Financing Durable Assets

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Effect of Durability on Financing

- **Durability essential feature of capital**
  - Fixed assets comprise as much as 72% of aggregate capital stock
  - Includes structures (both residential and non-residential), infrastructure, equipment, and consumer durable goods
  - Focus on tangible durable assets but intangible assets similar

- **Variation in durability**
  - Depreciation rates vary from as low as 1% (new residential structures) to as high as 31% (computing equipment)

- **How does durability affect financing?**
  - Can durable assets serve as collateral and facilitate financing?
  - “Intuitively, as the assets become more durable, they provide the creditor with the security to wait longer before being repaid. … And hence the debtor need not set aside as much of his initial borrowing to finance early debt repayments, leaving more to finance the initial investment” – Hart/Moore (1994)
Effect of Durability on Financing (Cont’d)

- To the contrary: **durable assets are harder to finance**
  - Durability increases resale/collateral value allowing more borrowing
  - But durability also raises price of asset and hence financing need
  - Net effect: raises down payments

**Implications**

- **Technology adoption**: dominated technologies may be adopted
  - **Legal enforcement** affects composition of investment
- Choice between **vintages**: new, durable vs. used, non-durable assets
  - **Trade** in used capital goods
- **Rent** vs. buy decision

**Critical distinction between durability and pledgeability**

- Durability impedes financing
- Pledgeability facilitates financing
- Hart/Moore (1994) is about pledgeability not durability
Model of Durable Asset Financing

- **Discrete time, infinite horizon, deterministic productivity**

- **Entrepreneurs** ("borrowers" or "firms")
  - Measure $\rho \in (0, 1)$ enter each period; survive with probability $1 - \rho$; measure 1 alive
  - Preferences: $\sum_{t=0}^{\infty} \beta^t d_t$ with $\beta \in (0, 1)$, $d_t \geq 0$, and $\beta \equiv \hat{\beta}(1 - \rho)$
  - Initial net worth $w_0$ (cash-in-hand)

- In equilibrium, well capitalized entrepreneurs are financiers
  - Equilibrium (gross) interest rate $R = \beta^{-1}$

- **Limited enforcement**
  - Borrowers can default and abscond with all cash flows and fraction $1 - \theta$ of assets without exclusion where $\theta \in [0, 1)$
  - Rampini/Viswanathan (2010, 2013, 2016)
    - Optimal dynamic contract can be implemented with one-period ahead debt subject to **collateral constraints**
Modeling Durability: Two Approaches

- **Neoclassical capital with geometric depreciation at rate** $\delta$
  - Depreciation rate $\delta$ measures durability (lower $\delta$, more durable)

- **Capital with finite useful life (one-horse shay depreciation)**
  - Capital provides same service flow for finite number of periods
  - New capital assets are more durable than used ones
Financing and Durability: Neoclassical Investment

- Suppose **neoclassical capital** depreciates at rate $\delta$ per period
  \[ k' = k(1 - \delta) + q^{-1}i \]
  where $q$ is price of capital and $i$ investment

- Output $Af(k)$ where $A$ is TFP and $f$ is (strictly) increasing, concave, and $\lim_{k \to 0} f_k(k) = +\infty$

**Firm’s problem with neoclassical investment:** Given $w$, solve

\[
v(w) \equiv \max_{d,k,b,w' \in \mathbb{R}_+^2 \times \mathbb{R}^2} d + \beta v(w')
\]

subject to budget constraints and collateral constraint

\[
w + b \geq d + qk
\]

\[Af(k) + qk(1 - \delta) \geq Rb + w'
\]

\[\theta qk(1 - \delta) \geq Rb
\]

- Endogenous state variable: net worth $w$
Effect of Durability on Down Payment and User Cost

- **Effect of durability on down payment**
  - When collateral constraint binds, \( b = R^{-1} \theta q k (1 - \delta) \)
  - Minimal **down payment** per unit of capital
    \[ \phi = q - R^{-1} \theta q (1 - \delta) \]
    \[ \frac{\partial \phi}{\partial \delta} = R^{-1} \theta q > 0; \text{ more durable capital has lower down payment?} \]
  - But keeping \( q \) fixed, lower \( \delta \) reduces frictionless user cost of capital!
    \[ u \equiv R^{-1} q (r + \delta) \]
Effect of Durability on Down Payment and User Cost

