Constrained-Efficient Capital Reallocation

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Motivation

- Financial frictions (specifically, **collateral constraints**) distort
  - level of aggregate investment
  - (re)allocation of capital across firms

- Analyze efficiency of capital allocation **subject to** financial frictions

- Is resale price of capital (collateral) “too high” or “too low”? 
This Paper

- Efficiency analysis in equilibrium model with
  (macro) investment and capital reallocation
  (heterogeneity) heterogeneous firms facing idiosyncratic shocks
  (finance) collateral constraints

- Two types of pecuniary externalities through resale price of capital
  Collateral externality
    - Higher collateral value facilitates new investment
  Distributive externality
    - More constrained buy old capital from less constrained
    - Lower price of old capital facilitates purchases of old capital

- Insight: Distributive externality dominates collateral externality
  New investment has positive externality: reduces price of old capital
  Facilitates reallocation of old capital to more constrained firms
  Both analytical and quantitative results
Related Literature

- **Capital Reallocation**
  - Eisfeldt/Rampini (2006, 2007); Lanteri (2018); Rampini (2019); Ma/Murfin/Pratt (2019); Gavazza/Lanteri (forthcoming)

- **Pecuniary Externalities**
  - Lorenzoni (2008); Dávila/Hong/Krusell/Ríos-Rull (2012); Dávila/Korinek (2018); Bianchi/Mendoza (2018); Itskhoki/Moll (2019); Jeanne/Korinek (2019)

- **Financial Frictions and Misallocation**
  - Kiyotaki/Moore (1997); Buera/Kaboski/Shin (2011); Midrigan/Xu (2014); Moll (2014)
Outline

(1) Stylized Model: Analytical Results

(2) Quantitative Analysis
(1) Stylized Model
Capital Reallocation and Pecuniary Externalities

Roadmap

» Environment

» First Best

» Competitive Equilibrium with Financial Frictions

» Constrained Efficiency
  » Distributive externality > collateral externality in comp. eqm.
  » Sustaining First Best

» Three Generalizations

» Essential Role of Heterogeneity and Reallocation
Environment

- Time is discrete and horizon infinite
- Infinitely-lived representative household
  - Linear preferences
    \[ \sum_{t=0}^{\infty} \beta^t C_t \]
- Continuum of firms born at each date \( t \); live for two dates
  - Owned by representative household
  - Each firm draws initial net worth \( w \)
    - \( w \in W \equiv [w_{min}, w_{max}] \) with distribution \( \pi(w) \) (mass 1)
  - Invest at \( t \), produce output at \( t + 1 \)
  - Maximize present value of dividends (net of financing costs)
Capital Goods and Technology

- Capital goods
  - Last for two periods (so “new” and “old”)
  - New capital produced using output with linear technology at cost 1
  - (Standard) one period time to build
  - New and old capital perfect substitutes in production

- Firm production
  \[ y_t(w) = f\left(k_{t-1}^N(w) + k_{t-1}^O(w)\right) \]
  with \( f_k > 0 \) and \( f_{kk} < 0 \)

- Resource constraint (frictionless economy)
  \[ \int y_t(w) d\pi(w) = C_t + \int k_t^N(w) d\pi(w) \]

- Evolution of aggregate old capital
  \[ \int k_{t-1}^N(w) d\pi(w) = \int k_t^O(w) d\pi(w) \]
Social planner maximizes household utility subject to resource constraints

First-Best allocation satisfies

\[ 1 = \beta \left( f_k(k_t^FB) + q_t^FB \right) \]

\[ q_t^FB = \beta f_k(k_t^FB) \]

Steady state

\[ q_t^FB = \frac{1}{1+\beta} \quad k_t^FB = f_k^{-1} \left( \frac{1}{\beta(1+\beta)} \right) \]

Allocation of new vs. old capital at firm level is indeterminate
Financial Frictions

- Collateral constraint
  \[ \theta q_{t+1} k_t^N \geq \beta^{-1} b_t \]
  with \( \theta \in [0, 1) \)

