

# Asset Prices and Wealth Inequality\*

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## Abstract

I document a strong interplay between asset prices and wealth inequality (i) when stock returns are high, inequality increases (ii) higher inequality predicts lower stock returns. This corresponds to the basic prediction of a model where agents have heterogeneous preferences. Quantitatively, however, the model cannot match the excess volatility of asset prices without implying a wealth distribution with a tail thicker than the data. I suggest two parsimonious deviations to resolve this tension: (i) “live-fast-die-young dynamics”, in which risk-tolerant investors remain levered only for a short period of time, (ii) time-varying investment opportunities for the rich relative to the rest.

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# 1 Introduction

Recent empirical studies have documented important fluctuations in wealth inequality over the past century.<sup>1</sup> Volatile stock market returns potentially account for these fluctuations. Conversely, a large theoretical literature in asset pricing examines the role of household heterogeneity in shaping asset prices, but seldom considers its implication on wealth inequality. In this paper, I use recently available data on wealth inequality to examine empirically and theoretically the relationship between asset prices and the wealth distribution.

I focus on the following mechanism. Risk-tolerant investors hold more risky assets, accumulate more wealth, and disproportionately end up at the top of the wealth distribution. As a consequence, in periods when stocks enjoy large realized returns, investors at the top of the wealth distribution gain more than the rest, i.e. wealth inequality increases. In turn, as a larger share of wealth is owned by risk-tolerant households, aggregate demand for risk increases, which lowers risk premia and pushes up asset prices, i.e. higher wealth inequality predicts lower future excess returns. I confirm empirically each step of this mechanism: (i) when stock market returns are high, wealth inequality increases (ii) higher wealth inequality predicts lower future excess stock returns.

I then evaluate whether this mechanism can quantitatively explain the excess volatility of asset prices in equilibrium. I use the reduced-form evidence I documented earlier to estimate a state-of-the-art model where agents have heterogeneous preferences. I find that, to match the excess volatility of asset prices, the model requires such a large degree of preference heterogeneity that it gives rise to a wealth distribution close to Zipf’s law, i.e. with a right tail thicker than the data. To solve this tension, I propose two possible parsimonious deviations from the model: (i) “live-fast-die-young dynamics”, in which risk-tolerant investors remain highly levered only for a short period of time, or (ii) time-varying investment opportunities for rich households compared to the rest of the distribution.

The paper proceeds in four stages. First, I present two stylized facts on the relationship between asset prices and the wealth distribution. I first show that stock market returns generate large fluctuations in wealth inequality. More precisely, I use the series of top wealth shares constructed from tax filings by [Kopczuk and Saez \(2004\)](#) and from Forbes 400 to estimate the exposure of

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<sup>1</sup>See, for instance, [Kopczuk and Saez \(2004\)](#), [Piketty \(2014\)](#), and [Saez and Zucman \(2016\)](#).

the top percentiles to stock market returns. In response to a realized stock return of 10%, I find that the total wealth in the economy increases by 4%, while the total wealth for the top 0.01% increases by 9.5% (the wealth share of the top 0.01% increases by a difference of 4.5%). The flip side of this relationship is that in an economy where inequality is high, the share of wealth owned by risk-tolerant investors is high, and therefore, in equilibrium, risk premia are low. Thus, higher inequality should predict lower future returns. Indeed, in the data, I find that a 10% increase in the wealth share of the top 0.01% predicts lower future excess returns by one percentage point over the following year.

Second, I examine those facts through the lens of an asset pricing model with heterogeneous agents. Specifically, I study the joint dynamics of asset prices and wealth inequality in a continuous-time, overlapping generations framework where agents differ with respect to their relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS), following [Gârleanu and Panageas \(2015\)](#). The model can qualitatively generate my two stylized facts. One key contribution of my analysis is to show that, in this environment in which the *dynamics* of the wealth distribution fluctuate over time, the wealth distribution exhibits a thick right tail. I offer an analytical characterization for its tail index: it is determined by the *average logarithmic wealth growth rate* of top households relative to the economy.

Third, I ask whether the model can explain the excess volatility of asset prices in equilibrium. I find that, quantitatively, the model cannot generate volatile asset prices without implying an excessive level of inequality. Intuitively, to match the excess volatility of asset prices, the model requires a large degree of preference heterogeneity. In turn, this generates a wealth distribution with a right tail that is *too thick* compared to the data. This tension arises independently of the source of preference heterogeneity: it is present whether households differ with respect to their relative risk aversion, intertemporal elasticity of substitution, or/and subjective discount rates. Allowing for heterogeneous labor income or idiosyncratic wealth shocks, which are realistic ingredients for the wealth distribution, does not help resolve this tension.

Fourth, given this negative result, I propose two potential deviations from the baseline model that help resolve this tension. In the first departure, I assume that risk-tolerant households “live-fast-die-young”, i.e. that risk-tolerant households transition to risk-averse after a short period of time. This makes the right tail of the wealth distribution thinner compared to the baseline model,

which helps resolve the tension described above. Consistent with this departure, I document that older households in the right tail of the wealth distribution tend to be less levered than younger households. In the second departure, I augment the model with *time-varying investment opportunities* for risk-tolerant households relative to risk-averse households, i.e. purely redistributive shocks between households. Consistent with this departure, I show that certain dynamics in wealth inequality cannot be fully accounted for by stock market returns, in particular the decrease in wealth inequality after the Great Depression, or the rise in wealth inequality at the end of the 20th century. In equilibrium, these small but persistent redistributive shocks can generate large fluctuations in asset prices in equilibrium. Therefore, the augmented model can match the high volatility of asset prices with a reduced degree of preference heterogeneity, which helps resolve the tension discussed above.

**Related Literature.** This paper contributes to the growing literature on wealth inequality. On the empirical side, I rely critically on the recent series of top wealth shares constructed by [Kopczuk and Saez \(2004\)](#). On the theoretical side, random growth theories of the wealth distribution include [Wold and Whittle \(1957\)](#), and, more recently, [Benhabib et al. \(2011\)](#), [Benhabib et al. \(2015b\)](#), [Benhabib et al. \(2016\)](#), [Jones \(2015\)](#), and [Cao and Luo \(2016\)](#). While a number of studies focus on the role of the risk-free rate of return in shaping the wealth distribution ([Piketty \(2014\)](#), [Acemoglu and Robinson \(2015\)](#)), I document a more important role for the rate of return on risky assets. One key contribution relative to this literature is that I characterize the wealth distribution in an economy in which the *dynamics* of the wealth distribution is itself *time-varying*. I show that the tail index of the distribution is determined by the *average logarithmic* wealth growth rate of top households relative to the economy.<sup>2</sup> This extends the tools developed by [Luttmer \(2012\)](#) and [Gabaix et al. \(2016\)](#), that examine the transition of a distribution between two steady states.<sup>3</sup>

A large literature in household finance examines the heterogeneity in portfolio choice across the wealth distribution ([Guiso et al. \(1996\)](#), [Carroll \(2000\)](#), [Campbell \(2006\)](#), [Wachter and Yogo \(2010\)](#), [Roussanov \(2010\)](#), [Calvet and Sodini \(2014\)](#), and [Fagereng et al. \(2016\)](#)). [Parker and](#)

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<sup>2</sup>The role of this object in shaping the wealth distribution ties this paper to the literature on growth-optimal portfolios, which is initiated by [Kelly \(1956\)](#). See also [Blume et al. \(1992\)](#), or, more recently, [Borovička \(Forthcoming\)](#).

<sup>3</sup>As in these papers, heterogeneity across agents is key to obtain fast transition dynamics. More precisely, a model with preference heterogeneity is a particular case of the “type-dependence” model explored in [Gabaix et al. \(2016\)](#).

Vissing-Jørgensen (2009) and Guvenen et al. (2017) study the exposure of the labor income of top households to aggregate shocks. Malloy et al. (2009) documents that the consumption of richer stockholders tends to be more exposed to stock market returns. Bach et al. (2015) stresses the heterogeneous exposure to aggregate risk at the top of the wealth distribution using Swedish data. Wolff (2002) and Kuhn et al. (2017) document the role of asset prices for the dynamics of the wealth distribution. Compared to these papers, I focus on the very top of the wealth distribution (i.e. top 1% and beyond), which is key to fully capture the heterogeneity in stock market exposure across the wealth distribution.

This paper also contributes to a large asset pricing literature where agents have heterogeneous preferences.<sup>4</sup> As pointed out by Dumas (1989), preference heterogeneity tends to imply exploding wealth distribution in infinite-horizon economies. To solve this issue, Chan and Kogan (2002) assumes “catching up with the Joneses” preferences, making all investors grow at the same average rate. While this makes the model particularly tractable, it also means that risk-tolerant investors do not disproportionately end up at the top of the distribution; therefore, the model cannot generate the two-way feedback between wealth inequality and asset prices that I observe in the data. Guvenen (2009) presents a model that combines preference heterogeneity with limited participation, where heterogeneity in EIS is key to generate volatile asset prices. To obtain a stationary wealth distribution, the model has to abstract from long-run growth, which makes it difficult to take it to the data. Another difference is that the paper focuses on the heterogeneity between stockholders and non-stockholders, whereas I focus on the heterogeneity *within* stock-holders. Most related to this paper, Gârleanu and Panageas (2015) studies the role of preference heterogeneity in an overlapping generation model. Because investors die after a certain time, the model naturally gives rise to a stationary wealth distribution. A key contribution of my paper is to test the model empirically using the data on wealth inequality. I show that the model cannot match asset prices without implying a wealth distribution close to Zipf’s law, i.e. with a right tail too thick compared to the data. I suggest two parsimonious deviations to resolve this tension.

The first deviation consists in “live-fast-die-young” dynamics for risk-tolerant households, i.e. that risk-tolerant investors transition to risk-averse investors after a certain amount of time. This

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<sup>4</sup>The literature is too large to be summarized exhaustively, so I focus on few models trying to match quantitatively asset prices in the U.S.

relates this paper to a growing literature suggesting that transition rates from high-growth individuals to low-growth individuals are important to capture certain aspects of the distribution. [Luttmer \(2011\)](#) argues that these transition dynamics help explain why top firms are relatively young even though the firm size distribution is close to Zipf's law. [Gabaix et al. \(2016\)](#) argues that they help generate faster transitions between two steady states. The second deviation consists in time-varying investment opportunities for rich households compared to the rest of the distribution. This ties this paper to a literature examining the role of redistributive shocks for asset prices, through displacement shocks ([Gârleanu et al. \(2012\)](#), [Gârleanu and Panageas \(2017\)](#)), fluctuating tax rates ([Pastor and Veronesi \(2016\)](#)), or fluctuating capital shares ([Lettau et al. \(2016\)](#), [Greenwald et al. \(2014\)](#)).

A growing literature studies the impact of investment heterogeneity on wealth inequality and asset prices. [Gollier \(2001\)](#) is an early example that examines theoretically the importance of the wealth distribution for asset prices. [Barczyk and Kredler \(2016\)](#) examines theoretically the role of inequality and incomplete markets on asset prices. [Eisfeldt et al. \(2016\)](#) examines the joint relation between the wealth distribution and asset prices across markets with different expertises. On a more empirical side, [Johnson \(2012\)](#) examines the role of income inequality shocks for the cross-section of returns, [Favilukis \(2013\)](#) examines the role of changes in participation cost and wage inequality on asset prices. [Kacperczyk et al. \(2018\)](#) studies the role of investor sophistication for the recent rise in capital income inequality. Most relevant to this paper, [Toda and Walsh \(2016\)](#) independently shows that fluctuations in income inequality negatively predict future excess stock returns, using the series on top income shares from [Piketty and Saez \(2003\)](#). My paper confirms these results using the series of top wealth shares from [Kopczuk and Saez \(2004\)](#). My main contribution is to examine the magnitude of this effect through a quantitative model. This leads me to highlight some other moments, more precisely estimated, that discipline the effect wealth inequality can have on asset prices (e.g. the exposure of top households to stock market returns and the tail index of the wealth distribution).

**Road Map** The rest of my paper is organized as follows. In Section 2, I document two key stylized facts about the relation between wealth inequality and asset prices. In Section 3, I present a standard asset pricing model with heterogeneous agents to interpret these findings. In Section 4, I characterize analytically the wealth distribution in the model. In Section 5, I highlight the key tension to match quantitatively the model to the data. In Section 6, I propose two parsimonious

deviations that help resolve this tension. Section 7 concludes.

## 2 Data and Facts

I now analyze data about the top of the wealth distribution to document two stylized facts predicted by heterogeneous agents models. I focus on the following mechanism. Risk-tolerant households invest more in risky assets and disproportionately end up at the top of the wealth distribution. In periods when stocks enjoy large realized returns, investors at the top of the wealth distribution gain more than the rest; thus, inequality increases. In turn, as a larger share of wealth falls into the hands of risk-tolerant households, the aggregate demand for risk increases, which lowers risk premia; thus, higher inequality predicts lower future returns. After introducing the data, I document facts reflecting each step of this mechanism.

### 2.1 Data

**Wealth Shares** I am interested in measuring changes in the wealth distribution and their relationship to stock returns. Therefore, I need yearly estimates of the wealth distribution that cover several business cycles. I use two datasets that, together, cover most of the last 100 years.

The first wealth series is the annual series of top wealth shares constructed by [Kopczuk and Saez \(2004\)](#). This series is constructed from estate tax returns, which report the wealth of deceased individuals, above a certain wealth threshold. From the wealth distribution of the deceased, [Kopczuk and Saez \(2004\)](#) estimates the wealth distribution of the living using the mortality multiplier technique, which amounts to re-weighting each estate tax return by the inverse probability of death (depending on age and gender). The series is constructed using the universe of estate tax returns during the 1916-1945 period, and a stratified sample of micro-files for 1965, 1969, 1972, 1975 and 1982-2000. Total wealth uses the household balance sheets of the US Financial Accounts after 1945. Before 1945, [Kopczuk and Saez \(2004\)](#) reports estimates based on the same concepts and methods as the Financial Accounts, following [Wolff and Marley \(1989\)](#).

Another data series about wealth inequality is [Saez and Zucman \(2016\)](#). They construct top wealth shares from income tax returns using a capitalization method. However, the series builds in smoothing over time, which makes it harder to examine the joint dynamics of asset prices and

top wealth shares. I compare the two series more thoroughly in Appendix A.

I supplement the series of top wealth shares with the list of the wealthiest 400 Americans constructed by Forbes Magazine every year since 1982, which offers an unparalleled view of the right tail of the wealth distribution. This list is created by a dedicated staff, based on a mix of public and private information.<sup>5</sup> The total wealth of individuals on the list accounts for approximately 1.5% of total aggregate wealth in 2010.<sup>6</sup>

**Asset prices** For asset prices, I use yearly stock market returns and risk-free rates from Shiller (2015).<sup>7</sup> The excess stock market return is measured as the log stock market return minus the log risk-free rate. Data on the price-dividend ratio and the price-payout ratio comes from Welch and Goyal (2008).

## 2.2 Wealth Exposure to the Stock Market Across the Wealth Distribution

The basic building block of heterogeneous agents models is that there is a group of investors that is disproportionately exposed to aggregate shock. Following a positive return, these households gain more relative to other households; therefore, the wealth distribution fluctuates.

To measure the heterogeneity in risk exposure across the wealth distribution, I estimate the wealth exposure to stock market returns at different percentiles of the distribution. More precisely, I define the exposure of households in a given group as the slope estimate of a regression of the growth of total wealth in the group on excess stock market returns, i.e.,

$$\log \left( \frac{W_{G,t-1+h}}{W_{G,t-1}} \right) - h \log R_{ft} = \alpha_{Gh} + \beta_{Gh} (\log R_{Mt} - \log R_{ft}) + \epsilon_{Ght} \quad (1)$$

where  $W_{G,t}$  denotes the wealth of households in the percentile group  $G$  in year  $t$ ,  $\log R_{Mt}$  denotes the log stock market return, and  $\log R_{ft}$  denotes the log risk-free rate. The dependent variable is

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<sup>5</sup>Forbes Magazine reports: “We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don’t pretend to know what is listed on each billionaire’s private balance sheet, although some candidates do provide paperwork to that effect.”

<sup>6</sup>Recent empirical studies examining the Forbes 400 list also include Klass et al. (2006) and Kaplan and Rauh (2013).

<sup>7</sup>Available at <http://aida.wss.yale.edu/~shiller/data/chapt26.xlsx>.

the log ratio of wealth in the percentile group  $G$  from year  $t - 1$  to year  $t - 1 + h$ . This variable starts in year  $t - 1$  because  $W_{Gt}$  denotes the *average* wealth owned by the group during year  $t$ , rather than the wealth owned at the beginning of the year.

A large share of wealth in top percentiles is held in privately-held assets, which are not typically traded. If the valuations of this non-traded wealth reacts sluggishly to changes in the stock market, top wealth shares may not react immediately to changes in stock market returns. This may introduce a negative bias in the estimate for  $\beta_{GH}$ . To handle this stale pricing problem, I examine the long-run reaction of top wealth shares to stock market returns, using a longer horizon  $h \geq 1$ .<sup>8</sup>

Figure 1 plots the estimate of  $\beta_h$  obtained by making the horizon  $h$  vary from  $h = 1$  to  $h = 8$  in Equation (1) for the top 0.01% and for the top 400. This traces the impulse response of top wealth shares to a stock market return shock.<sup>9</sup> The plot shows that the estimates increase up to  $h = 4$ , consistent with the idea that the valuation of wealth at the top lags public indices.<sup>10</sup>

Table 1 reports the results of the regression for  $h = 4$  for different groups. The first four columns (Panel A) report the estimates for  $\beta_{G4}$ , the wealth exposure to the stock market at the four year horizon, for four groups of households: all households, households in the top 1–0.1%, households in the top 0.1%–0.01%, and households in the top 0.01%. The estimated exposure  $\beta_{G4}$  increases with the top percentiles, from 0.48 for the average household, to 0.95 for households in the top 0.01%. The last column of Panel A in Table 1 reports the wealth exposure of the Top 400 from Forbes. The estimates for households in the extreme tail of the distribution are similar in magnitude to the estimates for households in the top 0.01% from tax data. This is reassuring, because these datasets are constructed from two completely different sources. In short, top households are twice as exposed to stock market returns as the representative household.

Panel B of Table 1 regresses top wealth *shares* on stock market returns. The estimate 0.48, which is statistically significant, corresponds to the difference of exposure between households at

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<sup>8</sup>The valuation of this private wealth depends on the source. For estate tax, valuation of non-tradable assets is done by an external appraiser. Forbes magazine uses the valuation implied by the most recent financing round, or the prevailing price-to-earnings ratios for similar public companies.

<sup>9</sup>See Jordà (2005).

<sup>10</sup>Relatedly, Brav et al. (2002), Malloy et al. (2009) show that reported consumption growth for richer households is more correlated to aggregate consumption growth at longer horizon. Similarly, Getmansky et al. (2004) and Jurek and Stafford (2015) stress the role of returns smoothing in hedge-fund returns.

the top and the average household ( $\approx 0.95 - 0.48$ ). The table shows directly that top wealth shares increase when stock market returns are high.

**Composition Changes** Top percentiles do not necessary include the same individuals over time — some people enter and drop from the top every year. This composition effect may bias the regressions above. For instance, if there are more entrants in a given top percentile when stock market returns are high, total wealth in the top percentile will increase more after high stock market returns than the wealth of the existing households in the top percentile. To address this concern, I use the panel dimension of Forbes 400. Each year, I construct the average wealth growth of households in the Top 400 (which includes the one that drop from the top in the following year). This yearly series differs from the total growth of the top 400, by removing the effect of compositional changes on the growth of top wealth shares.<sup>11</sup> Table 2 shows that I obtain very similar results with this new series, which means that compositional changes play little role in driving the stock market exposure of top percentiles. Appendix A shows that this is due to the fact that year-to-year changes in stock market returns are large compared to year-to-year changes in the idiosyncratic volatility of wealth.

