Dynamic Collateralized Finance

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Aim: Tractable Dynamic Model of Collateralized Financing

- Key friction: limited enforcement
  - Enforcement of repayment by borrower limited to tangible assets
  - Implication: collateral constraints
    - Promises are not credible unless collateralized
  - Implementation: complete markets in one-period Arrow securities
    - Tractable!

- Key substantive implications
  
  (1) Capital structure
    - Determinant: fraction tangible assets required for production
  
  (2) Risk management
    - Involves state contingent promises and needs collateral
    - Opportunity cost: forgone investment
    - Severely constrained firms do not hedge
  
  (3) Leasing and rental markets
    - Leasing has repossession advantage and permits greater borrowing
    - Severely constrained firms lease

- Useful laboratory to study dynamics of financial constraints
Papers on Dynamic Collateralized Finance

- **Corporate capital structure, risk management, and leasing**

- **Financial intermediation**

- **Household insurance and other applications**
(1) Capital Structure

- **Collateral key determinant of capital structure**
  - Enforcement of repayment by borrower limited to tangible assets
  - Nature of assets required for production determines financing

- Key papers: Rampini/Viswanathan (2010, 2013)
(Frictionless) Neoclassical Theory of Investment

- **Environment**
  - Discrete time, infinite horizon, deterministic (for now)
  - Investor/owner

- **Preferences**
  - Investor is risk neutral and discounts at rate $R^{-1} < 1$

- **Endowments**
  - Investor net worth $w \gg 0$, i.e., deep pockets

- **Technology**
  - Capital $k$ invested in current period
  - Payoff ("cash flow") next period $Af(k)$
    - Parameter $A > 0$ is “total factor productivity” (TFP)
  - Strict concavity $f_k(k) > 0$ and $f_{kk}(k) < 0$; also:
    - $\lim_{k \to 0} f_k(k) = +\infty$; $\lim_{k \to \infty} f_k(k) = 0$
  - Capital is durable and depreciates at rate $\delta \in (0, 1]$
    - Depreciated capital $k(1 - \delta)$ remains next period
Neoclassical Investment: Investor’s Problem

- **Investor’s objective**
  - Maximize “value” – present discounted value of dividends

- **Investor’s problem - recursive formulation**
  - Choose current dividend $d$ and invest capital $k$ to solve

$$\max_{\{d, w', k\}} \ d + R^{-1} v(w')$$

subject to budget constraints (but no limited liability constraints)

$$w \geq d + k$$

$$Af(k) + k(1 - \delta) \geq w'$$
Neoclassical Investment and User Cost of Capital

- First-order conditions (*FOC*s) (multipliers $\mu$ and $R^{-1}\mu'$)

\[
1 = \mu
\]
\[
R^{-1} = R^{-1}\mu'
\]
\[
\mu = R^{-1}\mu'[Af_k(k) + (1 - \delta)]
\]

- **Investment Euler Equation**
  - Optimal investment/capital $k^*$ solves (combining *FOC*s)

\[
1 = R^{-1}[Af_k(k) + (1 - \delta)]
\]
  
  or letting $R \equiv 1 + r$ and rewriting

\[
\underbrace{r + \delta}_{\text{user cost of capital}} = \underbrace{Af_k(k)}_{\text{marginal product of capital}}
\]

- **Jorgenson's (1963) user cost of capital** (paid at end of period)

\[
u \equiv \underbrace{r}_{\text{interest rate}} + \underbrace{\delta}_{\text{depreciation rate}}
\]
Collateral Constraints as in Rampini/Viswanathan

- Environment with frictions (otherwise as before)

- Two types of agents
  - Owner/borrower
  - Investor/lender

- Owner/borrower ("firm," "entrepreneur")
  - Preferences: risk neutral, impatient $\beta < R^{-1}$, subject to limited liability
  - Endowment: borrower has limited funds $w > 0$

