Week 5: Collateral, Rental Markets, and Repo Markets

• Collateral key determinant of capital structure
  ◦ Enforcement of repayment by borrower limited to tangible assets
  ◦ Nature of assets required for production determine financing
  ◦ Readings: Rampini/Viswanathan (2013), Collateral and capital structure

(Frictionless) Neoclassical Theory of Investment

• Environment
  ◦ Time 0 and 1
  ◦ Investor/owner

• Preferences
  ◦ Investor is risk neutral and discounts time 1 payoffs at rate $R^{-1} < 1$

• Endowments
  ◦ Investor net worth $w \gg 0$, i.e., deep pockets (Holmström/Tirole’s (1997) called this $A$)

• Technology
  ◦ Capital $k$ invested at time 0
  ◦ Payoff (“cash flow”) at time 1
    \[ Af(k) \]
    where parameter $A$ is “total factor productivity” (TFP)
  ◦ Strict concavity $f'(k) > 0$ and $f''(k) < 0$; also: $\lim_{k \to 0} f'(k) = +\infty$; $\lim_{k \to \infty} f'(k) = 0$
  ◦ Capital is durable and depreciates at rate $\delta$; capital $(1 - \delta)k$ remains at time 1
Neoclassical Investment and User Cost of Capital

- Investor’s problem (objective: maximize “value” – present discounted value of dividends)
  - Choose dividends $d_0$ at time 0 and $d_1$ at time 1 and invest capital $k$ to solve
    \[
    \max_{\{d_0,d_1,k\}} d_0 + R^{-1}d_1
    \]
    subject to budget constraints (but no limited liability constraints)
    \[
    w \geq d_0 + k \\
    Af(k) + k(1 - \delta) \geq d_1
    \]
  - First-order conditions (FOCs) (multipliers $\mu_0$ and $\mu_1$)
    \[
    1 = \mu_0 \\
    R^{-1} = \mu_1 \\
    \mu_0 = \mu_1 [Af'(k) + (1 - \delta)]
    \]

- Optimal investment/capital $k^*$ solves (combining FOCs)
  \[
  1 = R^{-1} [Af'(k) + (1 - \delta)]
  \]
  or letting $R \equiv 1 + r$ and rewriting
  \[
  \underbrace{r + \delta}_{\text{user cost of capital}} = \underbrace{Af'(k)}_{\text{marginal product of capital}}
  \]

- Jorgenson’s (1963) user cost of capital
  \[
  u \equiv \underbrace{r}_{\text{interest rate}} + \underbrace{\delta}_{\text{depreciation rate}}
  \]

Collateral Constraints as in Rampini/Viswanathan (2013)

- Environment with frictions (otherwise as before)
  - Environment: Two types of agents, owner/borrower and investor/lender
  - Borrower is risk neutral, impatient $\beta < R^{-1}$, and subject to limited liability
  - Borrower has limited funds $w > 0$; lender has deep pockets
  - Collateral constraints: Need to collateralize loan repayment with tangible assets

- Financing problem with collateral constraints (where $\theta < 1$)
  \[
  \max_{\{d_0,d_1,k,b\}} d_0 + \beta d_1
  \]
  subject to budget constraints and collateral constraint
  \[
  w + b \geq d_0 + k \\
  Af(k) + k(1 - \delta) \geq d_1 + Rb \\
  \theta k(1 - \delta) \geq Rb
  \]
  and limited liability $d_0, d_1 \geq 0$

- First-order conditions (FOCs) (multipliers $\mu_0, \mu_1$, and $\lambda$)
  \[
  1 \leq \mu_0, \quad \beta = \mu_1 \\
  \mu_0 = \mu_1 [Af'(k) + (1 - \delta)] + \lambda \theta (1 - \delta), \quad \mu_0 = \mu_1 R + \lambda R
  \]

- Optimal investment/capital $k$ solves (combining FOCs)
  \[
  [1 - R^{-1} \theta (1 - \delta)] \mu_0 = \beta [Af'(k) + (1 - \theta) (1 - \delta)]
  \]
Collateral Constraints: Tangible Assets and Capital Structure

