Financial Intermediary Capital*

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Abstract

We propose a dynamic theory of financial intermediaries that are better able to collateralize claims than households, that is, have a collateralization advantage. Intermediaries require capital as they have to finance the additional amount that they can lend out of their own net worth. The net worth of financial intermediaries and the corporate sector are both state variables affecting the spread between intermediated and direct finance and the dynamics of real economic activity, such as investment, and financing. The accumulation of net worth of intermediaries is slow relative to that of the corporate sector. The model is consistent with key stylized facts about macroeconomic downturns associated with a credit crunch, namely, their severity, their protractedness, and the fact that the severity of the credit crunch itself affects the severity and persistence of downturns. The model captures the tentative and halting nature of recoveries from crises.

Keywords: Collateral; Financial intermediation; Financial constraints; Investment

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1 Introduction

The capitalization of financial intermediaries is arguably critical for economic fluctuations and growth. We provide a dynamic theory of financial intermediaries that have a collateralization advantage, that is, are better able to collateralize claims than households. Financial intermediaries require net worth as their ability to refinance their loans to firms by borrowing from households is limited, as intermediaries need to collateralize their promises as well. Importantly, since both intermediaries and firms are subject to collateral constraints, the net worth of both plays a role in our model, in contrast to most previous work, and these two state variables jointly determine the dynamics of economic activity, investment, financing, and loan spreads. A key feature of our model is that the accumulation of the net worth of intermediaries is slow relative to that of the corporate sector. The slow-moving nature of intermediary capital results in economic dynamics that are consistent with key stylized facts about macroeconomic downturns associated with a credit crunch, namely, their severity, their protractedness, and the fact that the severity of the credit crunch itself affects the severity and persistence of downturns. Most uniquely, the model captures the tentative and halting nature of recoveries from crises.

In the model firms need to finance investment and can raise financing from both intermediaries and households. Firm financing needs to be collateralize with tangible assets and is subject to two types of collateral constraints, one for loans from households and one for loans from intermediaries. Firms require net worth as collateral constraints limit financing. Intermediaries are better able to enforce collateralized claims and thus can lend more to firms than households can. However, intermediaries have to collateralize their promises as well, and can borrow against their corporate loans only to the extent that households themselves can collateralize the assets backing these loans. Thus, intermediaries need to finance the additional amount that they can lend out of their own net worth, giving a role to financial intermediary capital. We show that these collateral constraints can be derived from an economy with limited enforcement that constrains firms' and intermediaries' ability to make credible promises.[1]

We focus our analysis on the deterministic version of the economy and start by analyzing the steady state. In the steady state, intermediaries are essential in our economy in the sense that allocations can be achieved with financial intermediaries, which cannot be achieved otherwise. Since intermediary net worth is limited, intermediated finance commands a positive spread over the interest rate charged by households. Moreover,

[1]We model limited enforcement à la Kehoe and Levine (1993) but without exclusion, as in Chien and Lustig (2010) and Rampini and Viswanathan (2010, 2013), and extend their results by introducing limited participation as well. Intermediaries, but not households, participate in markets at all times which affords intermediaries with an advantage in enforcing claims. This economy with limited enforcement without exclusion and with limited participation is equivalent to our economy with collateral constraints.

in the steady state, the equilibrium capitalization of both the representative firm and intermediary are positive. Steady state firm net worth is determined by the fraction of tangible assets that firms cannot pledge to intermediaries or households and thus have to finance internally, while steady state intermediary net worth is determined by the fraction of investment that intermediaries have to finance due to their collateralization advantage, that is, by the difference in the ability to enforce collateralized claims between intermediaries and households.

Away from the steady state, the equilibrium spread on intermediated finance is determined by firm and intermediary net worth jointly. Intermediary net worth increases intermediated loan supply and hence reduces the spread all else equal. In contrast, firm net worth has two opposing effects on intermediated loan demand: on the one hand, firm net worth increases investment, lowering the levered marginal product of capital, reducing firms' willingness to pay, and lowering the spread; on the other hand, firm net worth, by increasing investment, increases firms' collateralizable assets, which in turn raises loan demand, raising the spread. Hence, the spread can be high or low when firm net worth is low as it depends on the relative capitalization of firms and intermediaries. When intermediary net worth is relatively scarce, the collateral constraint on intermediated finance is slack and firm net worth reduces the spread. When firm net worth is relatively scarce instead, the collateral constraint on intermediated finance binds and firm net worth increases the spread as it increases firms' ability to pledge and hence loan demand. This interaction of loan supply and demand results in rich and subtle dynamics for intermediated finance and its spread.

In the dynamics of our model, two state variables, the net worth of firms and intermediaries, jointly determine the dynamic supply and demand for intermediated loans and the equilibrium intermediary interest rate. A key feature of the equilibrium dynamics is that intermediary net worth accumulation is slow relative to corporate net worth accumulation, at least early in a recovery; the reason is that intermediary net worth grows at the intermediary interest rate, which is at most the marginal levered product of capital, and may be lower than that when the collateral constraint on intermediated finance binds; in contrast, firms accumulate net worth at the average levered product of capital, which exceeds the marginal levered product of capital.

Several aspects of the dynamic response of the economy to drops in the net worth of firms, intermediaries, or both are noteworthy. First, consider the response of the economy to a drop in corporate net worth only. In such a downturn, corporate investment drops and the collateral constraints on intermediated finance implies a reduction in firms' loan demand for intermediated loans. Facing reduced corporate loan demand, intermediaries respond by paying dividends and may lend to households at an interest rate less than their rate of time preference, that is, hold "cash;" thus, initially, the intermediary interest rate drops. The reason that intermediaries conserve net worth despite temporarily low interest rates is that, since firms reaccumulate net worth faster, corporate loan demand is expected to recover relatively quickly and intermediary capital becomes scarce again, raising the intermediary interest rate.

Second, the recovery from a drop in intermediary net worth (a "credit crunch") is relatively slow making such episodes protracted. In a credit crunch, the spread on intermediary finance rises and investment drops even if the corporate sector remains well capitalized, as firms are forced to delever due to the limited supply of intermediated loans and thus have to finance a larger part of their investment internally. Indeed, deleveraging may mean that firms temporarily accumulate more net worth then they retain in the steady state. Moreover, and importantly, a credit crunch can have persistent real effects as corporate investment may not recover for a prolonged period of time, due to the slow recovery of intermediary capital.

Third, a drop in both corporate and intermediary net worth (a downturn associated with a credit crunch) makes the downturn more severe and the recovery more protracted, featuring a higher spread on intermediated finance. Moreover, the recovery can stall, after an initial relatively swift recovery, when firm net worth has partially recovered while intermediaries have yet to recover. When the economy stalls, firms may seem to be well capitalized because they are paying dividends, but the economy nevertheless has not fully recovered. The severity of the credit crunch itself significantly affects the depth and protractedness of the macroeconomic downturn and the spike in the spread on intermediary finance; indeed, the recovery from a downturn associated with a more severe credit crunch is especially slow and halting, with output depressed and the spread elevated for a prolonged period of time. Finally, our theory predicts that in a bank-oriented economy downturns associated with a credit crunch are more severe and more protracted, with longer stalls of the recovery at lower levels of investment. Thus, the recovery from crises in bank-oriented economies may be more sluggish than in economies with more market-oriented financial systems.

We revisit the evidence on the effect of financial crises from the vantage point of our theory. There are three main stylized facts about downturns associated with financial crises that emerge from prior empirical work: (i) downturns associated with financial crises are more severe; (ii) recoveries from financial crises are protracted and often tentative; and (iii) the severity of the financial crises itself affects the severity and protractedness of the downturn. Consistent with this evidence, our model predicts that the effects of a credit crunch on economic activity is protracted due to the slow accumulation of intermediary net worth. But perhaps most uniquely, our model captures the tentative and halting nature of recoveries from such episodes emphasized by Reinhart and Rogoff (2014) and allows the analysis of the severity of the credit crunch itself on the recovery. Thus, our model implies empirically plausible dynamics.

Few extant theories of financial intermediaries provide a role for intermediary capital. Notable is in particular Holmström and Tirole (1997) who model intermediaries as monitors that cannot commit to monitoring and hence need to have their own capital at stake to have incentives to monitor. In their static analysis, firm and intermediary capital are exogenous and the comparative statics with respect to these are analyzed. Holmström and Tirole conclude that "[a] proper investigation ... must take into account the feedback from interest rates to capital values. This will require an explicitly dynamic model, for instance, along the lines of Kiyotaki and Moore [1997a]." We provide a dynamic model in which the joint evolution of firm and intermediary net worth and the interest rate on intermediated finance are endogenously determined. Diamond and Rajan (2001) and Diamond (2007) model intermediaries as lenders which are better able to enforce their claims due to their specific liquidation or monitoring ability in a similar spirit to our model, but do not consider equilibrium dynamics. In contrast, the capitalization of intermediaries plays essentially no role in liquidity provision theories of financial intermediation (Diamond and Dybvig (1983)), in theories of financial intermediaries as delegated, diversified monitors (Diamond (1984), Ramakrishnan and Thakor (1984), and Williamson (1986)) or in coalition based theories (Townsend (1978) and Boyd and Prescott (1986)).

Dynamic models in which net worth plays a role, such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997a), typically consider the role of firm net worth only, although dynamic models in which intermediary net worth matters have recently been considered (see, for example, Gertler and Kiyotaki (2010) and Brunnermeier and Sannikov (2014)).[2] However, to the best of our knowledge, we are the first to consider a dynamic contracting model in which both firm and intermediary net worth are critical and jointly affect the dynamics of financing, spreads, and economic activity.

In Section 2 we describe the model with two types of collateral constraints, for intermediated and direct finance, respectively, and discuss how these collateral constraints can be derived in an economy with limited enforcement and limited participation. Section 3 shows that intermediation is essential in our economy and determines the capitalization of intermediaries and spreads on intermediated finance in the steady state. The dynamics of intermediary capital are analyzed in Section 4, focusing on the dynamic interaction between corporate and intermediary net worth, the two state variables in the model; specifically, we consider the effects of a downturn, a credit crunch, and a downturn associated with a credit crunch. In Section 5 we use the model to revisit three main stylized facts about downturns associated with financial crises. Section 6 considers risk management of financial intermediary capital. Section 7 concludes. All proofs are in Appendix A.

[2] Gromb and Vayanos (2002) and He and Krishnamurthy (2012) study the asset pricing implications of intermediary net worth in dynamic models.

2 Collateralized finance with intermediation

We propose a dynamic model of financial intermediaries that have a collateralization advantage, that is, are better able to collateralize claims than households. In the model firms can borrow from both intermediaries and households, and all financing needs to be collateralized. Firm financing is subject to two types of collateral constraints, one for loans from households and one for loans from intermediaries. Since intermediaries are better able to enforce collateralized claims, they can lend more than households, but the additional amount that they can lend has to be financed out of their own net worth, giving a role to financial intermediary capital. Thus, the net worth of both intermediaries and firms are state variables and jointly determine economic activity.

We show that these collateral constraints can be derived from an economy with limited enforcement that constrains firms' and intermediaries' ability to make credible promises. Intermediaries, but not households, participate in markets at all times which affords intermediaries with an advantage in enforcing claims. This economy with limited enforcement and limited participation is equivalent to our economy with collateral constraints.

2.1 Environment

Time is discrete and the horizon infinite. We focus on a deterministic environment here and study a stochastic environment in Section 6. There are three types of agents: entrepreneurs, financial intermediaries, and households; we discuss these in turn.

There is a continuum of entrepreneurs or firms with measure one which are risk neutral and subject to limited liability and discount the future at rate $\beta \in (0, 1)$. We consider an environment with a representative firm. The representative firm (which we at times refer to simply as the firm or the corporate sector) has limited net worth $w_0 > 0$ at time 0 and has access to a standard neoclassical production technology; an investment of capital k_t at time t yields output $Af(k_t)$ at time $t + 1$ where $A > 0$ is the total factor productivity and $f(\cdot)$ is the production function, which is strictly increasing and strictly concave and satisfies the Inada condition $\lim_{k\to 0} f_k(k) = +\infty$. Capital depreciates at rate $\delta \in (0, 1)$. The firm can raise financing from both intermediaries and households as we discuss below.

There is a continuum of financial intermediaries with measure one which are risk neutral, subject to limited liability, and discount future payoffs at $\beta_i \in (0, 1)$. We consider the problem of a representative financial intermediary with limited net worth $w_{i0} > 0$ at time 0.[3] Intermediaries can lend to and borrow from firms and households as described in more detail below.

[3] There is a representative intermediary in our model since intermediaries have constant returns to scale, making the distribution of intermediaries' net worth irrelevant and aggregation in the intermediation sector straightforward, and thus only the aggregate capital of the intermediation sector matters.

There is a continuum of households with measure one which are risk neutral and discount future payoffs at a rate $R^{-1} \in (0,1)$. Households have a large endowment of funds and collateral in all dates, and hence are not subject to enforcement problems but rather are able to commit to deliver on their promises. They are willing to provide any claim at a rate of return R as long as the claims satisfy the firms' and intermediaries' collateral constraints.

We assume that $\beta < \beta_i < R^{-1}$, that is, households are more patient than intermediaries which in turn are more patient than the firms. Since firms and intermediaries are financially constrained, they would have an incentive to accumulate net worth and save themselves out of their constraints. Assuming that firms and intermediaries are impatient relative to households is a simple way to ensure that their net worth matters even in the long run. Moreover, assuming that intermediaries are somewhat more patient than firms implies that the net worth of both the corporate sector and the intermediary sector are uniquely determined in the long run, too. We think these features are desirable properties of a dynamic model of intermediation and are empirically plausible.

Financial intermediaries in this economy have a collateralization advantage. Specifically, intermediaries are better able to collateralize claims than households; intermediaries can seize up to fraction $\theta_i \in (0,1)$ of the (resale value of) collateral backing promises issued to them whereas households can to seize only fraction $\theta < \theta_i$, where $\theta > 0$.