- Investor/lender has deep pockets (as before)

- Collateral constraints
  - Need to collateralize loan repayment with tangible assets
Collateral and Limited Enforcement

- Question: why does borrower need to collateralize loans?
  - Enforcement is limited and it has to be incentive compatible for borrower to repay

- Friction: limited enforcement without exclusion
  - Borrower can abscond with all cash flows and fraction $1 - \theta$ of (depreciated) capital
Limited Enforcement Implies Collateral Constraints

- **Enforcement constraint**
  - Ensure that borrower prefers to repay instead of absconding; heuristically,
    \[
    v(w') \geq v(Af(k) + (1 - \theta)k(1 - \delta))
    \]
    value when repaying \quad value when defaulting
  - and since \( v(\cdot) \) is strictly increasing
    \[
    w' \geq Af(k) + (1 - \theta)k(1 - \delta)
    \]
  - and using budget constraint to substitute for \( w' \) given borrowing \( b \)
    \[
    Af(k) + k(1 - \delta) - Rb = w' \geq Af(k) + (1 - \theta)k(1 - \delta)
    \]

- **Collateral constraint**
  - Canceling terms and rearranging enforcement constraint we obtain
    \[
    \theta k(1 - \delta) \geq Rb
    \]
Limited Enforcement – Collateral Constraints: Equivalence

- Proof (sketch) – see Rampini/Viswanathan (2013), Appendix B

- **Limited enforcement problem**
  - Start with limited enforcement problem in sequence formulation
  - [Step 1] Present value of remaining sequence of promises can never exceed current collateral value
    - Otherwise default and reissue same promises \(\Rightarrow\) borrower better off
  - [Step 2] Any sequence of promises satisfying this condition can be implement with one-period ahead state-contingent claims subject to collateral constraints
  - Results in collateral constraint problem in sequence formulation

- **Collateral constraint problem – recursive formulation**
  - Define state variable (net worth \(w\)) appropriately
Dynamic Financing Problem with Collateral Constraints

- **Firm’s problem**

\[ v(w) \equiv \max_{\{d,k,b,w'\}} d + \beta v(w') \]

subject to budget constraints and collateral constraint

\[ w + b \geq d + k \]
\[ Af(k) + k(1 - \delta) \geq w' + Rb \]
\[ \theta k(1 - \delta) \geq Rb \]

and limited liability \( d \geq 0 \)

- **Net worth next period** \( w' = Af(k) + k(1 - \delta) - Rb \)
First Order Conditions and Investment Euler Equation

- First-order conditions (multipliers $\mu$, $\beta \mu'$, and $\beta \lambda'$)

  $1 \leq \mu, \quad v_w(w') = \mu' \quad \mu = \beta \mu' R + \beta \lambda' R$

  $\mu = \beta \mu'[A f_k(k) + (1 - \delta)] + \beta \lambda' \theta (1 - \delta), \quad \mu = \beta \mu' R + \beta \lambda' R$

- Also: envelope condition $v_w(w) = \mu$

- Investment Euler Equation

  $1 = \beta \frac{\mu' A f_k(k) + (1 - \theta)(1 - \delta)}{\mu \left(1 - R^{-1} \theta (1 - \delta)\right)}$
“Minimal down payment” (per unit of capital)

\[ \phi \equiv 1 - R^{-1}\theta(1 - \delta) \]

PV of \( \theta \times \) resale value of capital

- **Capital structure**
  - In deterministic case, collateral constraints always bind
  - Debt per unit of capital
    \[ R^{-1}\theta(1 - \delta) \]
  - Internal funds per unit of capital
    \[ \phi = 1 - R^{-1}\theta(1 - \delta) \]
Investment Policy

- **Investment Euler Equation** for dividend paying firm

\[
1 = \beta \frac{Af_k(k)}{\theta} + (1 - \theta)(1 - \delta)
\]