- “Minimal downpayment” (per unit of capital)
  \[ \varphi \equiv 1 - \frac{R^{-1}\theta(1 - \delta)}{PV\ of\ \theta\times\ resale\ value\ of\ capital} \]

- Capital structure
  - Collateral constraints bind: Using FOCs and noting \( \beta R < 1 \)
    \[ \beta R + \lambda R = \mu_0 \geq 1 \quad \Rightarrow \quad \lambda > 0, \quad i.e., \quad Rb = \theta k(1 - \delta) \]
  - Debt per unit of capital
    \[ R^{-1}\theta(1 - \delta) \]
  - Internal funds per unit of capital
    \[ \varphi = 1 - R^{-1}\theta(1 - \delta) \]

- Collateralizability \( \theta \)
  - Structures more collateralizable than equipment (composition varies by industry)
  - Financial development may raise \( \theta \) and hence leverage

- Tangibility (includes mainly structures (incl. land) and equipment)
  - Suppose tangible assets are collateralizable (but not intangible assets)
  - Fraction tangible assets (\( \varphi \)) needed for production key: \( \varphi = 1 - R^{-1}\varphi\theta(1 - \delta) \)

Investment and Dividend Policy

- Investment policy
  - Investment FOC
    \[ 1 \leq \beta Af'(k) + (1 - \theta)(1 - \delta) \]
    with equality if \( d_0 > 0 \)
  - Dividend paying firm: Capital \( k \) solves equation above
    - Comparing FOCs can show \( k < k^* \) (underinvestment)
  - Non-dividend paying firm: \( k = \frac{1}{\mu}w \) (invest all net worth and lever as much as possible)

- Dividend policy (threshold policy)
  - Pay out dividends today \( (d_0 > 0) \) if \( w \geq \bar{w} \)
    - Can we show threshold is optimal? Suppose pay dividends at \( w \) but not at \( w^+ > w \)
    - At \( w \), invest \( \bar{k} \); if not paying dividends at \( w^+ \), must invest more; can FOC hold?

- Value of internal funds \( \mu_0 \) (remember the envelope condition?)
  - Premium on internal funds (unless firm pays dividends) since \( \mu_0 \geq 1 \)

- User cost \( u(w) \)
  - User cost such that \( u(w) = R\beta \frac{1}{\mu_0}Af'(k) \) where
    \[ u(w) \equiv r + \delta + R\frac{\lambda}{\mu_0}(1 - \theta)(1 - \delta) \quad \Rightarrow \quad u \]
Limited Enforcement Implies Collateral Constraints

- **Question**: Why does borrower need to collateralize loans?
  - Enforcement is limited and it has to be incentive compatible for borrower to repay

- **Friction**: Limited enforcement
  - Borrower can abscond with all cash flows and fraction $1 - \theta$ of (depreciated) capital

- **Limited enforcement implies collateral constraints**
  - **Enforcement constraint**
    - Ensure that borrower prefers to repay instead of absconding
    
    $Af(k) + k(1 - \delta) - Rb = d_1 \geq Af(k) + (1 - \theta)k(1 - \delta)$
    
    - **Collateral constraint**
      - Canceling terms and rearranging enforcement constraint we obtain
      
      $\theta k(1 - \delta) \geq Rb$

Dynamic Financing

- **Dynamics**
  - Suppose financing problem repeats itself at $t = 0, 1, 2, \ldots$ (infinite horizon)

- **Dynamic programming** (Bellman (1953))
  - Suppose function $v(w)$ summarizes value to borrower from having net worth $w$
  - Problem: Function $v(w)$ is unknown!
    - Richard Bellman’s key insight: $v(w)$ must solve a particular (functional) equation!
  - Financing problem with collateral constraints (Bellman equation)
    
    $v(w) \equiv \max_{\{d,k,b,w^\prime\}} d + \beta v(w^\prime)$
    
    subject to budget constraints and collateral constraint
    
    $w + b \geq d + k$
    
    $Af(k) + k(1 - \delta) \geq w^\prime + Rb$
    
    $\theta k(1 - \delta) \geq Rb$
    
    and limited liability $d \geq 0$
  - Net worth next period $w^\prime = Af(k) + k(1 - \delta) - Rb$
  - Note: Problem looks almost exactly as before!