One interpretation of the environment is that there are three types of capital, working capital, equipment (fraction $\theta_i - \theta$), and structures (fraction θ) (see Figure 1). Firms have to finance working capital entirely out of their own net worth. Only intermediaries can lend against equipment, but both households and intermediaries can lend against structures. Equipment loans have to be extended by intermediaries and have to be finance out of financial intermediary capital. We refer to these loans as intermediated finance. In contrast, structure loans can be provided by either intermediaries or households. We assume that these loans are provided by households and refer to such loans as direct finance. This is without loss of generality and we could equivalently assume that all corporate loans are extended by the intermediary who in turn borrows from households, which we refer to as the *indirect implementation*. However, we focus on the (equivalent) *direct implementation* in which households extend all structure loans directly throughout as it simplifies the notation and analysis.[4]

We assume that loans are one-period and the economy has markets in two types of one-period ahead claims, claims provided by intermediaries and claims provided by households, each subject to a collateral constraint. These collateral constraints are similar to the ones in Kiyotaki and Moore (1997a), except that there are different collateral

[4]Holmström and Tirole's (1997) model of financial intermediation also has two implementations – a direct one and an indirect one – which are equivalent and they, too, focus on the direct implementation.

constraints for promises to pay intermediaries and households.

Here we simply assume that there are only one-period ahead claims and that intermediaries provide the equipment loans, and only the equipment loans, and must finance these out of their own net worth. In Section 2.3 we provide an environment with limited enforcement and limited participation which is equivalent to the economy with collateral constraints described here. In that environment each period has two subperiods, morning and afternoon, and equipment can serve as collateral only in the morning. The key assumption affording intermediaries an enforcement advantage is that intermediaries, but not households, participate in markets at all times; thus, equipment loans must be provided by intermediaries. Moreover, limited enforcement of intermediaries' liabilities implies that intermediaries must finance such loans out of their own funds. Thus, the properties we assume here are in fact endogenous properties of optimal dynamic contracts.

2.2 Economy with collateral constraints

We write the firm's and intermediary's problems recursively by defining an appropriate state variable, *net worth*, for the firm (w) and intermediary (w_i).[5] The state of the economy $z \equiv \{w, w_i\}$ comprises these two endogenous state variables, the net worth of the corporate sector w and of the intermediary sector w_i. The interest rate on intermediated finance R_i' depends on the state z of the economy, as shown below, but we suppress the argument for notational simplicity.

The *firm's problem* stated recursively is, for given net worth w and aggregate state z, to maximize the discounted expected value of future dividends by choosing a dividend payout policy d, capital k, promises b' and b_i' to households and intermediaries, and net worth w' for the next period, taking the interest rate on intermediated finance R_i' and its law of motion as given, to solve

$$v(w, z) = \max_{\{d,k,b',b_i',w'\}} d + \beta v(w', z') \tag{1}$$

subject to the budget constraints for the current and next period

$$\begin{aligned} w &\geq d + k - b' - b_i', && (2) \\ A'f(k) + k(1-\delta) &\geq w' + Rb' + R_i'b_i', && (3) \end{aligned}$$

the collateral constraints for loans from intermediaries and households

$$\begin{aligned} (\theta_i - \theta)k(1-\delta) &\geq R_i'b_i', && (4) \\ \theta k(1-\delta) &\geq Rb', && (5) \end{aligned}$$

[5]In our model with collateral constraints net worth, properly defined, turns out to be the most convenient state variable, whereas the state variable is typically continuation utility in dynamic contracting models in the literature.

and the non-negativity constraints

$$d, k, b_i' \geq 0. \tag{6}$$

Next period the firm repays Rb' to households and $R_i'b_i'$ to financial intermediaries as the budget constraint for the next period, equation (3), shows. While equation (3) is stated as an inequality, which allows for free disposal, it binds at an optimal solution, and hence we can define the net worth of the firm (next period) as $w' \equiv A'f(k) + k(1-\delta) - Rb' - R_i'b_i'$, that is, cash flows plus assets (net of depreciation) minus liabilities. The budget constraint for this period, equation (2), states that current net worth can be spent on dividends and purchases of capital net of the proceeds of the loans from households and intermediaries. The interest rate on loans from households R is constant as discussed above.

In the direct implementation which we focus on, equipment loans are provided by intermediaries and all structure loans are provided by households. Our economy has two types of collateral constraints (4) and (5), illustrated in Figure 1; these state that repayments to intermediaries and households cannot exceed the residual value of equipment and structures, respectively.[6] These collateral constraints are inequality constraints and may or may not bind.

The *intermediary's problem* stated recursively is, for given net worth w_i, to maximize the discounted value of future dividends by choosing a dividend payout policy d_i, loans to households l', intermediated loans to firms l_i', and net worth w_i' next period to solve

$$v_i(w_i, z) = \max_{\{d_i, l', l_i', w_i'\}} d_i + \beta_i v_i(w_i', z') \tag{7}$$

subject to the budget constraints for the current and next period

$$\begin{aligned} w_i &\geq d_i + l' + l_i', & (8) \\ Rl' + R_i'l_i' &\geq w_i', & (9) \end{aligned}$$

and the non-negativity constraints

$$d_i, l', l_i' \geq 0. \tag{10}$$

We can define the net worth of the intermediary (next period) as $w_i' \equiv Rl' + R_i'l_i'$, that is, the sum of the proceeds from loans to households and firms (as equation (9) binds at an optimal solution). Recall that we focus on the direct implementation in which the intermediary only lends the additional amount that it can take as collateral from firms to

[6] A model with two types of collateral constraints is also studied by Caballero and Krishnamurthy (2001) who consider international financing in a model in which firms can raise funds from domestic and international financiers subject to separate collateral constraints.

simplify the analysis (but this is without loss of generality). In this direct implementation, the intermediary can lend to households but not borrow from them ($l' \geq 0$); thus, the intermediary's collateral constraint reduces to a short-sale constraint.[7]

We now define an equilibrium for our economy which determines both aggregate economic activity and the cost of intermediated finance.

Definition 1 (Equilibrium)**.** *An equilibrium is an allocation $x \equiv [d, k, b', b'_i, w']$ for the representative firm and $x_i \equiv [d_i, l', l'_i, w'_i]$ for the representative intermediary for all dates and an interest rate process R'_i for intermediated finance such that (i) x solves the firm's problem in (1)-(6) and x_i solves the intermediary's problem in (7)-(10) and (ii) the market for intermediated finance clears in all dates, that is, $l'_i = b'_i$.*

Note that equilibrium promises are default free, as the promises satisfy the collateral constraints (4) and (5), which ensure that neither firms nor financial intermediaries are able to issue promises on which it is not credible to deliver.

The first-order conditions of the firm's problem in equations (1) to (6), which are necessary and sufficient, can be written as

$$\begin{aligned}
\mu &= 1 + \nu_d, && (11)\\
\mu &= \beta\left(\mu'\left[A' f_k(k) + (1-\delta)\right] + \left[\lambda'\theta + \lambda'_i(\theta_i - \theta)\right](1-\delta)\right), && (12)\\
\mu &= R\beta\mu' + R\beta\lambda', && (13)\\
\mu &= R'_i\beta\mu' + R'_i\beta\lambda'_i - R'_i\beta\nu'_i, && (14)\\
\mu' &= v_w(w', z'), && (15)
\end{aligned}$$

where the multipliers on the constraints (2) through (5) are μ, $\beta\mu'$, $\beta\lambda'$, and $\beta\lambda'_i$, and ν_d and $R'_i\beta\nu'_i$ are the multipliers on the non-negativity constraints on dividends and intermediated borrowing.[8] The envelope condition is $v_w(w, z) = \mu$, the marginal value of firm net worth, which by equation (11) exceeds 1 and equals 1 when the firm pays dividends, as does the marginal value of net worth next period, denoted μ' (see equation (15)).

Define the *down payment* $\wp$ when the firm borrows the maximum amount it can from households only as $\wp = 1 - R^{-1}\theta(1-\delta)$. Similarly, define the down payment when the firm borrows the maximum amount it can from both households (at interest rate R) and intermediaries (at interest rate R'_i) as $\wp_i(R'_i) = 1 - [R^{-1}\theta + (R'_i)^{-1}(\theta_i - \theta)](1-\delta)$ (illustrated at the bottom of Figure 1). Note that the down payment, at times referred

[7] In the indirect implementation, the intermediary's collateral constraint implies that the intermediary can borrow from households up to the value of corporate loans the intermediary has extended against structures (see Section 2.3 and Appendix B, especially equation (B.24), for details).

[8] We ignore the constraints that $k \geq 0$ and $w' \geq 0$ as they are redundant, due to the Inada condition and the fact that the firms can never credibly promise their entire net worth in next period (which can be seen by combining (3) at equality with (4) and (5)).

to as the margin requirement, is endogenous in our model. Using this definition and equations (12) through (14) the firm's investment Euler equation can be written as

$$1 \geq \beta \frac{\mu'}{\mu} \frac{A' f_k(k) + (1-\theta_i)(1-\delta)}{\wp_i(R_i')}, \tag{16}$$

which obtains whether or not the collateral constraints bind.

The first-order conditions of the intermediary's problem in equations (7) to (10), which are necessary and sufficient, can be written as

$$\begin{aligned} \mu_i &= 1 + \eta_d, & (17)\\ \mu_i &= R\beta_i\mu_i' + R\beta_i\eta', & (18)\\ \mu_i &= R_i'\beta_i\mu_i' + R_i'\beta_i\eta_i', & (19)\\ \mu_i' &= v_{i,w}(w_i', z'), & (20) \end{aligned}$$

where the multipliers on the constraints (8) and (9) are μ_i and $\beta_i\mu_i'$, and η_d, $R\beta_i\eta'$, and $R_i'\beta_i\eta_i'$ are the multipliers on the non-negativity constraints on dividends and direct and intermediated lending. The envelope condition is $v_{i,w}(w_i, z) = \mu_i$, the marginal value of intermediary net worth, which exceeds 1 by equation (17) and equals 1 when dividends are paid, as does the marginal value of net worth next period, denoted μ_i' (see equation (20)). Equations (18) and (19) imply that intermediaries lend to households only when $R_i' = R$.

2.3 Deriving collateral constraints from limited enforcement

This section describes an economy with limited enforcement which is equivalent to the economy with collateral constraints described above. We first describe the environment with limited enforcement, allowing for long-term contracts, and then sketch our equivalence result which we formally state and prove in Appendix B.[9] This equivalence is significant for three reasons; it shows that (i) intermediaries must provide equipment loans, that is, loans against the additional amount of collateral they can seize; (ii) intermediaries must finance these loans out of their own net worth; and (iii) the restriction to one-period ahead contracts is without loss of generality. Thus, the economy with limited enforcement endogenizes three key properties of the model with collateral constraints that we have simply assumed so far. That said, a reader, who is primarily interested in the dynamic implications of our model, may choose to skip this derivation and proceed directly to Section 3.

[9] Appendix B establishes this equivalence in an economy with stochastic total factor productivity, as described in Section 6, of which the economy considered here is a special case. In online Appendix C, we establish this result and characterize the equilibrium in a static economy, which simplifies the analysis.

Suppose that the environment is as before, but that each period has two subperiods which we refer to as morning and afternoon. The economy has limited participation by households. All types of agents participate in markets in the afternoon. In the morning, however, only entrepreneurs and intermediaries participate in markets but not households. This is the key assumption affording intermediaries an enforcement advantage.

The economy has limited enforcement in the spirit of Kehoe and Levine (1993) except that firms or intermediaries that default cannot be excluded from participating in financial and real asset markets going forward. Rampini and Viswanathan (2010, 2013) study this class of economies but consider an economy with only one type of lender with deep pockets and hence take the interest rate as given. We build on their work by considering an economy with two types of lenders, intermediaries and households, of which one has limited net worth, and extend their analysis by determining the interest rates on intermediated finance in dynamic general equilibrium with aggregate fluctuations.

Specifically, enforcement is limited as follows: Firms can abscond both in the morning and in the afternoon. In the morning, after cash flows are realized, firms can abscond with all cash flows and a fraction $1-\theta_i$ of depreciated capital, where $\theta_i \in (0,1)$. In the afternoon, firms can abscond with cash flows net of payments made in the morning and a fraction $1-\theta$ of depreciated capital, where $\theta \in (0,1)$. Critically, we assume that $\theta_i > \theta$, which means that firms can abscond with less capital in the morning than in the afternoon. Intermediaries, too, can abscond in both subperiods, although there is no temptation for intermediaries to do so in the morning, as they will at best receive payments, and so we can ignore this constraint and focus just on the afternoon. In the afternoon, intermediaries can abscond with any payments received in the morning. To reiterate, neither firms nor intermediaries are excluded from markets after default.

The timing is summarized as follows (see Figure 2): Each afternoon, firms and intermediaries first decide whether to make their promised payments or default. Then, firms, intermediaries, and households consume, invest, and borrow and lend. The next morning, cash flows are realized. Firms decide whether to make their promised morning payments or default. Firms carry over the cash flows net of payments made and intermediaries carry over any funds received until the afternoon.

Loans backed by the additional amount of collateral that can be seized in the morning, that is, $\theta_i - \theta$, must be repaid in the morning, as by the afternoon firms can abscond with that additional amount of capital and these payments are no longer enforceable. This implies property (i); such loans must be extended by intermediaries, as only they participate in markets in the morning when the claims need to be enforced. Moreover, it means that intermediaries must finance such loans out of their own net worth, that is, property (ii), as they cannot in turn finance them by borrowing from households because they could simply default on promises to repay the households in the afternoon and abscond with the payments received in the morning.[10] In contrast, financial intermediaries could refinance corporate loans that they make to firms, which are repaid in the afternoon, up to a fraction θ of collateral by borrowing from households. Loans beyond that, for fraction $\theta_i - \theta$, have to be financed out of financial intermediary capital.

The limits on enforcement for the three types of capital (see Figure 1) are as follows: firms can always abscond with working capital; firms cannot abscond with equipment in the morning, but can abscond with equipment in the afternoon; and firms can never abscond with structures. Structure loans can be provided by either intermediaries or households. In contrast, equipment loans have to be extended by intermediaries, have to be repaid in the morning, and have to be finance out of intermediary net worth.