- Cost of equity issuance \( \phi(-d) \)
  - increasing and convex for \( d < 0 \)
  - zero otherwise
New firm’s problem at time $t$ in competitive equilibrium

$$\max \{d_{0t}, d_{1,t+1}, b_t, k^N_t, k^O_t\} \in \mathbb{R}^3 \times \mathbb{R}_+^2$$

$$d_{0t} - \phi(-d_{0t}) + \beta d_{1,t+1}$$

subject to budget constraints of new firm at $t$ and old firm at $t+1$

$$w_{0t} + b_t = d_{0t} + k^N_t + q_t k^O_t$$

$$f(k^N_t + k^O_t) + q_{t+1} k^N_t = d_{1,t+1} + \beta^{-1} b_t$$

and collateral constraint

$$\theta q_{t+1} k^N_t \geq \beta^{-1} b_t$$
Firm Optimality

- Firms maximize present value of dividends net of cost $\phi$ subject to
  - budget constraints
  - collateral constraint ($\beta \lambda_t$)
  - non-negativity constraints on $k^N_t, k^O_t (\nu^N_t, \nu^O_t)$

- First-order conditions w.r.t. $k^N_t, k^O_t, b_t$

\[ 1 + \phi_{d,t} = \beta (f_k(k_t) + q_{t+1}) + \beta \theta \lambda_t q_{t+1} + \nu^N_t \]

\[ q_t (1 + \phi_{d,t}) = \beta f_k(k_t) + \nu^O_t \]

\[ 1 + \phi_{d,t} = 1 + \lambda_t \]

- Marginal value of net worth $1 + \phi_{d,t}$
Stationary Competitive Equilibrium

- **Definition: Stationary Competitive Equilibrium**
  - Policy functions $d_0(w)$, $d_1(w)$, $k^N(w)$, $k^O(w)$, and $b(w)$
  - Price of old capital $q$

such that

- **Individual optimality**
- **Goods market clearing (including costs of equity issuance)**
  \[
  \int y(w) d\pi(w) = C + \int k^N(w) d\pi(w) + \int \phi(d_0(w)) d\pi(w)
  \]
- **Capital goods market clearing**
  \[
  \int k^O(w) d\pi(w) = \int k^N(w) d\pi(w)
  \]
Characterization

Proposition 1

Stationary competitive equilibrium is characterized as follows

(i) New capital has higher down payment than old capital, but (weakly) lower user cost from perspective of unconstrained firm

(ii) Price of old capital exceeds price in frictionless economy: \( q \geq q^{FB} \)

(iii) If \( q > q^{FB} \), thresholds \( \underline{w}_N < \underline{w}_O < \underline{w} \) such that

- firms with \( w \leq \underline{w}_N \) invest only in old capital
- firms with \( w \in (\underline{w}_N, \underline{w}_O) \) invest \( k \); invest in both new & old capital
- firms with \( w \geq \underline{w}_O \) invest only in new capital
- firms with \( w > \underline{w} \) pay dividends and invest \( \bar{k} > k^{FB} > k \)
Choice between New and Old Capital

► New and old capital differ in terms of
  ▶ down payments $\varphi_N$ and $\varphi_O$
    \[
    \varphi_N \equiv 1 - \beta \theta q > \varphi_O \equiv q
    \]
  ▶ user cost (for unconstrained firm)
    \[
    u_N \equiv 1 - \beta q \leq u_O \equiv q
    \]
► Investment Euler equations for new and old capital
  \[
  u_N(w) \equiv u_N + \phi_d \varphi_N \geq \beta f_k(k)
  \]
  \[
  u_O(w) \equiv u_O + \phi_d \varphi_O \geq \beta f_k(k)
  \]
► Sufficiently (un)constrained firms invest in old (new) capital
Policy Functions

Parameters
Constrained-Efficient Allocation

- **Planner** chooses allocations and price to maximize household utility subject to
  - technological constraints
  and
  - individual budget and financial constraints
  - market clearing condition for old capital ($\eta_t$)

- First-order conditions w.r.t. $k_t^N, k_t^O, b_t$

\[
1 + \phi_{d,t} = \beta (f_k(k_t) + q_{t+1}) + \beta \theta \lambda t q_{t+1} + \nu^N + \beta \eta_{t+1}
\]

\[
q_t(1 + \phi_{d,t}) = \beta f_k(k_t) + \nu^O - \eta_t
\]

\[
1 + \phi_{d,t} = 1 + \lambda_t
\]
Constrained-Efficient Price

- First-order condition w.r.t. price $q_t$

\[
\int k_t^O(w) (1 + \phi_{d,t}(w)) \, d\pi(w) = \int k_{t-1}^N(w) (1 + \theta \lambda_{t-1}(w)) \, d\pi(w)
\]

or

\[
\int k_t^O(w) (1 + \phi_{d,t}(w)) \, d\pi(w) - \int k_{t-1}^N(w) \, d\pi(w) = \theta \int k_{t-1}^N(w) \lambda_{t-1}(w) \, d\pi(w)
\]