- **More durable capital requires larger down payment**
  - Fixing frictionless user cost \( u^* \) instead, using \( u^* = R^{-1} q(\delta)(r + \delta) \)
    
    The price is
    
    \[
    q(\delta) = \frac{Ru^*}{r + \delta}
    \]

  - Down payment keeping user cost fixed
    
    \[
    \phi(\delta) = q(\delta) - R^{-1} \theta q(\delta)(1 - \delta)
    \]
    
    \[
    \frac{\partial \phi(\delta)}{\partial \delta} = -q(\delta) \frac{1-\theta}{r+\delta} < 0
    \]

  - Down payment – decomposition
    
    \[
    \phi(\delta) = u^* + R^{-1} q(\delta)(1 - \theta)(1 - \delta)
    \]
    
    - **Jorgensonian user cost**
    - **Non-pledgeable part of resale value**
Adopting Dominated Technologies

- Suppose **two types of neoclassical capital of different durability**
  - depreciation rates $\delta_d < \delta_{nd}$
  - prices $q_d > q_{nd}$ (determined by technology – exogenous)
  
  such that

- frictionless user costs $u_d < u_{nd}$, where $u_j \equiv R^{-1} q_j (r + \delta_j)$
- down payments $\varphi_d > \varphi_{nd}$, where $\varphi_j \equiv q_j - R^{-1} \theta q_j (1 - \delta_j)$

- Perfect substitutes in production

» Firm’s problem
Constrained Firms Adopt Dominated Technologies

- Unconstrained never use less durable “low quality” capital
  - Since $\lambda' = 0$, so
    \[
    u_j(w) = u_j + \beta \frac{\lambda'}{\mu} q_j (1 - \theta)(1 - \delta_j) = u_j
    \]
  - Unconstrained firms simply compare user costs

- Severely constrained adopt dominated technology
  - As $w \to 0$, $\sum_j k_j \to 0$, and using the investment Euler equation,
    \[
    1 \geq \beta \frac{\mu'}{\mu} A f_k (\sum_j k_j) + (1 - \theta) q_j (1 - \delta_j) \]
    \[
    \beta \frac{\mu'}{\mu} \to 0, \text{ so }
    u_j(w) = \varphi_j - \beta \frac{\mu'}{\mu} q_j (1 - \theta)(1 - \delta_j) \to \varphi_j
    \]
  - Severely constrained firms simply compare down payments

Continuum of technologies  Neoclassical model with vintage capital
Dynamics of Firm Financing, Payout, and Investment

- Assume initial net worth $w_0$ low

- Young (severely constrained) firms
  - Pay no dividends
  - Compare down payments and invest only in less durable capital

- Unconstrained firms
  - Pay dividends
  - Compare user costs and invest only in more durable capital
Adopting Dominated Technologies and Legal Enforcement

- Two countries with strong and weak legal enforcement: $\theta_H > \theta_L$

- Technology choice
  - As before two types of capital with $\delta_d < \delta_{nd}$ and $q_d > q_{nd}$ such that $u_d < u_{nd}$ but $\varphi_d > \varphi_{nd}$; perfect substitutes

- More dominated technology use with weak legal enforcement
  - Firm indifferent between two types of capital
    \[
    \frac{A f_k (\sum_j k_j) + (1 - \theta) q_d (1 - \delta_d)}{\varphi_d} = \frac{A f_k (\sum_j k_j) + (1 - \theta) q_{nd} (1 - \delta_{nd})}{\varphi_{nd}},
    \]
  - Lower $\theta$ implies indifferent at higher total investment $k = \sum_j k_j$ and hence higher net worth
Suppose two types of capital are imperfect substitutes

Aggregator for capital with constant elasticity of substitution (CES)

\[ k \equiv \left( \sum_j \sigma_j k_j^\gamma \right)^{1/\gamma} \]