### 2.3 Top Wealth Shares and Future Excess Returns

The previous evidence suggests that wealthy households are more willing to take on aggregate risk. The flip side of this relationship is that, as top wealth shares increase, wealth is rebalanced from risk-averse households to risk-tolerant households; therefore, the total demand for risk in the economy increases. In equilibrium, the compensation for holding risk decreases. Hence, higher top wealth shares should predict lower future excess returns.

To test this prediction, I regress future excess stock returns on the wealth share of the top 0.01%:

$$\sum_{1 \leq h \leq H} \log R_{M,t+h} - \log R_{f,t+h} = \alpha + \beta_H \log \text{Wealth Share Top } 0.01\%_t + \epsilon_{Ht} \quad (2)$$

where  $h$  denotes the horizon,  $\log R_{Mt}$  denotes the log stock market return, and  $\log R_{ft}$  denotes the log risk-free rate.

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<sup>11</sup>See Gomez (2018) for more details about the methodology.

The first line in Table 3 reports the results of the predictability regression at the one-year and three-year horizons using the wealth share of the top 0.01% as a predictor. The estimate is negative and significant at the 10% level. Quantitatively, a 10% increase in the wealth share of the top 0.01% is associated with a decrease of excess returns by one percentage point over the next year.

It is well known that for a predictor that is persistent and correlated with returns, like top wealth shares, conventional t-statistics are misleading.<sup>12</sup> To address this concern, I rely on a test developed in Campbell and Yogo (2006), which is valid even when the predictor variable has a root close to or larger than one.<sup>13</sup> As reported in Table 4, the hypothesis that the top wealth share has a unit-root cannot be rejected. Still, even after allowing for explosive dynamics in top wealth shares, the wealth share of the top 0.01% is found to significantly predict returns. Another way to correct for the persistence of the predictor is to use a detrended version of the predictor (see Hodrick (1992)). Table 3 also reports the predictability regression using both the five-year difference and the detrended version of the top wealth share. The predictive power of top wealth shares remains significant.

Finally, I examine whether the information in the wealth share of the top 0.01% is subsumed by the price-dividend ratio, a widely known predictor of excess returns.<sup>14</sup> To do so, I run the following bivariate predictive regression:

$$\sum_{1 \leq h \leq H} \log R_{M,t+h} - \log R_{f,t+h} = \alpha + \beta_H \log \text{Wealth Share Top } 0.01\%_t + \gamma_H dp_t + \epsilon_{Ht} \quad (3)$$

Table 3 reports that the estimates for  $\beta_H$  and  $\gamma_H$  at the one year and three year horizon. The predictive power of top wealth shares remains substantial even after adding the dividend price as a predictor.

I have shown that fluctuations in stock prices generate fluctuations in inequality, and that in turn, the level of inequality determines future excess returns. Those facts are at the heart of asset pricing models with heterogeneous agents. I now examine these facts within a quantitative model.

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<sup>12</sup>For instance, Elliott and Stock (1994) and Stambaugh (1999).

<sup>13</sup>The test can only be done for the restricted sample without gaps in the predictor, i.e. 1917-1951.

<sup>14</sup>See, for instance, Campbell and Shiller (1988).

### 3 Asset Pricing Model with Heterogeneous Preferences

I consider a continuous-time pure-exchange economy. I present a model where overlapping generations of households differ in their preferences, which follows [Gârleanu and Panageas \(2015\)](#). I derive the dynamics of individual wealth, as well as the dynamics of the asset prices in this model. I show how the excess volatility of asset prices in the model relates to the degree of preference heterogeneity between agents.

#### 3.1 Setup

**Demographics** The specification of demographics follows [Blanchard \(1985\)](#). Each agent faces a constant hazard rate of death  $\delta > 0$ . Total population size,  $N_t$ , grows at rate  $n$ . During a short time period  $dt$ , a mass  $\delta dt$  of the population dies and a new cohort of mass  $(\delta + n)dt$  is born.

**Endowment** I consider a pure exchange economy. The aggregate endowment per capita exhibits i.i.d. growth, i.e. its law of motion is

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dZ_t \quad (4)$$

where  $Z = \{Z_t \in \mathbb{R} \mid \mathcal{F}_t, t \geq 0\}$  is a standard Brownian motion defined on a probability space  $(\Omega, P, \mathcal{F})$ , equipped with a filtration  $\mathbb{F} = \{\mathcal{F}_t, t \geq 0\}$  with the usual conditions.

An agent  $i$  born at time  $s(i)$  is endowed with the labor income process  $L_i = \{L_{it} : t \geq s(i)\}$ , given by

$$L_{it} = \omega Y_t \times \chi_i \times G(t - s(i)) \quad (5)$$

The first term of this formula,  $\omega Y_t$ , corresponds to the fraction of the aggregate endowment distributed as labor income. The second term,  $\chi_i$ , is an individual specific level of income, which is realized at birth, with mean one. This component captures the heterogeneity in labor income within a generation. The third term,  $G(t - s)$ , captures the life-cycle profile of earnings of households. The function  $G$  is a sum of exponentials normalized so that aggregate earnings equal  $\omega Y_t$  at each point in time, i.e.

$$\int_{-\infty}^t (\delta + \eta) e^{-(\delta + \eta)(t - s)} G(t - s) ds = 1$$

The rest of the endowment  $(1 - \omega)Y_t$  is distributed as dividends by the representative firm.

**Preferences** Agents have recursive preferences as defined by [Duffie and Epstein \(1992\)](#).<sup>15</sup> For an agent  $i$  with a consumption process  $C_i = \{C_{it} : t \geq 0\}$ , their utility  $U_i = \{U_{it} : t \geq 0\}$  is defined recursively by:

$$U_{it} = E_t \int_t^{+\infty} f_i(C_{is}, U_{is}) ds \quad (6)$$

$$f_i(C, U) = \frac{1}{1 - \frac{1}{\psi}} \left( \frac{C^{1 - \frac{1}{\psi}}}{((1 - \gamma)U)^{\frac{\gamma - \frac{1}{\psi}}{1 - \gamma}}} - (1 - \gamma)(\rho + \delta)U \right) \quad (7)$$

These preferences are characterized by three parameters: the subjective discount rate  $\rho$ , the coefficient of relative risk aversion (RRA)  $\gamma$ , and the elasticity of intertemporal substitution (EIS)  $\psi$ .

There are two types of agents, labeled A and B, that can differ with respect to their relative risk aversion  $\gamma_j$ , and their elasticity of intertemporal substitution is  $\psi_j$ . I denote A the risk-tolerant agent, i.e.  $\gamma_A < \gamma_B$ . In calibrations explored below, this agent will also be more willing to substitute consumption over time, i.e.  $\psi_A \geq \psi_B$ . Finally, at every point in time a proportion  $\pi_A$  of newly born agents are of type A.

**Markets.** Households can trade two assets: claims to the representative firm and instantaneous risk-free claims (in zero net supply). The price of both of those claims is determined in equilibrium. Because markets are dynamically complete, there is a unique stochastic discount factor  $\Lambda_t$ , with law of motion

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \kappa_t dZ_t$$

where  $r_t$  is the risk-free rate and  $\kappa_t$  is the market price of risk. The cumulative return of the representative firm can be written:

$$\frac{dR_t}{R_t} = (r_t + \kappa_t \sigma_{Rt}) dt + \sigma_{Rt} dZ_t$$

**Household Problem.** Denote  $A_{it}$  the financial wealth of agent  $i$  at time  $t$ . As in [Blanchard \(1985\)](#), agents can access a market for annuities. There are life insurance companies that collect the agents' financial wealth when they die. In exchange, agents receive an income stream equal to  $\delta A_{it}$  per unit of time.

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<sup>15</sup>They are a continuous-time version of the recursive preferences of [Epstein and Zin \(1989\)](#).

The problem of households is as follows. An household  $i$  born at time  $s(i)$  chooses a consumption path  $C_i = \{C_{it} : t \geq s(i)\}$  and an amount of dollars invested in the representative firm  $\theta_i = \{\theta_{it} : t \geq s(i)\}$  to maximize his lifetime utility

$$V_{it} = \max_{C_i, \theta_i} U_{it}(C_i)$$

subject to the dynamic budget constraint

$$A_{is(i)} = 0 \tag{8}$$

$$dA_{it} = (L_{it} - C_{it} + (r_t + \delta)A_{it} + \theta_{it}(\mu_{Rt} - r_t))dt + \theta_{it}\sigma_{Rt}dZ_t \text{ for all } t \geq s(i) \tag{9}$$

**Human Capital.** Denote  $H_{it}$  the human capital of household  $i$ , i.e. the present value at time  $t$  of the labor income of agent  $i$ :

$$H_{it} = E_t \left[ \int_t^{+\infty} \frac{\Lambda_u}{\Lambda_t} e^{-\delta(u-t)} L_{iu} du \right] \tag{10}$$

Define  $W_{it}$ , the total wealth of household  $i$  as the sum of his financial wealth  $A_{it}$  and his human capital  $H_{it}$ . The wealth at birth,  $W_{is(i)}$ , is given by  $W_{is(i)} = H_{is(i)} = \chi_i \phi_{s(i)} Y_{s(i)}$  where  $\phi_t$  is defined as

$$\phi_t = E_t \left[ \int_t^{+\infty} \frac{\Lambda_u}{\Lambda_t} e^{-\delta(u-t)} \omega \frac{Y_u}{Y_t} G(u-t) \right] \tag{11}$$

The household problem can now be reformulated as follows. Household  $i$  chooses a consumption rate  $c_i = \{c_{it} = C_{it}/W_{it} : t \geq s(i)\}$  and a wealth exposure to aggregate shocks  $\sigma_{W_i} = \{\sigma_{W_{it}} : t \geq s(i)\}$  such that for all  $t \geq s(i)$

$$V_{it} = \max_{c_i, \sigma_i} U_{it}(c_i W_{it}) \tag{12}$$

$$\text{s.t. } \frac{dW_{it}}{W_{it}} = \mu_{W_{it}} dt + \sigma_{W_{it}} dZ_t$$

$$\text{with } \mu_{W_{it}} = r_t + \delta + \kappa_t \sigma_{W_{it}} - c_{it} \tag{13}$$

### 3.2 Law of Motion of Household Wealth

I first characterize the law of motion of households' wealth. This object is important to study because it will be a key determinant of the dynamics of the wealth distribution, studied in Section 4.

Households with the same preference parameters face the same trade-off, irrespective of their wealth or age, due to the homogeneity of the utility function and the constant death rate. Denote  $p_{jt}$  the wealth-to-consumption ratio of an agent in group  $j$ . Conjecture that the process  $p_{jt}$  follows a diffusion process:

$$\frac{dp_{jt}}{p_{jt}} = \mu_{p_{jt}} dt + \sigma_{p_{jt}} dZ_t \quad (14)$$

**Proposition 1** (Law of Motion for Households Wealth). *The wealth of households in group  $j \in \{A, B\}$  follows the law of motion*

$$\frac{dW_{jt}}{W_{jt}} = \mu_{W_{jt}} dt + \sigma_{W_{jt}} dZ_t \quad (15)$$

where  $\mu_{W_{jt}}$  and  $\sigma_{W_{jt}}$  are given by

$$\sigma_{W_{jt}} = \frac{\kappa_t}{\gamma_j} + \underbrace{\frac{\frac{1}{\gamma_j} - 1}{\psi_j - 1} \sigma_{p_{jt}}}_{H_{jt}} \quad (16)$$

$$\mu_{W_{jt}} = \psi_j(r_t - \rho) + \frac{1 + \psi_j}{2\gamma_j} \kappa_t^2 + \underbrace{\frac{\frac{\psi_j}{\gamma_j} - \psi_j}{\psi_j - 1} \kappa_t \sigma_{p_{jt}} + \frac{\frac{1}{\gamma_j} - \psi_j}{2(\psi_j - 1)} \sigma_{p_{jt}}^2}_{\Phi_{jt}} + \mu_{p_{jt}} \quad (17)$$

The geometric volatility of wealth  $\sigma_{W_{jt}}$  is the sum of two terms. The first term, the myopic demand, equals the ratio of the market price of risk to the relative risk aversion  $\gamma_j$ . The lower the relative risk aversion  $\gamma_j$ , the higher the myopic demand. The second term, the intertemporal hedging demand  $H_{jt}$ , captures deviations from the mean-variance portfolio due to changes in investment opportunities. If expected returns are countercyclical, this term is positive as long as  $\gamma_j > 1$ .

The geometric drift of wealth  $\mu_{W_{jt}}$  is the sum of three terms. The first term is a standard term due to intertemporal substitution, determined by the EIS  $\psi_j$  and the difference between the interest rate  $r_t$  and the subjective discount rate  $\rho$ :  $\psi_j(r_t - \rho)$ . Note that this term does not depend on the death rate  $\delta$ . While  $\delta$  increases the effective subjective discount rate of agents,  $\rho + \delta$ , it also increases the effective interest rate because of annuities,  $r + \delta$ .

The second term corresponds to the effect of higher risk exposure on wealth growth. Agents with lower risk aversion invest disproportionately in risky assets. Due to the compensation for holding more risk, they earn, on average, higher returns. This affects their consumption rate, through a combination of an income and a substitution effect. As  $\psi_j$  rises, the substitution effect becomes

increasingly important, which magnifies the effect of risk aversion on total wealth growth. Finally, the third term  $\Phi_{jt}$  captures changes in investment opportunities.

### 3.3 Prices

**Market Clearing** The market clearing for consumption is

$$\int_{i \in \mathbb{I}_{At}} C_{it} di + \int_{i \in \mathbb{I}_{Bt}} C_{it} di = Y_t N_t \quad (18)$$

Denoting  $p_t$  the ratio of total wealth to total consumption, i.e.

$$p_t = \frac{\int_{i \in \mathbb{I}_{At}} W_{it} di + \int_{i \in \mathbb{I}_{Bt}} W_{it} di}{\int_{i \in \mathbb{I}_{At}} C_{it} di + \int_{i \in \mathbb{I}_{Bt}} C_{it} di} \quad (19)$$

The market clearing for consumption can be rewritten as

$$\int_{i \in \mathbb{I}_{At}} W_{it} di + \int_{i \in \mathbb{I}_{Bt}} W_{it} di = p_t Y_t N_t \quad (20)$$

**Markov Equilibrium** For the purpose of determining prices, we only need to keep track of the share of aggregate wealth that belongs to the agent in group  $A$ :

$$x_t = \frac{\int_{i \in \mathbb{I}_{At}} W_{it} di}{\int_{i \in \mathbb{I}_{At}} W_{it} di + \int_{i \in \mathbb{I}_{Bt}} W_{it} di} \quad (21)$$

Because agents in group  $A$  choose a different wealth exposure compared to agents in group  $B$ , the process  $x_t$  is stochastic. The next proposition characterizes the law of motion of  $x_t$ .

**Proposition 2.** *The law of motion of  $x$  is*

$$dx_t = \mu_{xt} dt + \sigma_{xt} dZ_t \quad (22)$$

where  $\mu_{xt}$  and  $\sigma_{xt}$  are given by

$$\sigma_{xt} = x_t (\sigma_{W_{At}} - \sigma - \sigma_{p_t}) \quad (23)$$

$$\mu_{xt} = x_t (\mu_{W_{At}} - \mu - \mu_{p_t} - \sigma \sigma_{p_t}) - \sigma_{xt} (\sigma + \sigma_{p_t}) + (\delta + n) (\pi_A \frac{\phi_t}{p_t} - x_t) \quad (24)$$

The volatility of  $x_t$  is given by the difference between the wealth volatility of agents in group  $A$  and the volatility of aggregate wealth. The drift of  $x_t$  is the sum of three terms. The first term is the difference between the wealth drift of agents in group  $A$  and the wealth drift of the economy.

The second term is an Ito correction term. The third term is due to the OLG setup. It is due to the difference between the average wealth of newborns  $\phi_t/p_t$  and the average wealth of households in group  $A$  that die,  $x_t/\pi_A$ . This OLG term ensures that  $x_t$  is stationary: because the volatility of  $x$ ,  $\sigma_x$ , is zero at the boundaries 0 and 1, and its drift  $\mu_x$  is positive at 0 and negative at 1, no group of agents dominates the economy in the long run.

**Market Price of Risk** To gain some intuition on the role of wealth inequality for the market price of risk in the model, I now consider the determination of the equilibrium price of risk  $\kappa_t$ .

Applying Ito's lemma on Equation (20), one obtains that the wealth-weighted average of individual wealth volatility equals the total quantity of risk:

$$x_t\sigma_{W_{At}} + (1 - x_t)\sigma_{W_{Bt}} = \sigma + \sigma_{p_t} \quad (25)$$

Substituting the volatility of individual wealth from Equation (16), one obtains the market price of risk in terms of individual RRA:

$$\kappa_t = \Gamma_t(\sigma + \sigma_{p_t}) - H_t \quad (26)$$

where  $\Gamma_t$  corresponds to the aggregate RRA and  $H_t$  corresponds to the aggregate hedging demand:

$$\frac{1}{\Gamma_t} = \frac{x_t}{\gamma_A} + \frac{1 - x_t}{\gamma_B} \quad (27)$$

$$H_t = x_t H_{At} + (1 - x_t) H_{Bt} \quad (28)$$

The market price of risk  $\kappa_t$  is the product of the aggregate RRA  $\Gamma_t$  times the total quantity of risk  $\sigma + \sigma_{p_t}$ , minus the total demand for risk due to the hedging.

The aggregate RRA  $\Gamma_t$  is a wealth-weighted harmonic mean of individual RRAs. The higher the share of wealth owned by the agents in group  $A$ ,  $x_t$ , the lower the aggregate risk aversion  $\Gamma_t$ . Ignoring for a moment the hedging demand, an increase in the fraction hold by  $x_t$  decreases the market price of risk  $\kappa_t$ .

**Risk-Free Rate** Applying Ito's lemma on Equation (20), one obtains that the wealth-weighted average of individual wealth growth, plus a OLG term, equals the endowment growth:

$$x_t\mu_{W_{At}} + (1 - x_t)\mu_{W_{Bt}} + (\delta + n) \left( \frac{\phi_t}{p_t} - 1 \right) = \mu + \mu_{p_t} + \sigma\sigma_{p_t} \quad (29)$$

The OLG term depends on the death rate and population growth times the difference between the relative wealth of newborn households  $\phi/p_t$  and 1. Intuitively, because the wealth of newborns does not equal the wealth of deceased households, there is a wedge between the average individual wealth growth and the aggregate growth.

Substituting the drift of individual wealth from Equation (17), one obtains the risk-free rate in terms of individual EIS:

$$r_t = \rho + \frac{1}{\Psi_t} \left( \mu + \mu_{p_t} + \sigma \sigma_{p_t} - (\delta + n) \left( \frac{\phi_t}{p_t} - 1 \right) - \left( x \frac{1 + \psi_A}{2\gamma_A} + (1 - x) \frac{1 + \psi_B}{2\gamma_B} \right) \kappa_t^2 - \Phi_t \right) \quad (30)$$

where  $\Psi_t$  corresponds to the aggregate EIS and  $\Phi_t$  corresponds to aggregate changes in investment opportunities:

$$\Psi_t = x_t \psi_A + (1 - x_t) \psi_B \quad (31)$$

$$\Phi_t = x_t \Phi_{At} + (1 - x_t) \Phi_{Bt} \quad (32)$$

The higher the share of wealth owned by the agents in group A,  $x_t$ , the closer the aggregate elasticity of substitution to  $\psi_A$ . If agents in group A are more willing to substitute inter-temporally compared to agents in group B, the risk-free rate tends to decrease as their wealth share  $x_t$  increases.

**Volatility of Asset Prices** Models with heterogeneous agents can potentially explain the excess volatility of asset prices. As the share of wealth owned by agents in group A,  $x_t$ , fluctuates, both the market price of risk and the interest rate fluctuate, which generates fluctuations in the price-dividend ratio.