- Dividend paying firm: capital $\bar{k}$ solves equation above
  - Comparing FOC's can show $\bar{k} < k^*$ (underinvestment)

- Non-dividend paying firm: $k = \frac{1}{\phi}w$ (invest all net worth and lever as much as possible)
Dividend Policy

- Threshold policy

- Pay out dividends today \( (d' > 0) \) if \( w \geq \bar{w} \)

- Can we show threshold is optimal?
  - Suppose pay dividends at \( w \) but not at \( w^+ > w \)
  - At \( w \), invest \( \bar{k} \)
  - If not paying dividends at \( w^+ \), must invest more; can IEE hold?
Value of Internal Funds

- **Value of internal funds** $\mu$ (remember the envelope condition?)
  - Premium on internal funds (unless firm pays dividends) since $\mu \geq 1$

- **User cost** $u(w)$
  - User cost such that $u(w) = R\beta \frac{\mu}{\mu} A_f(k)$ where

\[ u(w) \equiv r + \delta + R\beta \frac{\lambda}{\mu} (1 - \theta)(1 - \delta) > u \]

internal funds require premium
Net Worth Accumulation and Firm Growth

- **Dividend policy and net worth accumulation**
  - Dividend policy is threshold policy
  - For \( w \geq \bar{w} \), pay dividends \( d = w - \bar{w} \)
  - For \( w < \bar{w} \), pay no dividends and reinvest everything ("retain all earnings")

- **Investment policy and firm growth**
  - For \( w \geq \bar{w} \), keep capital constant at \( \bar{k} \) (no growth)
  - For \( w < \bar{w} \), invest everything \( k = \frac{1}{\sigma} w \) resulting in net worth \( w' > w \) next period

- **Firm age**
  - Young firms \( (w < \bar{w}) \) do not pay dividends, reinvest everything, grow
  - Mature firms \( (w \geq \bar{w}) \) pay dividends and do not grow
Dynamic Debt Capacity Management: Stochastic Case

- Environment as before but here with uncertainty
  - Uncertainty: Markov chain state $s' \in S$ next period – transition probability $\Pi(s, s')$
  - Two types of agents, owner/borrower and investor/lender

- Preferences
  - Borrower is risk neutral, impatient $\beta$, and subject to limited liability
  - Lender is risk neutral and discounts at $R^{-1} \in (\beta, 1)$

- Endowments
  - Borrower has limited funds $w > 0$
  - Lender has deep pockets
Dynamic Debt Capacity Management (Cont’d)

■ **Technology**

  ■ Capital $k$ invested in current period yields stochastic payoff ("cash flow") in state $s'$ next period

  $$A(s')f(k)$$

  where $A' \equiv A(s')$ is realized "total factor productivity" (TFP)

  ■ Strict concavity $f_k(k) > 0; f_{kk}(k) < 0$; also: $\lim_{k \to 0} f_k(k) = +\infty$; $\lim_{k \to \infty} f_k(k) = 0$

  ■ Capital is durable and depreciates at rate $\delta$

    ■ Depreciated capital $k(1 - \delta)$ remains next period

■ **Collateral constraints**

  ■ Need to collateralize all promises to pay with tangible assets

  ■ Can pledge up to fraction $\theta < 1$ of value of depreciated capital
Firm’s Dynamic Debt Capacity Management Problem

- **State-contingent borrowing** \( b' \equiv b(s') \)
  - Collateral constraint for state-contingent borrowing \( b' \)
    \[ \theta k (1 - \delta) \geq R b' \]

- Firm’s debt capacity use problem
  \[
  \max \{ d, w', k, b' \} \quad d + \beta \sum_{s' \in S} \Pi(s, s') v(w', s')
  \]
  subject to budget constraints and collateral constraints, \( \forall s' \in S, \)
  \[
  w + \sum_{s' \in S} \Pi(s, s') b' \geq d + k \\
  \underbrace{A' f(k) + k (1 - \delta)} \geq R b' + w' \\
  \theta k (1 - \delta) \geq R b'
  \]
  and limited liability \( d \geq 0 \)
Dynamic Debt Capacity Choice – Optimality Conditions