Finally, let us sketch our main equivalence result (see Appendix B for the formal statement and proof). The economic intuition for the equivalence of the economy with limited enforcement and the economy with collateral constraints, both described in detail in Appendix B, is based on two main insights. First, limited enforcement implies that the present value of any sequence of promises, that is, long-term contract, can never exceed the current value of collateral, as otherwise delivering on these promises would not be optimal and the borrower would default. Indeed, limited enforcement constraints are equivalent to a type of collateral constraint on the present value of sequences of promises (see Theorem B.1). Second, any sequence of promises satisfying these collateral constraints on present values can be implemented with one-period ahead morning and afternoon claims subject to collateral constraints for the firm and the intermediary (see Theorem B.2). Hence, the economy with collateral constraints is tractable, in part because we can restrict attention, without loss of generality, to complete markets in one-period ahead morning and afternoon Arrow securities, that is, property (iii).

This economy with limited enforcement and limited participation therefore endogenizes three key properties that we previously simply assumed in the economy with collateral constraints. Henceforth, we work with the equivalent, recursive formulation of the economy with collateral constraints.

[10]Limited participation is essential as a foundation for the economy with two types of collateral constraints. If households participated in the morning as well, they would have the same ability to enforce claims as intermediaries, and since households have deep pockets, there would be no role for intermediaries. If instead there were no subperiods and default decisions were simultaneous, and intermediaries nevertheless retained their repossession advantage, intermediaries would have no incentive to default before receiving any payments, and thus could finance all corporate loans, including equipment loans, with loans from households, obviating the need for intermediary capital.

3 Intermediary capital and steady state

Intermediary capital is scarce in the model. We first show that, as a consequence, intermediated finance carries a premium or spread and that this spread affects investment and real economic activity. We then show that intermediaries are essential in our economy, that is, allow the economy to achieve allocations that would not be achievable in their absence. Finally, we show that in a steady state intermediary finance carries a positive spread over direct finance and determine the steady state capitalization of intermediaries.

3.1 Cost of intermediated finance

Internal funds and intermediated finance are both scarce in our model and command a premium as collateral constraints drive a wedge between the cost of different types of finance. Since the firm would never be willing to pay more for intermediated finance than the shadow cost of internal funds, the premium on internal finance is higher than the premium on intermediated finance. Define the premium on internal funds ρ as $1/(R+\rho) \equiv \beta\mu'/\mu$, where the right-hand side is the firm's discount factor. Define the premium on intermediated finance ρ_i as $1/(R+\rho_i) \equiv (R_i')^{-1}$, so $\rho_i = R_i' - R$, the spread over the household interest rate. In equilibrium, intermediaries lend at the intermediary interest rate R_i' and thus, using equations (18) and (19), it must be that $R_i' \geq R$, and since firms borrow from intermediaries, equation (14) implies that $R_i' \leq (\beta\mu'/\mu)^{-1}$; therefore:

Proposition 1 (Premia on internal and intermediated finance)**.** *The premium on internal finance ρ (weakly) exceeds the premium on intermediated finance ρ_i, that is, $\rho \geq \rho_i \geq 0$, and the two premia are equal, $\rho = \rho_i$, iff the collateral constraint for intermediated finance does not bind, that is, $\lambda_i' = 0$. Moreover, the premium on internal finance is strictly positive, $\rho > 0$, iff the collateral constraint for direct finance binds, that is, $\lambda' > 0$.*

When all collateral constraints are slack, there is no premium on either type of finance, but typically the inequalities are strict and both premia are strictly positive, with the premium on internal finance strictly exceeding the premium on intermediated finance.

The scarcity of internal and intermediated finance affects investment and in turn real economic activity. To see this, we can adapt Jorgenson's (1963) definition of the user cost of capital to our model with intermediated finance, and rewrite the investment Euler equation (16) as $u = R\beta\frac{\mu'}{\mu}A'f_k(k)$, where we define the user cost of capital u as

$$u \equiv r + \delta + \frac{\rho}{R+\rho}(1-\theta_i)(1-\delta) + \frac{\rho_i}{R+\rho_i}(\theta_i - \theta)(1-\delta), \tag{21}$$

where $r + \delta$ is the frictionless user cost derived by Jorgenson and $r \equiv R - 1$. The user cost of capital exceeds the user cost in the frictionless model, because part of investment needs to be financed with internal funds at premium ρ (the second term on the right hand side) and part is financed with intermediated finance at premium ρ_i (the last term on the right hand side). The premium on intermediated finance thus affects investment; scarcer intermediary capital reduces corporate investment.

3.2 Intermediation is essential

Intermediary capital is positive in equilibrium, that is, intermediaries always keep strictly positive net worth and never choose to pay out their entire net worth as dividends.

Proposition 2 (Positive intermediary net worth)**.** *Financial intermediaries always have strictly positive net worth in equilibrium.*

The intuition is that if intermediary net worth went to zero, the marginal value of intermediary net worth in equilibrium would go to infinity, because intermediaries would earn a positive spread forever; thus, intermediaries never pay out all their net worth.

Since intermediaries always have positive net worth, in equilibrium the intermediary interest rate R_i' must be such that the representative firm would never want to lend at that rate (that is, $\nu_i' = 0$ in equation (14)), as otherwise there would be no demand for intermediated finance, as the following lemma shows:

Lemma 1. *In any equilibrium, **(i)** the cost of intermediated funds (weakly) exceeds the cost of direct finance, that is, $R_i' \geq R$; **(ii)** the multiplier on the collateral constraint for direct finance (weakly) exceeds the multiplier on the collateral constraint for intermediated finance, that is, $\lambda' \geq \lambda_i'$; **(iii)** the constraint that the representative firm cannot lend at R_i' never binds, that is, $\nu_i' = 0$ w.l.o.g.; **(iv)** the constraint that the representative intermediary cannot borrow at R_i' never binds, that is, $\eta_i' = 0$; and **(v)** the collateral constraint for direct financing always binds, that is, $\lambda' > 0$.*

We define the essentiality of intermediaries as follows:

Definition 2 (Essentiality of intermediaries)**.** *Intermediaries are **essential** if an allocation can be supported with financial intermediaries but not without.*[11]

The above results together imply that financial intermediaries must always be essential. First note that firms are always borrowing the maximal amount from households, since direct finance is relatively cheap. If firms moreover always borrow a positive amount from intermediaries, then they must achieve an allocation that would not otherwise be feasible. If $R_i' = R$, then the firm must be collateral constrained in terms of intermediated finance, too, that is, borrow a positive amount. If $R_i' > R$, then intermediaries lend

[11] This definition is analogous to the definition of essentiality of money (see Hahn (1973)).

all their funds to the corporate sector and in equilibrium firms must be borrowing from intermediaries. We have proved that intermediation always plays a role in our economy:

Proposition 3 (Essentiality of intermediaries). *Financial intermediaries are always essential in equilibrium.*

3.3 Intermediary capitalization and spreads in steady state

Consider a steady state defined as follows:

Definition 3 (Steady state). *A steady state equilibrium is an equilibrium with constant allocations, that is,* $x^* \equiv [d^*, k^*, b'^*, b_i'^*, w'^*]$ *and* $x_i^* \equiv [d_i^*, l'^*, l_i'^*, w_i'^*]$*, and a constant interest rate on intermediated finance* $R_i'^*$.

In a steady state, intermediary capital and the spread on intermediated finance are positive:

Proposition 4 (Steady state). *There exists a unique steady state with the following properties: Intermediaries are essential, have positive net worth, and pay positive dividends. The spread on intermediated finance is strictly positive:* $\rho_i^* \equiv R_i'^* - R = \beta_i^{-1} - R > 0$. *Firms' collateral constraint for intermediated finance binds. The relative (ex dividend) intermediary capitalization is*

$$\frac{w_i^*}{w^*} = \frac{\beta_i(\theta_i - \theta)(1-\delta)}{\wp_i(\beta_i^{-1})}.$$

The relative (ex dividend) intermediary capitalization, that is, the ratio of the representative intermediary's net worth (ex dividend) relative to the representative firm's net worth (ex dividend), is the ratio of the intermediary's financing (per unit of capital) to the firm's down payment requirement (per unit of capital). In a steady state, the shadow cost of internal funds of the firm is β^{-1} while the interest rate on intermediated finance $R_i'^* = \beta_i^{-1}$, the shadow cost of internal funds of the intermediary. Since $\beta_i > \beta$, intermediated finance is cheaper than internal funds for firms in the steady state, and firms borrow as much as they can from intermediaries. The spread on intermediated finance is strictly positive in the steady state because intermediaries are less patient than households. In the analysis of the equilibrium dynamics in the next section we find that the spread on intermediated finance depends on the net worth of both firms and intermediaries, and can be higher or lower than the steady state spread.

In a steady state equilibrium, financial intermediaries have positive capital and pay out the steady state interest income as dividends $d_i^* = (R_i'^* - 1)l_i'^*$. Both firms and intermediaries have positive net worth in the steady state despite the fact that their rates of time preference differ and both are less patient than households. The reason is that firms have access to investment opportunities, but face collateral constraints and hence need to finance part of their investment internally, and intermediaries can finance part of firms' investment more cheaply, but face collateral constraints themselves.

The determinants of the capital structure of firms and intermediaries are distinct. In a steady state, firm leverage, that is, the total value of debt relative to total tangible assets, is $1-\wp_i(R_i'^*) = (R^{-1}\theta+(R_i'^*)^{-1}(\theta_i-\theta))(1-\delta)$ and is determined by the extent to which the firm can collateralize tangible assets, as emphasized in Rampini and Viswanathan (2013). In contrast, intermediary leverage can be defined in our indirect implementation as the value of total direct finance divided by the total value of debt, that is, $R^{-1}\theta(1-\delta)$ divided by $(R^{-1}\theta + (R_i'^*)^{-1}(\theta_i - \theta))(1 - \delta)$, which is approximately equal to θ/θ_i. Intermediary leverage is therefore determined by the relative enforcement ability of households and intermediaries. The substantial difference in leverage between firms and intermediaries in practice may simply be a consequence of their different determining factors. Thus, the model provides consistent guidance on the financial structure of firms and intermediaries.

Financial intermediaries are essential in our economy. Intermediated finance is costly and the spread on intermediated finance affects investment and aggregate economic output. Equilibrium determines the capitalization of both firms and intermediaries as well as the spread on intermediated finance; in a steady state equilibrium financial intermediary capital is positive as is the spread on intermediated finance. Next we consider the dynamics of our economy with intermediated finance, including the dynamics of firm and intermediary net worth and the spread on intermediated finance.

4 Dynamics of intermediary capital

Our model allows the analysis of the joint dynamics of the capitalization of the corporate and intermediary sector. The net worth of firms and intermediaries are the key state variables determining dynamic intermediated loan demand and supply and the interest rate on intermediated finance. The interaction between firms and intermediaries which are both subject to financial constraints leads to subtle dynamics with several compelling features. For example, spreads on intermediated finance are high when both firms' and intermediaries' net worth is low and intermediaries are poorly capitalized even relative to firms. A key feature is that intermediary capital accumulation is slow relative to corporate net worth accumulation, at least early in a recovery. One reflection of this is that the recovery from a credit crunch, that is, a drop in intermediary net worth, is relatively slow. Another reflection is that a simultaneous drop in the net worth of both firms and intermediaries, that is, a downturn associated with a credit crunch, results in an especially slow recovery, and that such recoveries can stall, with firm investment and output remaining depressed for an extended period of time. We relate the dynamic properties of our economy to stylized facts in the next section.

4.1 Dynamics of intermediary capital and spreads

To characterize the deterministic dynamics in an equilibrium converging to the steady state in general, consider the recovery of the economy from an initial, low level of net worth of firms and/or intermediaries, say after a downturn or credit crunch. We show that the equilibrium dynamics evolve in two main phases, an initial one in which the corporate sector pays no dividends and a second one in which the corporate sector pays dividends. Intermediaries do not pay dividends until the steady state is reached, except for an initial dividend, if they are initially well capitalized relative to the corporate sector.

Before stating these results formally (see Proposition 5 below, illustrated in Figure 3), we provide an intuitive discussion. Suppose both firms and intermediaries are constrained, that is, the marginal value of net worth strictly exceeds 1; then neither firms nor intermediaries pay dividends (Region ND in the proposition below). If the firms' collateral constraint on intermediated finance is slack, the intermediary interest rate equals firms' marginal levered return on capital (and exceeds the corporate discount rate β^{-1}), that is,

$$R_i' = \frac{A' f_k\left(\frac{w+w_i}{\wp}\right) + (1-\theta)(1-\delta)}{\wp},$$

where we use the investment Euler equation (16), that is, $1 = \beta\frac{\mu'}{\mu}\frac{A' f_k(k)+(1-\theta_i)(1-\delta)}{\wp_i(R_i')}$, and substitute out the discount factor using equation (14) and the fact that the collateral constraint for intermediated finance is slack, which implies that $\beta\frac{\mu'}{\mu} = (R_i')^{-1}$, and then rearrange. This case obtains when corporate net worth is sufficiently high so that firms' loan demand exceeds intermediaries loan supply, which is constrained by intermediary net worth. Intermediaries then lend their entire net worth w_i to firms, which in turn use their own net worth w plus loans from intermediaries to finance the fraction of investment not financed by households, that is, $k = \frac{w+w_i}{\wp}$. We observe that in this case the intermediary interest rate decreases in both firm and intermediary net worth since increased investment reduces the marginal return on capital.

If the firms' collateral constraint on intermediated finance binds instead, that constraint determines the interest rate which is then strictly lower than firms' marginal levered return on capital; specifically,

$$R_i' = (\theta_i - \theta)\frac{\frac{w}{w_i}+1}{\wp}(1-\delta),$$

where we use the collateral constraint (4) at equality, that is, $R_i' b_i' = (\theta_i - \theta)k(1-\delta)$ together with the fact that $k = \frac{w+w_i}{\wp}$ and in equilibrium $b_i' = l_i' = w_i$. Notice that in this case, the ratio of the net worth of firms relative to intermediaries matters, and remarkably the intermediary interest rate increases in firms' net worth keeping intermediary net worth the same; the economic intuition is that higher firm net worth raises investment and thus the collateral firms are able to pledge, increasing the equilibrium interest rate.

