The paper analyzes capital structure from the perspective of control rights - who owns the asset controls it - in a framework of incomplete contracts with wealth constraints. The existence of wealth constraints is a key difference from the original Grossman-Hart (1986) paper. Note that these approaches always lead to debt, never equity. For some recent work on equity financing, see Myers, *Journal of Finance*, 1998.

The Model Setup

In this bilateral contracting setting, the two parties are:

- the entrepreneur/manager: needs setup costs $K$ for a project, has zero wealth, and has all the bargaining power (i.e. he can make a take-it-or-leave-it offer);
- the wealthy investor: needs an expected payoff of $K$. 
The timing of the game is as follows: at time $t = 0$ investment $K$ is taken, at time $t = 1$ the state $\theta$ is realized and the signal $s$ is observed, then an action is taken, and finally at time $t = 2$, the returns $r$ are realized.

Both the entrepreneur (manager) and the investor are risk-neutral, with utility functions

$$
U_E(r, a, \theta) = r + l(a, \theta) \\
U_I(r, a, \theta) = r
$$

where $r$ is the realization of the returns and $l(a, \theta)$ is a non-monetary payoff to the entrepreneur.

Note that in a complete contracts framework, the contract specifies a mapping $\alpha : \Theta \rightarrow A$, so that $\alpha(\theta) \rightarrow a$, where $\theta \in \Theta$ and $a \in A$. Then this needs to be enforced.

Assumption 1: The state of nature $\theta$ cannot be contracted on. Ex-post, $\theta$ is known, but it is not contractible.

Assumption 2: There exists a publicly verifiable signal $s$ that is imperfectly correlated with $\theta$.

Assumption 3: All monetary payoffs are verifiable and contractible (i.e. $r$).

Other assumptions:

- There are two states of nature, so that $\Theta = \{\theta_g, \theta_b\}$;
• There are two actions in the action set: \( A = \{a_g, a_b\} \). These actions are the state-dependent optimal actions, i.e. \( a_g = a^*(\theta_g) \) and \( a_b = a^*(\theta_b) \);

• The signal \( s \in \{0, 1\} \). Letting \( \beta = P[s = 1|\theta] \), we assume that

\[
\beta^g = \Pr[s = 1|\theta = \theta_g] > \frac{1}{2} \\
\beta^b = \Pr[s = 1|\theta = \theta_b] < \frac{1}{2}
\]

Given \( \beta^g \) and \( \beta^b \) defined above, let \( d(\tilde{\beta}, (1, 0)) = [|1 - \beta^g| + |0 - \beta^b|] \). Then \( d \) measures the degree of ”contract incompleteness”. If \( d = 0 \) there is perfect correlation between the state of nature \( \theta \) and the signal \( s \). If \( d = 1 \) there is no correlation.

• The return \( r \in \{0, 1\} \)

Let \( y^j_i \) denote the expected final-period return in state \( \theta_i \) when action \( a_j \) is chosen, and \( l^j_i \) the private benefit of the entrepreneur in state \( \theta_i \) when action \( a_j \) is chosen. It follows that

\[
y^j_i = E(r|\theta = \theta_i, a = a_j) = \Pr[r = 1|\theta = \theta_i, a = a_j]
\]

The First Best

\[
qg_g^g + (1 - q)g_b^b > K \quad \text{(feasibility)} \\
g_g^g + l_g^g > \bar{g}_g^g + \bar{l}_g^g \quad \text{\(a_g \) when \( \theta_g \)} \\
g_b^b + l_b^b > \bar{g}_b^b + \bar{l}_b^b \quad \text{\(a_b \) when \( \theta_b \)}
\]

1 \ where \( q = \Pr[\theta = \theta_g] \).

The Contract

2 A contract consists of:

(i) Compensation schedule for the entrepreneur/manager

\[
t(s, r) \geq 0 \\
t(a, s, r) \geq 0 \text{ if actions are verifiable}
\]
(ii) Control allocation rule

- Individual control: \((\alpha_0, \alpha_1) \in [0, 1]^2\) where \(\alpha_s \equiv \text{probability that the entrepreneur gets the right to decide what action to choose when } s = 1 \text{ or } s = 0, \text{ and } (1 - \alpha_s) \equiv \text{probability that the investor gets control}

- Joint control: \((\mu_{0I}, \mu_{1I}) \in (0, 1)^2\) and \((\mu_{0E}, \mu_{1E}) \in (0, 1)^2\) where \(\mu_{0I} + \mu_{0E} > 1\) for some \(s \in \{0, 1\} \).

