise makes it more costly for the current owner to motivate the agent to work, thus increasing the cost of earnings manipulation. As a result, the current owner finds it desirable to engage in less earnings management, and investors rationally incorporate this into the price, resulting in a lower level of earnings management in equilibrium.

Figure 5 provides a graphical representation of the optimal \( \Delta \) under the unbiased standard and under the conservative standard. It shows that when \( \Gamma \) increases from zero to positive, the optimal \( 1 - \Delta^* - \Gamma \) increases.
CONSERVATISM IN ACCOUNTING STANDARDS

The effect of conservative accounting on contracting efficiency is captured by \( \Gamma \)'s net impact on the current owner's expected payment to the agent, that is,

\[
\frac{dE(s^\Gamma)}{d\Gamma} = \frac{\partial E(s^* (Z, \Gamma))}{\partial Z} \frac{dZ}{d\Gamma} + \frac{\partial E(s^* (Z, \Gamma))}{\partial \Gamma}.
\]

The first term is the indirect effect of conservatism on the expected payment to the agent: It reduces the noise introduced by earnings management (i.e., \( \Delta \)) and thus reduces the expected payment (as \( \frac{dZ}{d\Gamma} > 0 \) and \( \frac{\partial E(s^* (Z, \Gamma))}{\partial Z} < 0 \)). The second term is the direct effect of conservatism on the expected payment. Expanding the second term, we have

\[
\frac{\partial E(s(Z, \Gamma))}{\partial \Gamma} = -(s_h - s_l) + \left[ \eta \frac{\partial s_h}{\partial \Gamma} + (1 - \eta) \frac{\partial s_l}{\partial \Gamma} \right].
\]

The \(-(s_h - s_l)\) term captures the fact that a higher \( \Gamma \) makes it more likely that the agent is paid the lower salary, \( s_l \), hence reducing the expected payment. The last two terms capture the fact that conservatism itself introduces noise into the accounting system and hence reduces efficient risk sharing, which leads to higher expected compensation. The following proposition outlines a sufficient condition under which the net of these effects is in favor of conservatism.

**PROPOSITION 3.** Suppose the conditions of Proposition 2 are satisfied. Then, when \( \Gamma \) is relatively small, a sufficient condition for the expected payment to the agent to be decreasing in \( \Gamma \) is

\[
\frac{1}{U_h} \eta + \frac{1}{U_l} (1 - \eta) \leq \frac{s_h - s_l}{U_h - U_l}.
\]
Certainly the additional (sufficient) condition in Proposition 3 is not entirely without teeth in the sense that Proposition 3 does not apply in all cases where Proposition 2 applies. However, it is not overly restrictive either. For example, it can easily be verified that this condition can be met for various standard preference representations. For example, it is met when the agent’s utility function belongs to the CARA family (constant absolute risk aversion), provided that the absolute risk aversion is not too low. Similarly, for utility functions in the CRRA family (constant relative risk aversion), the condition is satisfied as long as the risk in the agent’s contract is not too high.

We know from common intuition that it is more costly to impose risk on agents with higher risk aversion. This suggests that when the agent is relatively risk averse, a conservative accounting standard achieves a higher efficiency than an unbiased standard. Intuitively, when the agent is risk averse, more informative accounting reports become more valuable because they improve risk sharing. To the extent that conservatism reduces the equilibrium earnings management and improves the informativeness of accounting reports (as established in Proposition 2), it has a more pronounced cost saving effect for the current owner when the contract inefficiency introduced by earnings management is large.

Proposition 3 shows that conservatism can help firms escape the inefficient equilibrium. This does not, of course, imply that conservatism is the only way this can be done. It is, for example, possible that, if the inefficiency identified in the model is sufficiently severe, firms may have incentives to create separate internal performance measures for the purpose of evaluating managerial performance. However, it is unlikely that such alternatives would completely substitute for the beneficial effects of conservatism we have identified. Rather, we conjecture that conservatism would always be part of the mixture of solutions. This conjecture is based on the following two observations.