- Using market clearing for capital goods ($\int k_t^O \, d\pi = \int k_{t-1}^N \, d\pi$)

\[
\int k_t^O(w) \phi_{d,t}(w) \, d\pi(w) = \theta \int k_{t-1}^N(w) \lambda_{t-1}(w) \, d\pi(w)
\]

- Two types of pecuniary externalities
  
  - Distributive externality: $k_t^O \phi_{d,t}$
  
  - Collateral externality: $\theta k_{t-1}^N \lambda_{t-1}$
Externalities in Competitive Equilibrium

Proposition 2

*In stationary competitive equilibrium*

- Distributive externality is larger than collateral externality

\[
\int k^O(w)\phi_d(w)d\pi(w) > \theta \int k^N(w)\lambda(w)d\pi(w)
\]

- Competitive-equilibrium price of old capital is **higher** than constrained-efficient one

- Intuition: (recall \( \lambda(w) = \phi_d(w) \))
  - cov. between mrg. value of net worth and old capital investment exceeds
  - cov. between mrg. value of net worth and new capital investment
Stationary constrained-efficient allocation achieves First-Best welfare

\[ q^* = \frac{w_{\text{min}}}{k^{FB}} < q^{FB} \leq q \]
Constrained-Efficient Allocation: Implementation

- Competitive equilibrium with taxes $\tau_t^N(w), \tau_t^O(w)$

  $$\tau^N = -\beta \eta = -\beta (q^{FB} - q^*) < 0$$

  $$\tau^O = \frac{\eta}{q^*} = \frac{q^{FB}}{q^*} - 1 > 0$$

- Tax rates independent of net worth $w$

- Taxes rebated lump-sum so as to respect each budget constraint

- Under additional restriction $\tau_t^O(w) = 0$, we show $\tau_t^N(w) < 0$
Three Generalizations

Sign of inefficiency obtains in three generalizations of the model:

- Risk-averse entrepreneurs (Proposition 3)
  \[ u(c_{0t}) + \beta u(c_{1,t+1}), \; u_c > 0, \; u_{cc} < 0 \]

- Heterogeneity in productivity (Proposition 4)
  \[ y_t(w) = s f(k_{t-1}(w)) \]
  \[ \frac{\partial \phi_d(w,s)}{\partial s} \geq 0 \]

- Long-lived firms and capital (Proposition 5)
Long-Lived Firms and Capital

- Stochastic firm life cycle
  - Probability of firm death $\rho$
  - Net worth is endogenous state variable

- Long-lived capital
  - Fraction $\delta^N$ of new capital becomes old
  - Fraction $\delta^O$ of old capital is destroyed
  - Both new and old capital serve as collateral

- Stylized model is special case: $\rho = \delta^N = \delta^O = 1$

Proposition 5

In stationary competitive equilibrium

- Distributive externality is larger than collateral externality
Essential Role of Heterogeneity and Reallocation

- Distributive externality hinges on reallocation in equilibrium
  - Stationary equilibrium with reallocation

- Representative entrepreneur in steady state – Kiyotaki/Moore (1997)
  - Assets in fixed supply (land)
  - Entrepreneur has constant amount of land in steady state
  - Misallocation, but no reallocation
  - Change in price of land has no effect on budget constraints
  - Only collateral externality

- Our result obtains with assets in fixed supply and OLG firms
  - Heterogeneity between young and old firms
  - Reallocation of land from old to young firms
  - Distributive externality dominates collateral externality
(2) Quantitative Analysis
Quantitative Model

We generalize assumptions as follows:

- Stochastic life cycle (prob. of death $\rho$)
- Long-lived capital ($\delta^N, \delta^O$)
- Persistent idiosyncratic productivity shocks $s$
- New and old capital imperfect substitutes in production
  \[ y = s f \left( g(k^N, k^O) \right) \]
  where $g$ CES aggregator
- Scrappage value of old capital $q \geq 0$
Calibration Strategy