In the case of joint control, both parties have control in some state of the world, and they have to agree on an action, otherwise they both get zero payoffs. It is assumed that the entrepreneur has all the bargaining power and thus makes a take-it-or-leave-it offer. If the investor accepts, the proposed action is taken; if he rejects, the payoff vector is \((0, 0)\).

Non-verifiable Actions

Three types of control allocation rules are considered: unilateral control, contingent control and joint control. The latter is shown to be (weakly) dominated.

Entrepreneur Control

Since there are only two possible returns \((r \in \{0, 1\})\), we can always write:

\[
\begin{align*}
t(s,r) &= (t(s,1) - t(s,0))r + t(s,0) \\
&= t_s r + t_s' \\
\therefore \quad \text{the contract is affine with only two states}
\end{align*}
\]
The entrepreneur chooses an action $a^E(\theta_i, s)$ such that:

$$a^E(\theta_i, s) = \arg \max_{a_j \in A} \{t_s y^i_j + t'_s + l^i_j\}$$

**Renegotiation**

Suppose a contract that specifies $(t_s, t'_s)$ results in entrepreneur’s decision that for any $\theta$

$$a_g \text{ when } s = 1$$

$$a_b \text{ when } s = 0$$

Consider the case $(\theta_b, s = 1)$. In this case, the optimal action is $a_b$. If the contract is not renegotiated, the investor gets:

$$U_I = y_b^b (1 - t_1) - t'_1$$

$$= y_b^b - (t_1 y_b^b + t'_1)$$

If the contract is renegotiated, the entrepreneur offers the investor $y_g^b (1 - t_1) - t'_1$ but takes action $a_b$ and gets:

$$(y_b^b + l_b^b) - y_g^b (1 - t_1) + t'_1$$

$$(y_b^b + l_b^b) - y_g^b (1 - t_1) + t'_1 = t_1 y_g^b + t'_1 + l_b^b$$
A similar argument applies to the case \((\theta_g, s = 0)\).

Implication 1  
If the entrepreneur has full control, ex-post renegotiation guarantees achieving the first best.

Implication 2  
This is optimal if the investor’s ex-ante individual rationality constraint can be met.

3 Proposition 1  
If private benefits \(l\) are comonotonic with \((y + l)\), i.e. if \(l^g_y > l^b_y\) and \(l^b_y > l^g_y\), entrepreneur control is always feasible.

4 Proof. Choose \(t(s, r) = t \forall s, r \Rightarrow \) the entrepreneur chooses first best because

\[
\begin{align*}
    t + l^g_y &> t + l^b_y \\
    t + l^b_y &> t + l^g_y
\end{align*}
\]

5 Now choose \(t\) such that \([qy^g_y + (1 - q)y^b_y] - t = K\). Feasibility ensures such a \(t\) exists. ■

6 Suppose now that \(l^b_y < l^g_y\), i.e. comonotonicity is lost. Define:

\[
\begin{align*}
\Delta^b &\equiv (y^b_y + l^b_y) - (y^g_y + l^g_y) > 0 \\
\Delta^b_y &\equiv y^b_y - y^g_y > 0 \\
\pi_1 &\equiv [qy^g_y + (1 - q)y^b_y] \frac{\Delta^b}{\Delta^b_y} \\
\pi_2 &\equiv qy^g_y + (1 - q)y^b_y \\
\pi_3 &\equiv q[\beta^g y^g_y + (1 - \beta^g)y^b_y] \frac{\Delta^b}{\Delta^b_y} + (1 - q)[\beta^b y^g_y + (1 - \beta^b)y^b_y] \frac{\Delta^b}{\Delta^b_y}
\end{align*}
\]
7 Proposition 2   **Entrepreneur control is feasible and implements the first best solution if and only if** \(\max(\pi_1, \pi_2, \pi_3) \geq K\). If \(K \in [\max(\pi_1, \pi_2, \pi_3), qy_g + (1-q)y_b]\) entrepreneur control is not feasible.

**Example** Let \(Pr[\theta = \theta_g] = \frac{1}{2}, y_g = 100, t_g = 150, y_b = 200, t_b = 0, y_b = 50, t_b = 0, y_b = 49\).

Restrict attention to contracts of the form \(t'_s = 0, t_s \leq 1\). We can do this because if \(t'_s > 0\), then we can lower \(t'_s\), so that the investor’s IC is relaxed while incentives are not affected. Then we can reallocate in a way not to affect incentives \(\therefore\) consider \(t(s, r) = t_s r\) where \(t_s \leq 1\).