Type-\(j\) capital \(k_j\), \(j \in \{d, nd\}\); factor shares \(\sigma_j > 0, \forall j; \sum_j \sigma_j = 1\)

Substitution coeff. \(\gamma; -\infty < \gamma < 1\); elasticity of subst. \(1/(1 - \gamma)\)

First order conditions yield

\[ \frac{u_{nd}(w)}{u_d(w)} = \left( \frac{k_d}{k_{nd}} \right)^{1-\gamma} \frac{\sigma_{nd}}{\sigma_d} \]

Composition of investment determined by

... (frictionless) user costs \(u_j(w) = u_j\) for unconstrained firms

... down payments \(u_j(w) = \wp_j\) for severely constrained firms

Constrained firms substitute away from durable assets
Legal enforcement affects investment composition

- Ratio of down payments $\varphi_{nd}/\varphi_d$ and hence $k_d/k_{nd}$ increasing in $\theta$
- No effect on unconstrained firms
“We say that the assets become longer lived, or more durable, if [the liquidation value] $L(t)$ rises for all $0 \leq t \leq T$.”

Durability $\approx$ liquidation value (price and use value fixed)

Interpretation of their liquidation value $L$ in our model

$$L_k \equiv \theta q(\delta)k(1 - \delta)$$

Effect of $L$ is effect of pledgeability $\theta$ in our model not durability
Durability and Financing: Two-Period Assets

- **Technology**
  - New assets last for two periods (one-horse shay depreciation)
  - **New, durable assets** $k_d$ last two periods; price $q_d = 1$ (exogenous)
  - **Used, non-durable assets** $k_{nd}$ one period of useful life left; price $q_{nd}$
  - Perfect substitutes in production; output $Af(k_d + k_{nd})$ where $A$ is TFP and $f$ is (strictly) increasing, concave, and $\lim_{k \to 0} f_k(k) = +\infty$

- Price of used capital $q_{nd}$ determined in **equilibrium**

- In equilibrium,
  - **New assets have larger down payments:** $\varphi_d > \varphi_{nd} (= q_{nd})$
  - **New assets have lower frictionless user cost:** $u_d \leq u_{nd}$

- **Properties**
  - (Un)constrained firms buy used (new) assets
  - Weak legal enforcement economies are net importers of used assets
Entrepreneur’s Problem with Two-Period Assets

- Given net worth $w$, entrepreneur solves

$$v(w) \equiv \max_{d, k_d, k_{nd}, b, w' \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w')$$

subject to budget constraints for current and next period

$$w + b \geq d + k_d + q_{nd} k_{nd}$$

$$Af(k_d + k_{nd}) + q_{nd} k_d \geq Rb + w'$$

and the collateral constraint

$$\theta q_{nd} k_d \geq Rb$$

- Endogenous state variable: net worth $w$
Down Payment

- When collateral constraint binds, $b = R^{-1} \theta q_{nd} k_d$ and so
  \[ \phi_d \equiv 1 - R^{-1} \theta q_{nd} \]
  
is minimal **down payment** required per unit of new, durable assets

Equilibrium

- **Down payment on new, durable assets exceeds used capital price**
  \[ \phi_d > \phi_{nd} = q_{nd} \iff \frac{R}{\theta + R} > q_{nd} \]
- Otherwise new, durable assets would dominate (arbitrage)

Why is down payment for durable assets larger? – Decomposition

\[ \phi_d = 1 - R^{-1} \theta q_{nd} = 1 - R^{-1} q_{nd} + \underbrace{R^{-1}(1 - \theta)q_{nd}}_{\text{non-pledgeable part of resale value}} + \underbrace{R^{-1} \theta q_{nd}}_{\text{user cost } u_d} \]

- New, durable assets down payment includes $1 - \theta$ of resale value
## Durability and Financing Need