- First-order conditions (multipliers $\mu$, $\Pi(s, s')\beta\mu(s')$, and $\Pi(s, s')\beta\lambda(s')$)
  
  $$1 \leq \mu, \quad v_w(w', s') = \mu'$$

  $$\phi \mu = \sum_{s' \in S} \Pi(s, s')\beta\mu'[A' f_k(k) + (1 - \theta)(1 - \delta)], \quad \mu = \beta\mu' R + \beta\lambda' R$$

- Investment Euler equation
  
  $$1 = \sum_{s' \in S} \Pi(s, s')\beta\frac{\mu'}{\mu} A' f_k(k) + (1 - \theta)(1 - \delta)$$

- Firms do not exhaust debt capacity against all states
  - Debt capacity use/leverage: $\theta(1 - \delta) \geq R \sum_{s' \in S} \Pi(s, s')b' / k$
  - Recall: equality in deterministic case
Stationary Distribution of Net Worth

- **Induced transition function** $P$
  - Optimal policy together with Markov process induce transition function $P$ on $(W, W)$
  - Induced state space of net worth $W = [\varepsilon_w, w_{bnd}] \subset \mathbb{R}$
  - Operator on bounded, cont. functions $T : B(W, W) \to B(W, W)$
  - Operator on probability measures $T^* : P(W, W) \to P(W, W)$
  - Show that $P$ satisfies properties such that $\exists$! stationary distribution

- **Stationary distribution allows computation of moments**
  - Computation of steady-state moments
  - Characterization of cross-sectional and time-series properties
  - Simulation and analysis using simulated data
Structural/Quantitative Work: Li/Whited/Wu (2015)

- Li, S., T.M. Whited, and Y. Wu, 2015, Collateral, taxes, and leverage, working paper.

Structural estimation of Rampini/Viswanathan (2013)

- Simulated Method of Moments (SMM)
- Data: non-financial Compustat firms; 1965-2012
- Assumptions:
  - \( f(k) = k^\alpha; \beta \) calibrated; 12 steady-state moments matched
  - \( z \equiv \log(A) \) with \( z' = \rho_z z + \varepsilon' \); discrete-state approximation to AR(1)

**Estimated parameter values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \delta )</th>
<th>( \alpha )</th>
<th>( \rho_z )</th>
<th>( \sigma_z )</th>
<th>( R^{-1} - \beta )</th>
<th>( \hat{\theta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.081</td>
<td>0.782</td>
<td>0.631</td>
<td>0.418</td>
<td>0.032</td>
<td>0.365</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.034)</td>
<td>(0.027)</td>
<td>(0.019)</td>
<td>(0.016)</td>
<td>(0.007)</td>
</tr>
</tbody>
</table>

- Firms conserve some debt capacity, albeit limited amount
  - Simulated debt (incl. interest) is 0.304; roughly 90% of debt capacity

- Remarkable: adding taxes to model leaves capital structure largely unchanged
Collateralizability vs. Tangibility

**Collateralizability** $\theta$
- Structures more collateralizable than equipment (composition varies by industry)
- Financial development may raise $\theta$ and hence leverage

**Tangibility** $\varphi$
- Includes mainly structures (incl. land) and equipment
- Suppose tangible assets are collateralizable (but not intangible assets)
- Fraction tangible assets ($\varphi$) needed for production key

\[
\varphi(\varphi) = 1 - R^{-1}\varphi\theta(1 - \delta)
\]

- Interpretation of $\hat{\theta}$ in Li/Whited/Wu (2015)
  - $\hat{\theta}$ should be interpreted as $\varphi\theta$
  - Substantial variation in estimated $\hat{\theta}$ across 24 industries
  - Correlation of estimated $\hat{\theta}$ with industry asset tangibility $\varphi$: 0.53
    - Slope in cross-industry regression: 0.99
Conclusions for Capital Structure