If the ratio $\frac{w}{w_i}$ is sufficiently low, loan demand can be so low that the interest rate on intermediated finance is below not just the firms' discount rate but also intermediaries' discount rate (β_i^{-1}). Indeed, the interest rate on intermediated finance can be as low as R, the discount rate of households; this can happen when intermediaries save net worth by lending to households, because current corporate loan demand is very low but expected to increase as firms recover. Throughout Region ND, firm net worth must accumulate faster than intermediary net worth because firms' net worth grows at their average levered return on capital (which exceeds their marginal levered return on capital) whereas intermediaries accumulate net worth at the intermediary interest rate, which, as just argued, is weakly below firms' marginal levered return on capital.

Suppose now that firms pay dividends but not intermediaries (Region D in the proposition below). If the firms' collateral constraint on intermediated finance is slack, the intermediary interest rate must again equal firms' marginal levered return on capital which in this case equals firms' discount rate, that is,

$$R_i' = \left(\beta\frac{\mu'}{\mu}\right)^{-1} = \beta^{-1} = \frac{A' f_k(\bar{k}) + (1-\theta)(1-\delta)}{\wp},$$

as $\mu = \mu' = 1$ since firms pay dividends. This case obtains when firms' net worth is relatively high while intermediaries' net worth does not suffice to meet the corporate loan demand at the intermediary interest rate $R_i' = \beta^{-1}$. In this phase, investment is constant at $\bar{k}$ and financed with firms' ex dividend net worth w_{ex} and intermediary loans, that is, $\wp\bar{k} = w_{ex} + w_i$; as intermediaries accumulate net worth with the law of motion $w_i' = \beta^{-1}w_i$ and progressively meet the corporate loan demand, firms gradually relever and draw down their (ex dividend) net worth by paying dividends. Therefore, $\frac{w_i'}{w_i} = \beta^{-1} > 1 > \frac{w_{ex}'}{w_{ex}}$, as intermediaries accumulate net worth while firms draw it down; this is the time when financial intermediaries are "catching up." If firms' collateral constraint binds, which happens once intermediaries' net worth is sufficient to meet loan demand at β^{-1}, the collateral constraint (4) and firms' investment Euler equation (16) jointly determine the intermediary interest rate, and, as intermediary net worth increases, the intermediary interest rate falls and investment increases. From the collateral constraint (4), $R_i' = (\theta_i - \theta)\frac{\frac{w_{ex}}{w_i}+1}{\wp}(1-\delta)$, we see that as the intermediary interest rate falls, the (ex dividend) net worth of firms relative to intermediaries must fall, too. Thus, in this phase, while firms' and intermediaries both accumulate net worth, intermediaries accumulate net worth faster than firms, as firms continue to relever; intermediaries continue to "catch up" until the steady state is reached.

Intermediaries do not pay dividends until the steady state is reached with one exception. If the initial corporate net worth is so low, that intermediaries are well capitalized relative to the corporate sector and the interest rate is below intermediaries' discount rate due to the limited corporate loan demand, then intermediaries may pay an initial dividend if they expect corporate loan demand to be depressed for an extended period of time. But after such an initial dividend, intermediaries do not resume payout until such time as the steady state is reached. We emphasize, however, that, in contrast, firms do initiate payout before the economy reaches the steady state. The firm and intermediary's first-order conditions for intermediated borrowing and lending, respectively, (14) and (19), imply that as long as $R_i' > \beta^{-1}$, $\mu \geq R_i'\beta\mu' > \mu' \geq 1$, so $\mu > 1$, and similarly when $R_i' > \beta_i^{-1}$, $\mu_i > 1$; thus, firms do not pay dividends until the intermediary interest rate reaches β^{-1}, while intermediaries wait to pay dividends until the intermediary interest rate reaches $\beta_i^{-1} < \beta^{-1}$.

The following proposition and lemma state these results formally and Figure 3 illustrates the pertinent regions of firm net worth w and intermediary net worth w_i:

Proposition 5 (Dynamics)**.** *Given w and w_i, there exists a unique deterministic dynamic equilibrium which converges to the steady state characterized by a no dividend region (ND) and a dividend region (D) (which is absorbing) as follows:*

Region ND *$w_i \leq w_i^*$ (w.l.o.g.) and $w < \bar{w}(w_i)$, and (i) $d = 0$ ($\mu > 1$), (ii) the cost of intermediated finance is*

$$R_i' = \max\left\{R, \min\left\{(\theta_i - \theta)\frac{\frac{w}{w_i}+1}{\wp}(1-\delta), \frac{A'f_k\left(\frac{w+w_i}{\wp}\right)+(1-\theta)(1-\delta)}{\wp}\right\}\right\},$$

(iii) investment $k = (w + w_i)/\wp$ if $R_i' > R$ and $k = w/\wp_i(R)$ if $R_i' = R$, and (iv) $w'/w_i' > w/w_i$, that is, firm net worth increases faster than intermediary net worth.

Region D *$w \geq \bar{w}(w_i)$ and (i) $d > 0$ ($\mu = 1$).*

For $w_i \in (0, \bar{w}_i)$, (ii) $R_i' = \beta^{-1}$, (iii) $k = \bar{k}$ which solves $1 = \beta[A'f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$, (iv) $w_{ex}'/w_i' < w_{ex}/w_i$, that is, firm net worth (ex dividend) increases more slowly than intermediary net worth, and (v) $\bar{w}(w_i) = \wp\bar{k} - w_i$.

For $w_i \in [\bar{w}_i, w_i^*)$, (ii) $R_i' = (\theta_i - \theta)(1-\delta)k/w_i$, (iii) k solves $1 = \beta[A'f_k(k) + (1-\theta)(1-\delta)]/(\wp - w_i/k)$, (iv) $w_{ex}'/w_i' < w_{ex}/w_i$, that is, firm net worth (ex dividend) increases more slowly than intermediary net worth, and (v) $\bar{w}(w_i) = \wp_i(R_i')k$.

For $w_i \geq w_i^*$, $\bar{w}(w_i) = w^*$ and the steady state of Proposition 4 is reached with $d = w - w^*$ and $d_i = w_i - w_i^*$.

The representative intermediary's dividend policy is characterized as follows:

Lemma 2 (Initial intermediary dividend)**.** *The representative intermediary pays at most an initial dividend and no further dividends until the steady state is reached. If $w_i > w_i^*$, the initial dividend is strictly positive.*

It is worth emphasizing the predictions of our model for the relative speed of adjustment of the two endogenous state variables. In Region ND firm net worth accumulation is faster while in Region D, when firms pay dividends, intermediaries accumulate net worth more quickly. Both of these features are in a sense a consequence of the fact that intermediaries accumulate net worth more slowly in our model. To understand the economic intuition consider Region ND when both firms and intermediaries are constrained and do not pay dividends. The dynamics of financial intermediary net worth are relatively simple, since as long as they do not pay dividends (which is the case until the steady state is reached), the intermediaries' net worth evolves according to the law of motion $w_i' = R_i' w_i$, that is, intermediary net worth next period is simply intermediary net worth this period plus interest income. When no dividends are paid, intermediaries lend out all their funds at the (equilibrium) intermediary interest rate R_i', and hence their net worth grows at the (gross) rate R_i'. Moreover, the intermediary interest rate is (weakly) less than firms' marginal levered return on capital, which in turn in less than firms' average levered return on capital which equals the growth rate of firms' net worth, that is,

$$\frac{w_i'}{w_i} = R_i' \leq \frac{A' f_k(k) + (1-\theta)(1-\delta)}{\wp} < \frac{w'}{w}.$$

This is why the net worth of the corporate sector grows faster than the net worth of the intermediary sector in this phase.

One reflection of this difference in net worth growth is that the corporate sector recovers faster, in the sense that firms initiate dividends before intermediaries do. Once firms pay dividends (in Region D), it is now the intermediary sector that accumulates net worth faster as it catches up; so the difference in the growth rate of net worth accumulation across the two sectors switches sign. Indeed, as the intermediaries continue to accumulate net worth, the corporate sector relevers and its net worth may temporarily shrink.

Below we use this analytical characterization of the dynamic equilibrium to study the dynamics of the economy in response to an initial drop in corporate net worth (a "downturn," see Section 4.2), an initial drop in the net worth of financial intermediaries (a "credit crunch," see Section 4.3), and a downturn associated with a credit crunch, that is, a simultaneous drop in both corporate and intermediary net worth (see Section 4.4).

4.2 Dynamics of a downturn without credit crunch

Suppose the economy experiences a downturn, which we model as an unanticipated one-time drop in corporate net worth w. In this subsection, we consider a downturn without a credit crunch, that is, assume that intermediaries' net worth remains unchanged at its ex-dividend steady state level. Figure 4 illustrates the dynamics of firm and intermediary net worth, the interest rate on intermediated finance, intermediary lending, and investment following such a downturn (at time 0). In terms of Proposition 5, the recovery evolves in several phases, as the net worths of the two sectors transit through various parts of first Region ND and then Region D.

On impact the drop in corporate net worth results in a drop in corporate loan demand, leaving intermediaries initially relatively well capitalized. Indeed, as can be seen in Panel A and Panel B2 of Figure 4, intermediaries respond by paying an initial dividend, as their previous net worth is much more than firms' reduced loan demand can accommodate. Moreover, intermediaries do not pay out all excess funds but conserve some net worth to meet future loan demand by lending some of their funds to households; in fact, the intermediaries' lending to households exceeds their lending to the corporate sector early on (see Panel B3). Because intermediaries lend to households at the margin, the equilibrium spread on intermediated finance is zero, that is, $R_i' = R$ (see Panel B1); in this sense, intermediaries are holding "cash" at an interest rate below their discount rate for some time. The reason why intermediaries are willing to hold cash at a rate of return below their rate of time preference is because they anticipate an eventual rise in the intermediary interest rate above their rate of time preference, at least for some time.

Intermediaries accumulate net worth at rate R in this phase while the corporate sector accumulates net worth at a faster rate, given the high marginal levered return on capital; thus, the net worth of the corporate sector rises relative to the net worth of intermediaries. As firms' net worth rises, so do corporate investment $k = w/\wp_i(R)$ as well as corporate loan demand. In Figure 4, this phase last from time 0 to time 4.

Eventually, the increased net worth of the corporate sector raises loan demand to the point where intermediated finance becomes scarce and the intermediary interest rate rises (time 5 in the figure). In the case considered in Figure 4, since the initial drop in net worth is not too large, it turns out that the cost of intermediated finance stays below β^{-1} and firms happen to initiate dividend payments at the same time; that is, the economy reaches Region D at time 5. Firms' collateral constraint continues to bind and, together with firms' investment Euler equation, jointly determines the intermediary interest rate and investment. Although firms are paying dividends from time 5 onwards, neither investment nor intermediaries have fully recovered. As intermediaries continue to accumulate net worth between time 5 and time 8, intermediary loan supply increases and the intermediary interest rate drops. In response, firms continue to increase investment. Intermediaries' net worth reaches its (ex dividend) steady state value at time 8 and the economy is back in steady state (as described in Proposition 4) from then on, with the cost of intermediated finance equal to β_i^{-1}, the unconstrained intermediaries' shadow cost for providing corporate loans.

We emphasize two key aspects of the dynamics of intermediary capital illustrated by the recovery from a downturn traced out here, beyond the fact that intermediary and firm net worth affect the dynamics jointly. First, intermediary capital accumulates more slowly than corporate net worth as long as both firms and intermediaries are constrained. Second, when the corporate sector is temporarily relatively poorly capitalized, the interest rate on intermediated finance is low and intermediaries conserve net worth by lending to households at a low interest rate to meet the higher subsequent corporate loan demand. Of course, the second observation is a reflection of the relatively slow pace of intermediary capital accumulation as well.

4.3 Dynamics of a credit crunch

Suppose the economy experiences a *credit crunch*, which we model here as an unanticipated one-time drop in intermediary net worth w_i. In this subsection, we consider a pure credit crunch and assume that the economy is in steady state when the credit crunch hits with corporate net worth at its (ex-dividend) steady state value. Figure 5 illustrates the effects of such a credit crunch on interest rates, net worth, intermediary lending, and investment. The drop in intermediary net worth results in a reduction in lending and an increase in the spread on intermediated finance on impact (see time 0 in Panel B1).[12] Moreover, the higher cost of intermediated finance increases the user cost of capital (21) as the premium on internal finance increases as well and so the investment Euler equation implies that investment drops (see Panel B4). Hence, a credit crunch has real effects in our model. Due to the limited supply of intermediated finance firms are forced to delever at time 0, replacing reduced intermediary loans with internal funds. Given their limited internal funds, firms are moreover forced to downsize.

At time 1, the corporate sector reaccumulates net worth leading to an initial partial recovery in investment and output; indeed, firms reinitiate dividend payments at time 1. However, the economy and especially intermediaries have not fully recovered at this point, as intermediary net worth remains well below its steady state level. The recovery stalls, potentially for a long time (from time 1 to time 14 in Figure 5), in the sense that the interest rate on intermediated finance remains elevated at $R_i' = \beta^{-1} > R_i'^* = \beta_i^{-1}$ and

[12] If the credit crunch would hit before dividends are paid, then firms and intermediaries could absorb the shock by cutting dividends, at least partially, reducing its impact.

investment remains constant below its steady state level at $\bar{k} < k^*$. The reason is that firms' user cost of capital remains elevated despite the fact that the corporate sector is well capitalized and paying dividends, as intermediaries' capacity to extend relatively cheap financing is reduced. Thus, the real effects in our model are persistent, even if the corporate sector recapitalizes relatively quickly, and the full recovery of the real economy is delayed. We emphasize that while firms seem to be well capitalized because they are paying dividends at this point, the economy has not fully recovered.

The recovery only resumes once intermediaries accumulate sufficient capital to meet corporate loan demand at $R_i' = \beta^{-1}$. At that point, intermediary interest rates start to fall and investment begins to recover further (time 15 to 18); as the interest rate on intermediated finance is now below the shadow cost of internal funds of the corporate sector, the collateral constraint binds again. Investment increases due to the reduced cost of intermediated financing and the recovery resumes. Eventually (at time 18), intermediaries accumulate their steady state level of net worth and the cost of intermediated finance reaches β_i^{-1}, the unconstrained intermediaries' shadow cost of providing loans, and investment recovers fully; and finally the steady state is reached (at time 19).