- Technology and shocks
  - Evidence on investment and reallocation dynamics of US firms (Khan/Thomas, 2013; Lanteri, 2018)

- Financial frictions
  - Estimates of financing costs from corporate-finance literature (Hennessy/Whited, 2007; Catherine/Chaney/Huang/Sraer/Thesmar, 2020; Li/Whited/Wu, 2016)

- Capital reallocation
  - Joint distribution of firm age and capital age (Ma/Murfin/Pratt, 2020)
Cross Section of Externalities

\[ |\text{Distributive externality}| \approx 2.3 \times |\text{Collateral externality}| \]

Thick red: High productivity. Thin blue: low productivity.
Constrained-Efficient Reallocation


Implementation
## Aggregate Outcomes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Comp. Eqm.</th>
<th>Constr. Eff.</th>
<th>Constr. Eff. $\left(\tau^O = 0\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.899</td>
<td>0.973</td>
<td>0.921</td>
</tr>
<tr>
<td>Investment</td>
<td>0.857</td>
<td>0.962</td>
<td>0.893</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.933</td>
<td>0.983</td>
<td>0.943</td>
</tr>
<tr>
<td>Price $q$</td>
<td>1.010</td>
<td>0.184</td>
<td>0.987</td>
</tr>
<tr>
<td>Average tax $\tau^N$</td>
<td>0</td>
<td>-8.6%</td>
<td>-0.6%</td>
</tr>
<tr>
<td>Average tax $\tau^O$</td>
<td>0</td>
<td>103.8%</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

- Allocations and price expressed as fractions of First-Best value
Additional Analyses and Robustness

- Transition dynamics
  - Compute optimal “simple” policy starting from competitive eqm.
  - Optimal time-invariant $\tau^N$ for all firms $\approx -0.3\%$

- Benchmarking gains from capital reallocation
  - Consider restriction: $\frac{k^O(s^a)}{k^N(s^a)} = \omega$
  - Going from restricted to unrestricted competitive equilibrium
  - Approximately 0.4% consumption gain

- Sensitivity analysis
  - Collateralizability $\theta$, elasticity of substitution $\epsilon$, scrap value $q$
Conclusion

- Gains from reallocation of old capital
  - High-MPK firms buy old capital

- **Price of old capital in competitive equilibrium is too high**
  - Distributive externality dominates collateral externality

- New investment today makes old capital less scarce in future
  - Positive externality on constrained firms in future
  - Novel rationale for subsidies on new investment
Extra
Lagrangian for Constrained Efficiency

\[ \mathcal{L} \equiv \sum_{t=0}^{\infty} \beta^t \left\{ \int (d_{0t} - \phi(-d_{0t}) + d_{1t}) \, d\pi \\ - \int \mu_{0t} \left( d_{0t} - w + k_t^N + q_t k_t^O - b_t \right) \, d\pi \\ - \int \mu_{1t} \left( d_{1t} - f \left( k_{t-1}^N + k_{t-1}^O \right) - q_t k_{t-1}^N + \beta^{-1} b_{t-1} \right) \, d\pi \\ + \int \lambda_t \left( \beta \theta q_{t+1} k_t^N - b_t \right) \, d\pi \\ + \int \nu_t^N k_t^N \, d\pi + \int \nu_t^O k_t^O \, d\pi - \eta_t \left( \int k_t^O \, d\pi - \int k_{t-1}^N \, d\pi \right) \right\} \]
Lagrangian for Constrained Efficiency: New Subsidies Only

\[
\mathcal{L} \equiv \sum_{t=0}^{\infty} \beta^t \left\{ \int \left( d_{0t} - \phi(-d_{0t}) + d_{1t} \right) d\pi \\
- \int \mu_{0t} \left( d_{0t} - w + k_t^N + q_t k_t^O - b_t \right) d\pi \\
- \int \mu_{1t} \left( d_{1t} - f \left( k_{t-1}^N + k_{t-1}^O \right) - q_t k_{t-1}^N + \beta^{-1} b_{t-1} \right) d\pi \\
+ \int \lambda_t \left( \beta \theta q_{t+1} k_{t+1}^N - b_t \right) d\pi \\
+ \int \psi_t \left( q_t (1 + \phi_{d,t}) - \beta f_k(k_t) \right) d\pi \\
+ \int \nu_t^N k_t^N d\pi + \int \nu_t^O k_t^O d\pi - \eta_t \left( \int k_t^O d\pi - \int k_{t-1}^N d\pi \right) \right\}
\]
### Numerical Example