The first derives from the relative credibility of conservatism vis-à-vis alternative mechanisms. One key reason for conservative accounting standards to be beneficial in our model is that they reduce the perceived benefits of earnings management; they can do so because accounting standards are publicly observable, and therefore credible. This contrasts with many other alternatives, such as using separate internal performance measures, which due to being private in nature are unlikely to be credible to outsiders. The inability of firms to credibly commit to unobservable actions is precisely the reason they are trapped in the inefficient equilibrium to begin with in our model.12

Second, and equally if not more importantly, as long as the alternative performance measures are not sufficient statistics for the external financial

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12 Otherwise firms can always announce that they will commit not to managing earnings. If the credibility of such commitment is not an issue, then there is indeed no role for conservatism in accounting standards.
With respect to the agent’s effort, it is always optimal for firms to use the external financial reports for the purpose of evaluating the agent. As discussed in footnote 3, in general, the fact that accounting numbers are useful for valuation purposes suggests that they also have stewardship value. Accordingly, it seems hard to imagine circumstances under which the external financial reports would not carry the dual role underlying our analysis. This implies that the value of conservatism as established in Proposition 3 would persist even if we were to allow explicitly in our model that the current owner can rely on other measures.

Separately, we acknowledge that our model is a static one and thus does not explicitly consider the dynamic aspect of accrual accounting. While a full-scale analysis of the effects of accrual reversal lies outside the scope of this paper, we offer some preliminary intuitions here. If the second-generation shareholders need to hire an agent to produce so they can sell to a third generation before the new output is realized, there is no fundamental difference between the second generation’s problem and the first generation’s problem. That is, the second-generation shareholders would still be tempted to manage earnings on their own and hence be trapped in the inefficient equilibrium. As a result, conservative accounting standards for the second generation still help reduce the inefficiency, which is the main point of this paper. This does not depend on the source of the baseline properties of the accounting system, including reversals of prior periods’ accruals.\(^\text{13}\)

If the second-generation shareholders do not need to sell to a third generation but still need to hire an agent to produce, they would take into account the effects of the reversal and pay a lower price to the current owner (because reversal introduces noise and makes it costly to motivate the agent to work). While this may reduce the current owner’s incentive to manage earnings, it does not necessarily eliminate the incentive. Thus, as long as there is a positive amount of earnings management in equilibrium, conservatism has the potential for helping to reduce such earnings management.

5. Conclusion

In this paper, we explore the role of conservative accounting standards in combating (unobservable) earnings management. When accounting information serves dual purposes (i.e., valuation and stewardship purposes), our model shows that the current owner of a firm has an incentive to engage in earnings manipulation activities in hope of boosting the market price of his firm. However, potential investors rationally expect and price protect against the earnings manipulation. Lacking the tools for committing not to manage earnings, the current owner is trapped by such expectations and

\(^{13}\) Also note that because conservatism in our setting reduces the equilibrium (liberal) earnings management, it is not clear that the net outcome of a conservative system is conservative or liberal. Therefore, it is not clear that the reversal of total accruals under a conservative system leads to a liberal accounting report in the next period either.
therefore has to manage earnings in order to fulfill potential investors’ rational conjecture. As a result, inefficient earnings manipulation exists, adding not just bias but also noise to the performance measure used to motivate the firm’s employees.

More significantly, however, our analysis also shows that employing conservative accounting standards can actually reduce the equilibrium amount of both noise and bias and thus help achieve a more efficient equilibrium outcome. This occurs despite the fact that conservative accounting itself introduces undesirable biases/noises and that the current owner maintains all manipulation options. As shown, the reason it works is that adding some known (stochastic) conservative bias reduces the impact of news on share prices and consequently reduces the benefits of earnings management, given potential investors’ belief. In addition, conservative bias also increases the marginal cost of earnings manipulation. Surprisingly, adding one type of bias can reduce the other so much that the total equilibrium amount of noise in the accounting system is diminished.

Although accounting conservatism is an active area of research in empirical accounting, theories explaining its existence and prominence have been sparse. Our paper formally establishes the link between accounting conservatism and earnings management. In particular, by demonstrating the ability of conservative distortions to reduce incentives for earnings management to the point where the noise introduced by conservatism is more than offset by the reduction in the noise introduced by earnings management, we highlight the potential for economic benefits of observable conservative reporting distortions, even if such distortions are themselves inefficient when viewed in isolation.