- **Tangible assets as collateral**
  - If debt needs to be collateralized, type of assets required determines capital structure

- **Dynamics of financing**
  - Accumulate net worth over time
  - Young firms grow and retain all earnings
  - Mature firms pay dividends and grow less
  - Firms conserve debt capacity to some extent
(2) Corporate Risk Management

- Financial constraints give rationale for corporate risk management
  - If firms’ net worth matters, then firms are as if risk averse
  - Collateral constraints link financing and risk management
  - More constrained firms hedge less and often not at all

- Key papers: Rampini/Viswanathan (2010, 2013)
  - Rampini/Sufi/Viswanathan (2014) consider input price risk management (see below)
Collateral and Corporate Risk Management

- Why should firms hedge?
  - Firms are risk neutral, why hedge?
  - Financial constraints make firms risk averse
    - Firms’ value function concave in net worth

- Financing vs. risk management trade-off
  - Limited enforcement: need to collateralize promises to financier and counterparties
  - Collateral constraints link financing and risk management
  - More constrained firms hedge less as financing needs dominate hedging concerns

- Relatedly for households: financing vs. insurance trade-off
  - “The poor can’t afford insurance”
  - Rampini/Viswanathan (2015b) (see (5) below)
Corporate Risk Management Problem

- **Equivalent risk management formulation**
  - Collateral constraint for state-contingent borrowing $b'$
    \[
    \theta k(1 - \delta) \geq Rb'
    \]
  - Equivalently, borrow as much as possible and hedge
    \[
    h' \equiv \theta k(1 - \delta) - Rb' \geq 0
    \]

- **Firm’s risk management problem**
  \[
  \max_{\{d, w', k, h'\}} d + \beta \sum_{s' \in S} \Pi(s, s') v(w', s')
  \]
  subject to budget constraints and short sale constraints, $\forall s' \in S$,
  \[
  w \geq d + \varphi k + R^{-1} \sum_{s' \in S} \Pi(s, s') h'
  \]
  cost of hedging portfolio

  \[
  A'f(k) + (1 - \theta)k(1 - \delta) + h' \geq w'
  \]

  and limited liability $d \geq 0$
Financing vs. Risk Management Trade-off

- **Investment Euler equation**

\[
1 = \sum_{s' \in S} \Pi(s, s') \beta \frac{\mu'}{\mu} A' f_k(k) + (1 - \theta)(1 - \delta) \frac{\varphi}{\varphi} \\
\geq \Pi(s, s') \beta \frac{\mu'}{\mu} A' f_k(k) + (1 - \theta)(1 - \delta) \frac{\varphi}{\varphi}
\]

- As \( w \to 0 \), capital \( k \to 0 \) and marginal product \( f_k(k) \to \infty \)
- Therefore, marginal value of net worth in state \( s' \) (relative to current period) \( \mu'/\mu \to 0 \)
- Using first order condition for hedging

\[
\lambda'/\mu = (\beta R)^{-1} - \mu'/\mu > 0
\]

so severely constrained firms do not hedge at all

- **Financing vs. risk management trade-off**

- Hedging uses up net worth which is better used to purchase additional capital/downsize less

- IID case: if firms hedge, they hedge states with low net worth due to low cash flows
Why Was This Not Previously Recognized?