**Table: Parameter Values**

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Net worth</td>
<td>Uniform $\pi(w)$</td>
<td>$w_{\text{min}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$w_{\text{max}}$</td>
</tr>
<tr>
<td>Technology</td>
<td>Curvature of $f$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Financial constraints</td>
<td>Collateralizability</td>
<td>$\theta$</td>
</tr>
<tr>
<td></td>
<td>Cost of equity</td>
<td>$\phi_0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\phi_1$</td>
</tr>
</tbody>
</table>

- Functional forms: $f(k) = k^\alpha$; $\phi(-d) = \phi_0(-d)^{\phi_1}$ for $d < 0$
Covariance Interpretation

Interpretation: covariance between net expenditure and marginal value of net worth

- We show that \( \text{Cov}(k^O, \phi_d) > \text{Cov}(k^N, \phi_d) \)

- Also, \( \text{Cov}(k^N, \phi_d) < 0 \)

- Typically (and in our example), \( \text{Cov}(k^O, \phi_d) > 0 \)
Implementation

- Competitive equilibrium with
  - proportional taxes on new and old capital $\tau_t^N(w), \tau_t^O(w)$
  - rebated lump-sum to each firm, so as to not to redistribute resources

- Budget constraint

$$w + b_t = d_{0t} + k_t^N (1 + \tau_t^N) + q_t k_t^O (1 + \tau_t^O) - T_t$$

- Lump-sum transfer

$$T_t = \tau_t^N k_t^N + \tau_t^O q_t k_t^O$$
Constrained-Efficient Reallocation

Introduce additional constraint: old capital cannot be taxed

\[ q_t (1 + \phi_{d,t}) \geq f_k(k_t) \]

with multiplier \( \psi_t \)
Constrained Efficiency: New-Capital Subsidies Only

- FOC wrt $k_{t+1}^N$

\[ 1 + \phi_{d,t} = \beta (f_k(k_t) + q_{t+1}) + \beta \theta \lambda_t q_{t+1} + \beta \eta_{t+1} \]

\[ + \psi_t (q_t \phi_{dd,t} - \beta f_{kk}(k_t)) \]

- FOC wrt $q_t$

\[ \int k_t^O (1 + \phi_{d,t}) d\pi = \int k_{t-1}^N (1 + \theta \lambda_{t-1}) d\pi \]

\[ + \int \psi_t (1 + \phi_{d,t} + q_t \phi_{dd,t} k_t^O) d\pi \]
Constrained-Eff. Reallocation: New-Capital Subsidies Only

Entrepreneurs maximize $u(c_{0t}) + \beta u(c_{1,t+1})$, $u_c > 0$, $u_{cc} < 0$

**Proposition 6**

*In stationary competitive equilibrium*

- *Distributive externality is larger than collateral externality*

\[
\int k^O(w) u_c(c_0(w)) \, d\pi(w) > \int k^N(w) [u_c(c_1(w)) + \theta \lambda(w)] \, d\pi(w)
\]
Heterogeneity in Productivity

- Joint distribution of net worth $w$ and productivity $s$, $\pi(w, s)$
- Production $y_t = s f(k_{t-1}^N + k_{t-1}^O)$
- We show $\frac{\partial \phi_d(w, s)}{\partial s} \geq 0$

Proposition 7

$\text{In stationary competitive equilibrium}$

- Distributive externality is larger than collateral externality

$$\int k^O(w, s)\phi_d(w, s)d\pi(w, s) > \theta \int k^N(w, s)\lambda(w, s)d\pi(w, s)$$
Quantitative Model: Net Worth and Collateral Constraint

- Net worth evolution

\[ w_t(s^a) = s_a f(k_{t-1}(s^{a-1})) + (1 - \delta^N(1 - q_t))k^N_{t-1}(s^{a-1}) \]
\[ + q_t(1 - \delta^O)k^O_{t-1}(s^{a-1}) - \beta^{-1}b_{t-1}(s^{a-1}) \]

- Collateral constraint

\[ \theta \left[ (1 - \delta^N(1 - q_{t+1}))k^N_t(s^a) + q_{t+1}(1 - \delta^O)k^O_t(s^a) \right] \geq \beta^{-1}b_t(s^a) \]
Quantitative Model: Market Clearing