Denote  $R_t$  the cumulative return of the representative firm, and denote  $p_{Dt}$  the price-dividend ratio in this economy. One can write the cumulative return of the representative firm as:

$$\frac{dR_t}{R_t} = \underbrace{(r_t + \kappa_t \sigma_{Rt})}_{\mu_{Rt}} dt + \underbrace{(\sigma + \sigma_{p_{Dt}})}_{\sigma_{Rt}} dZ_t \quad (33)$$

The excess volatility of the return is given by  $\sigma_{p_{Dt}}$ , the geometric volatility of the price-dividend ratio. This excess volatility can be decomposed using a continuous-time version of [Campbell and Shiller \(1988\)](#) decomposition:

**Proposition 3** (Campbell-Shiller in Continuous-Time Model). *The volatility of the price-dividend*

ratio,  $\sigma_{p_{Dt}}$ , is given by:

$$\sigma_{p_{Dt}} \approx \underbrace{-\int_0^{+\infty} e^{-\alpha t} \frac{\partial E[r_t | x_0 = x]}{\partial x} \sigma_x}_{\text{Risk-Free Rate Channel}} - \underbrace{\int_0^{+\infty} e^{-\alpha t} \frac{\partial E[\kappa_t \sigma_{Rt} - \frac{1}{2} \sigma_{Rt}^2 | x_0 = x]}{\partial x} \sigma_x}_{\text{Excess Returns Channel}} \quad (34)$$

with  $\alpha = \exp(-E[\log p_t^D])$ .

The volatility of the price-dividend ratio can be decomposed into two terms: the first term depends on the volatility of expectations of future risk-free rates (a “risk-free rate” channel). The second term depends on the volatility of expectations of future expected excess log-returns (a “market price of risk” channel).

By exchanging the expectation and the derivative operations in the RHS of Equation (34), one obtains:

$$\sigma_{p_{Dt}} \approx \underbrace{-\int_0^{+\infty} e^{-\alpha t} E \left[ \partial_x r \frac{\partial x_t}{\partial x_0} \Big| x_0 = x \right] \sigma_x}_{\text{Risk-Free Rate Channel}} - \underbrace{\int_0^{+\infty} e^{-\alpha t} E \left[ \partial_x (\kappa \sigma_{Rt} - \frac{1}{2} \sigma_{Rt}^2) \frac{\partial x_t}{\partial x_0} \Big| x_0 = x \right] \sigma_x}_{\text{Excess Returns Channel}}$$

where  $\partial x_t / \partial x_0$ , denotes the first-variation of the process  $x_t$ .<sup>16</sup> This equation shows that the excess volatility of asset prices increases with the derivative of the risk-free rate with respect to  $x$ , as well as the derivative of expected excess returns with  $x$ . In turn, as shown in (26) and (30), the derivatives of the risk-free rate and of the market price of risk depend critically on the derivative of the aggregate RRA  $\Gamma$  and of the inverse of the aggregate EIS  $1/\Psi$  with respect to the group share  $x$ :

$$\frac{\partial \Gamma}{\partial x} = \Gamma^2 \left( \frac{1}{\gamma_B} - \frac{1}{\gamma_A} \right) \quad (37)$$

$$\frac{\partial 1/\Psi}{\partial x} = \Psi^{-2} (\psi_B - \psi_A) \quad (38)$$

Together, these equations relate the volatility of asset prices to the degree of preference heterogeneity across households. Importantly, these derivatives themselves decrease in  $x$  (in absolute value); this suggests the volatility of asset prices in the model is higher for low value of the group share  $x$ .

<sup>16</sup>Formally, the first-variation process  $D_t = \partial x_t / \partial x_0$  associated to the diffusion  $x_t$  satisfies:

$$D_0 = 1 \quad (35)$$

$$\frac{dD_t}{D_t} = \frac{\partial \mu_x}{\partial x}(x_t) dt + \frac{\partial \sigma_x}{\partial x}(x_t) dZ_t \quad (36)$$

See for instance Fournié et al. (1999).

An important question is whether there is enough heterogeneity between households in the data to fully match the excess volatility of asset prices. The next section examines how moments about the wealth distribution help discipline household heterogeneity.

## 4 Wealth Distribution

In this section, I examine the wealth distribution implied by the model, which is a key theoretical contribution of this paper. The key difference from the existing literature on wealth inequality is that, in the model, the dynamics of the wealth distribution depends on the aggregate state of the economy. I show that the distribution exhibits a thick right tail, which is driven by the high average returns of top households.

### 4.1 Dynamics of Wealth Density

Denote  $w_{it}$  the wealth of agent  $i$  relative to the per-capita wealth in the economy, i.e.  $w_{it} = W_{it}/(p_t Y_t)$ . Applying Ito's lemma, the law of motion of the relative wealth  $w_{it}$  is

$$\frac{dw_{it}}{w_{it}} = \mu_{w_{it}} dt + \sigma_{w_{it}} dZ_t \quad (39)$$

where  $\mu_{w_{it}}$  and  $\sigma_{w_{it}}$  are given by

$$\sigma_{w_{it}} = \sigma_{W_{it}} - \sigma - \sigma_{p_t} \quad (40)$$

$$\mu_{w_{it}} = \mu_{W_{it}} - \mu - \mu_{p_t} - \sigma \sigma_{p_t} - \sigma_{w_{it}} (\sigma + \sigma_{p_t}) \quad (41)$$

I first characterize the dynamics of the wealth density in the model. Denote  $g_{jt}$  the density of relative wealth within each group of agent  $j \in \{A, B\}$ , and  $g_t$  the density of relative wealth across all households, i.e.

$$g_t = \pi_A g_{At} + (1 - \pi_A) g_{Bt} \quad (42)$$

Finally, denote  $g_{\chi_t}$  the relative distribution of human capital for newborn agents.<sup>17</sup> The next proposition characterizes the law of motion of the wealth density within each group  $g_{jt}$  for  $j \in$

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<sup>17</sup> Denoting  $g_{\chi}$  the density of  $\chi_i$ , we have

$$g_{\chi_t}(w) = \frac{p_t}{\phi_t} g_{\chi} \left( \frac{p_t}{\phi_t} w \right). \quad (43)$$

$\chi_i$  and  $\phi_t$  are respectively defined by (5) and (34).

$\{A, B\}$ :

**Proposition 4** (Kolmogorov-Forward Equation with Aggregate Shocks). *The law of motion of  $g_{jt}$  is given by*

$$dg_{jt} = \left( -\mu_{w_{jt}} \partial_w (wg_{jt}) + \frac{1}{2} \sigma_{w_{jt}}^2 \partial_{ww} (w^2 g_{jt}) + (\delta + n)(g_{\chi t} - g_{jt}) \right) dt - \sigma_{w_{jt}} \partial_w (wg_{jt}) dZ_t \quad (44)$$

Given the evolution of individual wealth  $(\mu_{w_{jt}} dt, \sigma_{w_{jt}} dZ_t)$ , this equation gives the evolution of the wealth density  $g_{jt+dt} - g_{jt}$ . The key difference from the existing literature is that, because households choose different exposures to aggregate shocks (i.e.  $\sigma_{w_{jt}} \neq 0$ ), the wealth density is stochastic.

## 4.2 Tail Index

While a full characterization of the entire wealth distribution is not feasible, I show that one can characterize analytically its right tail.

**Tail Index** By analogy with the case of a static distribution, I define the tail index of a stochastic distribution as the smallest number  $\zeta$  for which moments of order higher than  $\zeta$  converge to infinity.

**Definition 1** (Thick-Tail). *For a density  $g_t$  and  $\xi > 0$ , denote  $m_t^\xi$  the moment of order  $\xi$ :*

$$m_t^\xi = \int_0^{+\infty} w^\xi g_t(w) dw \quad (45)$$

*A density  $g_t$  is thick-tailed with tail index  $\zeta$  if there exists  $0 < \zeta < +\infty$  such that moments of order higher than  $\zeta$  converge to infinity almost surely, i.e.*

$$\zeta = \inf \left\{ \xi \in \mathbb{R}_+ \text{ s.t. } \lim_{t \rightarrow +\infty} m_t^\xi = +\infty \text{ a.s.} \right\}$$

The next proposition characterizes the tail index of the wealth distribution in the economy.

**Proposition 5** (Tail Index). *Denote*

$$\zeta = \frac{\delta + n}{\mathbb{E} \left[ \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2 \right]} \quad (46)$$

*where  $\mathbb{E}$  denotes the expectation with respect to the invariant probability measure of  $x$ . If the following conditions are satisfied:*

1. Agents in group  $A$  grow on average faster than the economy and than agents in group  $B$ , i.e.

$$\mathbb{E} \left[ \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2 \right] > \max \left( 0, \mathbb{E} \left[ \mu_{w_{Bt}} - \frac{1}{2} \sigma_{w_{Bt}}^2 \right] \right) \quad (47)$$

2. The distribution of human capital  $\chi_i$  has a tail thinner than  $\zeta$ .

Then the wealth distribution is thick-tailed with tail index  $\zeta$ . The distribution of financial wealth is also thick-tailed with tail index  $\zeta$ .<sup>18</sup> Moreover, the wealth distribution of agents in group  $A$  is thick-tailed with tail index  $\zeta$ , whereas the wealth distribution of agents in group  $B$  has a thinner tail (which is the same as the distribution of human capital).

The tail index of the wealth distribution  $\zeta$  can be expressed as the ratio of two terms. The numerator is the sum of the death rate and of the population growth rate,  $\delta+n$ . The denominator is the average logarithmic wealth growth rate of agents in group  $A$  relative to the rest of the economy,  $\mathbb{E}[d \ln w_{At}]/dt = \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2$ .

This formula extends existing results in the literature on two dimensions.<sup>19</sup> First, it gives the tail index of the distribution in an economy where the dynamics of individual wealth *vary over time* (in this economy, the interest rate and the market price of risk vary over time, therefore the dynamics of individual wealth vary over time). In this setup, the formula shows that the tail index depends on the *average* wealth growth of top households. Second, it gives the tail index of the distribution in an economy where households have *heterogeneous exposure to aggregate shocks*. In this setup, the formula shows that the tail index depends on the average growth of the *logarithmic* wealth  $\mathbb{E}[\mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2]$ . While the exposure of top households to aggregate risk increases their average average geometric growth  $\mathbb{E}[\mu_{w_{At}}]$ , this has a dampened effect on the tail index due to the associated excess volatility  $-\frac{1}{2} \mathbb{E}[\sigma_{w_{At}}^2]$ .

It is enlightening to compare this result to the case of infinite-horizon economies. In infinite-horizon economies, a stationary wealth distribution obtains only when the logarithmic relative wealth growth rate of agents in group  $A$  is zero on average.<sup>20</sup> OLG models break this equivalence. What Proposition 5 shows, however, is that the growth rate of agents in group  $A$  is still a key

<sup>18</sup>The financial wealth of an household is defined as his total wealth minus the capitalized value of his future labor income, following Section 3.

<sup>19</sup>Wold and Whittle (1957) is the first paper studying the tail index of the wealth distribution.

<sup>20</sup>See for instance Blume et al. (1992).

driver of the wealth distribution in OLG models: it controls the thickness of the tail of the wealth distribution.

Proposition 5 is derived under the condition that the tail index of the labor income distribution is higher than the tail index of wealth distribution. This is the case empirically: the wealth distribution has a tail index of 1.5 while the distribution of labor income has a tail index between 2 and 3.<sup>21</sup> In this case, Proposition 5 says that the distribution of financial assets has the same tail index as the distribution of wealth. This is due to the fact that, for households in the right tail of the distribution, human capital represents a negligible share of total wealth.<sup>22</sup> Moreover, Proposition 5 says that the tail index of the wealth distribution does not depend on the exact distribution of human capital.<sup>23</sup> These two properties are why I will focus on the tail index when matching the model to the data, as opposed to other measures of wealth inequality, such as the Gini index, or the variance of log wealth.

**Law of motion of  $x_t$**  As shown in Section 3, the growth of  $x_t$  is the sum of the relative wealth growth of agents in group  $A$  and an OLG term, that depends on  $x_t$  as well as the relative wealth of a newborn agent  $\phi_t/p_t$ . Because  $x_t$  is ergodic, this gives a relationship between the relative wealth growth of agents in group  $A$  and the average level of  $x$ .

**Proposition 6** (Relation Between Tail Index and Group Share). *Under the assumptions of Proposition 5, the tail index  $\zeta$  of the wealth distribution is given by:*

$$\zeta = \frac{1}{1 - \mathbb{E} \left[ \frac{\pi_A \phi_t}{x_t p_t} \right]} \quad (48)$$

where  $\mathbb{E}$  denotes the expectation with respect to the stationary density of  $x$ . In particular  $\zeta \geq 1$ , i.e. the tail of the distribution is less thick than Zipf's law.

The formula shows that the tail index  $\zeta$  is directly related to  $\mathbb{E} \left[ \frac{\pi_A \phi_t}{x_t p_t} \right]$ , i.e. the average ratio of the relative wealth of the relative wealth of a newborn agent ( $\phi_t/p_t$ ) to the average wealth of an agent in group  $A$  ( $x_t/\pi_A$ ). This means that, in equilibrium, the tail index  $\zeta$ , which is measure of

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<sup>21</sup>See for instance Toda (2012).

<sup>22</sup>See also Appendix D.1.

<sup>23</sup>This is a classic result for deterministic economies. See, for instance, Achdou et al. (2016) or Gabaix et al. (2016). Proposition 5 extends this result to Markovian economies.

inequality *within* agents in group  $A$ , is directly related to  $x_t/\pi_A$ , which is a measure of inequality *between* the average household in group  $A$  and the average household in group  $B$ .

Small changes in the tail index correspond to large changes in this ratio. For instance, a tail index close to 1 (i.e. Zipf's law) corresponds to an economy where the average wealth of an agent in group  $A$  is infinitely large compared to the average human capital of a newborn. In contrast, a tail index closer to 1.5 corresponds to an economy where the average wealth of an agent in group  $A$  is only three times higher than the average human capital of a newborn. Small changes in the tail index correspond to large differences in the distribution of wealth.<sup>24</sup>

### 4.3 Dynamics of Top Wealth Shares

I now relate the dynamics of the wealth in a top percentile to the dynamics of individual wealth. In particular, this allows me to relate the exposure of top wealth shares to the exposure of individual wealth.

Let  $\alpha$  be a top percentile, e.g. top 1%. Denote  $q_t$  the  $\alpha$ -quantile, i.e.,

$$\alpha = \int_{q_t}^{+\infty} g_t(w)dw \quad (49)$$

$q_t$  corresponds to the wealth of an agent exactly at the  $\alpha$  percentile threshold of the distribution.

We can now define the wealth share owned by the top percentile  $\alpha$ ,  $S_t$ :

$$S_t = \int_{q_t}^{+\infty} w g_t(w)dw \quad (50)$$

The following proposition characterizes the dynamics of the wealth share of the top percentile:

**Proposition 7** (Law of Motion of Top Wealth Shares). *The law of motion of the top wealth share  $S_t$  is*

$$\frac{dS_t}{S_t} = \mu_{S_t}dt + \sigma_{S_t}dZ_t \quad (51)$$

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<sup>24</sup>Note that  $x/\pi_A$  is *not* the wealth share of the top  $\pi_A$  of households in the economy. This is because some households in group B are born with a high amount of initial human capital. Therefore, some of them end up in top percentiles. Conversely, some households in group A are born with little human capital. Therefore, they end up at the bottom of the wealth distribution.

where  $\mu_{S_t}$  and  $\sigma_{S_t}$  are given by:<sup>25</sup>

$$\sigma_{S_t} = E^{g_t^w}[\sigma_{w_{it}} | w_{it} \geq q_t] \quad (52)$$

$$\begin{aligned} \mu_{S_t} = & E^{g_t^w}[\mu_{w_{it}} | w_{it} \geq q_t] + \frac{q_t^2 g_t(q_t)}{2S_t} Var^{g_t}[\sigma_{w_{it}} | w_{it} = q_t] \\ & + (\delta + n) \left( \frac{\alpha q_t}{S_t} - 1 \right) + \frac{\delta + n}{S_t} \int_{q_t}^{\infty} (w - q_t) g_{\chi^t}(w) dw \end{aligned} \quad (53)$$

where the expectation  $E^{g_t^w}$  refers to the wealth-weighted, cross-sectional average with respect to the wealth density  $g_t$ ,

The geometric volatility of the top wealth share,  $\sigma_{S_t}$ , is the wealth-weighted average geometric volatility of individuals in the top percentile. As  $\alpha$  denotes an increasingly high percentile, there are relatively more agents of type  $A$  in the top percentile, and therefore the exposure of the wealth share of top percentiles converges to  $\sigma_{w_{A_t}}$ . The exposure of top wealth shares reflects the exposure of individuals in group  $A$ . The model therefore generates the empirical evidence found in Section 2 (Table 1).

The geometric drift of the top wealth share,  $\mu_{S_t}$ , is the sum of three terms. The first term corresponds to the average, wealth-weighted, geometric drift of individuals at the top. The second term is due to the heterogeneous exposure of households at the threshold. It depends on the variance of risk exposures across households at the quantile  $q_t$ . When a *negative* shock hits the economy, top wealth shares decrease a bit *less* than the wealth of households inside the top percentile, because some households from group  $B$  enter the top. Conversely, when a *positive* shock hits the economy, top wealth shares increase a bit *more* than the wealth of households inside the top percentile, because some households from group  $A$  enter the top. Therefore, heterogeneous exposure to aggregate shocks tends to increase the growth of top wealth shares due to a composition effect. However, it does not change the exposure of top wealth shares to stock market returns. The third term is due to the death of individuals at the top, as well as the eventual birth of households with human capital higher than  $q_t$ .

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<sup>25</sup>  $E^{g_t^w}$  denotes the wealth-weighted average and  $Var^{g_t}$  denotes the variance with respect to the wealth density  $g_t$ .

## 5 Fitting the Model to the Data

I now bring the model to the data. Qualitatively, the model generates the two key facts about wealth inequality and the wealth distribution: (i) top wealth shares increase when stock market returns are high, and (ii) top wealth shares predict future excess returns. However, I highlight a key tension between the model and the data: to match the high volatility of asset prices, the model requires such a large degree of heterogeneity that it generates a wealth distribution with a right tail thicker than the data.

### 5.1 Estimation Method

**Method** I estimate the parameters of the model by minimizing the distance between moments from the data and those implied by the model. I proceed as follows. I select a vector of moments  $m$  computed from the actual data. Given a candidate set of parameters  $\Theta$ , I solve the model, and compute the moments  $\hat{m}(\Theta)$  using simulated data from the model. I use 500 simulated samples with 100 years of data to match the length of the actual data.<sup>26</sup> I search the set of parameters  $\hat{\Theta}$  that minimizes the weighted deviation between the actual and model-implied moments  $J(\Theta)$ , i.e.

$$J(\Theta) = (m - \hat{m}(\Theta))' W (m - \hat{m}(\Theta)) \quad (54)$$

$$\hat{\Theta} = \arg \min_{\Theta} J(\Theta) \quad (55)$$

where  $W$  is a weight matrix, computed as the inverse of the variances of moments as measured in the data.

**Asset Prices** Following [Gârleanu and Panageas \(2015\)](#), I use four asset price moments, corresponding to the average and standard deviation of the risk-free rate and of stock market returns (in real terms). The data for the average equity premium, the volatility of returns, and the average interest rate are from [Shiller \(2015\)](#). The volatility of the real risk-free rate is inferred from the yields of 5-year constant maturity TIPS following [Gârleanu and Panageas \(2015\)](#).

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<sup>26</sup>For each sample, I simulate the model for 300 years, starting from a random draw from the stationary distribution of  $x$ , and I throw away the first 200 years. Increasing the number of years, increasing the number of samples, or diminishing the frequency do not materially change the results.

Following the approach of Barro (2006), I compare the equity premium and the standard deviation of the stock market return in the data to the returns of a firm with an asset-equity ratio equal to the historically observed asset-equity ratio for the U.S. non financial corporate sector,  $\lambda \approx 1.5$ .<sup>27</sup>

**Wealth Inequality** Compared to Gârleanu and Panageas (2015), I add two moments to discipline the wealth dynamics of the households in the right tail of the distribution.