We emphasize two key additional aspects of the dynamics of our economy. First, the recovery from a credit crunch can be much delayed, or can stall, due to the slow accumulation of intermediary net worth; thus, one feature of a credit crunch in our model is the persistence of the real effects. Second, the response of firms' to a credit crunch is also striking. In response to a credit crunch, firms cut dividends to substitute retained earnings for intermediated loans. That is, firms delever and temporarily accumulate more net worth than they retain in the steady state, and only as the intermediaries recover, do firms gradually relever. The interaction between the two sectors which are both subject to financial constraint leads to rich implications for economic dynamics.

4.4 Effects of severity of credit crunch

Suppose the economy simultaneously experiences a downturn in corporate net worth and a credit crunch. This subsection examines the effect of the severity of the credit crunch itself on the depth and protractedness of the macroeconomic downturn, the joint dynamics of corporate and intermediary net worth accumulation, and spreads on intermediated finance. We find that downturns associated with a credit crunch are more severe and more protracted, and feature higher spreads and slower corporate net worth accumulation.

Figure 6 traces out three scenarios: a downturn in corporate net worth without a credit crunch (solid), and downturns in corporate net worth associated with a moderate and a severe credit crunch (dotted and dashed, respectively). The downturn in corporate net worth without a credit crunch is the baseline scenario analyzed in Section 4.2 and Figure 4 above. As noted there, in this scenario, the intermediary is initially well-capitalized and can hence accommodate the reduced corporate loan demand easily; spreads are initially low, as intermediaries conserve net worth by lending to households. As corporate loan demand increases and the economy recovers, spreads begin to rise and eventually fall back to the steady state level. The corporate sector accumulates net worth faster and reinitiates dividends sooner than the intermediary sector (corporate and intermediary dividend initiations are marked with triangles pointing up and down, respectively).

A moderate credit crunch slows the recovery of intermediary lending and investment (see Panel B3 and B4, respectively), and raises spreads earlier and to a higher level (see Panel B1).[13] Corporate net worth accumulation is also slowed somewhat and the reinitiation of corporate dividends is delayed; moreover, the corporate sector first accumulates net worth, and then partially draws down its net worth as intermediaries recover (see Panel B2). In addition, the recovery stalls, albeit briefly, when the corporate sector has recovered to the point where it initiates dividends, but the intermediary sector still has not accumulated sufficient net worth to meet loan demand at $R_i' = \beta^{-1}$ (see the leveling off of spreads and investment from time 6 to 8 in Panels B1 and B4). Once intermediaries accumulate enough net worth to accommodate loan demand at an interest rate equal to the corporate sector's discount rate, the recovery resumes.

A severe credit crunch magnifies all these effects, amplifying the downturn, making it more protracted, leading to longer stalls in the recovery, and raising spreads. Investment initially drops more, and then recovers more slowly, stalling for an extend period of time (time 7 through 16) before eventually recovering (see Panel B4). The spread on intermediated finance shoots up on impact, because intermediated finance is limited not by corporate loan demand but by the supply of intermediated funds;[14] the spread is hence determined by the levered marginal product of capital, which is high because the corporate sector is financially constrained.[15] As both sectors accumulate net worth, the spread comes down from its initial highs, but stalls once the corporate sector starts to pay dividends. Indeed, the spread stays at this elevated level while the intermediary sector reaccumulates net worth, and returns to its steady state level only later on (see

[13]Indeed, the spread rises over a couple of periods (from time 1 to time 3); at time 2 the corporate sector borrows all the funds intermediaries are able to lend and invests $k = (w + w_i)/\wp$. The interest rate on intermediated finance is determined by the collateral constraint, which is binding, and equals $R_i' = (\theta_i - \theta)(w/w_i + 1)(1 - \delta)/\wp$; and since corporate net worth increases faster than intermediary net worth, the interest rate on intermediated finance rises in this phase. As the corporate sector accumulates net worth, it can pledge more and the equilibrium interest rate rises. As the net worth and investment of the corporate sector continues to rise faster than intermediary net worth, eventually the increase in firms' collateral means that firms' ability to pledge no longer constrains their ability to raise intermediated finance, at which point the spread starts to drop.

[14]Since intermediaries lend out their entire net worth w_i to firms, investment is $k = (w + w_i)/\wp$.

[15]That is, the equilibrium interest rate on intermediated finance is $R_i' = [A' f_k(k) + (1 - \theta)(1 - \delta)]/\wp$.

Panel B1). Intermediary lending is also noteworthy, as it remains substantially below the steady state level for an extended amount of time (see Panel B3). Finally, note that the corporate sector temporarily accumulates more net worth than in the steady state, a result of substantial corporate deleveraging, as firms substitute internal funds for the lack of intermediated finance for some time.

In the scenarios considered above, downturns associated with a credit crunch are also larger drops in total net worth and hence larger shocks to the economy. An alternative is to hold the total drop in net worth constant and consider scenarios in which intermediaries are more affected while the firms are less affected. In unreported results, we find similar effects in terms of persistence and spreads; when the drop in intermediary net worth is larger, investment is depressed for longer, stalls are at a lower level, and spreads are higher on impact and elevated for longer. That said, since the net worth of the corporate sector is less affected, the initial drop in investment may be smaller and firms may initiate dividends sooner; corporate deleveraging, however, is substantially more pronounced, with firms accumulating net worth well above the steady state value. However, we consider our baseline scenarios more empirically relevant and more consistent with the measurement of the stylized facts documented in the literature and discussed in Section 5.

4.5 Net worth dynamics in bank-oriented economy

Suppose now that the economy is more bank-oriented. An economy can be more bank-oriented (as opposed to market-oriented) because banks play a larger role whereas either direct finance by households, say in the form of public debt markets, or internal finance by the corporate sector plays a smaller role; the former corresponds to an economy with lower θ and the latter to an economy with higher θ_i. We consider an economy with a lower value of θ, as this may be the more typical interpretation of a bank-oriented financial system, although an economy with a higher value of θ_i behaves very similarly.[16] Recall from Proposition 4 that this implies that the ratio of intermediary net worth to corporate net worth in the steady state is higher, that is, a larger fraction of investment is financed by intermediaries. How does such a bank-oriented economy respond to downturns with and without a credit crunch? The dynamic response of such an economy is illustrated in Figure 7. To facilitate the comparison, we keep all the parameters unchanged from the previous figures, except for two: first, we reduce θ from 0.6 to 0.5 and second, we raise the productivity A' to keep steady state investment k^* unchanged. We consider the same three scenarios as in Figure 6, a downturn without a credit crunch (solid), as well as downturns with a moderate and a severe credit crunch (dotted and dashed, respectively).

Several features of the response are worth noting: First, the severity of the credit

[16] See the working paper version, Rampini and Viswanathan (2017), for the dynamics when θ_i is higher.

crunch itself has a more significant impact on the initial drop in investment (see Panel B4). Moreover, investment is more substantially reduced both early in the downturn and the recovery stalls at a lower level of investment. Further, and perhaps most significantly, the downturn is also more protracted with investment reduced, intermediated finance depressed (see Panel B3), and spreads elevated (see Panel B1) for longer. The effects on net worth of the two sectors are also magnified: intermediary net worth takes a long time to recover and the corporate sector temporarily accumulates substantially more net worth than it has in the steady state, again in an effort to substitute internal funding for the lack of intermediated finance (see Panel B2).[17]

Our theory predicts that in a bank-oriented economy, downturns associated with a credit crunch are more severe and more protracted, with slower net worth accumulation and longer stalls of the recovery at lower levels of investment. Thus, the recovery from crises in bank-oriented economies such as Europe or Japan may be substantially different from, and more sluggish than, that in market-oriented economies such as the U.S., where it may be typically more swift.

5 Revisiting evidence on crises with theory

Our model provides guidance on the joint dynamics of the macro economy and the financial sector. We use the analysis of the dynamics in the previous section to revisit three main stylized facts about downturns associated with financial crises, namely their severity, their protractedness, and the relation between the severity of the financial crisis itself and the severity and persistence of the downturn, and to discuss the evidence on spreads.

5.1 Fact 1: Severity of downturns associated with financial crises

Prior empirical research shows that downturns associated with financial crises are more severe. Reinhart and Rogoff (2014) find that "the average peak-to-trough decline for the US real per capita GDP across nine major crises is about 9 percent." Similarly,

[17] If we consider scenarios in which intermediaries are more affected and the firms are less affected, that is, holding the total drop in net worth constant, as discussed in Section 4.4, we also find that in bank-oriented economies recoveries stall at lower levels of investment, downturns are more protracted and spreads elevated for longer; moreover, corporate deleveraging is even more pronounced as firms accumulate even more net worth to substitute for reduced intermediated finance is such economies. The response of the bank-oriented economy thus shows more persistence. That said, since firm net worth drops less in these scenarios, the initial impact may not be more severe and spreads may not be much more elevated initially, although as emphasized spreads are elevated for longer. Again, we think our baseline scenarios are the empirically more relevant ones.

using panel data for a large set of countries, Cerra and Saxena (2008) find that "banking crises lead to severe output loss [even in high-income countries]" (page 442). Finally, Krishnamurthy and Muir (2016) conclude that "[o]ur results affirm that financial crises do result in deeper and more protracted recessions" (page 15).[18]

Thus, the evidence shows a first stylized fact, namely that downturns associated with financial crises are more severe in the sense that output losses are larger, suggesting amplification of macroeconomic shocks. The evidence also shows that such downturns are more severe in the sense of being more protracted, but while these two notions of severity are often commingled in the literature, we treat their persistence as a separate stylized fact which we discuss in the next subsection. Comparing the dynamics of our model without and with a credit crunch in Figures 4 and 6, we find that downturns associated with a financial crisis are more severe, in the sense that output drops by more on impact and that output is lower at any fixed horizon. That said, the initial effect of the credit crunch on output can be muted in the model if the drop in corporate net worth reduces loan demand so much that loan demand drops below intermediary net worth.

5.2 Fact 2: Slow recoveries after financial crises

The evidence also shows that recoveries after financial crises are protracted. Reinhart and Rogoff (2014) state that "a significant part of the costs of [systemic banking] crises lies in the protracted and halting nature of the recovery" (page 50). Moreover, they argue that "[t]he halting, tentative nature of the post-crisis recoveries (even in cases where there is a sharp—but not sustained—growth rebound) is evidenced in the relatively high incidence of double dips (or secondary downturns before the previous peak is reached)" (page 52). They find that "[o]n average it takes about eight years to reach the pre-crisis level of income," with a median time of about 6.5 years, and that about 45% of the 100 crises they study are associated with a double dip, and about two-thirds of the most severe crises involve double-dips. Cerra and Saxena (2008) find "that the large output loss associated with financial crises ... is highly persistent" (page 456); specifically, they show that the

[18] We recognize that the conclusions regarding these stylized facts in part turn on the assumptions made for measurement, for example, the severity cutoff used in defining a crisis, the definition of a recovery (return to the previous output level or trend), adjustment for population growth, the definition of the start of the episode (previous output peak or start of the financial disruption), and finally whether the data includes emerging economies and pre-war episodes. Indeed, some recent studies provide a contrarian view. For example, Bordo and Haubrich (2012) find "generally ... rapid recoveries" with the exception of the 1930s, 1990s, and the present recovery. Similarly, Romer and Romer (2015) find that "output declines following financial crises in modern advanced countries are highly variable, on average only moderate, and often temporary," (abstract) using a new, qualitative measure of financial distress with a relatively fine scale, thus, challenging the conventional wisdom that "the aftermath of financial crises is typically severe and long-lasting" (page 1). We match the stylized facts that are the mainstream view.

output loss on impact of a banking crisis is about 7.5 percent and the output loss at a ten-year horizon exceeds 6 percent, that is, is extremely persistent.

Our second stylized fact is therefore the persistence of downturns associated with financial crises; recoveries are slow and frequently stall, at least temporarily. The dynamics of our model in Figure 5 highlight the protractedness of a credit crunch: the recovery of output after a credit crunch is slower than after a regular downturn (compare to Figure 4). In addition, spreads remain elevated for an extended period of time. This is due to the relative sluggishness of intermediary net worth accumulation.

Moreover, when both corporate and intermediary net worth drop at the same time, that is, in a downturn associated with a credit crunch, downturns are more prolonged (see Figure 6) and the recovery can stall, potentially for an extended period of time, due to the joint dynamics of corporate and intermediary net worth; note that such stalls can occur even in cases in which the initial rebound of output is quite brisk, consistent with the data. The model therefore captures what Reinhart and Rogoff call the "halting, tentative nature" of such recoveries. We are not aware of other models that predict such stalls in the economic recovery post crises.

5.3 Fact 3: Impact of severity of financial crises

Further, the evidence suggests that how severe the financial crises per se is plays an important role. Romer and Romer (2015) find that "one factor that appears to be important to the variation [in the aftermath of crises] is the severity and persistence of the crisis itself" (page 41). For example, "Japan's dismal economic performance ... [may be] related to the fact that the distress was particularly severe and long-lasting" (page 38), and for the 2008 Crisis they argue that "the extreme nature of the recent distress is one likely reason for the severe and continuing economic weakness" (page 41-42).

Thus, the third stylized fact is that the severity of the financial crises itself in turn affects both the severity and protractedness of the downturn; in other words, Fact 1 ("severity") and Fact 2 ("protractedness") are a matter of degree. Our model captures this fact as demonstrated in Figure 6: the more severe the drop in intermediary net worth, the larger the drop in output and the more protracted the recovery; recoveries from severe financial crises are more protracted in the sense that (i) the initial recovery is slower, (ii) the recovery stalls for longer, and (iii) the economy takes longer to recover fully.