- Financing need of new, durable and used, non-durable assets

<table>
<thead>
<tr>
<th>Time</th>
<th>$t$</th>
<th>$t + 1$</th>
<th>$t + 2$</th>
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<tr>
<td><strong>Used, non-durable assets</strong></td>
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<tr>
<td>Value</td>
<td>$q_{nd}$</td>
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<tr>
<td>Collateral value</td>
<td>$0$</td>
<td>0</td>
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<tr>
<td>Borrowing</td>
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<tr>
<td>Internal funds required ($\varphi_{nd}$)</td>
<td>$q_{nd}$</td>
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<td><strong>New, durable assets</strong></td>
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<td>Value</td>
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</tr>
<tr>
<td>Collateral value</td>
<td>$\theta q_{nd}$</td>
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<td>Borrowing</td>
<td>$R^{-1} \theta q_{nd}$</td>
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<tr>
<td>Internal funds required ($\varphi_d$)</td>
<td>$1 - R^{-1} \theta q_{nd}$</td>
<td></td>
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</tr>
</tbody>
</table>

- New, durable assets require more internal funds $\varphi_d > \varphi_{nd}$
  - ... despite ability to borrow $R^{-1} \theta q_{nd} > 0$
  - ... because they cost more $1 > q_{nd} \Rightarrow$ larger financing need
User Cost of Capital for Unconstrained Firm

- **Equilibrium user costs with collateral constraints**
  - User cost of used, non-durable assets: \( u_{nd} \equiv q_{nd} \)
  - User cost of new, durable assets (for unconstrained borrowers):
    \[
    u_d \equiv 1 - R^{-1} q_{nd}
    \]

- **Equilibrium**
  - User cost of new, durable assets less than that of used ones
    \[
    u_d = 1 - R^{-1} q_{nd} \leq u_{nd} = q_{nd} \quad \iff \quad q_{nd} \geq q_{nd}^* = u^* = \frac{R}{1 + R}
    \]
  - Otherwise used, non-durable assets would dominate

- **Frictionless user cost (rental rate) of capital** \( u^* \) would be
  \[
  1 = u^* + R^{-1} u^* \quad \Rightarrow \quad q_{nd}^* = u^* = \frac{R}{1 + R}
  \]
Dividend Paying Firm

- Once firm starts to pay dividends, it always pays dividends and is unconstrained
  - Marginal value of net worth $v_w(w) = \mu = 1$
  - Firm is unconstrained going forward $v_w(w') = \mu' = 1$ and $\lambda' = 0$

- If $q_{nd} > \frac{R}{1+R}$, unconstrained firm purchases only new assets
  
  $$
  u_d = \beta Af_k(k_d + k_{nd}) + v_d
  $$
  
  $$
  u_{nd} = \beta Af_k(k_d + k_{nd}) + v_{nd}
  $$

- Unconstrained firms simply compare user costs
- Unconstrained firms sell assets once they are old/used

- Capital stock of unconstrained firm $\bar{k}_d$ solves
  
  $$
  1 = \beta [Af_k(k_d) + q_{nd}]
  $$
Severely constrained firm purchases only used capital

As \( w \to 0 \) so does \( k_d + k_{nd} \) and hence \( f_k \to +\infty \)

Using Investment Euler equations

\[
1 = \beta \frac{\mu'}{\mu} A f_k (k_d + k_{nd}) + (1 - \theta) q_{nd} + \frac{\nu_d}{\mu \varphi_d}
\]

\[
1 = \beta \frac{\mu'}{\mu} A f_k (k_d + k_{nd}) + \frac{\nu_{nd}}{\mu q_{nd}}
\]

so as \( w \to 0 \), \( \beta \frac{\mu'}{\mu} \to 0 \) and combining Euler equations \( \Rightarrow \nu_d > 0 \)

\[
\varphi_d - \varphi_{nd} = \beta \frac{\mu'}{\mu} (1 - \theta) q_{nd} + \frac{\nu_d}{\mu} - \frac{\nu_{nd}}{\mu}
\]

Severely constrained firms simply compare down payments
Equilibrium

- **Marginal investor in used capital**
  - Indifferent between investing in new and used capital: \( \nu_d = \nu_{nd} = 0 \)
  - Investment Euler equations imply

\[
\begin{align*}
\text{Down payment } \varphi_d &= 1 - R^{-1} \theta q_{nd} - q_{nd} = \beta \frac{\mu'}{\mu} (1 - \theta) q_{nd} \\
\text{Incr. cost of new capital} &\quad \text{Valuation of resale value}
\end{align*}
\]