The first moment is the tail index of the distribution of financial wealth. As shown in Proposition 5, the moment disciplines the average logarithmic wealth growth rate of households in group *A* relative to the economy. I use a MLE method to estimate the moment in the model and in the simulated data.<sup>28</sup> In the data, I measure a tail index of  $\zeta = 1.5$ , consistent with previous studies.<sup>29</sup> Graphically, Figure 2 plots the log percentile as a function of the log net worth for the U.S. distribution, in the SCF and in Forbes 400 data. The linear slope is characteristic of a distribution with a Pareto tail.

The second moment is the exposure of financial wealth of the top 0.01% to stock market returns, measured in Table 1. This moment disciplines the wealth exposure of households in group *A* to the stock market, as indicated by Proposition 7. There is one caveat, however: whereas I measure the wealth exposure of *financial* wealth, Proposition 7 characterizes the exposure of *total* wealth. As shown in Appendix D.1, financial wealth accounts for most of the wealth of households in the right tail of the distribution, so this difference is likely to be small. In any case, to avoid any discrepancy between the model and the data, the moment in the model is computed exactly as in the data, by regressing the log growth of financial wealth in the top 0.01% over a four year horizon on excess stock market returns.<sup>30</sup>

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<sup>27</sup>The cumulative return of the levered firm  $R_t^\lambda$  follows the law of motion:

$$\frac{dR_t^\lambda}{R_t^\lambda} = (r_t + \kappa_t \lambda \sigma_{Rt})dt + \lambda \sigma_{Rt} dZ_t$$

<sup>28</sup>See Goldstein et al. (2004).

<sup>29</sup>See, for instance, Klass et al. (2006) or Vermeulen (2018).

<sup>30</sup>As explained in the discussion of asset price moments above, the stock market return is the return of the representative levered firm.

**Calibrated Parameters** The law of motion of the endowment process per capita is  $\mu = 2\%$  and  $\sigma = 4.1\%$ . The death rate is  $\delta = 2\%$ . The population growth rate is  $n = 1\%$ . Finally, human capital is calibrated based on U.S. data about life-cycle labor income, following [Gârleanu and Panageas \(2015\)](#).<sup>31</sup>

**Estimated Parameters** The model has 6 remaining parameters that I estimate. 2 parameters correspond to the preference parameters specific to households in group A ( $\gamma_A, \psi_A$ ) and 2 parameters correspond to the preference parameters specific to households in group B ( $\gamma_B, \psi_B$ ). I impose sensible restrictions on the preference parameters of agents in group B, i.e.  $\psi_B \geq 0.05$  and  $1/\gamma_B \geq 0.05$ . The two remaining parameters are the subjective discount rate  $\rho$  and the population share of the agents in group A,  $\pi_A$ .

## 5.2 Results

**Estimation on Asset Prices Only** I first estimate the model on asset price moments, which is a very similar to the exercise conducted in [Gârleanu and Panageas \(2015\)](#). Column (2) of Table 5 reports the result of the estimation. Figure 3 plots the market price of risk, the interest rate, the stationary density of the state variable, and the Campbell-Shiller decomposition of the volatility of the price-dividend ratio.<sup>32</sup> The model matches asset price moments very accurately: it generates a high equity premium, with a high volatility, together with a low risk-free rate with low volatility.

The model matches the high volatility of asset prices because it has a high degree of preference heterogeneity, both in terms of RRA ( $\gamma_A \approx 1.5$  vs  $\gamma_B \approx 17$ ) and in terms of EIS ( $\psi_A \approx 0.8$  vs  $\psi_B = 0.05$ ). Due to this high degree of preference heterogeneity, the market price of risk and the risk-free rate are very sensitive to the wealth share of group A (as explained in Section 3). Indeed, Figure 3 show that both prices decrease sharply in the wealth share of group A. This sensitivity, combined with the fact that the model spends a large amount of time in the region where  $x$  is low, i.e. where the sensitivity of these prices is high, generates a large volatility of asset prices: Figure 3 plots the excess volatility of returns, as a sum of a risk-free rate channel and an excess returns

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<sup>31</sup>See Appendix D.

<sup>32</sup>The only difference is that I consider an economy where population grows at rate 1%. That being said, the result of the estimation is very similar to [Gârleanu and Panageas \(2015\)](#), both in terms of preference parameters and in terms of moments.

channel. Overall, 75% of excess volatility is due to the expected excess-return channel, while 25% is due to the risk-free rate channel. The possibility that agents in group  $A$  may be “wiped out” after a series of negative shocks, i.e. that their wealth share  $x$  may approach zero, drives a large amount of volatility, even in normal times.

I now assess whether the model fits moments about the wealth distribution, that were not targeted in the first estimation. I find that the model slightly overestimates the exposure of top wealth shares to stock market returns: it is 1.0 in the model compared to 0.95 in the data. More importantly, however, the model vastly overestimates the thickness of the tail of the distribution: in the model, the tail index of the wealth distribution is 1.1, whereas it is close to 1.5 in the data. Visually, Figure 2 shows that the right tail of the wealth distribution is much thicker in the model than in the data. The large heterogeneity in preferences, necessary to match the excess volatility of asset prices, generates a model in which agents in group  $A$  grow much faster than the rest of the economy, and therefore in which the right tail of the wealth distribution is much thicker than the data (Proposition 5).

**Adding Wealth Inequality Moments** To ask whether the model can match asset prices without implying a counter-factual wealth distribution, I re-estimate the model in Column (3) of Table 5, targeting jointly the four asset prices moments and the two moments about the wealth distribution. The model cannot jointly match asset prices and the wealth distribution: the J-statistic, which measures the distance between the model and the data, jumps from 0.05 to 15.

The reason the model fails to match the new sets of moments is that there is a tension between matching the volatility of asset prices and the tail index of the wealth distribution. To match a thinner tail index, the model needs to decrease the average wealth growth of households in group  $A$ . Because the wealth-weighted average growth of individual households has to sum up to aggregate wealth growth, this must be compensated either by an increase of the average wealth growth of agents in group  $B$ , or by an increase in the average group share of agents in group  $A$ . In either case, the average sensitivity of aggregate RRA  $\Gamma$  and of the inverse of the aggregate EIS  $1/\Psi$  to the group share  $x$  must decrease.<sup>33</sup> In turn, this decreases the average sensitivity of the market price of risk and of the risk-free rate to the group share, which ultimately decreases the volatility

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<sup>33</sup>Indeed, Equation (37) shows that the sensitivity of aggregate RRA and of the inverse of the aggregate EIS is high when  $x$  is close to zero and when  $\gamma_B$  and  $1/\psi_B$  are high.

of asset prices.<sup>34</sup>

More precisely, the model underestimates the equity premium (3.6% in the model compared to 5.2% in the data). The reason the model misses the equity premium, rather than the volatility of returns, is that this moment is less precisely estimated, and therefore is given less weight in the J-statistic. Moreover, having such a low equity premium allows the model to lower the average wealth growth of households in group *A* (by decreasing the compensation for holding aggregate risk), and therefore to match the thinner tail of the wealth distribution, without substantially changing the degree of preference heterogeneity. Moreover, the model overestimates the interest rate (3.2% in the model compared to 2.8% in the data). This is because, in the model, both agents have a very low EIS. A low EIS for agent *A* is required to match the thinner tail of the wealth distribution, while a low EIS for agent *B* is required to match the high volatility of returns. Finally, the last line of the table reports the decomposition of the excess volatility of returns defined in Proposition 3: half of the fluctuations in asset prices are driven by fluctuations in the expected risk-free rate, rather than fluctuations in expected excess stock returns, which is counter-factual.<sup>35</sup>

### 5.3 Adding Realistic Ingredients

The environment is highly stylized and one could consider additional ingredients that would make the wealth distribution more realistic. However, I show that adding realistic ingredients such as idiosyncratic shocks (Benhabib et al. (2015a)), heterogeneous subjective discount rates (Krusell and Smith (1998), Carroll et al. (2017)), or bounded lives do not help the model to resolve the tension described above.

**Idiosyncratic Shocks** In the baseline model, there are no idiosyncratic wealth shocks. The position of households in the wealth distribution depends on their human capital, their group, and their age. Yet, in the data, idiosyncratic shocks play an important role in driving the right tail of the distribution.<sup>36</sup>

Augmenting the model with idiosyncratic wealth shocks does not help the model to better match the data. Intuitively, this thickens the right tail of the distribution compared to the baseline

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<sup>34</sup>As seen in Proposition 3.

<sup>35</sup>See, for instance, Campbell and Shiller (1988).

<sup>36</sup>See, for instance, Benhabib et al. (2015a).

model (Lemma 3). Since the baseline model already gives a right tail that is too thick compared to the data, adding idiosyncratic wealth shocks only worsens the tension between matching asset prices and the right tail of the distribution. Formally, Column (2) Table 6 re-estimates the model with an idiosyncratic wealth volatility of 10%. For simplicity, I assume that the presence of these idiosyncratic shocks does not affect the saving decisions of households.<sup>37</sup> As predicted, the fit worsens: the J-statistic increases to 21.

**Heterogeneity in Discount Rates** In the model, households have the same subjective discount rate  $\rho$ . Allowing heterogeneous subjective discount rate  $\rho$ , as in Krusell and Smith (1998) or Carroll et al. (2017), may help the model to better fit the data. This would capture, in a reduced form way, heterogeneity in the propensity to save (for instance due to different tax rates or bequest motives). To check whether this changes the results, I estimate a model where I allow different subjective discount rates for households:  $\rho_A$  and  $\rho_B$ . The results, reported in Column (3) of Table 6, show that it does not help the model to better fit the data. This is because in the estimated model, the EIS of agents in group B  $\psi_B$  is so low that the exact value of their subjective discount rate  $\rho_B$  has very little impact on their saving decision (Proposition 1), and therefore on aggregate quantities.

**Bounded Lives** To get analytical results on the tail index of the wealth distribution, the model assumes that the death probability of households does not depend on age. In particular, this means that in the model, certain households live for a large amount of time and end up with a disproportional amount of wealth compared to the rest of the households. In reality, households usually die after a certain age, which limits how rich households can really be.

In a static model, where the wealth distribution is in a steady state, one can show that this assumption has no impact on the tail index of the wealth distribution.<sup>38</sup> This is because the tail index is a local property that characterizes the *slope* of the wealth density. Imposing that households die after a certain age only results in a truncated Pareto distribution, with the same tail index as the original model.

To check that this still holds true in a model with aggregate shocks, I rely on simulations.

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<sup>37</sup>Precautionary saving would tend to increase the saving rate of households, which corresponds to a rise in their subjective discount rate  $\rho$ .

<sup>38</sup>See, for instance, Steindl (1965).

More precisely, I simulate the wealth distribution in the model, with the added assumption that households older than 70 years die with probability 1. I find that incorporating this more realistic “upper bound” on age does not change the tail index of the wealth distribution.<sup>39</sup>

## 6 Resolving the Tension

The previous section showed that the baseline model has difficulty to jointly match asset prices and wealth inequality. This tension comes from the combination of two facts: (i) matching the high volatility of asset prices requires a large degree of preference heterogeneity, and (ii) a large degree of preference heterogeneity gives rise to a wealth distribution with a tail thicker than the data.

I now explore two potential deviations from the baseline model that help resolve this tension. In Section 6.1, I assume that risk tolerant agents only remain levered for a short period of time. This weakens link (ii) between preference heterogeneity and the tail index of the wealth distribution, thereby resolving the tension in the baseline model. In Section 6.2, I augment the model with redistributive shocks between households. This weakens link (i) between preference heterogeneity and the volatility of asset prices, which helps the model to jointly match asset prices and wealth inequality.

### 6.1 Live-Fast-Die-Young

**Augmented Model** I now consider a model in which risk-tolerant households become risk-averse after some time, i.e. in which households transition from group  $A$  to group  $B$ . Formally, I assume that households transition from group  $A$  to group  $B$  with hazard rate  $\tau$ : during a short period of time  $dt$ , a proportion  $\tau dt$  of agents switches from group  $A$  to group  $B$ . For the sake of simplicity, I assume that these transitions are unexpected, so the agent’s problem remains the same as in the baseline model.<sup>40</sup>

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<sup>39</sup>A video of the wealth distribution in an economy with fluctuating drift is available at <http://www.matthieugomez.com/wealthinequality.html>. Even though the wealth distribution is not in a steady state, the slope of the log density with respect to log wealth remains stable over time.

<sup>40</sup>A model in which households expect these transitions to take place would be substantially more complex, since the utility of an agent in group  $A$  would not be homogeneous in wealth anymore. In any case, it is not clear that agents should take the transitory nature of preferences into account in their optimization problem.

The law of motion of the state variable  $x_t$ , the share of wealth owned by agents in group  $A$ , must be modified to account for the transition from group  $A$  to group  $B$ :

$$\mu_{xt} = x_t (\mu_{W_{At}} - \mu - \mu_{p_t} - \sigma \sigma_{p_t}) - \sigma_{xt} (\sigma + \sigma_{p_t}) + (\delta + n) (\pi_A \frac{\phi_t}{p_t} - x_t) - \underbrace{\tau x_t}_{\text{transition}} \quad (56)$$

The transition rate  $\tau$  tends to pull down the state variable  $x_t$ . This suggests that the average value of  $x_t$  will be smaller compared to the baseline model. Similarly, the right tail of the wealth distribution will be thinner.

**Proposition 8** (Tail Index with Transition). *Denote*

$$\zeta = \frac{\delta + n + \tau}{\mathbb{E} \left[ \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2 \right]} \quad (57)$$

where  $\mathbb{E}$  denotes the expectation with respect to the invariant probability measure of  $x$ . If the following conditions are satisfied:

1. Agents in group  $A$  grow on average faster than the economy and than agents in group  $B$ , even after adjusting for transition i.e.

$$\mathbb{E} \left[ \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2 \right] > \max \left( 0, \frac{\delta + n + \tau}{\delta + n} \mathbb{E} \left[ \mu_{w_{Bt}} - \frac{1}{2} \sigma_{w_{Bt}}^2 \right] \right) \quad (58)$$

2. The distribution of human capital  $\chi_i$  has a tail thinner than  $\zeta$ .

Then the wealth distribution is thick-tailed with tail index  $\zeta$ . The distribution of financial wealth is also thick-tailed with tail index  $\zeta$ . Moreover, when  $\tau > 0$ , both the wealth distributions of agents in group  $A$  and in group  $B$  are thick-tailed with tail index  $\zeta$

The transition rate  $\tau$  introduces a wedge between the tail index of the wealth distribution and the average growth rate of agents in group  $A$ . For a given level of preference heterogeneity (i.e. for a given average level of the growth rate of agents in group  $A$ ), a higher transition rate implies a wealth distribution with a thinner tail. Therefore, a high transition rate will help the model to better fit the data.

The intuition is as follows. In contrast to the baseline model, the distribution in group  $B$  “inherits” the tail index of the distribution of group  $A$ , i.e. *both* types of agents are present at the top of the distribution. One key consequence is that, in the augmented model, moments about

the right tail of the wealth distribution (such as its tail and the exposure of top wealth shares to stock market returns) reflect the wealth dynamics of *both* types of agents, rather than the wealth dynamics of agents in group  $A$  only.

**Reduced-Form Evidence** A central prediction of the augmented model is that households in the right tail of the distribution tend to become less levered as they grow older. This is because, every period of time, some of these households transition from being risk-tolerant to being risk averse. I now test this key prediction in the data by estimating separately the stock market exposure of young and old households in the right tail of the distribution.

To do so, I use the panel data constructed from Forbes 400. Every year, I construct the average wealth growth of households with an age below the median ( $g = 1$ ), and the average wealth growth of households with an age above the median ( $g = 2$ ). I then test whether the stock market exposure of older households differs from the stock market exposure of younger households, by estimating the following regression:

$$\sum_{0 \leq h \leq 3} \log R_{g,t+h} - h \log R_{f,t} = \alpha + \delta 1_{\text{Age} \geq \text{Median}} + (\beta + \gamma 1_{\text{Age} \geq \text{Median}})(\log R_{M,t} - \log R_{f,t}) + \epsilon_{g,t}$$

where  $R_{gt}$  the return of households in group  $g$  at year  $t$ . This specification follows the aggregate specification from Table 1, while allowing the stock market exposure to be differ for young and old households.

Table 7 reports the result in a specification without year fixed effects (Column (1)), and a specification with year fixed effects (Column (2)). I find that  $\gamma = -0.31$  in both specifications, i.e. the beta of older households is 0.31 less than the beta of younger households. This is consistent with the augmented model: risk-tolerant households grow quickly, reach the right tail of the distribution, before becoming risk-averse after a certain amount of time.

**Results** I now estimate the model augmented with a transition rate  $\tau = 2\%$ . This choice means that the average life of a risk-tolerant agent is roughly 25 years, after which the household dies or becomes risk-averse.<sup>41</sup>

I estimate the augmented model on the six moments described earlier. The first column of Table 8 shows that the augmented model can jointly match asset prices and the wealth distribution.

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<sup>41</sup> I take this value for the sake of example. I obtain similar results using transition rates between 2% and 4%.

In particular, the model can generate a high volatility of returns  $\sigma_R = 18\%$  together with a high tail index  $\zeta = 1.5$ . The J-statistic is equal to 0.15, which is 100 times smaller than the J-statistic of the baseline model.

The estimated model has a higher degree of preference heterogeneity compared to the baseline model estimated in Column (1) of Table 5: the RRA of households in group  $A$  equals 1.2 (v.s. 1.5 in the baseline model), while their EIS equals 1.0 (vs 0.7 in the baseline model). Because the state variable in the augmented model is less persistent than in the baseline model (see (56)), a higher degree of preference heterogeneity is required to match the excess volatility of asset prices (see Proposition 3). Even with this high degree of preference heterogeneity, the distribution has a thinner tail compared to the baseline model. In conclusion, augmenting the model with transition rates from group  $A$  to group  $B$  helps to resolve the tension between the volatility of asset prices and the tail index of the distribution.

## 6.2 Time-Varying Investment Opportunities

I now explore a second potential departure from the standard model. I augment the model with time-varying investment opportunities for the rich relative to the poor. More precisely, I introduce fluctuations in the financial returns available to the agents in group  $A$  compared to the financial returns available to the agents in group  $B$ . These shocks create additional, low-frequency fluctuations in wealth inequality, which increase the volatility of asset prices. The augmented model can match the excess volatility of asset prices with a reduced degree of preference heterogeneity.

**Augmented Model** Formally, I modify the model by introducing a mean-reverting process  $\nu_t$  that fluctuates around zero:<sup>42</sup>

$$d\nu_t = -\kappa_\nu \nu_t dt + \sigma_\nu dZ_t \tag{59}$$

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<sup>42</sup>When  $\sigma_\nu = 0$ , the model reverts to the baseline model. I consider  $\sigma_\nu \geq 0$ .

This process drives a wedge between the financial return available to the agents in group  $A$  and  $B$ , i.e., the drift of financial wealth becomes:

$$\mu_{W_{At}} = r_t + \delta + \kappa\sigma_{At} - c_{At} + \underbrace{(1 - x_t)\nu_t}_{\neq \text{inv. opport.}} \quad (60)$$

$$\mu_{W_{Bt}} = r_t + \delta + \kappa\sigma_{Bt} - c_{Bt} - \underbrace{x_t\nu_t}_{\neq \text{inv. opport.}} \quad (61)$$

Note that the total growth of wealth ( $x_t\mu_{W_{At}} + (1 - x_t)\mu_{W_{Bt}}$ ) does not depend on  $\nu_t$ , i.e. these changes are purely redistributive. This can be seen as a fluctuating wealth tax: the government levies a wealth tax  $\nu_t$  on agents in group  $A$  and immediately redistributes the proceeds to all agents in proportion to their wealth.