7.1 No Renegotiation

Note the investor always chooses \(a_g\) in state \(\theta_g\). In state \(\theta_b\), we need \(t_s\) big enough so

\[
\begin{align*}
t_s y_b + t_b & \geq t_s y_g + t_g \\
t_s 50 & \geq 49 \\
t_s & \geq \frac{49}{50}
\end{align*}
\]

So for \(t_s = \frac{49}{50}\) no renegotiation occurs. Investor’s maximum payoff is:

\[
\pi_1 = \frac{1}{2 \ 50} \cdot 100 + \frac{1}{2 \ 50} \cdot 500 = \frac{3}{2}
\]

\(\therefore \ K < \frac{3}{2}\)
7.2 Full Renegotiation in state $\theta_b$

7.3 In this case the contract is such that

\[
\begin{align*}
t_s y_b^b + t_b^b &< t_s y_g^b + l_g^b \\
t_s y_b^b &< l_g^b \\
t_s &< \frac{49}{50}
\end{align*}
\]

7.4 Since $t_s < \frac{49}{50}$, the entrepreneur chooses $a_g$ without renegotiation, yielding a payoff of zero for the investor $\implies$ on renegotiation, the investor gets zero as well. The ex-ante expected payoffs from the contract for the two parties are:

\[
\begin{align*}
\text{Entrepreneur : } &\quad \left(\frac{1}{2}t_s 100 + \frac{1}{2} \cdot \frac{50}{100}\right) + \frac{1}{2} \cdot \frac{50}{100} \\
\text{Investor : } &\quad \frac{1}{2}(1 - t_s) 100
\end{align*}
\]

7.5 Therefore, the investor expected payoff is $\pi_2 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{50}{100} = 1$ if $t_s = \frac{49}{50}$, and $\pi_2 = \frac{1}{2} \cdot 100 = 50$ if $t_s = 0 \implies$ the range of investments that can be supported in this case is $K \in (1, 50)$. If $K \in (1, \frac{3}{2})$, full renegotiation yields the same outcome in terms of efficiency as no renegotiation (i.e. the first best is implemented).

7.6 Partial renegotiation in state $\theta_b$

7.7 In this case, we have $t_1 \neq t_0$. By setting $t_1 < \frac{49}{50}$, renegotiation occurs in state $\theta_b$ only when $s = 1$, so that $a_b$ is implemented after renegotiation. By setting $t_0 > \frac{49}{50}$, the action $a_b$ is implemented in $\theta_b$ when $s = 0$. With these constraints, the values for $t_s$ that maximize investor’s payoff are: $t_1 = 0, t_0 = \frac{49}{50}$. The investor’s expected payoff is:

\[
\pi_3 = \frac{1}{2} \left[ \beta^g 100 + (1 - \beta^g)(1 - \frac{49}{50})100 \right] + \frac{1}{2} \left[ \beta^g 0 + (1 - \beta^g)(1 - \frac{49}{50})50 \right] = 50\beta^g + 1 - \beta^g + \frac{1 - \beta^b}{2}
\]
7.8 As $\beta^g \to 1$ and $\beta^b \to 0$, $\pi_3$ gets larger and we get $\pi_3 \to 50.5 > 50$. For $K \in (50, 50.5)$ only partial renegotiation can occur.

Investor Control

8 Proposition 3 When monetary benefits are comonotonic with total revenues ($y^g_g > y^g_b$ and $y^b_b > y^b_g$) the first best can be achieved under investor control.

9 Proof: Let $t(s, r) = t \cdot r$. Then:

$$(1-t)y^g_g > (1-t)y^g_b \Rightarrow \pi^g(a_g) > \pi^g(a_b) \quad \text{q.e.d.}$$

10 Set $t$ such that the investor’s participation constraint holds.

11 Without loss of generality, consider now the case when $y^g_g < y^g_b$, so that comonotonicity fails. Set $t'_s = 0$. Since $(1-t_s)y^g_g < (1-t_s)y^g_b$, the investor does not take the first-best action $a_g$ in state $\theta_g$.