- **If constrained firm prices used assets, then** \( q_{nd} > q^*_{nd} = \frac{R}{1+R} \)
  - \( \beta \frac{\mu'}{\mu} < R^{-1} \) implies \( q_{nd} > \frac{R}{1+R} \) (or \( u_d < u_{nd} \))
  - Moreover, \( \beta \frac{\mu'}{\mu} > 0 \) so \( \frac{R}{\theta + R} > q_{nd} \) (or \( \varphi_d > \varphi_{nd} \))

- **Market clearing condition in used capital market**

\[
\sum p(w)(1 - \rho) k_d(w) = \sum p(w) k_{nd}(w)
\]

where \( p(w) \) is stationary net worth distribution
Trade

- World market for used capital (at price \( q_{nd} > \frac{R}{1+R} \))

- Strong vs. weak legal enforcement economies: \( \theta_H > \theta_L \)

- Weak legal enforcement economies net used capital importers
  - Unconstrained firms investment identical: \( 1 = \beta[Af(\bar{k}_d) + q_{nd}] \)
  - Constrained firms indifferent between new and used assets when
    \[
    \frac{Af_k(k) + (1 - \theta)q_{nd}}{\varphi_d} = \frac{Af_k(k)}{q_{nd}}
    \]
  - With weak legal enforcement, firms indifferent
    - when their capital stock is larger \( k_L > k_H \)
    - at higher level of net worth
  - More firms use used assets in weak legal enforcement economies
Renting Durable Assets

- **Renting/leasing**
  - Benefit: higher leverage due to repossession advantage
  - Cost: monitoring cost

- Two types of capital $k_j$ (different $\delta_j$) required (fixed proportions)

- Severely constrained firms rent both types of assets

- Moderately constrained rent only more durable assets
Literature

- Hart/Moore (1994)
  - See above

- Eisfeldt/Rampini (2007)
  - Used capital is cheaper but requires ex-post maintenance cost
  - Durability plays no role

- Empirical evidence on used capital
  - Legal enforcement: Benmelech/Bergman (2011)
Conclusion

- **Durable assets are harder to finance**
  - Durability raises financing need and required down payments

- **Financial constraints especially salient for durable assets**
  - Structures (residential and non-residential) and infrastructure
  - Equipment, especially aircraft and ships
  - Consumer durables, especially motor vehicles and household durables
  - Similarly, durable intangible capital ($\theta = 0$), e.g., organization capital

- Constrained firms (and **households**) buy used assets, adopt dominated technologies, and rent durable assets

- Weak **legal enforcement** economies invest in dominated technologies and import used assets

- **Distinguish durability** $1 - \delta$ **from pledgeability** $\theta$
Characterization of Entrepreneur’s Problem

- Well-behaved dynamic program
  - Return function concave; constraint set convex
  - Operator defined by (1) to (4) satisfies Blackwell’s sufficient conditions
  - Solution: ∃! value function $v$; strictly increasing; concave

- First order conditions
  - Denote multipliers on (2) and (3) by $\mu$ and $\beta \mu'$ and on (4) by $\beta \lambda'$
    \begin{align*}
    \mu &= 1 + \nu \\
    \mu q &= \beta \mu' [Af_k(k) + q(1 - \delta)] + \beta \lambda' \theta q(1 - \delta) \\
    \mu &= \mu' + \lambda' \\
    \mu' &= v_w(w')
    \end{align*}

- Envelope condition: $v_w(w) = \mu$
  - Value function continuously differentiable
User cost depends on firm’s discount factor

\[ \psi(w)(\delta) = \varphi(\delta) - \beta \frac{\mu'}{\mu} q(\delta) (1 - \theta) (1 - \delta) \]

\[ = u^* + \beta \frac{\lambda'}{\mu} R u^* (1 - \theta) (1 - \delta) \]