Fluctuations in  $\nu_t$  introduce fluctuations in the growth rate of the wealth share of group  $A$ ,  $x_t$ . The law of motion of the  $x_t$  in the augmented model becomes:

$$\mu_{x_t} = x_t(\mu_{W_{At}} - \mu - \mu_{p_t} - \sigma\sigma_{p_t}) - \sigma_{x_t}(\sigma + \sigma_{p_t}) + (\delta + n)(\pi_A \frac{\phi_t}{p_t} - x_t) + \underbrace{x_t(1 - x_t)\nu_t}_{\neq \text{inv. opport.}} \quad (62)$$

The higher  $\nu_t$ , the higher the growth rate of agents in group  $A$  relative to the economy.

Moreover, these shocks amplify the excess volatility of asset prices. This is because the price-dividend ratio of the representative firms reacts to news about the level of inequality  $x_t$ , but also to news about the future growth of inequality  $\nu_t$ :

$$\sigma_R = \sigma + \frac{\partial \log p_D}{\partial x} \sigma_x + \underbrace{\frac{\partial \log p_D}{\partial \nu} \sigma_\nu}_{\neq \text{inv. opport.}} \quad (63)$$

The third term, the semi-elasticity of the price-dividend ratio to  $\nu$ , is a novel source of asset price volatility compared to the baseline model. It is positive for two reasons. First, because agents in group  $A$  have a higher EIS than agents in group  $B$ , a rise in  $\nu_t$  increases the aggregate demand for assets, which pushes up the price-dividend ratio. Second, a rise in  $\nu_t$  also increases the growth rate of  $x_t$ , the share of wealth owned by the agents in group  $A$ , (see (62)) and is therefore associated with a lower interest rate and a lower market price of risk in the future. This further pushes up asset prices.

The fact that purely redistributive shocks have an impact on asset prices comes from the fact that agents have heterogeneous preferences. If all agents had the same preference parameters, fluctuations in  $\nu_t$  would have no effect on asset prices: the rise in savings from one group of agents would be exactly compensated by the decrease in savings from the other group of agents.

**Reduced-Form Evidence** A key prediction of the augmented model is that some fluctuations in top wealth shares cannot be accounted for by stock market returns. To test this prediction, I compare the actual time series of the wealth share of the top 0.01% to a synthetic version constructed using past stock market returns. More precisely, I run the following regression in the simulated data from the model estimated in Section 5:

$$\log(\text{top share})_{t+1} = \alpha + \rho \times \log(\text{top share})_t + \beta \times (\log R_{t+1}^M - \log R_{t+1}^f) + \epsilon_{t+1} \quad (64)$$

The regression has a very high  $R^2 \approx 97\%$ : in the model, the dynamics of top wealth shares is well approximated by a simple weighted sum of past excess stock market returns. Therefore, I construct the synthetic version of top wealth shares as the predicted values of this regression using the *actual* realization of stock market returns during the time period, i.e.<sup>43</sup>.

$$\widehat{\log\text{topshare}}_{t+1} = \alpha + \rho \times \widehat{\log\text{topshare}}_t + \beta \times (\log R_{t+1}^M - \log R_{t+1}^f) \quad (65)$$

Figure 4 plots the synthetic version of the wealth share of the top 0.01%, as well as the actual one from Estate Tax Returns and Forbes 400.<sup>44</sup> This synthetic version of top wealth follows roughly the one measured in the data. Small but persistent changes in stock market returns can account for large fluctuations of top wealth shares over time.

However, the graph shows a few periods of “disconnect” between wealth inequality and asset prices. In particular, excess stock market returns cannot fully explain the decline in inequality in the 1930s, or the rise in inequality beginning in 1980s.<sup>45</sup> In the augmented model, this would correspond to changes in  $\nu_t$ . While I do not take a stand on the origin of these differential investment opportunities, the literature suggests some potential causes. These fluctuations could be generated by changes in taxes (Piketty and Zucman (2015), Hubmer et al. (2016), Pastor and Veronesi (2016)), changes in inflation (Doepke and Schneider (2006)), changes in housing prices (Kuhn et al. (2017)), or changes in technology (Gârleanu et al. (2012), Kogan et al. (Forthcoming)). In the rest of the section, I examine whether these periods of disconnect between asset prices and wealth inequality can help the model to better match asset prices in equilibrium.

<sup>43</sup>An alternative construction would be to simulate the model using actual endowment shocks during the time period, but this would require the model to generate accurate stock market returns

<sup>44</sup>The wealth share of the top 0.01% implied by Forbes 400 is obtained using the fact that the ratio of top wealth shares between two percentiles equals  $(p/p')^{\frac{1}{\zeta}} - 1$  for a distribution with tail index  $\zeta$ .

<sup>45</sup>The fact that the rise in wealth inequality after 1980 cannot be fully explained by changes in asset prices was also noted by Saez and Zucman (2016) using a different methodology.

**Results.** I assume that the process  $\nu_t$  has a mean reversion parameter  $\kappa_\nu = 0.05$ , i.e. shocks have a half life of 15 years. The instantaneous volatility is  $\sigma_\nu = 0.006$ . This calibration corresponds to small but persistent changes in investment opportunities, which fits the reduced form evidenced discussed above. This relates this paper to the literature on long-run risk (Bansal and Yaron (2004)), except that I only consider the role of purely redistributive shocks.

The last column of Table 8 demonstrates that the augmented model can jointly match asset prices and the wealth distribution. In particular, the model can generate a high volatility of returns  $\sigma_R = 18.1\%$ , together with a high tail index  $\zeta = 1.5$ . The J-statistic is equal to 1.6, which is 10 times smaller than the J-statistic of the baseline model. The augmented model cannot be rejected at a 5% significant level.

The estimated model has a reduced degree of preference heterogeneity compared to the baseline model. In the augmented model, the RRA of households in group A equals 2.0 (vs 1.5 in the baseline model), whereas their EIS equals 0.6 (vs 0.8 in the baseline model). Therefore, households in group A grow slower than in the baseline model, which allows the model to match the tail index of the wealth distribution. Even with this reduced degree of preference heterogeneity, however, the augmented model matches the high volatility of asset prices: small but persistent changes in investment opportunities  $\nu_t$  can generate large fluctuations on asset prices in equilibrium.

## 7 Conclusion

The paper documents a strong interplay between asset prices and the wealth distribution. This interplay is consistent with asset pricing models with heterogeneous agents. Heterogeneity in risk aversion explains key facts about the level and dynamics of wealth inequality. Because rich households hold more risky assets, they earn higher returns on average. Therefore, the wealth distribution has a thick right tail, as in the data. Moreover, realized stock returns generate large fluctuations in wealth inequality over time.

In turn, wealth inequality is an important driver of asset prices. In periods of high inequality, more wealth is in the hands of rich households, the risk-tolerant investors, which depresses risk premia and increases asset prices. Empirically, high wealth inequality predicts lower future excess returns. Quantitatively, however, a standard OLG model where agents have heterogeneous pref-

erences cannot fully explain the excess volatility of asset prices in equilibrium. I suggested two parsimonious deviations from the baseline model to resolve this tension: (i) transitory preferences, which limit the role of heterogeneity on the right tail of the wealth distribution, and (ii) additional redistributive shocks, which magnify the role of preference heterogeneity on the volatility of asset prices.

The implications of my analysis extend beyond asset pricing. The interplay I put forward can have effects on real quantities as well, through two channels. First, because the level of inequality affects the cost of capital, this may lead to changes in corporate investment policies. Second, a recent literature has also emphasized the role of inequality in aggregate demand ([Mian et al. \(2013\)](#), [Kaplan et al. \(2016\)](#)). Exploring these channels requires moving away from an endowment economy, which I leave for future research.

Table 1: The Exposure to Stock Returns Increases Across the Wealth Distribution

	Wealth Growth within Percentile Thresholds				
	Flow of Funds	Kopczuk and Saez (2004)			Forbes 400
	All Households	1 – 0.1%	0.1 – 0.01%	Top 0.01%	Top 400
	(1)	(2)	(3)	(4)	(5)
<i>Panel A: Wealth</i>					
Excess Stock Returns	0.44*** (0.07)	0.64*** (0.17)	0.81*** (0.15)	0.95*** (0.20)	0.93*** (0.25)
$R^2$	0.44	0.22	0.36	0.31	0.31
Period	1917-1997	1917-1997	1917-1997	1917-1997	1983-2013
$N$	53	53	53	53	31
<i>Panel B: Wealth Share</i>					
Excess Stock Returns		0.17*** (0.05)	0.34*** (0.06)	0.48*** (0.13)	0.46** (0.21)
$R^2$		0.20	0.39	0.23	0.13
Period		1917-1997	1917-1997	1917-1997	1983-2013
$N$		53	53	53	31

*Notes:* This table reports the results of the regression of the excess wealth growth of households in a given percentile group on the excess stock returns, i.e. Equation (1):

$$\log\left(\frac{W_{G,t+3}}{W_{G,t-1}}\right) - 4\log R_{ft} = \alpha_G + \beta_G(\log R_{Mt} - \log R_{ft}) + \epsilon_{Gt}$$

The dependent variables are the growth of wealth in Panel A and the growth of wealth *shares* in Panel B.

Each column corresponds to a different group of households. The first column corresponds to all U.S households. Columns (2) to (4) corresponds to increasing top percentiles in the wealth distribution, using data from [Kopczuk and Saez \(2004\)](#). Column (5) corresponds to the Top 0.0003%. This last percentile is chosen so that the group include the 400 wealthiest individuals in 2015.

Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 4 lags. \*, \*\*, \*\*\* indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Table 2: The Exposure to Stock Returns Across the Wealth Distribution:  
Controlling for Composition

	Forbes 400	
	All	Within
	(1)	(2)
Excess Stock Returns	0.93*** (0.25)	0.89*** (0.25)
$R^2$	0.31	0.29

*Notes:* In the first column, the table regresses the total growth of the wealth in the top 400 on excess stock market returns, as in Table 1. In the second column, the table regresses the “within growth” on excess stock market returns. The “within growth” corresponds to the yearly growth of households in the top 400, whether or not they drop out of the top by the end of the year. See Gomez (2018) for more details on the construction of the “within” term.

Table 3: The Share of Wealth Owned by the Top 0.01% And Future Excess Returns

	$\sum_{1 \leq h \leq H} \log R_{t+h}^M - \log R_{t+h}^f = \alpha + \beta_H \text{Log Top Wealth Shares}_t + \gamma_H X_t + \epsilon_{tH}$					
	Excess Returns at Horizon $H = 1$			Excess Returns at Horizon $H = 3$		
	$\beta_1$	$\gamma_1$	Adjusted $R^2$	$\beta_3$	$\gamma_3$	Adjusted $R^2$
	(1)	(2)	(3)	(4)	(5)	(6)
Log Top Share	-0.109*		0.031	-0.244*		0.058
$\Delta$ Log Top Share (5 years difference)	-0.271**		0.066	-0.787***		0.184
Log Top Share and Linear Trend	-0.264***	-0.003*	0.071	-0.620***	-0.007**	0.145
Log Top Share and Dividend Price	-0.172**	0.116*	0.077	-0.452***	0.384***	0.254
Log Top Share and Dividend Payout	-0.141**	0.094	0.033	-0.334**	0.268	0.088

*Notes:* This table reports the results of the regressions of future excess returns on the share of wealth owned by the Top 0.1% (row 1). Each row corresponds to a different set of regressors. Columns (1), (2), and (3) report the results when the dependent variable is the one year excess-return. Columns (4), (5), and (6) report the results when the dependent variable is the three-year excess-return.

Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 4 lags. \*, \*\*, \*\*\* indicate significance at the 0.1, 0.05, 0.01 levels.

Table 4: The Share of Wealth Owned by the Top 0.01% And Future Excess Returns  
(Campbell and Yogo (2006) test)

Confidence Interval for $\beta \in [\underline{\beta}, \bar{\beta}]$ in the Predictability Regression (2)				
	Case with $\underline{\rho} = 0.89$		Case with $\bar{\rho} = 1.02$	
	$\underline{\beta}$	$\bar{\beta}$	$\underline{\beta}$	$\bar{\beta}$
Log Top Share	-0.39	-0.07	-0.34	-0.02

*Notes:* The time period is 1917-1951, the longest period where the wealth share of the top 0.01% is available without missing years. This table uses the test developed by Campbell and Yogo (2006) that jointly takes into account the persistence of the predictor as well as its correlation with stock returns to compute the 90% confidence interval for  $\beta$ . The autoregressive lag length for the DF-GLS statistic is estimated to be 1, using the Bayes information criterion (BIC).

Table 5: Fitting the Baseline Model to the Data

		Data	Baseline Model Estimated on:	
			Asset Prices	Asset Prices + Inequality
		(1)	(2)	(3)
Parameters	RRA $\gamma_A$		1.5	1.5
	RRA $\gamma_B$		17	17
	EIS $\psi_A$		0.8	0.7
	EIS $\psi_B$		0.05	0.05
	Subjective Discount Rate $\rho$		0.1%	0.1%
	Population share $\pi_A$		1%	9.0%
Moments	Equity Premium	5.2%	5.1%	3.6%
	STD Market Return	18.2%	18.1%	16.2%
	Average interest rate	2.8%	2.7%	3.2%
	STD interest rate	0.9%	0.7%	0.8%
	Exposure Top Percentile $\beta$	0.95	1.0	0.97
	Tail Index $\zeta$	1.5	1.1	1.5
	$\beta_{\log S_t \rightarrow R_{Mt+1} - R_{ft+1}}$	-0.1	-0.08	-0.06
	Risk-Free Rate Channel %		25%	49%
J-statistic		0.05	15.2	

*Notes:* Column (1) corresponds to the moments in the data. Column (2) reports the corresponding moments in the model estimated on asset prices only. Column (3) reports the moments in the model estimated on asset prices and inequality moments. Numbers in bold indicate that these moments have been used in the estimation.

Table 6: Fitting the Baseline Model to the Data (Realistic Ingredients)

		Data	Model	
			Idiosyncratic Volatility	$\neq$ Discount Factors
		(1)	(2)	(3)
Parameters	RRA $\gamma_A$		1.4	1.5
	RRA $\gamma_B$		17	17
	EIS $\psi_A$		0.6	0.7
	EIS $\psi_B$		0.05	0.05
	Population share $\pi_A$		11.2%	9.0%
	Discount Rate $\rho_A$		0.1%	0.1%
	Discount Rate $\rho_B$			2%
Moments	Equity Premium	5.2%	3.4%	3.6%
	STD Market Return	18.2%	16.3%	16.2%
	Average interest rate	2.8%	3.7%	3.1%
	STD interest rate	0.9%	1.0%	0.8%
	Exposure Top Percentile $\beta$	0.95	0.96	1.0
	Tail Index $\zeta$	1.5	1.5	1.5
	$\beta_{\log S_t \rightarrow R_{Mt+1} - R_{ft+1}}$	-0.1	-0.03	-0.06
	Risk-Free Rate Channel %		56%	49%
J-statistic		21.0	15.0	

*Notes:* Column (1) corresponds to the moments in the data. Columns (2) and (3) correspond to different estimations of the extended model. All models are estimated on six moments: four moments about asset prices and two moments about the wealth distribution (the wealth exposure of the 0.01% to stock market returns and the tail index). Column (2) reports the model estimated with idiosyncratic volatility  $\nu = 10\%$ . Column (3) reports the model estimated with heterogeneous subjective discount rates.

Table 7: Heterogeneous Exposures to Stock Market Returns Within Forbes 400

	Exposure to Stock Market Returns $\beta$	
	(1)	(2)
Excess Stock Returns	1.02*** (0.25)	
Excess Stock Returns $\times \{\text{Age} \geq \text{Median}\}$	-0.31 (0.36)	-0.31*** (0.12)
Fixed Effects		Year
$R^2$	0.31	0.92
Period	1983-2013	1983-2013
$N$	62	62

*Notes:* For every year, I construct the (wealth-weighted) average return of households in Forbes 400 within two groups: those with age below the median ( $g = 1$ ), and those with age above the median ( $g = 2$ ). The median corresponds to 60 years. Denote  $R_{gt}$  the return of households in group  $g$  at year  $t$ . Column (1) reports the results of estimating the model:

$$\sum_{0 \leq h \leq 3} \log R_{g,t+h} - 4 \log R_{f,t} = \alpha + \delta 1_{\text{Age} \geq \text{Median}} + \beta (\log R_{Mt} - \log R_{ft}) + \gamma 1_{\text{Age} \geq \text{Median}} (\log R_{Mt} - \log R_{ft}) + \epsilon_{gt}$$

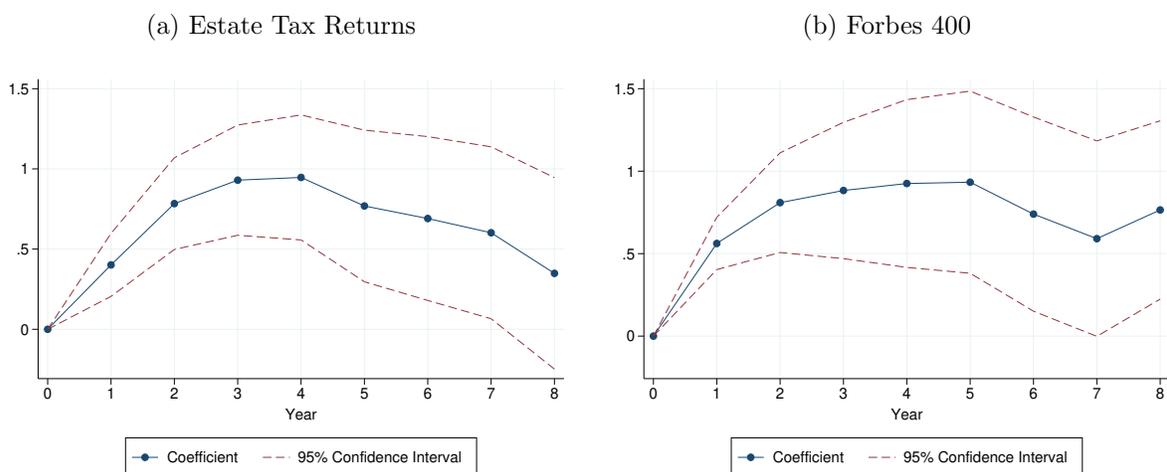
A negative  $\gamma$  says that the stock market exposure of old households is lower than the stock market exposure of young households. The four year horizon follows the specifications in Table 1. Column (2) adds year fixed effects. Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 4 lags. \*, \*\*, \*\*\* indicate significance at the 0.1, 0.05, 0.01 levels.

Table 8: Fitting the Augmented Model to the Data

		Data	Augmented Model	
			Transition Rate	$\neq$ Inv. Opport.
		(1)	(2)	(3)
Calibration	Transition Rate $\tau$		0.02	
	Persistence $\kappa_\nu$			0.05
	Volatility $\sigma_\nu$			0.006
Parameters	RRA $\gamma_A$		1.2	2.0
	RRA $\gamma_B$		18	17
	EIS $\psi_A$		1.0	0.6
	EIS $\psi_B$		0.05	0.06
	Population share $\pi_A$		0.09	0.08
	Discount Rate $\rho$		1.0%	0.1%
Moments	Equity Premium	5.2%	5.1%	5.0%
	STD Market Return	18.2%	18.3%	18.1%
	Average interest rate	2.8%	2.8%	2.8%
	STD interest rate	0.9%	0.7%	1.1%
	Exposure Top Percentile $\beta$	0.95	0.90	0.85
	Tail Index $\zeta$	1.5	1.5	1.5
	$\beta_{\log S_t \rightarrow R_{Mt+1} - R_{ft+1}}$	-0.1	-0.08	-0.04
	Risk-Free Rate Channel %		30%	45%
J-statistic		0.15	1.6	

*Notes:* Column (1) corresponds to the moments in the data. Column (2) corresponds to estimations of the model with transition rate. All models are estimated on six moments: four moments about asset prices, and two moments about the wealth distribution (the wealth exposure of the 0.01% to stock market returns and the tail index).