5.4 Evidence on spreads and financial crises

In a similar spirit, Krishnamurthy and Muir (2016) measure the severity of a crisis using the spread between high-yield and low-yield bonds, and conclude that "recessions in the aftermath of financial crises are severe and protracted" and that furthermore "the severity of the subsequent crisis can be forecast by the size of [changes in the spread]" (abstract). Moreover, they find that "the start of a crisis is associated with a spike in spreads" and "a spike in spreads shifts down the conditional distribution of output growth, fattening the lower tail" (page 17).[19]

Our model captures these observations in several ways: first, more severe downturns are associated with a higher spread; second, the spread spikes or "blows out" at the beginning of very severe crises; and third, the spread recovers relatively quickly initially, but then remains elevated and recovers fully only after an extended period of time. Thus, the dynamics of the spread in our model have empirically plausible features.

The measurement in the aforementioned studies is predicated on the assumption that there are two separate driving forces, one for downturns and one for financial crises. Considering the effects of a credit crunch or financial crisis on aggregate dynamics implicitly assumes that the financial disruption is a separate, exogenous force, rather than simply an endogenous reflection of a single aggregate source of fluctuations. Understanding these facts therefore calls for a model with two state variables, one for the corporate sector and one for financial intermediaries, which is the type of model that we provide.

6 Intermediary risk management

The focus of our analysis thus far has been on the deterministic dynamics following unanticipated drops in net worth. However, our model encompasses economies with uncertainty enabling the analysis of equilibrium risk management of shocks to net worth. Taking uncertainty explicitly into account allows firms and intermediaries to hedge shocks to net worth if they choose. Our model and the evidence in the previous section show that drops in net worth have significant macroeconomic consequences, especially when corporate and intermediary net worth are low at the same time. The stochastic version of the model allows us to study how firms and intermediaries respond to such shocks ex ante and whether their net worths potentially drop at the same time, that is, comove.

We now sketch a stochastic version of the economy. The source of uncertainty is stochastic total factor productivity $A(s')$ which depends on the state s' realized next period and follows a Markov process with transition function $\Pi(s, s')$, where s is the current state. As we show in Appendix B, the equivalence between economies with limited enforcement and with collateral constraints obtains in this stochastic economy as well. Thus, without loss of generality, we can consider markets in two types of one-period ahead claims, claims provided by intermediaries and claims provided by households, each subject to collateral constraints; the key difference is that now the claims provided by

[19] The spread between intermediated and direct finance in our model may be captured by commonly used empirical measures such as the spread on interbank loans or high-yield debt.

intermediaries and households are state-contingent Arrow securities, and thus we have complete markets subject to state-by-state collateral constraints.[20]

The state of the economy $z \equiv \{s, w, w_i\}$ now includes the exogenous state s in addition to the two endogenous state variables w and w_i. The interest rates on state-contingent intermediated finance R_i' now also depend on the state s' next period, that is, we have a vector of equilibrium state-dependent interest rates; the state price for one unit of intermediated finance in state s' next period is thus $\Pi(s, s')(R_i')^{-1}$. Note that a prime now denotes dependence of the variable on the state s' next period. A promise to pay Rb' to households in state s' next period, raises $\Pi(s, s')b'$ this period, as such claims are priced by households that are risk-neutral and have deep pockets, and thus the total proceeds from issuing such claims to households are $\sum_{s' \in S} \Pi(s, s')b' = E[b'|z]$; analogously, a promise to pay $R_i'b_i'$ to intermediaries in state s' next period at equilibrium state-dependent interest rate R_i', raises $\Pi(s, s')b_i'$ this period, and thus the total proceeds from issuing claims to intermediaries are $\sum_{s' \in S} \Pi(s, s')b_i' = E[b_i'|z]$. With these changes, the firm's and intermediary's problems can be straightforwardly adapted to the stochastic case.[21] In the stochastic economy, equilibrium now requires that the market for state-contingent intermediated finance clears for all states next period, that is, $l_i' = b_i'$, $\forall s' \in S$.

To illustrate how our model allows the analysis of equilibrium risk management by the corporate and intermediary sector, consider a stochastic economy which is deterministic from time 1 onward. Importantly, both firms and intermediaries can engage in risk management at time 0 and hedge the net worth available to them in different states $s' \in S$ at time 1. We show that the collateral constraint for direct finance against at least one state $s' \in S$ must bind. We also provide sufficient conditions for the marginal value of net worth of the firm and the intermediary to comove.

Proposition 6 (Equilibrium firm and intermediary risk management)**.** *In an economy that is deterministic from time* 1 *onward and has constant expected productivity,* ***(i)*** *the firm must be collateral constrained for direct finance against at least one state at time* 1*;* ***(ii)*** *the marginal value of firm and intermediary net worth comove, in fact* $\mu(s')/\mu(s'_+) = \mu_i(s')/\mu_i(s'_+)$, $\forall s', s'_+ \in S$, *if* $\lambda_i(s') = 0$, $\forall s' \in S$. ***(iii)*** *Suppose moreover that there are just two states, that is,* $S = \{\hat{s}', \check{s}'\}$. *If only one of the collateral constraints for direct finance binds,* $\lambda(\check{s}') > 0 = \lambda(\hat{s}')$, *then the marginal values must comove,* $\mu(\hat{s}') > \mu(\check{s}')$ *and* $\mu_i(\hat{s}') \geq \mu_i(\check{s}')$.

[20]While we study this implementation throughout, we emphasize that the contingent claims traded in our economy may be implemented in practice with non-contingent claims on which issuers are expected and in equilibrium indeed do default (see Kehoe and Levine (2008) for an implementation in this spirit).

[21]To adapt the firm's problem, we add an expectation operator to the objective, replace b' and b_i' in (2) with $E[b'|z]$ and $E[b_i'|z]$, respectively, and note that (3) through (6) hold state-by-state. Proceeding similarly, and replacing l' and l_i' in (8) by analogously defined $E[l'|z]$ and $E[l_i'|z]$, respectively, we obtain the stochastic version of the intermediary's problem. See the proof of Proposition 6 for further details.

Proposition 6 implies that the marginal values of firm and intermediary net worth comove, for example, when the intermediary has very limited net worth and hence the collateral constraints for intermediated finance are slack for all states. They also comove if the firm hedges one of two possible states by saving with households, as then the intermediary effectively must be hedging that state, too. Thus, the marginal value of intermediary net worth may be high exactly when the marginal value of firm net worth is high, too. The marginal values may however move in opposite directions, for example, if a high realization of productivity raises firm net worth substantially, which lowers the marginal product of capital and hence the marginal value of firm net worth, while raising loan demand substantially and hence raising the marginal value of intermediary net worth.

Moreover, since the state-dependent intermediary interest rate typically exceeds the interest rate households charge, firms that hedge by carrying additional net worth into some state next period, because their net worth in that state would otherwise be too low, will do so by reducing their borrowing from intermediaries against that state. This means that the net worth of intermediaries in that state must be lower as well, as intermediaries' net worth equals firms' repayments of intermediated loans. Thus, to the extent that intermediaries provide insurance to firms, there is a force in the model towards intermediary net worth dropping exactly when corporate net worth drops, too.

Comovement between the value of corporate and intermediary net worth is thus plausible but not a foregone conclusion. The nature of economic shocks affecting net worth may also play a role. So far we have considered stochastic total factor productivity, which directly affects corporate net worth only. Alternatively, one could easily adapt the model to consider stochastic depreciation of capital, which would directly affect both corporate and intermediary net worth; in fact, stochastic depreciation shocks would arguably affect intermediary net worth relatively more, as the assets of intermediaries are collateralized claims. Depreciation shocks are one way to model shocks to the value of collateral. A shock to total factor productivity may then result in a downturn without a credit crunch, whereas a depreciation shock to the value of capital may result in a downturn associated with a credit crunch. We leave an explicit analysis of a model with both stochastic productivity and stochastic depreciation to future work.

7 Conclusion

We develop a dynamic theory of financial intermediation and show that the capital of both the financial intermediary and corporate sector affect real economic activity, such as firm investment, financing, and the spread between intermediated and direct finance. We derive collateral constraints from an explicit model of limited enforcement in which financial intermediaries have a collateralization advantage due to limited participation by households. Financial intermediaries are able to enforce claims when more of the capital is collateralizable. This advantage in enforcement enables intermediaries to lend more than households, but they cannot in turn finance such loans by borrowing from households and hence have to finance these loans out of their own net worth; we argue that this is why financial intermediaries need capital. In our view, the enforcement of payment is a key rationale for the existence of financial intermediaries, in addition to the monitoring and liquidity provision motives emphasized in the previous literature.

The determinants of capital structure of firms and intermediaries in the model are distinct. Firms' capital structure is determined by the extent to which firms can pledge their tangible assets required for production as collateral to intermediaries and households. In contrast, intermediaries' capital structure is determined by the extent to which their collateralized loans can be collateralized themselves, that is, by the difference between the intermediaries' and households' ability to collateralize claims. Thus, firms issue promises against tangible assets whereas intermediaries issue promises against collateralized claims, which are in turn backed by tangible assets. Given these distinct determinants, firm leverage may be substantially lower than intermediary leverage as is the case in practice.

Central to the aggregate implications is the fact that our model features two state variables, the net worth of the corporate and intermediary sector, and their joint dynamics result in the compelling dynamics of the model. In downturns associated with a credit crunch, while the initial recovery can be quite brisk, the recovery can stall as the intermediary sector is slower to recover; thus, investment and macroeconomic activity may remain depressed, and spreads elevated, potentially for a prolonged period of time, even if the corporate sector seems to have recovered and pays dividends. In a downturn without a credit crunch, corporate loan demand falls, and intermediaries may hold cash to meet future corporate loan demand and thus spreads may initially be very low. A key factor driving these results is that intermediary net worth accumulates more slowly than corporate net worth in the model.

Our model is consistent with three key stylized facts about macroeconomic downturns associated with a credit crunch, namely, their severity, their protractedness, and the fact that the severity of the credit crunch itself in turn affects the severity and the persistence of the downturn. Most uniquely, our model captures the tentative and halting nature of recoveries from such episodes. Our model provides a useful framework for the analysis of the dynamic interaction between financial structure and economic activity.

Appendix A: Proofs

Proof of Proposition 1. Using (14) and the fact that $\nu_i' = 0$ (proved below in Lemma 1, part (iii)), we have $(R_i')^{-1} = \beta\mu'/\mu + \beta\lambda_i'/\mu$ and

$$\frac{1}{R+\rho_i} \equiv (R_i')^{-1} = \frac{1}{R+\rho} + \beta\frac{\lambda_i'}{\mu}$$

and hence $\rho \geq \rho_i$ with equality iff $\lambda_i' = 0$. Moreover, since $R_i' \geq R$ (proved below in Lemma 1, part (i)), $\rho_i \geq 0$. Finally, using (13), we have $1/(R+\rho) \equiv \beta\mu'/\mu = 1/R - \beta\lambda'/\mu$, implying that $\rho > 0$ iff $\lambda' > 0$. $\square$

Proof of Proposition 2. Suppose intermediaries pay out their entire net worth at some point. From that point on, the firm's problem is as if there is no intermediary. We first characterize the solution to this problem and then show that the solution implies shadow interest rates on intermediated finance at which it would not be optimal for intermediaries to exit.

To characterize the solution in the absence of intermediaries, consider a steady state at which $\mu = \mu' \equiv \bar{\mu}$ and note that (13) implies $\bar{\lambda}' = ((R\beta)^{-1} - 1)\bar{\mu} > 0$. The investment Euler equation (16) simplifies to $1 = \beta[A'f_k(k) + (1-\theta)(1-\delta)/\wp$ which defines $\bar{k}$. The firm's steady state net worth is $\bar{w}' = A'f(\bar{k}) + (1-\theta)\bar{k}(1-\delta)$ and the firm pays out

$$\begin{aligned}
\bar{d} &= \bar{w}' - \wp\bar{k} = A'f(\bar{k}) - \bar{k}[1 - (R^{-1}\theta + (1-\theta))(1-\delta)] \\
&> A'f(\bar{k}) - \beta^{-1}\bar{k}[1 - (R^{-1}\theta + \beta(1-\theta))(1-\delta)] \\
&= \int_0^{\bar{k}} [A'f_k(k) - \beta^{-1}(1 - (R^{-1}\theta + \beta(1-\theta))(1-\delta))]dk > 0.
\end{aligned}$$

Therefore, $\bar{\mu} = 1$. Investment $\bar{k}$ is feasible as long as $w \geq \bar{w} = \bar{w}' - \bar{d}$. Whenever $w < \bar{w}$, $k < \bar{k}$ and hence using (16) we have $\mu/\mu' = \beta[A'f_k(k) + (1-\theta)(1-\delta)]/\wp > 1$. The shadow interest rate on intermediated finance is $R_i' = \beta^{-1}\mu/\mu' \geq \beta^{-1}$ for all values of w. But then it cannot be optimal for intermediaries to pay out all their net worth as keeping $\varepsilon > 0$ net worth for one more period improves the objective by $(\beta_i R_i' - 1)\varepsilon > 0$. $\square$

Proof of Lemma 1. Part (i): If $R_i' < R$, then using (13) and (14) we have $0 < (R - R_i')\beta\mu' \leq R_i'\beta\lambda_i'$ and thus $b_i' > 0$. But (18) and (19) imply that $0 < (R - R_i')\beta\mu_i' \leq R_i'\beta\eta_i'$ and thus $l_i' = 0$, which is not an equilibrium.

Part (ii): Given $\nu_i' = 0$ (see part (iii)), (13) and (14) imply that $\lambda' = (R_i'/R - 1)\mu' + R_i'/R\lambda_i' \geq \lambda_i'$.

Part (iii): First, suppose to the contrary that $\nu_i' > 0$. Then $\lambda_i' = 0$ as $b_i' = 0 < (R_i')^{-1}(\theta_i - \theta)k(1-\delta)$ implies that (4) is slack. Using (14) and (13) we have $\beta\mu'R_i' > \mu \geq$

$\beta\mu' R$ and thus $R_i' > R$. Equations (18) and (19) imply that $R\eta' - R_i'\eta_i' = (R_i' - R)\mu_i' > 0$ and thus $\eta' > 0$ and $l' = 0$. But if $w_i' > 0$, which is always true under the conditions of Proposition 2, we have $l_i' = (R_i')^{-1}w_i' > 0 = b_i'$, which is not an equilibrium. If instead $w_i' = 0$, then $l_i' = 0$ and we can set $R_i' = (\beta\mu'/\mu)^{-1}$ and $\eta_i' = 0$ w.l.o.g.