  - 5 reasons provided (beyond “transactions costs”)
    - (i) market power; (ii) serial correlation of profits; (iii) aggregate risk; (iv) asymmetric information; (v) incentives
  - Fact that hedging uses up net worth is not listed
    - That said, Holmström/Tirole (2000) come close

- **No financing risk management trade-off in previous models**
  - Models consider risk management using frictionless markets
    - Without imposing same frictions on financing and hedging, no trade-off
  - Models have no financing in first period where firms hedge
    - Without investment which requires financing, no trade-off

- **Intuitive, but counterfactual, prediction: more constrained firms hedge more**
  - Froot/Scharfstein/Stein (1993)

- In practice, more constrained (and smaller) firms hedge less!
Input Price Risk Management

- **Profit functions are convex in prices** – basic microeconomics
  - In practice, many firms hedge input prices (e.g., airlines)
  - Say additional input $x'$ needed for production with stochastic price $p'$
  - Induced within-period profit function (with $\hat{\alpha} > 0$, $\phi > 0$, $\hat{\alpha} + \phi < 1$)

  \[
  \pi(k) \equiv \max_{x'} \hat{A}'k^{\hat{\alpha}}x'^\phi - p'x' \equiv A'k^\alpha
  \]

  where $\alpha \equiv \frac{\hat{\alpha}}{1-\phi}$ and $A' \equiv \hat{A}'\frac{1}{1-\phi} (1-\phi)\phi^{\phi-1-\phi} p' - \frac{\phi}{1-\phi}$; convex in $p'$

- **But:** firms as if risk averse in net worth
  - Hedging does not change spot price $p'$; convexity irrelevant
  - Hedging shifts net worth across states; value function concave in $w'$

- **Ad-hoc approach to modeling risk management fails**
  - Ad-hoc model: hedging means buying input at expected price $E[p'|s]$
  - Fails given convexity of profit function!

- **Fuel price risk management** by airlines
- Why useful empirical laboratory? – Panel data on hedging intensity
  - Fraction of next year’s expected fuel expenses hedged
  - Most other studies
    - Dummies for derivatives use – extensive margin only
    - Single cross section – no within-firm variation
- Evidence in cross section and time series consistent with theory
  - More constrained airlines hedge less – across and within airlines
  - Hedging around distress – within-airline variation

![Graph showing the fraction of next year's fuel expenses hedged over time](image)
Ad-hoc Ex-ante Collateral Constraints

- **Ex-ante collateral constraints and limited enforcement**
  - Literature at times imposes ex-ante collateral constraints
    \[ \hat{\theta}_k \geq \sum_{s' \in S} \Pi(s, s')b' \]
    instead of our state-by-state ex-post constraints, \( \forall s' \in S \),
    \[ \theta_k(1 - \delta) \geq Rb' \]
  - Ex-ante limited enforcement: abscond ex ante with dividend and
    \( 1 - \hat{\theta} \) of capital and borrow from other lender
    \[ v(w) \geq v(d_0 + (1 - \hat{\theta})k) \]
    implies ex-ante collateral constraints using budget constraint

- **Equivalence in deterministic case or with non-contingent debt**
  - Setting \( \hat{\theta} \equiv R^{-1} \theta(1 - \delta) \) equivalent under these conditions

- **But: no constraints on risk management**
  - Only one collateral constraint so \( \mu = \beta \mu' R + \beta \lambda R \); all \( \mu' \) equalized
  - Counterfactual implications – complete hedging!
Conclusions for Corporate Risk Management

- Rationale for **corporate risk management**
  - Financial constraints make firms as if risk averse

- Trade-off between financing and risk management
  - Promises to financiers and hedging counterparties need to be collateralized
  - Severely constrained firms hedge less or not at all
    - ... both in theory and in practice
  - Such firms may be more susceptible to downturns
Leasing has repossession advantage and permits greater borrowing.

Severely constrained firms (and households) lease.