Figure 1: Exposure  $\beta$  of Top Wealth Group to Stock Market Returns at Different Horizons

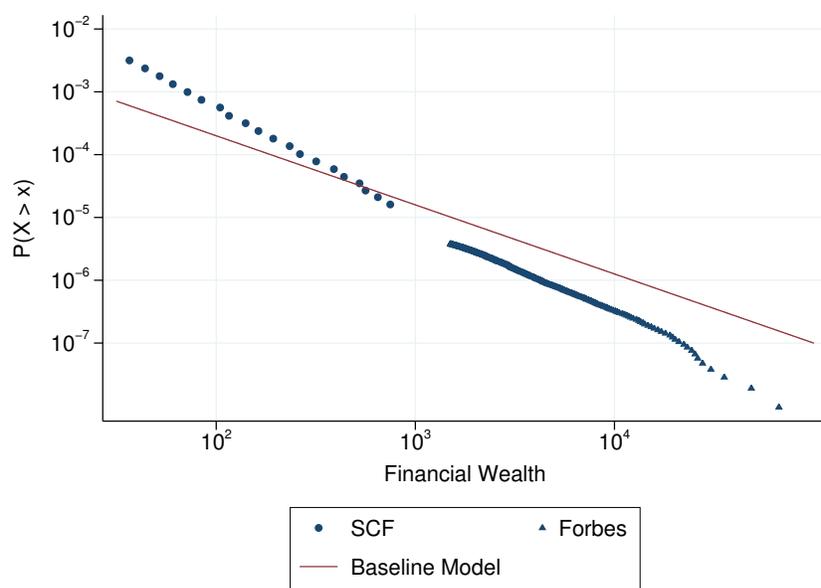


Notes: This figure reports the results of regressing excess wealth growth of households in the top 0.01% (left) and in the top 400 (right) on excess stock returns i.e.

$$\log\left(\frac{W_{G,t-1+h}}{W_{G,t-1}}\right) - h \log R_{ft} = \alpha_{Gh} + \beta_{Gh}(\log R_{Mt} - \log R_{ft}) + \epsilon_{Ght}$$

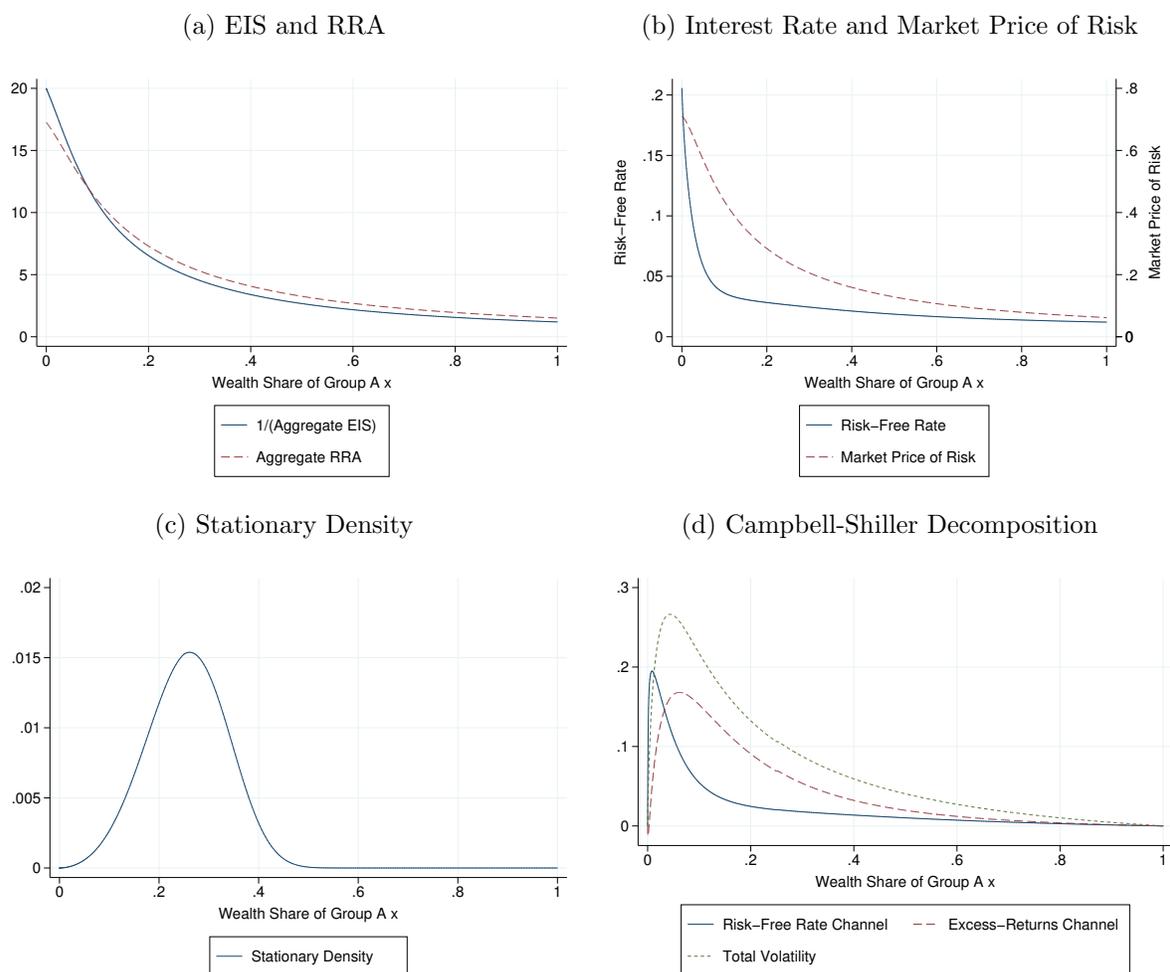
Each figure reports the estimates for  $\beta_{Gh}$ , from  $h = 0$  to  $h = 8$ , as well as the 5%-95% confidence interval using Newey-West with 4 lags.

Figure 2: The Right Tail of the Wealth Distribution: Data vs Baseline Model



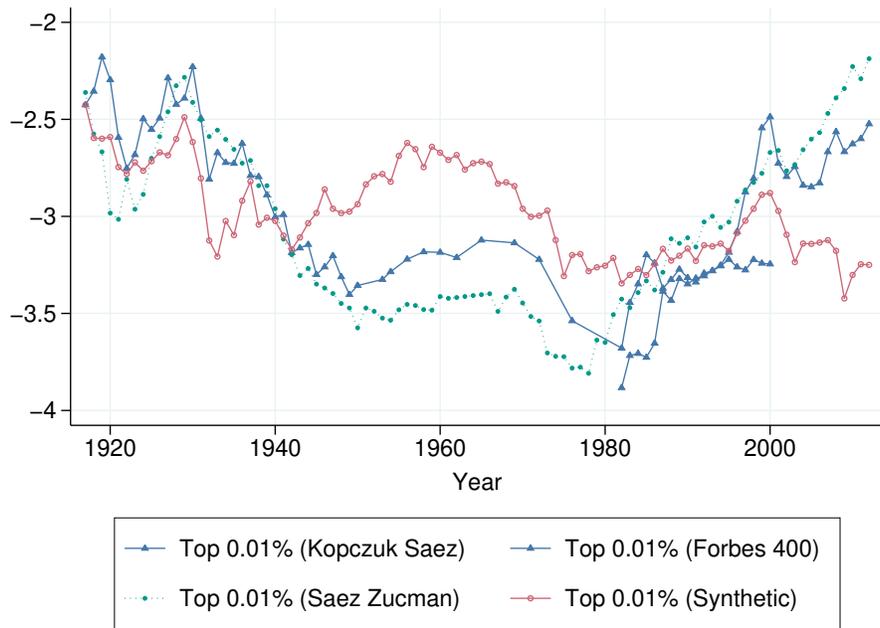
*Notes:* This figure compares the log net worth (relative to the average net worth) to the log percentile in SCF and Forbes. More precisely, the figure plots the average log net worth within 40 logarithmically spaced percentile bins in SCF. The figure plots the average log net worth for each position in Forbes 400. The (opposite of) the slope estimate gives  $\zeta \approx 1.5$  for SCF and for Forbes 400. In red, the figure plots the distribution implied by the model estimated on asset prices (see Table 5).

Figure 3: Asset Prices in the Baseline Model



*Notes:* This figure plots the interest rate, the market price of risk, the stationary density, and the Campbell-Shiller Decomposition of Proposition 3 for the model estimated on asset prices only (Columns (2) in Table 5). The Campbell-Shiller Decomposition is estimated by constructing  $E[r_t|x_0 = x]$  and  $E[\kappa_t\sigma_{Rt} - \frac{1}{2}\sigma_{Rt}^2|x_0 = x]$  using the Kolmogorov-Backward equation

Figure 4: Time Series of the Top 0.01% vs  $\widehat{\text{Top 0.01\%}}$



# Appendix A

## A.1 Cross Sectional Evidence

To understand better what drives the heterogeneous exposure of top households to aggregate shocks, I also examine the heterogeneity in equity holding across the wealth distribution using the Survey of Consumer Finances (SCF). The survey is a repeated cross-section of about 4,000 households per survey year, including a high-wealth sample. The survey is conducted every three years, from 1989 to 2013. The respondents provide information on their net worth, including their investments in public and private equity. I define the equity share as the total investment in equity over net worth. I define the set of entrepreneurs as the households with equity held in an actively managed business.<sup>46</sup>

Figure A1a plots the average equity share within percentile bins across the wealth distribution. The average equity share of 0.4 masks a substantial heterogeneity across households. The equity share is essentially flat at 0.2 over the majority of the wealth distribution, but increases sharply within the top 1%. Figure A1b plots the equity share with respect to the *log* top percentiles, showing that the equity share is approximately linear in the log percentile at the top of the distribution. The figure suggests that the bulk of the heterogeneity is concentrated within the top percentiles, which justifies my focus on the top percentile.

A stylized fact in the household finance literature is that stock market participation increases with wealth (Vissing-Jørgensen (2002)). Therefore, the increase in the equity share within the top percentiles could be driven by an increase in the proportion of stockholders (i.e. the extensive margin). However, Panel B of Table A1 shows that the percentage of stockholders is constant within the top percentiles (90%). The increase in the equity share is entirely driven by the increase *within* stockholders. While the heterogeneity between stockholders and non-stockholders generates a lot of variations at the bottom of the wealth distribution, these variations account for a small share of total wealth.

Investment in risky assets comes mainly in two forms: public equity and private equity. Panel A of Table A1 decomposes the increase in equity share across the top percentiles between the two types of equity. The decomposition reveals that the increase in the equity share is mostly driven by an increase in the share of wealth invested in private equity. Panel C of Table A1 shows that the proportion of entrepreneurs increases sharply in the top percentile: the proportion of households with an actively managed business is 78.5% in the Top 0.01%, compared to 10% in the general population.<sup>47</sup> The wealth of these entrepreneurs is mostly invested in their private business. A potential concern is that, if entrepreneurs cannot trade or sell their firms easily, the heterogeneity in private equity holdings may have no impact on stock market prices. However,

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<sup>46</sup>The definition follows Moskowitz and Vissing-Jørgensen (2002).

<sup>47</sup>Similarly, Hurst and Lusardi (2004), using the Panel Study of Income Dynamics (PSID), show that the propensity of entrepreneurship increases sharply with wealth in the top percentiles.

Panel C of Table A1 shows that entrepreneurs hold substantial amounts of public equity (15%). Even with illiquid businesses, entrepreneurs can adjust their overall risky holdings at the margin.

## A.2 Saez-Zucman series

Saez and Zucman (2016) have recently proposed a new series for top wealth shares, which relies on Income Tax Returns. In Table A2, I estimate the stock market exposure of top wealth percentiles using this series. Qualitatively, the results of Section 2 hold true: the exposure of top wealth shares to stock market return increases with top percentile. However, the estimates are now uniformly lower compared to Kopczuk and Saez (2004). For instance, the stock market exposure of the Top 0.01% is 0.66 using Income Tax Returns, compared to 0.95 using Estate Tax returns or Forbes.

These results suggest that the methodology used in Saez and Zucman (2016) tends to smooth out the business cycle fluctuations of top wealth shares (even though they may capture more accurately the long run fluctuations in inequality). First, certain wealth categories, such as bonds, are reported in face value rather than in market value. While this follows the methodology of the US Financial Accounts, this choice tends to mute business cycles fluctuations in asset valuation. Second, certain wealth categories, such as private equity, are constructed using interpolations across years.<sup>48</sup> Finally, because they use realized capital gains to infer top wealth shares, this mechanically smooths out fluctuations in top shares if rich households disproportionately sell when stock market returns are low, which is what the model predicts.

## A.3 Composition Change

As shown in Table 2, compositional changes do not matter in driving the exposure of top wealth shares to stock market returns. To understand why, I use a formula in Gomez (2018), that shows that, in a large family of models, the growth of wealth in a given top percentile can be written as

$$\frac{W_{t+1}^G}{W_t^G} = r_{within} + \frac{\zeta - 1}{2} \nu_t^2$$

where  $r_{within}$  denotes the growth of wealth for individuals in the top percentiles at time  $t$ ,  $\zeta$  denotes the tail index of the wealth distribution, and  $\nu_t^2$  denotes the idiosyncratic variance of wealth at the top.

By projecting this equality on stock market returns, one can write the difference between the exposure of total wealth and the exposure of the wealth of individuals in the top percentile (i.e. the difference between

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<sup>48</sup>See <http://gabriel-zucman.eu/uswealth/> for a detailed construction of their dataset. Interpolation is particularly important before 1945.

estimates in Columns (1) and (2) Table 2) as an omitted variable bias:

$$\frac{\zeta - 1}{2} \frac{\text{cov}(\nu_t^2, \log R_{Mt})}{\text{var} \log R_{Mt}} = \frac{\zeta - 1}{2} \frac{\text{std}(\nu_t^2)}{\text{std}(\log R_{Mt})} \rho_{\nu_t^2, \log R_{Mt}}$$

This bias is the product of three terms: (i) a term that depends on the shape of the wealth distribution  $\frac{1}{2}(\zeta - 1)$ , (ii) a term that depends on the ratio between the standard deviation of idiosyncratic volatility compared to the standard deviation of the stock market return, and (iii) the correlation between idiosyncratic volatility and stock market returns. Quantitatively, because the wealth distribution is very concentrated ( $\zeta$  is close to one), and because yearly fluctuations in idiosyncratic volatility are small compared to the yearly fluctuations in stock market return, we can expect the effect of composition to be small.

## Appendix B

### B.1 Proof

*Proof of Proposition 1.* The HJB equation associated with the household's problem is

$$0 = \max_{c_{jt}, \sigma_{W_{jt}}} \{f(C_{jt}, V_{jt}) + E[dV_{jt}]\} \quad (\text{A1})$$

Given the homotheticity assumptions, the value function of the households in group  $j \in \{A, B\}$  with wealth  $W$  can be written:

$$V_{jt}(W) = \frac{W^{1-\gamma_j} p_{jt}^{\frac{1-\gamma_j}{\psi_j-1}}}{1-\gamma_j} \quad (\text{A2})$$

Applying Ito's lemma on HJB equation:

$$\begin{aligned} 0 = \max_{C_{jt}, \sigma_{W_{jt}}} & \left\{ \frac{1-\gamma_j}{1-\frac{1}{\psi_j}} \left( \frac{C_{jt}^{1-\frac{1}{\psi_j}}}{W_{jt}^{1-\frac{1}{\psi_j}} p_{jt}^{\frac{1}{\psi_j}}} - (\rho + \delta) \right) + (1-\gamma_j)\mu_{W_{jt}} + \frac{\gamma_j(\gamma_j-1)}{2}\sigma_{W_{jt}}^2 \right. \\ & \left. + \frac{1-\gamma_j}{\psi_j-1}\mu_{p_{jt}} + \frac{1}{2}\frac{1-\gamma_j}{\psi_j-1} \left( \frac{1-\gamma_j}{\psi-1} - 1 \right) \sigma_{p_{jt}}^2 + \frac{(1-\gamma_j)^2}{\psi_j-1}\sigma_{W_{jt}}\sigma_{p_{jt}} \right\} \end{aligned} \quad (\text{A3})$$

Substituting the expression for the wealth drift  $\mu_j$  using the budget constraint and dividing by  $1-\gamma_j$

$$\begin{aligned} 0 = \max_{C_{jt}, \sigma_{W_{jt}}} & \left\{ \frac{1}{1-\frac{1}{\psi_j}} \left( \frac{C_{jt}^{1-\frac{1}{\psi_j}}}{W_{jt}^{1-\frac{1}{\psi_j}} p_{jt}^{\frac{1}{\psi_j}}} - (\rho + \delta) \right) + r_t + \delta + \sigma_{W_{jt}}\kappa_t - \frac{C_{jt}}{W_{jt}} - \frac{\gamma_j}{2}\sigma_j^2 \right. \\ & \left. + \frac{1}{\psi_j-1}\mu_{p_{jt}} + \frac{1}{2(\psi-1)} \left( \frac{1-\gamma_j}{\psi_j-1} - 1 \right) \sigma_{p_{jt}}^2 + \frac{1-\gamma_j}{\psi_j-1}\sigma_{W_{jt}}\sigma_{p_{jt}} \right\} \end{aligned} \quad (\text{A4})$$

The FOC for aggregate risk exposure gives

$$\sigma_{W_{jt}} = \frac{\kappa_t}{\gamma_j} + \frac{\frac{1}{\psi_j} - 1}{\psi_j - 1} \sigma_{p_{jt}} \quad (\text{A5})$$

The FOC for consumption gives

$$C_{jt} = \frac{1}{p_{jt}} W_{jt} \quad (\text{A6})$$

that is,  $p_{jt}$  is the wealth-to-consumption ratio of the household.