Part (iv): Suppose to the contrary that $\eta_i' > 0$ (and hence $l_i' = 0$). Since intermediaries never pay out all their net worth, equation (9) implies $0 < w_i' \leq Rl'$ and hence $\eta' = 0$. But then (18) and (19) imply $\beta_i\mu_i'/\mu_i R = 1 > \beta_i\mu_i'/\mu_i R_i'$ or $R > R_i'$ contradicting the result of part (i). Thus, $\eta_i' = 0$ and $\mu_i' = (\beta_i R_i')^{-1}\mu_i$.

Part (v): Suppose $\lambda' = 0$. Then (13) reduces to $1 = \beta\mu'/\mu R$ and thus $1 \leq \mu = \beta R\mu' < \mu'$ and $d' = 0$. By part (ii), $\lambda_i' = 0$ and using (14) we have $R_i' = R$, $\mu_i' = (\beta R)^{-1}\mu_i > 1$, and $d_i' = 0$. The investment k^{**} solves $R = [A' f_k(k^{**}) + (1-\theta_i)(1-\delta)]/\wp_i(R)$ or $R - 1 + \delta = A' f_k(k^{**})$; this is the first best investment when dividends are discounted at R and it can never be optimal to invest more than that. To see this use (16) and note $[A' f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i') = \mu/(\beta\mu') \geq R = [A' f_k(k^{**}) + (1-\theta_i)(1-\delta)]/\wp_i(R)$, that is, $f_k(k) \geq f_k(k^{**})$. Note that the firm's net worth next period, using (3) and (16), is

$$\begin{aligned} w' &= A' f(k^{**}) + (1-\theta_i)(1-\delta)k^{**} - [Rb' - \theta(1-\delta)k^{**}] - [Rb_i' - (\theta_i - \theta)(1-\delta)k^{**}] \\ &> R\wp_i(R)k^{**} - [Rb' - \theta(1-\delta)k^{**}] - [Rb_i' - (\theta_i - \theta)(1-\delta)k^{**}] = R[k^{**} - b' - b_i'] \\ &= Rw_{ex}. \end{aligned}$$

Note that $d' = 0$, $d_i' = 0$, $k' \leq k^{**}$, and $w' > w_{ex}$, and from (2) next period, $k' = w' + b'' + b_i''$. If $R_i'' > R$, then $b_i'' = w_i'$ and $b'' = R^{-1}\theta(1-\delta)k'$. Therefore, $\wp k' = w' + w_i'$, but using (2) we have $\wp k^{**} \leq k^{**} - b' = w_{ex} + b_i' < w' + w_i' = \wp k'$, a contradiction. If $R_i'' = R$, then $b'' + b_i'' = k' - w' < k^{**} - w_{ex} = b' + b_i'$, that is, the firm is paying down debt, and $w'' > w'$ and $w_i'' > w_i'$. But then w and w_i grow without bound unless the firm or the intermediary eventually pay a dividend. But since μ and μ_i are strictly increasing as long as $R_i' = R$, if either pays a dividend at some future date, then $\mu < 1$ or $\mu_i < 1$ currently, a contradiction. $\square$

Proof of Proposition 4. First, note that $k^* > 0$ in a steady state due to the Inada condition and hence $w'^* \geq A' f(k^*) + k^*(1-\theta_i)(1-\delta) > 0$. Moreover, $d^* > 0$ since otherwise the value would be zero which would be dominated by paying out all net worth. Therefore, in a steady state $\mu^* = \mu'^* = 1$. By Proposition 2 intermediary net worth is positive and hence $d_i^* > 0$ as well (arguing as above), which implies $\mu_i^* = \mu_i'^* = 1$. But then $\eta'^* = (R\beta_i)^{-1} - 1 > 0$ and $l_i'^* > 0$ (and $\eta_i'^* = 0$), since otherwise intermediary net worth would be 0 next period. Therefore, $R_i'^* = \beta_i^{-1}$, and thus $\lambda_i'^* = (\beta_i^{-1}\beta)^{-1} - 1 > 0$, that is, the firm's collateral constraint for intermediated finance binds. Moreover, k^* uniquely solves $1 = \beta[A' f_k(k^*) + (1-\theta_i)(1-\delta)]/\wp_i(\beta_i^{-1})$ and d'^*, b'^*, $b_i'^*$, and w'^* are determined by (2)-(4) at equality. Specifically, $d^* = A'f(k^*) + k^*(1-\theta_i)(1-\delta) - \wp_i(\beta_i^{-1})k^* > 0$ and $b_i'^* = \beta_i(\theta_i - \theta)k^*(1-\delta)$. The net worth of the firm after dividends is $w^* = \wp_i(\beta_i^{-1})k^*$. Finally, $l_i'^* = b_i'^* = w_i^*$ and $d_i^* = (\beta_i^{-1} - 1)w_i^*$. Thus there exists a unique steady state that we have constructed. $\square$

Proof of Proposition 5. Consider first region D and take $w \geq \bar{w}(w_i)$ (to be defined below) and $d > 0$ forever ($\mu = \mu' = 1$). The investment Euler equation then implies $1 = \beta[A'f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i)$. If the collateral constraint for intermediated finance (4) does not bind, then $\mu = R_i'\beta\mu'$, that is, $R_i' = \beta^{-1}$, and investment is constant at $\bar{k}$ which solves $1 = \beta[A'f_k(\bar{k}) + (1-\theta_i)(1-\delta)]/\wp_i(\beta^{-1})$ or, equivalently, $1 = \beta[A'f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$. Define $\bar{w}(w_i) \equiv \wp\bar{k} - w_i$ and $\bar{w}_i = \beta(\theta_i - \theta)\bar{k}(1-\delta)$. At $\bar{w}_i$, (4) is just binding. For $w_i \in (0, \bar{w}_i)$, (4) is slack. Moreover, $w_i' = \beta^{-1}w_i$ and, if $w_i' \in (0, \bar{w}_i)$, the ex dividend net worth is $w_{ex} = \bar{w}(w_i)$ both in the current and next period, and we have immediately $w_{ex}'/w_i' < w_{ex}/w_i$. Further, using (3) and (16) we have

$$w' = A'f(\bar{k}) + (1-\theta)\bar{k}(1-\delta) - R_i'w_i > [A'f_k(\bar{k}) + (1-\theta)(1-\delta)]\bar{k} - R_i'w_i = R_i'\bar{w}(w_i).$$

But $w_{ex}' = \bar{w}(w_i') < \bar{w}(w_i)w_i'/w_i = R_i'w_{ex}$, so $d' = w' - w_{ex}' > 0$. For $w_i \in [\bar{w}_i, w_i^*)$, (4) binds and $k(w_i)$ solves $1 = \beta[A'f_k(k(w_i)) + (1-\theta_i)(1-\delta)]/[\wp - w_i/k(w_i)]$ and $R_i' = (\theta_i - \theta)k(w_i)/w_i(1-\delta)$. Note that the last two equations imply that $k(w_i) \geq \bar{k}$, $w_i/k(w_i) \geq \bar{w}_i/\bar{k}$, and $R_i' \leq \beta^{-1}$ in this region. As before, define $\bar{w}(w_i) = \wp k(w_i) - w_i$ and note that the ex dividend net worth is $w_{ex} = \bar{w}(w_i)$. Suppose $w_i^+ > w_i$ then $k(w_i^+) > k(w_i)$, $k(w_i^+)/w_i^+ < k(w_i)/w_i$, and $w_{ex}^+/w_i^+ = \wp k(w_i^+)/w_i^+ - 1 < w_{ex}/w_i$. Moreover, $w_i' = R_i'w_i > w_i$ and hence k (strictly) increases and R_i' (strictly) decreases in this region. Proceeding as before,

$$\begin{aligned} w' &= A'f(k(w_i)) + (1-\theta_i)k(w_i)(1-\delta) > [A'f_k(k(w_i)) + (1-\theta_i)(1-\delta)]k(w_i) \\ &\geq R_i'\beta[A'f_k(k(w_i)) + (1-\theta_i)(1-\delta)]k(w_i) = R_i'\bar{w}(w_i). \end{aligned}$$

But $w_{ex}' = \bar{w}(w_i') < \bar{w}(w_i)w_i'/w_i = R_i'w_{ex}$, so $d' = w' - w_{ex}' > 0$. Finally, if $w_i \geq w_i^*$ and $w \geq \bar{w}(w_i) = w^*$, the steady state of Proposition 4 is reached.

We now show that the above policies are optimal for both the firm and the intermediary given the interest rate process in region D and hence constitute an equilibrium. Since $R_i' > \beta_i^{-1}$ before the steady state is reached, the intermediary lends its entire net worth to the firm, $l_i' = w_i$, and does not pay dividends until the steady state is reached. Hence, the intermediary's policy is optimal. To see that the firm's policy is optimal in region D, suppose that the firm follows the optimal policy from the next period onward but sets $\tilde{d} = 0$ in the current period. If the firm invests the additional amount, then $\tilde{k} = (w_i + w)/\wp > k$ and $\tilde{w}' > w'$ (and therefore $\tilde{\mu}' = 1$). The investment Euler equation requires $1 = \beta/\tilde{\mu}[A' f_k(\tilde{k}) + (1-\theta_i)(1-\delta)]/\wp_i(R_i')$, but since $f_k(\tilde{k}) < f_k(k)$ and k satisfies the investment Euler equation at $\mu = \mu' = 1$, this implies $\tilde{\mu} < 1$, a contradiction. Suppose the firm instead invests the same amount $\tilde{k} = k$ but borrows less $\tilde{b}_i' < b_i'$. Then $\tilde{w}' > w'$, $\tilde{\mu}' = 1$, and from (16) $\tilde{\mu} = 1$. If $R_i' < \beta^{-1}$, then (4) is binding, a contradiction. If $R_i' = \beta^{-1}$, then the firm is indifferent between paying dividends in the current period or in the next period. But in equilibrium $b_i' = w_i$ and hence $\tilde{d} = d > 0$ for the representative firm. By induction starting at the steady state and working backwards, the firm's policy is optimal in region D. Further, we show in Lemmata A.1 and A.2 that the equilibrium in region D is the unique equilibrium converging to the steady state.

Consider now region ND with $w_i \leq w_i^*$ (as Lemma 2 shows) and $w < \bar{w}(w_i)$ as defined in the characterization of region D above and $d = 0$. Denote the firm's ex dividend net worth by $w_{ex} \leq w$. There are 3 cases to consider: $w_{ex}/w_i > \bar{w}/\bar{w}_i$, $w_{ex}/w_i \in [w^*/w_i^*, \bar{w}/\bar{w}_i]$, and $w_{ex}/w_i < w^*/w_i^*$.

First, if $w_{ex}/w_i > \bar{w}/\bar{w}_i$, then $w_{ex} + w_i < \bar{w}(w_i) + w_i = \bar{w} + \bar{w}_i$ and $k \leq (w_{ex} + w_i)/\wp < (\bar{w} + \bar{w}_i)/\wp = \bar{k}$. Note that since $b_i' \leq w_i - d_i \leq w_i$, we have $w_{ex}/b_i' \geq w_{ex}/w_i > \bar{w}/\bar{w}_i$. If (4) binds, then $R_i' = (\theta_i - \theta)(1-\delta)(w_{ex}/b_i' + 1)/\wp > (\theta_i - \theta)(1-\delta)(\bar{w}/\bar{w}_i + 1)/\wp = \beta^{-1}$. If (4) does not bind, then $R_i' = [A' f_k(k) + (1-\theta)(1-\delta)]/\wp > [A' f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp = \beta^{-1}$. In either case, $R_i' > \beta^{-1}$, and hence $d = 0$, $d_i = 0$, and $b_i' = w_i$.

Second, consider $w_{ex}/w_i \in [w^*/w_i^*, \bar{w}/\bar{w}_i]$. If $w_{ex}/b_i' > \bar{w}/\bar{w}_i$ we are in the first region and hence $d_i = 0$ and $b_i' = w_i$, a contradiction. Hence, w.l.o.g. $w_{ex}/b_i' \in [w^*/w_i^*, \bar{w}/\bar{w}_i]$. Take $\tilde{w}_i$ such that $w_{ex}/b_i' = \bar{w}(\tilde{w}_i)/\tilde{w}_i$. Note that (4) binds at $\tilde{w}_i$ and $\bar{w}(\tilde{w}_i)$, and thus $b_i' + w_{ex} < \tilde{w}_i + \bar{w}(\tilde{w}_i)$ and moreover $k < \hat{k}(\tilde{w}_i)$. If (4) does not bind, then

$$
\begin{aligned}
\hat{R}_i'(\tilde{w}_i) &= (\theta_i - \theta)(1-\delta)(\bar{w}(\tilde{w}_i)/\tilde{w}_i + 1)/\wp > (\theta_i - \theta)(1-\delta)(w_{ex}/b_i' + 1)/\wp > R_i' \\
&= [A' f_k(k) + (1-\theta)(1-\delta)]/\wp > [A' f_k(\hat{k}(\tilde{w})) + (1-\theta)(1-\delta)]/\wp.
\end{aligned}
$$

But since (4) binds at $\tilde{w}_i$ and $\bar{w}(\tilde{w}_i)$, $\hat{R}_i'(\tilde{w}_i) < [A' f_k(\hat{k}(\tilde{w})) + (1-\theta)(1-\delta)]/\wp$, a contradiction. Therefore, (4) binds and $R_i' = \hat{R}_i'(\tilde{w}_i)$. From (16), $\beta\mu'/\mu[A' f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i') = 1 = \beta[A' f_k(\hat{k}(\tilde{w}_i)) + (1-\theta_i)(1-\delta)]/\wp_i(\hat{R}_i'(\tilde{w}_i))$ and, since $k < \hat{k}(\tilde{w}_i)$, $\mu > \mu' \geq 1$, that is, $d = 0$. Further, if $w_{ex}/w_i \in (w^*/w_i^*, \bar{w}/\bar{w}_i]$, then $R_i' \in (\beta_i^{-1}, \beta^{-1}]$, and thus $d_i = 0$ and $b_i' = w_i$. If $w_{ex}/w_i = w^*/w_i^*$, then either $d_i > 0$ or $b_i' < w_i$ yields $R_i' > \beta_i^{-1}$ and therefore $d_i = 0$ and $b_i' = w_i$ at such w_{ex} and w_i as well.