Durable capital user cost sensitive to financial constraints

\[ \frac{\partial \psi(w)(\delta)}{\partial \delta} = -q(\delta) \frac{\lambda'}{\mu} \frac{1-\theta}{r+\delta} < 0 \]

\[ \psi(w)(1) = u^* \text{ independent of } w \text{ (also if } \theta = 1 \text{ but we assume } \theta < 1) \]
Adopting Dominated Technologies – Firm’s Problem

- **Firm’s problem with two types of capital:** Given $w$, solve

$$v(w) \equiv \max_{d,k_j,b,w' \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w')$$

subject to budget constraints and collateral constraint

$$w + b \geq d + \sum_j q_j k_j$$

$$Af\left(\sum_j k_j\right) + \sum_j q_j k_j (1 - \delta_j) \geq Rb + w'$$

$$\theta \sum_j q_j k_j (1 - \delta_j) \geq Rb$$
Continuum of Capital Types with Varying Durability

- **Continuum of technologies with durability** $1 - \delta$
  - Production cost (and price) for type $\delta$: $\phi(\delta)$ with $\phi_\delta(\delta) < 0 < \phi_{\delta\delta}(\delta)$

- **Unconstrained firms minimize frictionless user cost** $u(\delta)$
  
  $$\delta^* \in \arg \max_{\delta \in [0,1]} R^{-1} \phi(\delta) (r + \delta)$$

  - In frictionless economy (or if $\theta = 1$), only durability $1 - \delta^*$ produced

- **Constrained firms purchase less durable types of capital**
  - Possibly continuum of less durable types of capital produced
Neoclassical Model with Vintage Capital

- **Neoclassical vintage capital**
  - New capital: cost $q_d$; depreciation $\delta_d$; fraction $\eta$ becomes used
  - Used capital: price $q_{nd}$; depreciation $\delta_{nd} > \delta_d$
  - Previous vintage model: $\delta_d = 0$, $\delta_{nc} = 1$, $\eta = 1$, $q_d = 1$, $q_{nd} = q$

- **Endogenous price of used capital** $q_{nd}$
  - If marginal investor is unconstrained, $u_d = u_{nd}$; frictionless price
    \[
    q^*_{nd} = \frac{r + \delta_d + \eta(1 - \delta_d)}{r + \delta_{nd} + \eta(1 - \delta_d)} q_d < q_d
    \]
  - If marginal investor is constrained, $q_{nd} > q^*_{nd}$

- **Down payments** $\varphi_d > \varphi_{nd}$
  - $\varphi_d \equiv q_d - R^{-1} \theta q_d (1 - \delta_d) + R^{-1} \theta \eta (q_d - q_{nd})(1 - \delta_d)$
  - $\varphi_{nd} \equiv q_{nd} - R^{-1} \theta q_{nd} (1 - \delta_{nd})$
Firm’s problem with two types of capital: Given \( w \), solve

\[
v(w) \equiv \max_{d,k_j,b,w' \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w')
\]

subject to budget constraints and collateral constraint

\[
w + b \geq d + \sum_j q_j k_j
\]

\[
Af \left( \left( \sum_j \sigma_j k_j^{\gamma} \right)^{1/\gamma} \right) + \sum_j q_j k_j (1 - \delta_j) \geq Rb + w'
\]

\[
\theta \sum_j q_j k_j (1 - \delta_j) \geq Rb
\]

First order condition for type \( j \) capital

\[
u_j(w) = \beta \frac{\mu'}{\mu} Af_k(k) \frac{\partial k}{\partial k_j}
\]
Distortion of Financially Constrained Investment

- Investment distorted away from durable assets if

\[
\frac{u_{nd}}{u_d} > \frac{\varphi_{nd}}{\varphi_d} \iff \frac{\varphi_d}{u_d} > \frac{\varphi_{nd}}{u_{nd}}
\]

- Down payment to user cost ratio higher for durable assets

\[
\frac{\varphi_j}{u_j} = \frac{R^{-1}q_j(r + \delta_j) + R^{-1}q_j(1 - \theta)(1 - \delta_j)}{R^{-1}q_j(r + \delta_j)} = 1 + (1 - \theta) \frac{1 - \delta_j}{r + \delta_j}
\]