Plugging the optimal consumption rate into the HJB, we obtain an expression for the wealth drift in terms of  $r_t$  and  $\kappa_t$ :

$$\begin{aligned} \mu_{W_{jt}} &= r_t + \delta + \sigma_{W_{jt}} \kappa_t - \frac{1}{p_{jt}} \\ &= \psi_j(r - \rho) + \frac{1 + \psi_j}{2\gamma_j} \kappa_t^2 + \frac{\frac{\psi_j}{\gamma_j} - \psi_j}{\psi_j - 1} \kappa_t \sigma_{p_{jt}} + \frac{\frac{1}{\gamma_j} - \psi_j}{2(\psi_j - 1)} \sigma_{p_{jt}}^2 + \mu_{p_{jt}} \end{aligned} \quad (\text{A7})$$

□

*Proof of Proposition 3.* I derive a Campbell-Shiller decomposition in continuous-time. The cumulative return of a dollar invested in the asset follows the process:

$$d \log R_t = \frac{1}{p_{Dt}} dt + d \log p_{Dt} + d \log D_t \quad (\text{A8})$$

As in [Chacko and Viceira \(2005\)](#), I log-linearize the dividend-price ratio around the average value of  $\log p_D$ :

$$\frac{1}{p_{Dt}} \approx \alpha(1 - \log \alpha) - \alpha \log p_{Dt} \quad (\text{A9})$$

with  $\alpha = e^{-\mathbb{E}[\log p_D]}$ . Plugging this expression into Equation (A8), I obtain:

$$d \log R_t \approx \alpha(1 - \log \alpha) dt - \alpha \log p_{Dt} dt + d \log p_{Dt} + d \log D_t \quad (\text{A10})$$

Integrating, the price-dividend ratio can be written as:

$$\begin{aligned} \log p_{Dt} &\approx E_t \left[ \int_t^{+\infty} e^{-\alpha(s-t)} (\alpha(1 - \log \alpha) dt + d \log R_s - d \log D_s) \right] \\ &\approx 1 - \log \alpha + E_t \left[ \int_t^{+\infty} e^{-\alpha(s-t)} (d \log D_s - d \log R_s) \right] \end{aligned} \quad (\text{A11})$$

In the Markovian economy of Section 5, the expected growth rate of log-dividend is constant. Deriving with respect to the state variable  $x$ , one obtains:

$$\frac{\partial \log(p_D)}{\partial x} \approx - \int_t^{+\infty} e^{-\alpha(s-t)} \frac{\partial E[d \log R_s | x_t = x]}{\partial x} \quad (\text{A12})$$

□

*Proof of Proposition 2.* Applying Ito's lemma, the law of motion of the wealth per capita in group  $A$  is:

$$\begin{aligned} d \left( \int_{i \in \mathbb{I}_{At}} W_{it} / (\pi_A N_t) \right) &= d \left( \int_{-\infty}^t (\delta + n) e^{-(\delta+n)(t-s)} W_{Ats} ds \right) \\ &= \int_{-\infty}^t (\delta + n) e^{-(\delta+n)(t-s)} dW_{Ats} ds + \pi_A N_t (\delta + n) W_{Att} dt \\ &\quad - (\delta + n) dt \int_{-\infty}^t (\delta + n) e^{-(\delta+n)(t-s)} W_{Ats} ds \end{aligned} \quad (\text{A13})$$

Dividing by  $\int_{i \in \mathbb{I}_{A_t}} W_{it}/(\pi_A N_t)$  we obtain

$$\frac{d\left(\int_{i \in \mathbb{I}_{A_t}} W_{it}/(\pi_A N_t)\right)}{\int_{i \in \mathbb{I}_{A_t}} W_{it}/(\pi_A N_t)} = \frac{dW_{A_t s}}{W_{A_t s}} + (\delta + n) \left(\frac{\phi_t \pi_A}{p_t x_t} - 1\right) dt \quad (\text{A14})$$

This gives the law of motion of  $x_t$ . □

## B.2 Solving the Model

Assume that  $G(u)$  is a sum of  $K$  exponential

$$G_k(u) = B_k e^{-\delta_k u} \quad \forall 1 \leq k \leq K \quad (\text{A15})$$

$$G(u) = \sum_{1 \leq k \leq K} G_k(u) \quad (\text{A16})$$

where the coefficients  $(B_k)_{1 \leq k \leq K}$  are such that total aggregate earnings equal  $\omega Y_t$

$$1 = \sum_{1 \leq k \leq K} B_k \frac{\delta + n}{\delta + n + \delta_k} \quad (\text{A17})$$

Denote  $p_k^l$  the price-dividend of a claim with exponentially decreasing endowment at rate  $\delta + \delta_k$ , for  $1 \leq k \leq K$ . Conjecture that this process follows a diffusion process

$$\frac{dp_k^l}{p_k^l} = \mu_{p_k^l} dt + \sigma_{p_k^l} dZ_t \quad (\text{A18})$$

Using Equation (10), we obtain

$$\phi = \omega \sum_{1 \leq k \leq K} B_k p_k^l \quad (\text{A19})$$

We also note that

$$\frac{1}{p} = \frac{x}{p_A} + \frac{1-x}{p_B} \quad (\text{A20})$$

gives the first and second derivative of  $p$  with respect to  $x$  in terms of the first and second derivatives of  $p_A$  and  $p_B$  with respect to  $x$

**Solve for  $\sigma_x$**  Applying Ito's lemma, we have

$$\sigma_{p_j} = \frac{\partial_x p_j}{p_j} \sigma_x \quad \text{for } j \in \{A, B\} \quad (\text{A21})$$

$$\sigma_p = \frac{\partial_x p}{p} \sigma_x \quad (\text{A22})$$

$$\sigma_{p_k^l} = \frac{\partial_x p_k^l}{p_k^l} \sigma_x \quad \text{for } l \in 1 \leq k \leq K \quad (\text{A23})$$

Substituting the expression for  $\kappa$  in Equation (26) in Proposition 2, we can solve for  $\sigma_x$ :

$$\sigma_x = \frac{\frac{x(1-x)\Gamma}{\gamma_A \gamma_B} (\gamma_B - \gamma_A) \sigma}{1 - \frac{x(1-x)\Gamma}{\gamma_A \gamma_B} \left( (\gamma_B - \gamma_A) \frac{\partial_x p}{p} + \frac{1-\gamma_A}{\psi_A - 1} \frac{\partial_x p_A}{p_A} - \frac{1-\gamma_B}{\psi_B - 1} \frac{\partial_x p_B}{p_B} \right)} \quad (\text{A24})$$

**Solve for  $\mu_x$**  In terms of previously computed quantities:

$$\mu_x = x(1-x) \left( \kappa(\sigma_{W_{At}} - \sigma_{W_{Bt}}) - \frac{1}{p_A} + \frac{1}{p_B} \right) + (\delta + n) \frac{\phi}{p} (x - \pi_A) + x \sigma_{p_t Y_t}^2 - x \sigma_{W_{At}} \sigma_{p_t Y_t} \quad (\text{A25})$$

Using Ito's lemma, we can express the drift of all quantities to solve for:

$$\mu_{p_j} = \frac{\partial_x p_j}{p_j} \mu_x + \frac{1}{2} \frac{\partial_{xx} p_j}{p_j} \sigma_x^2 \text{ for } j \in \{A, B\} \quad (\text{A26})$$

$$\mu_p = \frac{\partial_x p}{p} \mu_x + \frac{1}{2} \frac{\partial_{xx} p}{p} \sigma_x^2 \quad (\text{A27})$$

$$\mu_{p_k^l} = \frac{\partial_x p_k^l}{p_k^l} \mu_x + \frac{1}{2} \frac{\partial_{xx} p_k^l}{p_k^l} \sigma_x^2 \quad (\text{A28})$$

**System of ODEs** After obtaining the interest rate with Equation (30), the budget constraint for  $A, B$  and the definition for  $(p_k^l)_{1 \leq k \leq K}$  gives a system of  $2 + K$  ODEs:

$$\frac{1}{p_j} + \mu_{W_j} = r + \delta + \kappa \sigma_{W_j} \text{ for } j \in \{A, B\} \quad (\text{A29})$$

$$\frac{1}{p_k^l} + \mu - \delta_k + \sigma_{p_k^l} \sigma = r + \delta + \kappa(\sigma + \sigma_{p_k^l}) \text{ for } 1 \leq k \leq K \quad (\text{A30})$$

where  $\mu_{W_j}$  and  $\sigma_{W_j}$  are given by Proposition 1.

**Price-Dividend Ratio** The ratio of total human capital to the total labor income at a given point in time is

$$p_h = \sum_{1 \leq k \leq K} B_k \frac{\delta + n}{\delta + n + \delta_k} \phi_k \quad (\text{A31})$$

Therefore, the price-dividend ratio  $pd$  for the representative firm is given by

$$pd = \frac{1}{1 - \omega} (p - \omega p_h) \quad (\text{A32})$$

Using Ito's lemma, one can then obtain the return dynamics of the representative levered firm, as

$$\frac{dR_t}{R_t} = (r + \kappa \lambda (\sigma + \sigma_{pd})) dt + (\sigma + \sigma_{pd}) dZ_t$$

### B.3 Computational Method

The system of PDEs is written on a state space grid and derivatives are substituted by finite difference approximations. Importantly, first order derivatives are upwinded.

Denote  $Y$  the solution and denote  $F(Y)$  the finite difference scheme corresponding to a model. The goal is to find  $Y$  such that  $F(Y) = 0$ . I solve for  $Y$  using a fully implicit Euler method. Updates take the form

$$\forall t \leq T \quad 0 = F(y_{t+1}) - \frac{1}{\Delta} (y_{t+1} - y_t) \quad (\text{A33})$$

Each update requires solving a non-linear equation. I solve this non-linear equation using a Newton-Raphson method. The Newton-Raphson method converges if the initial guess is close enough to the solution. Since  $y_t$  converges towards  $y_{t+1}$  as  $\Delta$  tends to zero, one can always choose  $\Delta$  low enough so that the inner steps converge. Therefore, I adjust  $\Delta$  as follows. If the inner iteration does not converge, I decrease  $\Delta$ . If the inner iteration converges, I increase  $\Delta$ . After a few successful implicit time steps,  $\Delta$  is large and, therefore the algorithm becomes like Newton-Raphson. In particular, the convergence is quadratic around the solution.

This method is most similar to a method used in the fluid dynamics literature, called the Pseudo-Transient Continuation method. Formal conditions for the convergence of the algorithm are given in [Kelley and Keyes \(1998\)](#). The algorithm with  $I = 1$  and  $\Delta$  constant corresponds to [Achdou et al. \(2016\)](#). Allowing  $I > 1$  and adjusting  $\Delta$  are important to ensure convergence for non-linear PDEs.

## Appendix C

**Lemma 1** (Kolmogorov-Forward with Aggregate Risk). *Suppose  $w_t$  is a process evolving according to*

$$\frac{dw_t}{w_t} = \mu_t dt + \sigma_t dZ_t \quad (\text{A34})$$

where  $Z_t$  is a standard aggregate Brownian Motion. Suppose that agents die with Poisson rate  $\delta$  and are born according to the density  $g_{\chi t}$ . Finally suppose that population grows with rate  $n$ . The density of  $w_t$ ,  $g_t$ , follows the law of motion:

$$dg_t = \left( -\mu_t \partial_w (w g_t) + \frac{1}{2} \sigma_t^2 \partial_{ww} (w^2 g_t) + (\delta + n)(g_{\chi t} - g_t) \right) dt - \sigma_t \partial_w (w g_t) dZ_t \quad (\text{A35})$$

*Proof of Lemma 1.* For any function  $f$ , we have

$$\int_{-\infty}^{+\infty} f(w) g_{t+dt}(w) dw = \int_{-\infty}^{+\infty} ((f(w) + df(w)) g_t(w) + f(w)(\delta + n) dt (g_{\chi t}(w) - g_t(w))) dw \quad (\text{A36})$$

Assuming that  $f$  is twice differentiable, Ito's lemma gives:

$$\begin{aligned} \int_{-\infty}^{+\infty} f(w) dg_t(w) dx &= \int_{-\infty}^{+\infty} \left( \mu_t w dt \partial_w f(w) + \frac{1}{2} \sigma_t^2 w^2 dt \partial_{ww} f(w) + \sigma_t w \partial_w f(w) dZ_t \right) g_t(w) dw \\ &\quad + \int_{-\infty}^{+\infty} f(w) (\delta + n) dt (g_{\chi t}(w) - g_t(w)) dw \end{aligned} \quad (\text{A37})$$

Assume that  $f$  decays fast enough as  $|x| \rightarrow +\infty$  and use integration by parts to obtain

$$\begin{aligned} \int_{-\infty}^{+\infty} f(w) dg_t(w) dw &= \int_{-\infty}^{+\infty} f(w) \left( -\mu_t \partial_w (w g_t) + \frac{1}{2} \sigma_t^2 \partial_{ww} (w^2 g_t) + (\delta + n) dt (g_{\chi t} - g_t) \right) dt dw \\ &\quad - \int_{-\infty}^{+\infty} f(w) \sigma_t \partial_w (w g_t) dZ_t dw \end{aligned} \quad (\text{A38})$$

This equality must hold for all  $f$  satisfying the conditions above. Therefore, we obtain

$$dg_t = \left( -\mu_t \partial_w (wg_t) + \frac{1}{2}(\sigma_t^2 + \nu_t^2) \partial_{ww} (w^2 g_t) + (\delta + n)(g_{\chi t} - g_t) \right) dt - \sigma_t \partial_w (wg_t) dZ_t \quad (\text{A39})$$

□

*Proof of Proposition 4.* The proposition follows from Lemma 1 applied to the particular model. □

**Lemma 2** (Stability of Linear Functional). *Let  $x_t \in \mathbb{R}$  be a continuous-time strong Markov process non-explosive, irreducible, positive recurrent with unique invariant probability measure.*

*Consider the process*

$$dM_t = (\mu(x_t)M_t + b(x_t)) dt + \sigma(x_t)M_t dW_t \quad (\text{A40})$$

with  $P(b(x) \geq 0) = 1$  and  $P(b(x) > 0) > 0$ .

(i) *If  $\mathbb{E}[\mu(x) - \frac{1}{2}\sigma(x)^2] > 0$ ,  $M_t$  converges to infinity a.s.*

(ii) *If  $\mathbb{E}[\mu(x) - \frac{1}{2}\sigma(x)^2] < 0$ ,  $M_t$  does not converge to infinity a.s.*

where  $\mathbb{E}$  denotes the expectation with respect to the invariant probability measure of  $x$ .

*Proof of Lemma 2.* The proof for the case  $b(x_t) = 0$  (purely multiplicative process) is in Maruyama and Tanaka (1959). To my knowledge, however, there exists no proof for the more general process (A40), so I extend their proof to handle the case  $b(x_t) \neq 0$ . The idea is to bound the continuous time process by a discrete time process, which can be examined using Vervaat (1979).

For  $\tau > 0$ , we have the following recurrence equation:

$$M_{t+\tau} = e^{\int_t^{t+\tau} ((\mu(x_u) - \frac{1}{2}\sigma(x_u)^2) du + \sigma(x_u) dW_u)} M_t + \int_t^{t+\tau} e^{\int_s^{t+\tau} ((\mu(x_u) - \frac{1}{2}\sigma(x_u)^2) du + \sigma(x_u) dW_u)} b(x_s) ds$$

Denote  $I$  the set of values that  $x_t$  can take. Take  $a < b$ , both in  $I$ . Define the sequence of stopping times  $S_0 = 0$  and

$$T_n \equiv \inf\{t > S_n; x_t = a\}$$

$$S_{n+1} \equiv \inf\{t > T_n; x_t = b\}$$

Define

$$Y_n = M_{T_n}$$

$$A_n = \exp \left( \int_{T_n}^{T_{n+1}} ((\mu(x_u) - \frac{1}{2}\sigma(x_u)^2) du + \sigma(x_u) dW_u) \right)$$

$$B_n = \int_{T_n}^{T_{n+1}} e^{\int_s^{T_{n+1}} ((\mu(x_u) - \frac{1}{2}\sigma(x_u)^2) du + \sigma(x_u) dW_u)} b(x_s) ds$$

The sequence  $Y_n$  satisfies the following recurrence relation:

$$Y_{n+1} = A_n Y_n + B_n$$

where  $A_n$  and  $B_n$  are i.i.d over time. Moreover,  $A_1$  is positive a.s.,  $B_1$  is non negative a.s. with  $P(B_1 > 0) > 0$  and  $E[\log(B_1)] < +\infty$ . As proven by [Vervaat \(1979\)](#),  $Y_n$  converges in distribution if  $E[\log A_1] < 0$ , i.e.  $E\left[\int_{T_1}^{T_2} (\mu(x_u) - \frac{1}{2}\sigma(x_u)^2) du\right] \geq 0$ , and converges a.s. to infinity if  $E[\log A_1] > 0$ . Finally, as shown in [Maruyama and Tanaka \(1959\)](#), any integrable function  $f$ ,  $E\left[\int_{T_1}^{T_2} f(x_u) du\right] \geq 0$  iff  $\mathbb{E}[f(x)] \geq 0$ . I conclude by noting that  $M_t$  converges to infinity a.s. if and only if  $Y_n$  converges to infinity a.s. because for  $t \in (T_n, T_{n+1}]$ ,  $Y_n \leq M_t \leq Y_{n+1}$ .

□

**Lemma 3** (Tail index). *Let  $x_t \in \mathbb{R}$  a continuous-time strong Markov process non-explosive, irreducible, positive recurrent with unique invariant probability measure.*

*Suppose the dynamics of individual wealth  $w_{it}$  has the following law of motion:*

$$\frac{dw_{it}}{w_{it}} = \mu(x_t)dt + \sigma(x_t)dZ_t + \nu(x_t)dW_{it} \quad (\text{A41})$$

*where  $W_{it}$  is an idiosyncratic Brownian Motion and  $Z_t$  is an aggregate Brownian Motion. Moreover, assume that individuals die with death rate  $\delta > 0$  and are re-injected according to a distribution  $g_{\chi t}$  with thin tails.<sup>49</sup>*

*If  $\mathbb{E}[\nu(x)^2] > 0$  or  $\mathbb{E}[\mu(x)] - \frac{1}{2}\mathbb{E}[\sigma(x)^2] > 0$ , the wealth distribution is thick-tailed with tail index  $\zeta$ , given by the positive root of*

$$\zeta \mathbb{E}[\mu(x)] - \frac{1}{2}\mathbb{E}[\sigma(x)^2] + \frac{\zeta(\zeta - 1)}{2}\mathbb{E}[\nu(x)^2] - \delta = 0 \quad (\text{A42})$$

*where  $\mathbb{E}$  denotes the expectation with respect to the stationary density of  $x$ .*

*Proof of Lemma 3.* Denote  $m_t^\xi$  the  $\xi$ -moment of the wealth distribution and  $m_{\chi t}^\xi$  the  $\xi$  cross-sectional moment of the distribution of wealth at birth. The law of motion of  $m_t^\xi$  is given by:

$$dm_t^\xi = \left( \xi \mu(x_t) + \frac{\xi(\xi - 1)}{2} (\sigma(x_t)^2 + \nu(x_t)^2) \right) m_t^\xi dt + (\delta + n)(m_{\chi t}^\xi - m_t^\xi) dt + \xi \sigma(x_t) m_t^\xi dZ_t \quad (\text{A43})$$

Compared to the dynamics of the wealth density, the dynamics of  $m_{j_t}^\xi$  can be examined in isolation for each  $\xi$ , since it does not depend on the derivative of the function  $\xi \rightarrow m_{j_t}^\xi$ . This insight is due to [Gabaix et al. \(2016\)](#) and [Luttmer \(2012\)](#), that show that, in setups where wealth dynamics are linear in wealth, it is often easier to work with higher-order moments of wealth.

Denote  $f$  the function defined as:

$$f(\xi) = \xi \left( \mathbb{E}[\mu(x)] - \frac{1}{2}\mathbb{E}[\sigma(x)^2] \right) + \frac{\xi(\xi - 1)}{2}\mathbb{E}[\nu(x)^2] - (\delta + n) \quad (\text{A44})$$

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<sup>49</sup>Formally, the  $\xi$ th moments of  $\psi$  exists for  $\xi \leq \zeta$ .

There exists a unique  $\zeta > 0$ , such that  $f(\zeta) = 0$ . Moreover,  $f(\xi)$  is negative for  $\xi \in [0, \zeta)$  and positive for  $\xi \in (\zeta + \infty)$ . Using Lemma 2, we have that  $\zeta = \inf\{\xi \in \mathbb{R}_+ | \lim_{t \rightarrow +\infty} m_t^\xi = +\infty \text{ a.s.}\}$ .  $\square$

*Proof of Proposition 5.* The first part of the proposition follows from Lemma 3 applied to the particular model.

I now prove that the distribution of financial wealth has the same tail index  $\zeta$ . For an individual with wealth  $W_{it} \geq 0$ , define  $H_{it} \geq 0$  the capitalized value of his human capital, and  $A_{it} = W_{it} - H_{it}$ . Denote  $A_{it}^+ = \max(A_{it}, 0)$ .

Because  $x \rightarrow x^\xi$  is an increasing function, we have the lower bound

$$E[(A_{it}^+)^\xi] \leq E[(A_{it} + H_{it})^\xi]$$

Moreover, we have the upper bound:

$$E[(A_{it} + H_{it})^\xi] \leq \max(1, 2^{\xi-1})(E[A_{it}^{+\xi}] + E[H_{it}^\xi])$$

This inequality obtains because, for  $\xi > 1$ ,  $x \rightarrow x^\xi$  is convex, and for  $\xi \in [0, 1]$ ,  $x \rightarrow x^\xi$  is sub-additive.