Market clearing for old capital

\[
\sum_{a=0}^{\infty} \gamma_a \sum_{s^a} p(s^a) k_t^O(s^a) = \sum_{a=0}^{\infty} \gamma_a \sum_{s^a} p(s^a) \left[ \delta^N k_{t-1}^N(s^a) + (1 - \delta^O) k_{t-1}^O(s^a) \right]
\]
Quantitative Model: Constrained-Efficient Allocation

- FOC wrt $k^N_t(s^a)$
  
  \[ 1 + \phi_{d,t} = \beta \mathbb{E}_t \left[ s_{a+1} f_k(k_t) g_{N,t} + (1 - \delta^{N}(1 - q_{t+1})) \right] (1 + (1 - \rho) \phi_{d,t+1}) \]
  \[ + \beta \theta \lambda_t (1 - \delta^{N}(1 - q_{t+1})) + \beta \delta^N \eta_{t+1} \]

- FOC wrt $k^O_t(s^a)$
  
  \[ q_t(1 + \phi_{d,t}) = \beta \mathbb{E}_t \left[ s_{a+1} f_k(k_t) g_{O,t} + (1 - \delta^{O}) q_{t+1} \right] (1 + (1 - \rho) \phi_{d,t+1}) \]
  \[ + \beta \theta (1 - \delta^{O}) \lambda_t q_{t+1} - \eta_t + \beta (1 - \delta^{O}) \eta_{t+1} \]

- FOC wrt $b_t(s^a)$
  
  \[ \phi_{d,t} = (1 - \rho) \mathbb{E}_t \phi_{d,t+1} + \lambda_t \]
Quantitative Model: Constrained-Efficient Price

- FOC wrt $q_t$

\[
\sum_{a=0}^{\infty} \gamma_a \sum_{s^a} p(s^a) k_t^O (1 + \phi_{d,t}) \geq \\
\sum_{a=0}^{\infty} \gamma_a \sum_{s^{a+1}} p(s^{a+1}) \left( \delta^N k_{t-1}^N + (1 - \delta^O) k_{t-1}^O \right) (1 + (1 - \rho) \phi_{d,t} + \theta \lambda_{t-1})
\]
Solving for Constrained Efficiency

- Guess shadow value of old capital $\eta$
  - Guess price $q$
  - Compute policy functions solving investment FOCs on grid for $(w, s)$
  - Compute stationary distribution $\pi(w, s)$
  - Check market clearing and update $q$

- Evaluate externalities (FOC wrt $q$) and update $\eta$
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\beta$ 0.96</td>
</tr>
<tr>
<td>Initial net worth</td>
<td>$w_0$ 5</td>
</tr>
<tr>
<td>Death probability</td>
<td>$\rho$ 0.1</td>
</tr>
<tr>
<td>Curvature of production function</td>
<td>$\alpha$ 0.6</td>
</tr>
<tr>
<td>CES elasticity of substitution</td>
<td>$\epsilon$ 5</td>
</tr>
<tr>
<td>CES new share</td>
<td>$\nu$ 0.5</td>
</tr>
<tr>
<td>Depreciation new</td>
<td>$\delta^N$ 0.2</td>
</tr>
<tr>
<td>Depreciation old</td>
<td>$\delta^O$ 0.2</td>
</tr>
<tr>
<td>Scrap value</td>
<td>$q$ 0.1</td>
</tr>
<tr>
<td>Productivity persistence</td>
<td>$\chi_s$ 0.7</td>
</tr>
<tr>
<td>Productivity st. dev. of innovations</td>
<td>$\sigma_s$ 0.12</td>
</tr>
<tr>
<td>Collateralizability</td>
<td>$\theta$ 0.5</td>
</tr>
<tr>
<td>Cost of raising equity</td>
<td>$\phi_0$ 0.1</td>
</tr>
<tr>
<td></td>
<td>$\phi_1$ 5</td>
</tr>
</tbody>
</table>

- Functional forms: $f(k) = k^\alpha$; $\phi(-d) = \phi_0(-d)^{\phi_1}$ for $d < 0$
Implementation

\[ \tau^N \]

\[ \tau^O \]

\[ w \]
Transition Dynamics

- $\tau_N$
- $q$
- $K^0$