- Dynamic programming is a remarkably powerful tool to solve dynamic problems
Net Worth Accumulation and Firm Growth

- First-order conditions (FOCs) for dynamic problem (multipliers $\mu$, $\mu'$, and $\lambda$)

\[
1 \leq \mu, \quad \beta v'(w') = \mu', \quad \mu = \mu'[Af'(k) + (1 - \delta)] + \lambda \theta (1 - \delta), \quad \mu = \mu' R + \lambda R
\]

- Note: FOCs look almost exactly as before!
- Also: Envelope condition $v'(w) = \mu$

- Dividend policy and net worth accumulation
  - Dividend policy is threshold policy
    - For $w \geq \bar{w}$, pay dividends $d = w - \bar{w}$
    - For $w < \bar{w}$, pay no dividends and reinvest everything ("retain all earnings")

- Investment policy and firm growth
  - For $w \geq \bar{w}$, keep capital constant at $\bar{k}$ (no growth)
  - For $w < \bar{w}$, invest everything $k = 1/p w$ resulting in net worth $w' > w$ next period

- Firm age
  - Young firms ($w < \bar{w}$) do not pay dividends, reinvest everything, and grow
  - Mature firms ($w \geq \bar{w}$) pay dividends and do not grow

Conclusions

- Tangible assets as collateral
  - If debt needs to be collateralized, type of assets required determines capital structure

- Dynamics of financing
  - Accumulate net worth over time
  - Young firms grow and retain all earnings
  - Mature firms pay dividends and grow less
Week 5: Collateral, Rental Markets, and Repo Markets
(Cont’d)

• Rental markets
  ◦ Leasing has repossession advantage and permits greater borrowing
  ◦ Severely constrained firms (and households) lease
  ◦ Readings: Rampini/Viswanathan (2013), Collateral and capital structure

• Repurchase (Repo) agreements
  ◦ Collateralized loans in which lender (temporarily) owns collateral
  ◦ Key aspect of financial crisis?
  ◦ Readings: Gorton/Metrick (2012), Securitized banking and the run on repo

Leasing as in Rampini/Viswanathan (2013)

• Environment with collateral constraints (as in last class) but firms can lease
  ◦ Environment: Two types of agents, owner/borrower, investor/lender, and lessor
  ◦ Borrower is risk neutral, impatient \( \beta < R^{-1} \), and subject to limited liability
  ◦ Borrower has limited funds \( w > 0 \); lender and lessor have deep pockets

• Borrowing subject to collateral constraints
  ◦ Need to collateralize promises to pay with tangible assets (due to limited enforcement)
  ◦ Promised repayment \( \leq \theta \times \) resale value of tangible assets

• Leasing: Borrower can rent capital
  ◦ Repossession advantage: Borrower cannot abscond with leased capital
    • In practice, repossession of rented capital easier than foreclosure on secured loan
    • Leasing allows borrower to borrow full resale value, not just fraction \( \theta \)
  ◦ Monitoring cost \( m \) (per unit of capital): Lessor needs to monitor to prevent abuse
    • Why? – Leasing separates ownership and control
  ◦ User cost of leased capital (assuming lessors, like lenders, discount at \( R^{-1} \))
    \[
    u_t = r + \delta + m
    \]
    needs to be paid in advance, i.e., at time 0
Lease or Buy?

- **Firm’s problem with leasing** \((k_o \text{ owned capital}; k_l \text{ leased capital})\)

\[
\max_{(d_0, d_1, k_o, k_l)} \ d_0 + \beta d_1
\]

subject to budget constraints and collateral constraint

\[
w + b \geq d_0 + k_o + R^{-1} u_l k_l \]

\[
Af(k_o + k_l) + k_o(1 - \delta) \geq d_1 + Rb
\]

\[
\theta k_o(1 - \delta) \geq Rb
\]

and non-negativity constraints \(k_o, k_l \geq 0\), as well as limited liability \(d_0, d_1 \geq 0\)