- Decreasing in \(\delta_j\): \(\frac{\partial}{\partial \delta_j} \left( \frac{\varphi_j}{u_j} \right) < 0\)

- Why? – Residual value is larger fraction of value

\[
\frac{\varphi_j - u_j}{\varphi_j} = \frac{1}{1 + \left( (1 - \theta) \frac{1 - \delta_j}{r + \delta_j} \right)^{-1}}
\]
“The Deacons Masterpiece: or the Wonderful ‘One-Hoss-Shay’”

Oliver Wendell Holmes (1858)
Characterization of Entrepreneur’s Problem

- Well-behaved dynamic program
  - Return function concave; constraint set convex
  - Operator defined by (5) to (8) satisfies Blackwell’s sufficient conditions
  - Solution: ∃! value function \( v \); strictly increasing; concave

**First order conditions**

- Denote multipliers on (6) and (7) by \( \mu \) and \( \beta \mu' \) and on (8) by \( \beta \lambda' \)

\[
\begin{align*}
\mu &= 1 + \nu \\
\mu &= \beta \mu' \left[ Af_k(k_d + k_{nd}) + q_{nd} \right] + \beta \lambda' \theta q_{nd} + \nu_d \\
\mu q_{nd} &= \beta \mu' Af_k(k_d + k_{nd}) + \nu_{nd} \\
\mu &= \mu' + \lambda' \\
\mu' &= v_w(w')
\end{align*}
\]

- **Envelope condition (marginal value of net worth):** \( v_w(w) = \mu \)
  - Value function continuously differentiable
# Durability and Financing Need

- Financing need of new, durable and used, non-durable assets

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<th>Time</th>
<th>t</th>
<th>t + 1</th>
<th>t + 2</th>
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<tbody>
<tr>
<td>Used, non-durable assets</td>
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</tr>
<tr>
<td>Value</td>
<td>( q_{nd} )</td>
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</tr>
<tr>
<td>Collateral value</td>
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<tr>
<td>Borrowing</td>
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<tr>
<td>Internal funds required (( \varphi_{nd} ))</td>
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</tr>
<tr>
<td>New, durable assets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>1</td>
<td>( q_{nd} )</td>
<td>0</td>
</tr>
<tr>
<td>Collateral value</td>
<td>( \theta q_{nd} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Borrowing</td>
<td>( R^{-1} \theta q_{nd} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal funds required (( \varphi_{d} ))</td>
<td>( 1 - R^{-1} \theta q_{nd} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **New, durable assets require more internal funds** \( \varphi_{d} > \varphi_{nd} \)
  - **... despite ability to borrow** \( R^{-1} \theta q_{nd} > 0 \)
  - **... because they cost more** \( 1 > q_{nd} \Rightarrow \) larger financing need

- Example: If \( R \approx 1, \theta = 0, \) and \( q_{nd} = 0.5 \), then \( \varphi_{d} = 1 \)
In first period, new assets depreciate at **depreciation rate**

\[
\delta_d \equiv \frac{1 - q_{nd}}{1} \leq 1 - \frac{R}{1 + R} = \frac{1}{1 + R} < 50\%
\]

... and in second period (when used) at rate

\[
\delta_{nd} \equiv \frac{q_{nd} - 0}{q_{nd}} = 100\%
\]
User cost of durable assets depends on firm’s discount factor

$$u_d(w) = \varphi_d - \beta \frac{\mu'}{\mu} (1 - \theta) q_{nd}$$

$$= u_d + \beta \frac{\chi'}{\mu} (1 - \theta) q_{nd}$$

User cost of new, durable assets for severely constrained firm

- As $w \to 0$, $\beta \frac{\mu'}{\mu} \to 0$ so

$$u_d(w) \to \varphi_d$$

- Severely constrained firms simply compare down payments
Composition of Investment with Two Period Assets