12 When $t_s < 1$, we need to renegotiate (i.e. agree on a payment $\hat{t}_s$) so that

$$(1-\hat{t}_s)y^g_g \geq (1-t_s)y^g_b$$

or $\hat{t}_s \leq 1 - (1-t_s)\frac{y^g_b}{y^g_g}$
13 So if $\frac{y_b}{y_g}$ is large or $t_s$ is small, the entrepreneur’s wealth constraint may be violated. We need to ensure that

$$i_s \geq 0$$

or $$(1 - t_s)\frac{y_b}{y_g} \leq 1$$

thus $t_s \geq 1 - \frac{y_b}{y_g}$

14 Proposition 4 If monetary benefits are not comonotonic with total returns, a necessary and sufficient condition for implementing the first-best action plan under investor control is:

$$\pi_4 = [qy_g + (1 - q)y_b] \frac{y_b}{y_g} \geq K$$

15 Example $y_b = 50; \quad y_g = 100 < y_b = 200$ ⇒ Investor control is first-best efficient if

$$\pi_4 = (\frac{1}{2}200 + \frac{1}{2}50)\frac{1}{2} = 62.5 > K.$$
To analyze this type of control, we assume that neither monetary nor private benefits are comonotonic with total benefits: \( y^g_g < y^g_b \) and \( l^b_g < l^b_g \). Consequently, in state \( \theta_b \), the investor chooses the first-best, while in state \( \theta_g \), the entrepreneur chooses the first-best. However, control cannot be made contingent on the state \( \theta \), but only on the signal \( s \). If \( s \) is highly correlated with \( \theta \), and setting \( t(s, r) = 0 \), the optimal contract will specify: \[
\begin{align*}
\begin{cases}
  & s = 1 \Rightarrow \text{Entrepreneur control} \\
  & s = 0 \Rightarrow \text{Investor control}
\end{cases}
\end{align*}
\]

With no renegotiation of contract and contingent control on \( s \), we have:

17.1 In state \( \theta_g \), \( a = \begin{cases} a_g & \text{if } s = 1 \text{ - entrepreneur (efficient)} \\
  a_b & \text{if } s = 0 \text{ - investor (inefficient)}
\end{cases} \)

17.2 In state \( \theta_b \), \( a = \begin{cases} a_g & \text{if } s = 1 \text{ - entrepreneur (inefficient)} \\
  a_b & \text{if } s = 0 \text{ - investor (efficient)}
\end{cases} \)

The investor’s expected payoff is:
\[
\pi_C = q\beta^g_g y^g_g + (1 - \beta^g_g) y^g_b + (1 - q)\beta^b_b y^b_b + (1 - \beta^b_b) y^b_b
\]

As \( \beta^g_g \rightarrow 1 \) and \( \beta^b_b \rightarrow 0 \), \( \pi_C \) approaches the first-best monetary payoff \( \Rightarrow \pi_C > \pi_1, \pi_C > \pi_2 \).

As \( \beta^g_g \rightarrow 1 \) and \( \beta^b_b \rightarrow 0 \), \( \pi_3 \) approaches the first best monetary payoff as well, but
\[
\pi_C - \pi_3 = q(1 - \beta^g_g)(y^b_b - y^g_b \Delta^b_g) + (1 - q)(1 - \beta^b_b)(1 - \frac{\Delta^b_b}{\Delta^b_b}) y^b_b
\]
21 Now $\frac{\Delta \beta}{\Delta \theta} < 1$ since $l_{b} < l_{g}$ and $y_{g} < y_{b}$, so $\pi_{C} > \pi_{3} \Rightarrow$ as $\beta^{g} \rightarrow 1$ and $\beta^{b} \rightarrow 0$ there exist values for $K$ for which contingent control is feasible but entrepreneurial control is not. Suppose $\pi_{4} < K$ (so the condition for Proposition 4 does not hold). Then investor control achieves $a_{b}$ in state $\theta_{g}$, not the first-best. Under contingent control, as $\beta^{g} \rightarrow 1$ and $\beta^{b} \rightarrow 0$ the first-best is implemented.

22 Proposition 5  When neither monetary nor private benefits are comonotonic with total benefits, there exist values of $K$ such that:

22.1 *(i)* entrepreneur control is not feasible

22.2 *(ii)* investor control is not first-best efficient

22.3 *(iii)* contingent control ($\alpha_{0} = 0, \alpha_{1} = 1$) dominates when ($\beta^{g}, \beta^{b}$) $\rightarrow (1, 0)$.

23 The alternative of joint control/ownership exhibits the ex-post hold-up problem as either party has veto power. Examples include joint ventures and alliances. The instruments that implement the control structures discussed in the paper are:

- Entrepreneur control $\rightarrow$ Preffered stock
- Investor control $\rightarrow$ Equity voting
- Contingent control $\rightarrow$ Debt
- Joint control $\rightarrow$ Partnership

24 Venture Capital applications seem to be consistent.