- First-order conditions \((FOCs)\) (multipliers \(\mu_0, \mu_1\), and \(\lambda\); let \(k \equiv k_o + k_l\)): As before,

\[
1 \leq \mu_0, \quad \beta = \mu_1, \quad \mu_0 = \mu_1 R + \lambda R
\]

and almost as before (except inequality as borrower might not own any assets)

\[
\mu_0 \geq \mu_1[Af'(k) + (1 - \delta)] + \lambda \theta(1 - \delta) \iff u(w) \geq R\beta \mu_0^{-1} Af'(k)
\]

and finally new

\[
R^{-1} u_l \mu_0 \geq \mu_1 Af'(k) \iff u_l \geq R\beta \mu_0^{-1} Af'(k)
\]

- **Leasing policy**

  - Lease if \(u_l < u(w)\) and buy otherwise (“choose capital with lower user cost”)
  
  - Recall: \(u_l = r + \delta + \frac{m}{\text{monitoring cost}}\) and \(u(w) = r + \delta + \frac{R\lambda}{\mu_0(1 - \theta)(1 - \delta)}\) premium on internal funds required

Leasing as Costly Way to Borrow More

- **Incremental cash flows of buying vs. leasing**

<table>
<thead>
<tr>
<th>Time</th>
<th>Buying (with secured loan)</th>
<th>Leasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 - (R^{-1} \theta(1 - \delta))</td>
<td>(R^{-1} u_l)</td>
</tr>
<tr>
<td>1</td>
<td>((1 - \theta)(1 - \delta))</td>
<td>((1 - \theta)(1 - \delta))</td>
</tr>
</tbody>
</table>

- **Implicit interest rate on additional amount borrowed when leasing**

\[
R_l \equiv \frac{(1 - \theta)(1 - \delta)}{\varphi - R^{-1} u_l} = R \frac{1 - \frac{m}{(1 - \theta)(1 - \delta)}}{1 - \frac{1}{(1 - \theta)(1 - \delta)}} > R
\]

  - Leasing is costly financing since \(R_l > R\)

- **Implicit “down payment” when leasing**

\[
R^{-1} u_l = 1 - \frac{R^{-1} \theta(1 - \delta)}{\text{financed at } R} - \frac{R^{-1} \theta(1 - \theta)(1 - \delta)}{\text{financed at } R_l}
\]

- **Who leases?**

  - Severely constrained firms do!
  
  - As \(w \to 0, k \to 0\) and \(f'(k) \to +\infty\); hence, using \(FOCs\), \(\mu_0 \to +\infty\) and

\[
R\lambda/\mu_0 = 1 - \frac{\beta}{\mu_0 R} \to 1 \Rightarrow u(w) \to r + \delta + (1 - \theta)(1 - \delta)
\]

  - Assuming \((1 - \theta)(1 - \delta) > m\), borrowers with sufficiently low \(w\) lease all their capital!
Repo Markets as discussed in Gorton/Metrick (2012)

- **Collateralized financing**
  - Securitization
    - Pooling of assets, sold to separate legal entities (special purpose vehicles (SPVs))
    - SPVs are financed with (mostly) debt of different seniority (tranching)
  - **Repo (“repurchase agreements”)**
    - Agreement to sell and repurchase security; form of (super-)collateralized financing

- **Cost of financing during crisis**
  - Spreads (on asset backed securities) and repo rates (interest rate on repos) “blow out”

- **Facts**
  - Spreads on various asset classes correlated with
    - ... LIBOR-OIS spread but not ABX (Asset-Backed subprime RMBS Index)
  - **Haircuts** (essentially “down-payment requirements”) on repos are correlated with
    - ... volatility but not LIBOR-OIS spread or ABX

- **Questions**
  - What type of collateralized financing are repos arguably (and remarkably) similar too?
  - Repos are collateralized – why would there be a run?
  - Is there a “run on repo” during the financial crisis and, if so, in what sense?

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**Conclusions**

- **Rental markets**
  - Renting capital facilitates repossession
  - Lessor is financier but retains ownership
  - Leasing permits greater leverage which is beneficial for severely constrained firms

- **Repo markets**
  - Collateralized loans in which lender owns collateral
  - Haircuts vary with volatility - but why?