Key papers: Rampini/Viswanathan (2013, 2015b); Eisfeldt/Rampini (2009)
Financing Subject to Collateral Constraints

- Environment with collateral constraints but firms can lease
  - Three types of agents, owner/borrower, investor/lender, and lessor
  - Borrower is risk neutral, impatient $\beta < R^{-1}$, and subject to limited liability
  - Borrower has limited funds $w > 0$
  - Lender and lessor have deep pockets, discount at $R^{-1}$
  - For simplicity, deterministic case here

Borrowing subject to collateral constraints

- Need to collateralize promises to pay with tangible assets (due to limited enforcement)
- Promised repayment $\leq \theta \times$ resale value of tangible assets
Leasing as in Eisfeldt/Rampini and Rampini/Viswanathan

- **Leasing**: borrower can rent capital

- **Repossession advantage**
  - Borrower cannot abscond with leased capital
  - In practice, repossession of rented capital easier than foreclosure on secured loan
  - Leasing allows borrower to borrow full resale value, not just fraction $\theta$

- **Monitoring cost** $m$ (per unit of capital)
  - Lessor needs to monitor to prevent abuse
  - Why? – Leasing separates ownership and control

- **User cost of leased capital**

$$u_l \equiv r + \delta + m$$

needs to be paid in advance (i.e., at beginning of period)
Firm’s Problem with Leasing and Secured Lending

- **Firm’s problem with leasing** \( (k_o\text{ owned capital}; k_l\text{ leased capital}) \)

\[
\max_{\{d,w',k_o,k_l,b\}} \quad d + \beta v(w')
\]

subject to budget constraints and collateral constraint

\[
\begin{align*}
w + b & \geq d + k_o + R^{-1}u_l k_l \\
Af(k_o + k_l) + k_o(1 - \delta) & \geq Rb + w' \\
\theta k_o(1 - \delta) & \geq Rb
\end{align*}
\]

non-negativity constraints \( k_o, k_l \geq 0 \), and limited liability \( d \geq 0 \)
First-order conditions (multipliers \( \mu, \beta \mu', \) and \( \beta \lambda; \) let \( k \equiv k_o + k_l \))

As before,

\[
1 \leq \mu, \quad v_w(w') = \mu', \quad \mu = \beta \mu' R + \beta \lambda R
\]

and almost as before (except inequality as borrower might not own any assets)

\[
\mu \geq \beta \mu'[Af_k(k) + (1-\delta)] + \beta \lambda \theta (1-\delta) \quad \Leftrightarrow \quad u(w) \geq R \beta \frac{\mu'}{\mu} Af_k(k)
\]

and finally new

\[
R^{-1} u_l \mu \geq \beta \mu' Af_k(k) \quad \Leftrightarrow \quad u_l \geq R \beta \frac{\mu'}{\mu} Af_k(k)
\]
Lease or Buy?

- Lease if $u_l < u(w)$ and buy otherwise ("choose capital with lower user cost")

- Recall

\[ u_l = r + \delta + m \]

monitoring cost

and

\[ u(w) = r + \delta + \beta R \lambda / \mu (1 - \theta)(1 - \delta) \]

premium on internal funds required
Leasing as Costly, Highly Collateralized Financing

- **Incremental cash flows of buying vs. leasing**

<table>
<thead>
<tr>
<th>Time</th>
<th>Buying (secured loan)</th>
<th>Leasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1 - R^{-1} \theta(1 - \delta)$</td>
<td>$R^{-1} u_l$</td>
</tr>
<tr>
<td>1</td>
<td>$(1 - \theta)(1 - \delta)$</td>
<td>$(1 - \theta)(1 - \delta)$</td>
</tr>
</tbody>
</table>

  **Diff buying - leasing**

  $$\text{extra funds required up front} = \frac{1}{\phi - R^{-1} u_l}$$

  **extra amount recovered**

- **Implicit interest rate on additional amount borrowed by leasing**

  $$R_l \equiv \frac{(1 - \theta)(1 - \delta)}{\phi - R^{-1} u_l} = R \frac{1}{1 - \frac{m}{(1-\theta)(1-\delta)}} > R$$

- Leasing is costly financing since $R_l > R$
Financially Constrained Firms Lease

Implicit “down payment” when leasing

\[
R^{-1} u_l = 1 - R^{-1} \theta (1 - \delta) - R_l^{-1} (1 - \theta) (1 - \delta)
\]

Where

Which leases?