Joining all these inequalities, and dividing by  $E[A_{it} + H_{it}]^\xi$ , we obtain:

$$\frac{E[A_{it}^{+\xi}]}{E[A_{it} + H_{it}]^\xi} \leq m_{it}^\xi \leq \max(1, 2^{\xi-1}) \frac{E[A_{it}^{+\xi}] + E[H_{it}^\xi]}{E[A_{it} + H_{it}]^\xi}$$

Finally, denote  $p_{Dt}$  the price-dividend ratio of the representative firm, and  $p_{Ht}$  the total value of human capital in the economy divided by the share of endowment distributed as labor income. We have

$$\begin{aligned} E[A_{it} + H_{it}]^\xi / Y_t^\xi &= p_t^\xi \\ E[A_{it}]^\xi / Y_t^\xi &= p_{Dt}^\xi \omega^\xi \\ E[H_{it}^\xi] / Y_t^\xi &= \omega^\xi p_{Ht}^\xi \mathbb{E}[\chi_i^\xi] \end{aligned}$$

Therefore the  $\xi$  moment of total wealth converges a.s. to infinity iff the  $\xi$  moment of financial wealth converges to infinity a.s., which means that both distribution have the same tail index.  $\square$

*Proof of Proposition 6.* Applying Ito's lemma on Proposition 2, we get

$$d \ln x_t = \left( \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2 + (\delta + n) \left( \frac{\pi_A \phi_t}{x_t p_t} - 1 \right) \right) dt + \sigma_{w_{At}} dZ_t \quad (\text{A45})$$

Since  $\ln(x_t)$  is ergodic, its drift must average to zero. This gives:

$$0 = \mathbb{E} \left[ \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2 \right] + (\delta + n) \left( \mathbb{E} \left[ \frac{\pi_A \phi_t}{x_t p_t} \right] - 1 \right) \quad (\text{A46})$$

This expression allows me to replace  $\mathbb{E} \left[ \mu_{w_{At}} - \frac{1}{2} \sigma_{w_{At}}^2 \right]$  in the expression for the tail index.  $\square$

*Proof of Proposition 7.* This proof adapts Gomez (2018) to the case of this model. Applying Ito's lemma on Equation (49) gives the law of motion of the quantile  $q_t$

$$0 = -g_t(q_t) \frac{dq_t}{dt} + \int_{q_t}^{+\infty} \frac{dg_t(w)}{dt} dw - \sigma[dg_t(q_t)]\sigma[dq_t] \quad (\text{A47})$$

where  $\sigma[dg_t(q_t)]$  and  $\sigma[dq_t]$  denote respectively the volatility of  $g_t(q_t)$  and  $q_t$ . Applying Ito's lemma on Equation(50) gives the law of motion of the top share  $S_t$ :

$$dS_t = -q_t g_t(q_t) dq_t + \int_{q_t}^{\infty} w dg_t(w) dw - q_t \sigma[dg_t(q_t)]\sigma[dq_t] dt - \frac{1}{2} g_t(q_t) \sigma[dq_t]^2 dt \quad (\text{A48})$$

Using the law of motion for  $q_t$  from Equation (A47), we obtain the law of motion of  $S_t$ :

$$\begin{aligned} dS_t &= \int_{q_t}^{\infty} (w - q_t) dg_t(w) dw - \frac{1}{2} g_t(q_t) \sigma[dq_t]^2 dt \\ &= \int_{q_t}^{\infty} (w - q_t) dg_t(w) dw - \frac{1}{2} \frac{1}{g_t(q_t)} \left( \int_{q_t}^{\infty} \sigma[dg_t(w)] dw \right)^2 dt \end{aligned} \quad (\text{A49})$$

where

$$dg_t = \pi_A dg_{At} + (1 - \pi_A) dg_{Bt} \quad (\text{A50})$$

and the law of motion of  $g_{jt}$  for  $j \in \{A, B\}$  is given by the Kolmogorov-Forward equation from Lemma 3

$$dg_{jt} = \left( -\mu_{w_{jt}} \partial_w (w g_{jt}) + \frac{1}{2} \sigma_{w_{jt}}^2 \partial_{ww} (w^2 g_{jt}) + (\delta + n)(g_{\chi t} - g_{jt}) \right) dt - \sigma_{w_{jt}} \partial_w (w g_{jt}) dZ_t \quad (\text{A51})$$

Plugging it into (A49), we obtain

$$\begin{aligned} dS_t &= \int_{q_t}^{\infty} (w - q_t) \left( \sum_{j \in \{A, B\}} \pi_j \left( \left( -\mu_{w_{jt}} \partial_w g_{jt}(w) + \frac{1}{2} \sigma_{w_{jt}}^2 \partial_{ww} g_{jt}(w) \right) dt - \sigma_{w_{jt}} \partial_w g_{jt}(w) dZ_t \right) \right) dw \\ &\quad + \int_{q_t}^{\infty} (w - q_t) \left( \sum_{j \in \{A, B\}} \pi_j (\delta + n) (g_{\chi t} - g_{jt}(w)) \right) dt \\ &\quad - \frac{1}{2g_t(q_t)} \left( \int_{q_t}^{+\infty} \sum_{j \in \{A, B\}} \pi_j \sigma_{w_{jt}} \partial_w (w g_{jt}(w)) \right)^2 dt dw \end{aligned} \quad (\text{A52})$$

Integrating by parts and rearranging:

$$\begin{aligned} dS_t &= \left( \int_{q_t}^{+\infty} \sum_{j \in \{A, B\}} \mu_{w_{jt}} w \pi_j g_{jt}(w) \right) dt + \left( \int_{q_t}^{+\infty} \sum_{j \in \{A, B\}} \sigma_{w_{jt}} w \pi_j g_{jt}(w) dw \right) dZ_t \\ &\quad + (\delta + n) dt \left( q_t \alpha - S_t + \int_{q_t}^{+\infty} (w - q_t) g_{\chi t}(w) dw \right) \\ &\quad + \frac{q_t^2 g_t(q_t)}{2} dt \left( \sum_{j \in \{A, B\}} \sigma_{w_{jt}}^2 \frac{\pi_j g_{jt}(q)}{g_t(q)} - \left( \sum_{j \in \{A, B\}} \sigma_{w_{jt}} \frac{\pi_j g_{jt}(q_t)}{g_t(q_t)} \right)^2 \right) \end{aligned} \quad (\text{A53})$$

Dividing by  $S_t$ , we obtain Proposition 7 □

## Appendix D

### D.1 The Exposure of Human Capital in Top Percentiles

The model gives sharp predictions for the wealth exposure of top percentiles, where wealth is defined using a concept that includes human capital. However, in the data, I only measure the exposure of financial wealth, which does not include human capital. In this section, I argue that for households at the top of the wealth distribution, the difference between the exposure of total wealth and the exposure of financial wealth is quantitatively small.

Formally, for a given household in the economy, denote  $a$  its financial wealth and  $h$  its human capital. Denote  $\omega = h/(a+h)$  the ratio of human capital wealth to total wealth. Following the log-linearization in [Campbell \(1996\)](#), the return on total wealth can be written as a weighted average of the return of financial assets and the return of human capital:

$$\log \frac{a_{t+1} + h_{t+1}}{a_t + h_t} \approx (1 - \omega) \log \frac{a_{t+1}}{a_t} + \omega \log \frac{h_{t+1}}{h_t} \quad (\text{A54})$$

Projecting this approximation on stock returns, the exposure of total wealth to stock market returns,  $\beta_{a+h}$ , can be written as a weighted average of the exposure of financial wealth,  $\beta_a$ , and the exposure of human capital,  $\beta_h$ :

$$\beta_{a+h} \approx (1 - \omega)\beta_a + \omega\beta_h \quad (\text{A55})$$

This allows me to express the relative bias between the exposure of total wealth and the exposure of financial wealth:

$$\frac{\beta_{a+h} - \beta_a}{\beta_a} = \omega \left( \frac{\beta_h - \beta_a}{\beta_a} \right) \quad (\text{A56})$$

The bias increases with the ratio of human capital to total wealth,  $\omega$ , and with the relative difference between the exposure of financial wealth and of human capital  $(\beta_h - \beta_a)/\beta_a$ .

I first proxy for  $\omega$ , the share of total wealth in human capital, is small. This appears to be the case for households at the very top of the wealth distribution. Indeed, the IRS reports that labor income represents 8.5% of total income for top households in the U.S.<sup>50</sup> Assuming the same capitalization rate for human capital and financial assets, this suggests that human capital represents less than one tenth of financial wealth for households at the top of the wealth distribution.<sup>51</sup>

I then proxy for  $\beta_h$ . In particular, [Guvonen et al. \(2017\)](#) reports the exposure of top labor income to stock market returns: for the top 0.1% of the income distribution, the beta of labor income growth to stock

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<sup>50</sup>See <https://www.irs.gov/pub/irs-soi/13intop400.pdf>.

<sup>51</sup>This result is linked to the fact that the distribution of financial wealth has a thinner tail than the distribution of financial wealth, see [Toda \(2012\)](#).

market returns is close to 0.45. Assuming a beta of the discount rate for human capital to stock market returns between 0 and 1, this corresponds to  $\beta_h \in (0.45, 1.45)$ .

Overall, I obtain that the bias is

$$0.1 \times (0.45/0.95 - 1) \leq \frac{\beta_{a+h} - \beta_a}{\beta_a} \leq 0.1 \times (1.45/0.95 - 1)$$

that is, the potential bias only accounts for less than 5% of the estimate.

## D.2 Calibrated Parameters

I calibrate the human capital using the life cycle evolution of labor income, following [Gârleanu and Panageas \(2015\)](#). The life cycle income of households  $G(u)$  is a sum of two exponentials approximating the hump shaped pattern of earnings observed in the data:

$$G(u) = B_1 e^{-\delta_1 u} + B_2 e^{-\delta_2 u} \tag{A57}$$

with  $B_1 = 30.72$ ,  $B_2 = -30.29$ .

I chose the share of endowment distributed as labor income as  $\omega = 92\%$ , following [Gârleanu and Panageas \(2015\)](#).<sup>52</sup>

## Appendix E Resolving the Tension

### E.1 Live-Fast-Die-Young

*Proof of Proposition 8.* Denote  $m_{At}^\xi$  the  $\xi$ -moment of relative wealth for households in group  $A$ , i.e.

$$\begin{aligned} m_{At}^\xi &= \left( \int_{i \in \mathbb{I}_{At}} w_{it}^\xi di \right) / N_t \\ &= \pi_A \int_{-\infty}^t (\delta + \eta) e^{-(\delta + \eta + \tau)(t-s)} w_{Ats}^\xi ds \end{aligned}$$

Applying Ito's lemma

$$dm_{At}^\xi = \left( \xi \mu_{w_{At}} + \frac{\xi(\xi - 1)}{2} \sigma_{w_{At}}^2 - \delta - n - \tau \right) m_{At}^\xi dt + (\delta + n) \pi_A m_{\chi t}^\xi dt + \xi \sigma_{w_{At}} m_{At}^\xi dZ_t \tag{A58}$$

Using Lemma 3, the distribution of wealth of households in group  $A$  has a tail index given by  $\zeta$ .

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<sup>52</sup>A lower value for the labor share would only strengthen my result: since a lower labor share decreases the wealth of arriving agents, it raises the average wealth growth of existing households. In equilibrium, this tends to increase the interest rate. But the model already tends to give an interest rate that is too high, as seen in Table 5.

Similarly, denote  $m_{Bt}^\xi$  the  $\xi$ -moment of wealth for households in group B

$$m_{Bt}^\xi = \left( \int_{i \in \mathbb{I}_{Bt}} w_{it}^\xi di \right) / N_t$$

We have, using Ito's lemma:

$$dm_{Bt}^\xi = \left( \xi \mu_{w_{Bt}} + \frac{\xi(\xi-1)}{2} \sigma_{w_{Bt}}^2 - \delta - n \right) m_{Bt}^\xi dt + (\delta+n)(1-\pi_A) m_{\chi t}^\xi dt + \tau m_{At}^\xi dt + \xi \sigma_{w_{Bt}} m_{Bt}^\xi dZ_t$$

The distribution of wealth for households in group B inherits the tail  $\zeta$  due to the transition from group A to group B.  $\square$

**Solving the Model** To solve the model, I follow the same steps as Appendix B.2. The wealth evolution Proposition 1 is not modified, because agents do not forecast any change in preferences.

## E.2 Time-Varying Investment Opportunities

**Solving the Model** To solve the augmented model, I look for a Markov equilibrium with two state variables  $(x_t, \nu_t)$ . The wealth of households in group  $j \in \{A, B\}$  follows the law of motion

$$\mu_{W_{At}} = \psi_A(r_t - \rho) + \frac{1 + \psi_A}{2\gamma_A} \kappa_t^2 + \frac{\psi_A - \psi_A}{\psi_A - 1} \kappa_t \sigma_{p_{At}} + \frac{1 - \psi_A}{2(\psi_A - 1)} \sigma_{p_{At}}^2 + \mu_{p_{At}} + \underbrace{\psi_A(1 - x_t)\nu_t}_{\neq \text{inv. opport.}} \quad (\text{A59})$$

$$\mu_{W_{Bt}} = \psi_B(r_t - \rho) + \frac{1 + \psi_B}{2\gamma_B} \kappa_t^2 + \frac{\psi_B - \psi_B}{\psi_B - 1} \kappa_t \sigma_{p_{Bt}} + \frac{1 - \psi_B}{2(\psi_B - 1)} \sigma_{p_{Bt}}^2 + \mu_{p_{Bt}} - \underbrace{\psi_B x_t \nu_t}_{\neq \text{inv. opport.}} \quad (\text{A60})$$

The expression for the interest rate becomes

$$r_t = \rho + \frac{1}{\Psi_t} \left( \mu + \mu_{p_t} + \sigma \sigma_{p_t} - (\delta + n) \left( \frac{\phi_t}{p_t} - 1 \right) - \left( x \frac{1 + \psi_A}{2\gamma_A} + (1 - x) \frac{1 + \psi_B}{2\gamma_B} \right) \kappa_t^2 - \Phi_t \right) - \underbrace{\frac{(\psi_A - \psi_B)x_t(1 - x_t)}{\Psi_t} \nu_t}_{\neq \text{inv. opport.}} \quad (\text{A61})$$

The interest rate decreases with  $\nu_t$  as long as  $\psi_A \geq \psi_B$ .

The law of motion for  $\nu_t$  is exogenously given by (59). The instantaneous volatility of  $x_t$  is now given by:

$$\sigma_x = \frac{\frac{x(1-x)\Gamma}{\gamma_A \gamma_B} ((\gamma_B - \gamma_A)\sigma + \frac{1-\gamma_A}{\psi_A-1} \frac{\partial_\nu p_A}{p_A} \sigma_\nu - \frac{1-\gamma_B}{\psi_B-1} \frac{\partial_\nu p_B}{p_B} \sigma_\nu)}{1 - \frac{x(1-x)\Gamma}{\gamma_A \gamma_B} ((\gamma_B - \gamma_A) \frac{p_x}{p} + \frac{1-\gamma_A}{\psi_A-1} \frac{\partial_x p_A}{p_A} - \frac{1-\gamma_B}{\psi_B-1} \frac{\partial_x p_B}{p_B})} \quad (\text{A62})$$

The growth rate of  $x$  is now given by:

$$\mu_x = x(1-x) \left( \kappa(\sigma_{W_{At}} - \sigma_{W_{Bt}}) - \frac{1}{p_A} + \frac{1}{p_B} + \nu_t \right) + (\delta+n) \frac{\phi}{p} (x - \pi_A) + x \sigma_{p_t Y_t}^2 - x \sigma_{W_{At}} \sigma_{p_t Y_t}$$

Given the law of motion of  $\nu_t$  and  $x_t$ , the model can be solved using the same method as Appendix B.2.

Table A1: The Equity Share Increases Across the Wealth Distribution

	Groups of Households Defined by Wealth Percentiles			
	All Households	1% – 0.1%	0.1% – 0.01%	Top 0.01%
<i>Panel A: All Sample</i>				
Equity Share	40.8%	55.8%	65.8%	73.9%
Public Equity	20.2%	22.0%	21.1%	19.6%
Private Equity	20.6%	33.9%	44.6%	54.4%
Non Actively Managed	2.4%	4.5%	6.3%	7.8%
Actively Managed	18.2%	29.4%	38.4%	46.6%
<i>Panel B: Stockholders</i>				
Is Stockholder	45.9%	90.7%	91.2%	91.0%
Equity Share among Stockholders	44.7%	56.0%	65.9%	76.0%
<i>Panel C: Entrepreneurs</i>				
Is Entrepreneur	10.5%	62.1%	69.8%	78.5%
Equity Share among non-Entrepreneurs	26.8%	40.7%	50.0%	57.9%
<i>Panel D: Stock Options Holders</i>				
Received Stock Options	6.4%	11.2%	11.5%	6.1%
Equity Share among non Stock Options Holders	44.7%	56.0%	65.9%	76.0%
Share of Total Wealth		20.7%	7.7%	3.8%
Labor Income / Wealth	12.6%	2.9%	1.6%	0.7%

*Notes:* Data from SCF 1989-2013. The variable Equity Share is defined as private equity + public equity over net worth:  $(\text{equity} + \text{bus}) / \text{net worth}$ . Stockholders are defined as the households that hold public equity. Entrepreneurs are defined as the households with an active management role in one of the company they invest in.

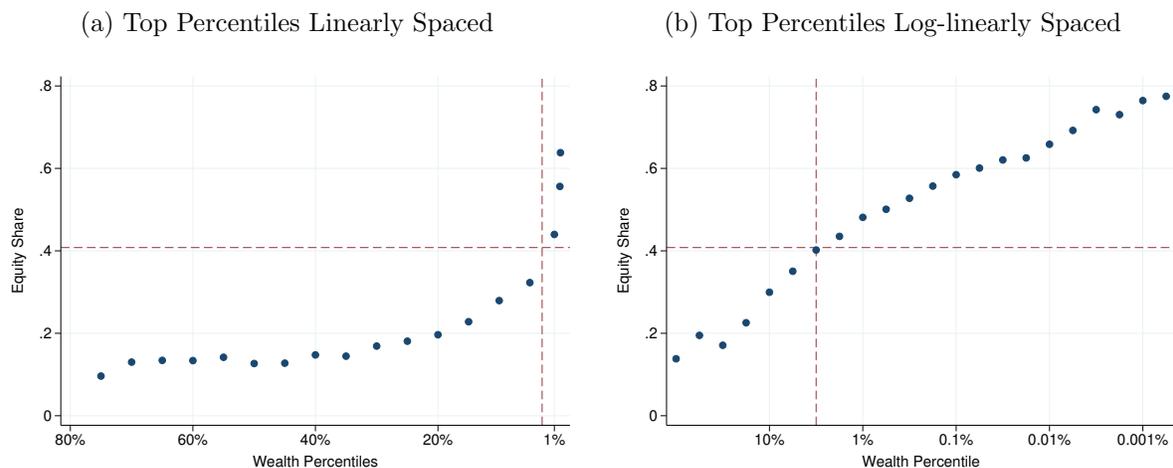
Table A2: The Exposure to Stock Returns Across the Wealth Distribution:  
Saez and Zucman (2016) Series

	Wealth Growth Within Percentile Thresholds		
	Saez and Zucman (2016)		
	1 – 0.1%	0.1 – 0.01%	Top 0.01%
	(1)	(2)	(3)
Excess Stock Returns	0.49*** (0.10)	0.49*** (0.11)	0.66*** (0.15)
$R^2$	0.19	0.17	0.18
Period	1914-2009	1914-2009	1914-2009
$N$	96	96	96

*Notes:* This table reports the results of the regression of the wealth growth of households in a given percentile group on asset returns.

Estimation is via OLS. Standard errors in parentheses and estimated using Newey-West with 4 lags. \*, \*\*, \*\*\* indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Figure A1: The Equity Share Increases Across the Wealth Distribution



*Notes.* Figure A1a plots the average equity share within 20 linearly spaced percentile bins in the wealth distribution. Figure A1b plots the average equity share within 20 logarithmically spaced percentile bins in the wealth distribution. The horizontal line represents the average equity share. The vertical line splits the set of households in two: households on either side of the vertical line own half of total wealth (this corresponds to top percentile  $\approx 3\%$ ). Data from the Survey of Consumer Finance (SCF), a cross-sectional survey of US households from 1989 to 2013. The equity share is constructed as  $(\text{equity} + \text{bus}) / \text{net worth}$ .

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