**APPENDIX**

**Proof of Lemma.**

\[
\frac{\partial E(s)}{\partial Z} = -(1 - p) (s_h - s_l) + \eta \frac{\partial s_h}{\partial Z} + (1 - \eta) \frac{\partial s_l}{\partial Z}
\]

\[
= -(1 - p) (s_h - s_l) + \eta \frac{\partial U^{-1}\left(\frac{D}{p} \left(1 + \frac{\Gamma}{Z}\right)\right)}{\partial Z}
\]

\[
+ (1 - \eta) \frac{\partial U^{-1}\left(\frac{D}{p} \left(1 + \frac{\Gamma - \frac{1}{Z}}{Z}\right)\right)}{\partial Z}
\]

\[
= -(1 - p) (s_h - s_l) + \eta \frac{D}{p} \frac{1}{U_h' Z^2} - \Gamma + (1 - \eta) \frac{D}{p} \frac{1}{U_l' Z^2} (1 - \Gamma)
\]

\[
\leq -(1 - p) (s_h - s_l) + \eta \frac{D}{p} \frac{1}{U_h' Z^2} - \Gamma + (1 - \eta) \frac{D}{p} \frac{1}{U_l' Z^2} (1 - \Gamma)
\]
The inequality uses the fact that $U$ is an increasing, concave function, i.e., $U'_i > U'_h > 0$. Further simplifying the above expression yields

$$\frac{\partial E(s)}{\partial Z} \leq -(1 - p)(s_h - s_l) + \frac{1}{U'_i} \frac{D}{p} \frac{1}{Z} \left[ (1 - \eta)(1 - \Gamma) - \eta \Gamma \right]$$

$$= -(1 - p)(s_h - s_l) + \frac{1}{U'_i} \frac{D}{p} (1 - p).$$

Substitute in $U_h - U_i = \frac{D}{p} Z$, we have

$$\frac{\partial E(s)}{\partial Z} \leq (1 - p) \left[ \frac{U_h - U_i}{U'_i} - (s_h - s_l) \right] < 0,$$

where the last inequality obtains because $U'' < 0$. ■

Proof of Proposition 2. The optimal $Z$ under the conservative standard is obtained by setting $FOC$ (below) to zero and substituting $\tilde{Z}$ (in $\tilde{P}_{e_h}$ and $\tilde{P}_{e_l}$) with $Z$.

$$FOC = \frac{\partial E(s)}{\partial Z} - (1 - p)(\tilde{P}_{e_h}^\Gamma - \tilde{P}_{e_l}^\Gamma).$$

Hence, using implicit function theorem, we have

$$\frac{d Z^*}{d \Gamma} = \frac{-\frac{\partial FOC}{\partial Z}}{\frac{\partial E(s)}{\partial Z}} \bigg|_{\tilde{Z}=Z} = \frac{\partial FOC}{\partial \Gamma} - (1 - p) \frac{\partial (\tilde{P}_{e_h}^\Gamma - \tilde{P}_{e_l}^\Gamma)}{\partial Z}.$$

Under the assumption that an interior optimal solution exists, the second order condition is $SOC = -\frac{\partial^2 E(s)}{\partial Z^2} < 0$, hence $\frac{\partial^2 E(s)}{\partial Z^2} > 0$. Since $\frac{\partial (\tilde{P}_{e_h}^\Gamma - \tilde{P}_{e_l}^\Gamma)}{\partial Z} \big|_{\tilde{Z}=Z} = \frac{1}{\eta} p(1 - p) x_h > 0$, the denominator is positive. Then, if we can show $\frac{\partial FOC}{\partial \Gamma} > 0$ under the condition in Proposition 2, we are done.

$$\frac{\partial FOC}{\partial \Gamma} = -\frac{\partial^2 E(s)}{\partial Z \partial \Gamma} + \frac{p(1 - p)^2 Z x_h}{\eta(1 - \eta)^2} (2\eta - 1).$$

When $p \geq 1/2$, $\eta = p + (1 - p) \Delta \geq 1/2$, therefore, the second term above is positive. In the following, we identify the condition under which $-\frac{\partial^2 E(s)}{\partial Z \partial \Gamma} \big|_{\Gamma=0} > 0$. Recall

$$E(s) = \eta s_h + (1 - \eta) s_l, \quad \text{where} \quad \eta = 1 - \Gamma - (1 - p) Z.$$  

Hence

$$\frac{\partial E(s)}{\partial \Gamma} = \eta \frac{\partial s_h}{\partial \Gamma} + (1 - \eta) \frac{\partial s_l}{\partial \Gamma} + \frac{\partial \eta}{\partial \Gamma} (s_h - s_l),$$
\[
\frac{\partial^2 E(s)}{\partial Z \partial \Gamma} = \frac{\partial \eta}{\partial Z} \frac{\partial s_h}{\partial \Gamma} + \frac{\partial^2 s_h}{\partial \Gamma^2} + \frac{\partial (1 - \eta)}{\partial Z} \frac{\partial s_l}{\partial \Gamma} + (1 - \eta) \frac{\partial^2 s_l}{\partial \Gamma^2} - \frac{\partial (s_h - s_l)}{\partial Z} \\
= -(1 - \rho) \frac{\partial (s_h - s_l)}{\partial \Gamma} - \frac{\partial (s_h - s_l)}{\partial Z} + \eta \frac{\partial^2 s_h}{\partial \Gamma^2} + (1 - \eta) \frac{\partial^2 s_l}{\partial \Gamma^2}.
\]