- Severely constrained firms do!
- As \( w \to 0, k \to 0 \) and \( f_k(k) \to +\infty \); using FOCs, \( \mu'/\mu \to 0 \) and

\[
\beta R \lambda/\mu = 1 - \beta R \mu'/\mu \to 1 \quad \Rightarrow \quad u(w) \to r + \delta + (1 - \theta)(1 - \delta)
\]

- Assuming \( (1 - \theta)(1 - \delta) > m \), borrowers with sufficiently low \( w \) lease all their capital!
Conclusions for Leasing and Rental Markets

- Renting capital facilitates repossession

- Lessor is financier but retains ownership

- **Leasing permits greater leverage – beneficial for severely constrained firms**

- Despite quantitative importance, rental markets largely ignored in theoretical and empirical economics (finance, macro, development)
(4) Financial Intermediation

- Rampini/Viswanathan (2015a)

- **Economy with limited enforcement and limited participation**
  - Two sub periods
    - Morning: cash flows realized; more $\theta_i$ capital collateralizable
    - Afternoon: investment/financing; only fraction $\theta < \theta_i$ collateralizable
  - Limited participation with two types of lenders
    - Households present only in afternoons; intermediaries always
  - Optimal contract implemented with two sets of one-period Arrow securities (for morning and afternoon)

- **Financial intermediaries as collateralization specialists**
  - Intermediaries need to enforce morning claims
  - Intermediaries need to finance morning claims out of own net worth
  - Intermediated finance is short term

- **Role for intermediary capital**
  - Economy with two state variables: firm and intermediary net worth
(5) Dynamic Household Insurance

- Rampini/Viswanathan (2015b)

- **Risk-averse household with stochastic income** $y'$

$$
\max_{\{c, w', h'\}} u(c) + \beta \sum_{s' \in S} \Pi(s, s') v(w', s')
$$

subject to budget constraints and **short sale constraints**, $\forall s' \in S$,

$$
w \geq c + R^{-1} \sum_{s' \in S} \Pi(s, s') h'
$$

$$
y' + h' \geq w'
$$

$$
h' \geq 0
$$

- Under stationary distribution, **household risk management is ...**
  - **incomplete** with probability 1; **absent** with positive probability
  - **globally increasing** in net worth and income
  - **precautionary** (increases when income gets riskier)

- **Insurance is state-contingent savings**
  - Insurance premia paid up front; intertemporal aspect to insurance
Durability facilitates financing – Hart/Moore (1994)

- Define higher durability as lower depreciation rate $\delta$
  
  $$\frac{\partial \phi}{\partial \delta} = \frac{\partial}{\partial \delta} \left\{ 1 - R^{-1} \theta (1 - \delta) \right\} = R^{-1} \theta > 0$$

- Durable assets easier to finance due to higher collateral value

To the contrary: durability impedes financing

- Keep frictionless user cost $u = r + \delta$ constant not price; so $q = \frac{u}{r+\delta}$
  
  $$\frac{\partial \phi}{\partial \delta} = \frac{\partial}{\partial \delta} \left\{ \frac{u}{r+\delta} (1 - R^{-1} \theta (1 - \delta)) \right\} = -q \frac{1 - \theta}{r+\delta} < 0$$

- Durable assets cost more and require larger down-payments

Implications for technology adoption, incidence of financial constraints, choice of capital vintage
Models of Dynamic Collateralized Financing – Conclusion

- Useful laboratory to study dynamic financing problems
  - Tractability allows explicit theoretical analysis of dynamics
  - Insights yielded so far
    - Capital structure/debt capacity
    - Risk management/insurance
    - Leasing
    - Intermediation
    - Durability
  - Dynamic models facilitate quantitative work/structural estimation

- Empirically/quantitatively plausible class of models