- Suppose two period assets are **imperfect substitutes**

- First order conditions yield (as in neoclassical case)

\[ \frac{u_{nd}}{u_d(w)} = \left( \frac{k_d}{k_{nd}} \right)^{1-\gamma} \frac{\sigma_{nd}}{\sigma_d} \]

- Composition of investment
  - \( \varphi_{nd}/\varphi_d (u_{nd}/u_d) \) determines composition for (un)constrained firms
  - For used, non-durable assets: \( \frac{\varphi_{nd}}{u_{nd}} = \frac{q_{nd}}{q_{nd}} = 1 \)
  - For new, durable assets: \( \frac{\varphi_d}{u_d} = \frac{1-R^{-1}\theta q_{nd}}{1-R^{-1}q_{nd}} > 1 \)
  - Constrained firms substitute away from new capital
  - Intuition: \( \frac{\varphi_d - u_d}{\varphi_d} = \frac{R^{-1}(1-\theta)q_{nd}}{1-R^{-1}q_{nd}} > 0 = \frac{\varphi_{nd} - u_{nd}}{\varphi_{nd}} \)

- Especially with weak legal enforcement
Renting Durable Assets

- **Capital goods with different durability**
  - Capital $k_j$, $j = \{d, nd\}$, with depreciation rate $\delta_d < \delta_{nd}$ and price $q_j$
  - Leontief aggregator $k = \min\{\frac{k_d + k_{ld}}{\sigma_d}, \frac{k_{nd} + k_{lnd}}{\sigma_{nd}}\}$
    - Implies $k_j + k_{jd} = \sigma_j k$ where $k$ is aggregate capital
  - Rental price $u_{lj} \equiv R^{-1} q_j (r + \delta_j + m)$ where $m$ is monitoring cost
  - Firm cannot abscond with rented capital (repossession advantage)

- **Entrepreneur’s problem with rentals**: Given $w$, solve
  \[
  v(w) \equiv \max_{d, k_j, k_{lj}, b, w' \in \mathbb{R}_+^5 \times \mathbb{R}_2^2} d + \beta v(w')
  \]  
  (9)

subject to budget, collateral, and technological constraints

\[
|w + b| \geq d + \sum_j q_j k_j + \sum_j u_{lj} k_{lj}
\]  
(10)

\[
Af(k) + \sum_j q_j k_j (1 - \delta_j) \geq Rb + w'
\]  
(11)

\[
\theta \sum_j q_j k_j (1 - \delta_j) \geq Rb
\]  
(12)

\[
k_j + k_{lj} \geq \sigma_j k
\]  
(13)
Renting Durable Assets (Cont’d)

- First order conditions with multipliers as before and $\beta \mu' \eta_j$ on (13)

\[
\sum_j \sigma_j \eta_j = Af_k(k) \\
\mu q_j = \beta \mu' \eta_j + \beta \mu' q_j (1 - \delta_j) + \beta \lambda' \theta q_j (1 - \delta_j) + \nu_j \\
\mu u'_l = \beta \mu' \eta_j + \nu'_j
\]

- Investment Euler equation (IEE)

\[
1 = \beta \frac{\mu' \eta_j + (1 - \theta) q_j (1 - \delta_j)}{\mu} \varphi_j + \frac{\nu_j}{\mu \varphi_j}
\]

- More constrained firms rent less durable assets
  - Suppose rent and own some type $j$ capital, so $\nu_j = \nu'_l = 0$ and IEE

\[
\frac{\lambda'}{\mu} (1 - \theta)(1 - \delta_j) = m
\]
  - If $\delta_j$ higher, so multiplier on collateral constraint $\frac{\lambda'}{\mu}$ higher
Literature: Durable Goods Theory

- **Optimal durability and planned obsolescence**
  - Conventional wisdom: monopolists produce less durable goods
  - Resuscitated: Barro (1972): impatience; Rust (1986): scrappage

- **Role of rental markets**
  - Coase (1972), Bulow (1982), Stokey (1981): Coasian dynamics
  - Hendel/Lizzeri (2002): leasing ameliorates adverse selection

- **Key driving force: market power**
  - Our theory: optimal competitive response to financial constraints
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- **Other features**
  - Illiquidity