(6)

From equations (3) and (4), we derive the detailed expressions for the partials listed above, evaluated at \(\Gamma = 0\). We have

\[
\frac{\partial s_h}{\partial \Gamma} = \frac{D}{pZ U_h}; \quad \frac{\partial s_h}{\partial Z} = \frac{\partial s_h - \Gamma}{Z \partial \Gamma}; \quad \bigg|_{\Gamma = 0} = 0;
\]

\[
\frac{\partial^2 s_h}{\partial \Gamma^2} = \left(-\frac{1}{Z} \frac{\partial s_h}{\partial \Gamma} + \frac{\Gamma}{Z} \frac{\partial^2 s_h}{\partial \Gamma^2}\right) \bigg|_{\Gamma = 0} = -\frac{1}{Z} \frac{\partial s_h}{\partial \Gamma};
\]

\[
\frac{\partial s_l}{\partial \Gamma} = \frac{D}{pZ U_l^2} \frac{\partial s_l}{\partial \Gamma^2} = \frac{D}{pZ} \frac{1}{U_l} \frac{\partial s_l}{\partial \Gamma} = \frac{1}{U_l} \left(\frac{\partial s_l}{\partial \Gamma}\right)^2;
\]

\[
\frac{\partial s_l}{\partial Z} = \left(\frac{D}{pZ U_l^2} \frac{1 - \Gamma}{Z} \right) \bigg|_{\Gamma = 0} = \left(\frac{1}{U_l} \frac{\partial s_l}{\partial \Gamma} \frac{1 - \Gamma}{Z}\right) \bigg|_{\Gamma = 0} = \frac{1}{Z} \frac{\partial s_l}{\partial \Gamma};
\]

\[
\frac{\partial^2 s_l}{\partial \Gamma^2} = \left(-\frac{1}{Z} \frac{\partial s_l}{\partial \Gamma} + 1 - \frac{\Gamma}{Z} \frac{\partial^2 s_l}{\partial \Gamma^2}\right) \bigg|_{\Gamma = 0} = -\frac{1}{Z} \frac{\partial s_l}{\partial \Gamma} + 1 - \frac{U_l''}{U_l} \left(\frac{\partial s_l}{\partial \Gamma}\right)^2.
\]

Substituting these expressions into equation (6) and collecting terms yields

\[
\left.\frac{\partial^2 E(s)}{\partial Z \partial \Gamma}\right|_{\Gamma = 0}
\]

\[
= -(1 - \rho) \frac{\partial(s_h - s_l)}{\partial \Gamma} - \frac{\partial(s_h - s_l)}{\partial Z} + \eta \left(-\frac{1}{Z} \frac{\partial s_h}{\partial \Gamma}\right)
\]

\[
+ (1 - \eta) \left[\frac{1}{Z} \frac{\partial s_l}{\partial \Gamma} + \frac{1 - U_l''}{U_l} \left(\frac{\partial s_l}{\partial \Gamma}\right)^2 \right]
\]

\[
= \frac{D}{pZ} \left(1 - \frac{U_l'}{U_l} \right) \left[\frac{1}{Z} + (1 - \rho) \frac{U_l''}{U_l} \frac{D}{pZ} \left(\frac{1}{U_l} \frac{1}{U_h - U_l}\right)^2 \right].
\]

Because \(Z < 1\), a sufficient condition for the above expression to be negative is \((*) \leq 1 - \rho\). The condition shown in Proposition 2 is obtained by replacing \(\frac{D}{pZ}\) with \(U_h - U_l\) and rearranging terms. \(\blacksquare\)

**Proof of Proposition 3.** Taking total derivative of \(E(s^*)\) with respect to \(\Gamma\), we have

\[
\frac{dE(s^* (Z, \Gamma))}{d\Gamma} = \frac{\partial E(s^* (Z, \Gamma))}{\partial Z} \frac{dZ}{d\Gamma} + \frac{\partial E(s^* (Z, \Gamma))}{\partial \Gamma}.
\]
Under the condition for Proposition 2, we have shown that \( \frac{dZ}{d\Gamma} = K > 0 \). Also, the lemma proves that \( \frac{\partial E(s^*(\Gamma, \Delta))}{\partial Z} < 0 \). So the first term in the above expression is negative. Writing out the detailed expressions, we have

\[
\frac{\partial E(s^*(Z, \Gamma))}{\partial Z} \bigg|_{\Gamma=0} = \left[ -(1 - p)(s_h - s_l) + \eta \frac{\partial s_h}{\partial Z} + (1 - \eta) \frac{\partial s_l}{\partial Z} \right] \bigg|_{\Gamma=0}
\]

and

\[
\frac{\partial E(s^*(Z, \Gamma))}{\partial \Gamma} \bigg|_{\Gamma=0} = \left[ -(s_h - s_l) + \eta \frac{\partial s_h}{\partial \Gamma} + (1 - \eta) \frac{\partial s_l}{\partial \Gamma} \right] \bigg|_{\Gamma=0}
\]

Substituting them in \( \frac{dE(s^*)}{d\Gamma} \) yields

\[
\frac{dE(s^*)}{d\Gamma} \bigg|_{\Gamma=0} = -(1 - p) K(s_h - s_l) + (1 - \eta) K \frac{D}{p Z} U'_h Z
\]

and

\[
\frac{dE(s^*(Z, \Gamma))}{d\Gamma} \bigg|_{\Gamma=0} = -(s_h - s_l) + \eta \frac{D}{p Z} U'_h + (1 - \eta) \frac{D}{p Z} U'_l Z
\]

where the last equality is obtained by substituting in \( (U_h - U_l) = \frac{D}{p Z} \) and \( (1 - \eta) \big|_{\Gamma=0} = (1 - p) Z \). Thus, to show \( \frac{dE(s^*)}{d\Gamma} < 0 \), we need to show

\[
\frac{1}{U'_h} \eta + \frac{1}{U'_l} (1 - \eta) < 1 + (1 - p) K
\]

which is equivalent to

\[
\frac{1}{U'_h} + \frac{1}{U'_l} (1 - \eta) < (1 - p) K \left( \frac{s_h - s_l}{U_h - U_l} \right)
\]

Collecting terms, we have

\[
\frac{1}{U'_h} + \frac{1}{U'_l} (1 - \eta) < \frac{s_h - s_l}{U_h - U_l} + \left( \frac{s_h - s_l}{U_h - U_l} - \frac{1}{U'_l} \right) (1 - p) K.
\]

Since \( K > 0 \) (from Proposition 2) and \( \frac{s_h - s_l}{U_h - U_l} - \frac{1}{U'_l} > 0 \) (by the concavity of the utility function), a sufficient condition for the above inequality is
\[
\frac{1}{U'_h} \eta + \frac{1}{U'_l} (1 - \eta) \leq \frac{s_h - s_l}{U_h - U_l}. \tag{8}
\]

Note that equation (8) can be further simplified by using the equilibrium equation (2) and substituting in the expression for \(\frac{\partial E(s^*)}{\partial (1 - \Delta)}\). Specifically, equation (2) implies

\[
s_h - s_l - \frac{U_h - U_l}{U'_l} = \frac{p}{1 - (1 - p) (1 - \Delta)} x_h > px_h
\]

\[
s_h - s_l - \frac{1}{U_h - U_l} > \frac{1}{U'_l} + \frac{px_h}{U_h - U_l}.
\]

Thus, a sufficient condition for equation (8) is

\[
\frac{1}{U'_h} \eta + \frac{1}{U'_l} (1 - \eta) \leq \frac{1}{U'_l} \frac{px_h}{U_h - U_l},
\]

which can be simplified as

\[
(U_h - U_l) \left( \frac{1}{U'_h} - \frac{1}{U'_l} \right) \leq px_h.
\]

Since both terms at the left-hand side of the above expression are increasing in \(s_h - s_l\), the above condition says that for a given \(p\) and \(x_h\), as long as the difference between \(s_h\) and \(s_l\) is not too large, equation (8) is satisfied.

In the case of CARA utility functions, note that the condition in Proposition 2 implies that

\[
(U_h - U_l) \left( \frac{1}{U'_h} - \frac{1}{U'_l} \right) \leq px_h.
\]

Thus, for a given \(p\) and \(x_h\), the above condition is satisfied as long as \(\frac{U'_l}{U_h} R\) is not too far away, and/or \(R\) is not too low.

\[\blacksquare\]

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