Third, consider $w_{ex}/w_i < w^*/w_i^*$. As before, w.l.o.g. $w_{ex}/b_i' < w^*/w_i^*$. Then from (4), $R_i' \leq (\theta_i - \theta)(1-\delta)(w_{ex}/b_i' + 1)/\wp < (\theta_i - \theta)(1-\delta)(w^*/w_i^* + 1)/\wp = \beta_i^{-1}$, that is, $R_i' < \beta_i^{-1}$. From (16), $\beta\mu'/\mu[A' f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i') = 1 = \beta[A' f_k(k^*) + (1-\theta_i)(1-\delta)]/\wp_i(\beta_i^{-1})$ and, since $k < k^*$ and $R_i' < \beta_i^{-1}$, $\mu > \mu' \geq 1$, that is, $d = 0$. Moreover, (4) binds, since otherwise $\beta_i^{-1} > R_i' = [A' f_k(k) + (1-\theta)(1-\delta)]/\wp > [A' f_k(k^*) + (1-\theta)(1-\delta)]/\wp$, but since in the steady state (4) binds $\beta_i^{-1} < [A' f_k(k^*) + (1-\theta)(1-\delta)]/\wp$, a contradiction.

Thus, we conclude that $d = 0$, (property (i) in the statement of the proposition),

$d_i = 0$ (except possibly in the first period (see Lemma 2), that R_i' satisfies the equation in property (ii) of the proposition), and that $b_i' = w_i$ and $k = (w + w_i)/\wp$ if $R_i' > R$ and $k = w/\wp_i(R)$ if $R_i' = R$ (property (iii)). Moreover, using (3) and (16) we have

$$\begin{aligned} w' &= A' f(k) + (1 - \theta_i)(1 - \delta)k - [R_i' b_i' - (\theta_i - \theta)(1 - \delta)k] \\ &> R_i' \wp_i(R_i')k - [R_i' b_i' - (\theta_i - \theta)(1 - \delta)k] \geq R_i' \wp k - R_i' b_i' = R_i' w, \end{aligned}$$

which, together with the fact that $w_i' = R_i' w_i$, implies that $w'/w_i' > w/w_i$ (property (iv)). Note that the equilibrium is thus unique in region ND as well. $\square$

Proof of Lemma 2. We first show that $d_i > 0$ when $w_i > w_i^*$. If $w \geq w^*$, the stationary state is reached and the result is immediate. Suppose hence that $w < w^*$. Suppose instead that $d_i = 0$. We claim that $R_i' < \beta_i^{-1}$ for such w_i and w. Either $R_i' = R$ and hence the claim is obviously true or $R_i' > R$, but then $b_i' = w_i$. Using (4) and (2) we have $R_i' \leq (\theta_i - \theta)(1 - \delta)k/b_i' \leq (\theta_i - \theta)(1 - \delta)(w/w_i + 1) < (\theta_i - \theta)(1 - \delta)(w^*/w_i^* + 1) = \beta_i^{-1}$, that is, $R \leq R_i' < \beta_i^{-1}$. But as long as $d_i = 0$, $w_i' = R_i' w_i \geq R w_i > w_i$, that is, intermediary net worth keeps rising. If eventually firm net worth exceeds w^*, then the steady state is reached and $\mu_i' = 1$ from then onward. But then $\mu_i = \beta_i R_i' \mu_i' = \beta_i R_i' < 1$, which is not possible. The intermediary must pay a dividend in the first period, because if it pays a dividend at any point after that, an analogous argument would again imply that $\mu_i < 1$ in the first period, which is not possible. Similarly, if $w < w^*$ forever, then $w > w_i^*$ forever and the firm must eventually pay a dividend in this region, as never paying a dividend cannot be optimal. But by the same argument again then the dividend must be paid in the first period.

To see that at most an initial dividend is paid and no further dividends are paid until the steady state is reached, note that in equilibrium once $R_i' > \beta_i^{-1}$, then this is the case until the steady state is reached. But as long as $R_i' > \beta_i^{-1}$, the intermediary does not pay a dividend (and this is true w.l.o.g. also at a point where $R_i' = \beta_i^{-1}$ before the steady state is reached). Before this region is reached, $R_i' < \beta_i^{-1}$, but then the intermediary would not postpone a dividend in this region, as other wise again $\mu_i = \beta_i R_i' \mu_i' = \beta_i R_i' < 1$, which is not possible. $\square$

Lemma A.1. *Consider an equilibrium with $R_i' \in [\beta_i^{-1}, \beta^{-1}]$ and $\mu = \mu' = 1$ and assume the equilibrium is unique from the next period onward. Consider another equilibrium interest rate $\tilde{R}_i'$, then $\tilde{k} \lesseqgtr k$ and $\tilde{R}_i' \lesseqgtr R_i'$ is impossible.*

Proof of Lemma A.1. Using (16) at the two different equilibria, if $\tilde{k} \lesseqgtr k$ and $\tilde{R}_i' \lesseqgtr R_i'$, then

$$\frac{\tilde{\mu}}{\tilde{\mu}'} = \beta \frac{A' f_k(\tilde{k}) + (1 - \theta_i)(1 - \delta)}{\wp_i(\tilde{R}_i')} \gtreqless \beta \frac{A' f_k(k) + (1 - \theta_i)(1 - \delta)}{\wp_i(R_i')} = 1 \qquad \text{(A.1)}$$

If $\tilde{k} < k$ and $\tilde{R}_i' < R_i' = \beta_i^{-1}$, then by (A.1) $\tilde{\mu} > \tilde{\mu}'$. Thus, $\tilde{\mu} > \tilde{\mu}'\tilde{R}_i'\beta_i$ implying that (4) must be binding. But then the firm must pay a dividend and $1 = \tilde{\mu} > \tilde{\mu}'$, a contradiction.

If $\tilde{k} > k$ and $\tilde{R}_i' > R_i'$ and the collateral constraint binds at the original equilibrium, then $\tilde{w}' \geq A'f(\tilde{k}) + (1-\theta)i)(1-\delta)\tilde{k} > A'f(k) + (1-\theta_i)(1-\delta)k = w'$. Since $\tilde{w}' > w'$, $\mu' = 1$, and the equilibrium is unique , $\tilde{\mu}' = 1$. By (A.1), $\tilde{\mu} < \tilde{\mu}' = 1$, a contradiction.

If $\tilde{k} > k$ and $\tilde{R}_i' > R_i'$ and the collateral constraint does not bind at the original equilibrium, the $R_i' = \beta^{-1}$ (using (14)). But then $\tilde{\mu}/\tilde{\mu}' \geq \tilde{R}_i'\beta > 1$ while (A.1) implies $\tilde{\mu}/\tilde{\mu}' < 1$, a contradiction. □

Lemma A.2. *The equilibrium in region D is the unique equilibrium converging to the steady state.*

Proof of Lemma A.2. The proof is by induction. First, note that if $w \geq w^*$ and $w_i \geq w_i^*$, then the unique steady state is reached. Consider an equilibrium interest rate R_i' in region D and suppose the equilibrium is unique from the next period on. Suppose $R_i' \in [\beta_i^{-1}, \beta^{-1})$ and consider another equilibrium with $\tilde{R}_i'$. If the collateral constraint (4) binds at this equilibrium, then $\tilde{R}_i' = (\theta_i-\theta)(1-\delta)\tilde{k}/w_i \gtrless (\theta_i-\theta)(1-\delta)k/w_i = R_i'$, which is impossible by Lemma (A.1). If the collateral constraint (4) does not bind at this equilibrium and $\tilde{k} < k$, then $\tilde{R}_i' < (\theta_i-\theta)(1-\delta)\tilde{k}/w_i < (\theta_i-\theta)(1-\delta)k/w_i = R_i'$, which is also impossible by Lemma (A.1). If the collateral constraint (4) does not bind at this equilibrium and $\tilde{k} > k$, by Lemma (A.1) $\tilde{R}_i' < R_i$. But then by (14) $\tilde{\mu}/\tilde{\mu}' = \beta\tilde{R}_i' < \beta R_i' < 1$. Since $\tilde{k} > k$ and the collateral constraint binds at R_i', $\tilde{w}' > w'$ implying $\tilde{\mu}' = 1$ and by above inequality $\tilde{\mu} < 1$, a contradiction. Thus for $R_i' \in [\beta_i^{-1}, \beta^{-1})$ the equilibrium is unique. Suppose $R_i' = \beta^{-1}$. By Lemma (A.1), we need only consider the two cases $\tilde{k} \gtrless k$ and $\tilde{R}_i' \lesseqgtr R_i' = \beta^{-1}$. If $\tilde{k} < k$ and $\tilde{R}_i' > \beta^{-1}$, (14) implies that $\tilde{\mu} > 1$ and hence the firm does not pay a dividend. But then the firm must be borrowing less from intermediaries, which cannot be an equilibrium as $l_i' = w_i$ at this interest rate. If $\tilde{k} > k$ and $\tilde{R}_i' < R_i' = \beta^{-1}$, and if (4) binds at $\tilde{R}_i'$, $\tilde{R}_i' = (\theta_i-\theta)(1-\delta)\tilde{k}/w_i > (\theta_i-\theta)(1-\delta)k/w_i \geq R_i'$, a contradiction; if (4) instead does not bind at $\tilde{R}_i'$, $\tilde{\mu}/\tilde{\mu}' = \beta\tilde{R}_i' < 1$. Since $\tilde{k} > k$ and $\tilde{R}_i'\tilde{b}_i' \leq R_i'w_i$, $\tilde{w}' > w'$ implying $\tilde{\mu}' = 1$ and by above inequality $\tilde{\mu} < 1$, a contradiction. Therefore the equilibrium in region D is unique. □

Proof of Proposition 6. We first provide some notation and key equations for the stochastic economy, to the extent that these differ from the deterministic economy. The firm's problem is to solve $v(w,z) = \max_{\{d,k,b',b_i',w'\}} d + \beta E\left[v(w',z')|z\right]$ subject to the budget constraint for the current period

$$w \geq d + k - E\left[b' + b_i'|z\right], \tag{A.2}$$

and (3) to (6), $\forall s' \in S$. Denote the transition probability on the induced state space for the economy by $\Pi(z,z')$ in a slight abuse of notation. Using the multipliers μ, $\Pi(z,z')\beta\mu'$,

$\Pi(z,z')\beta\lambda'$, $\Pi(z,z')\beta\lambda_i'$, ν_d, and $\Pi(z,z')R_i'\beta\nu_i'$, the first-order conditions are (11),

$$\mu = E\left[\beta\left(\mu'\left[A'f_k\left(k\right)+(1-\delta)\right]+\left[\lambda'\theta+\lambda_i'(\theta_i-\theta)\right](1-\delta)\right)|z\right], \tag{A.3}$$

and (13) to (15), $\forall s' \in S$.

The intermediary's problem is to solve $v_i(w_i,z) = \max_{\{d_i,l',l_i',w_i'\}} d_i + \beta_i E\left[v_i(w_i',z')|z\right]$ subject to the budget constraint for the current period

$$w_i \geq d_i + E[l' + l_i'|z], \tag{A.4}$$

and (9) and (10), $\forall s' \in S$. Using the multipliers μ_i, $\Pi(z,z')\beta_i\mu_i'$, η_d, $\Pi(z,z')R\beta_i\eta'$, and $\Pi(z,z')R_i'\beta_i\eta_i'$, the first-order conditions are (17) to (20).

Using the down payment defined as $\wp_i(R_i') = 1 - [R^{-1}\theta + E[(R_i')^{-1}|z](\theta_i-\theta)](1-\delta)$, the firm's investment Euler equation is

$$1 \geq E\left[\beta\frac{\mu'}{\mu}\frac{A'f_k\left(k\right)+(1-\theta_i)(1-\delta)}{\wp_i(R_i')}\middle| z\right]. \tag{A.5}$$

Finally, define the premium on internal funds ρ as $1/(R+\rho) \equiv E[\beta\mu'/\mu|z]$ and on intermediated finance ρ_i as $1/(R+\rho_i) \equiv E[(R_i')^{-1}|z]$.

Part (i): By assumption the expected productivity in the first period equals the deterministic productivity from time 1 onward (denoted $\bar{A}'$ here), that is, $E[A'] = \bar{A}'$. Define the first best level of capital k_{fb} by $r+\delta = \bar{A}'f_k(k_{fb})$.

Using the definition of the user cost of capital the investment Euler equation (A.5) for the deterministic steady state case can be written as

$$r+\delta+\frac{\rho}{R+\rho}(1-\theta_i)(1-\delta)+\frac{\rho_i}{R+\rho_i}(\theta_i-\theta)(1-\delta) = R\beta\bar{A}'f_k(k^*) < \bar{A}'f_k(k^*)$$

and thus $k^* < k_{fb}$. Now suppose that $\lambda(s') = 0$, $\forall s' \in S$. Part (ii) of Lemma 1 then implies that $\lambda_i(s') = 0$, $\forall s' \in S$, and (13) and (14) simplify to $\mu = R\beta\mu'$ and $\mu = R_i'\beta\mu'$, implying that $R_i' = R$, $\forall s' \in S$, and that $d' = 0$, $\forall s' \in S$, as otherwise $\mu < 1$. Moreover, (19) simplifies to $\mu_i = R\beta_i\mu_i'$ and thus $d_i' = 0$, $\forall s' \in S$, as well since otherwise $\mu_i < 1$. Investment Euler equation (A.5) reduces to $r+\delta = \bar{A}'f_k(k_{fb})$, that is, investment must be k_{fb}. We now show that this implies that the sum of the net worth of the intermediary and the firm exceeds their steady state (cum dividend) net worth in at least one state, which in turn implies that at least one of them pays a dividend, a contradiction. To see this note that $w' = A'f(k_{fb}) + k_{fb}(1-\delta) - Rb' - R_i'b_i'$ and $w_i' = Rl' + R_i'l_i' \geq R_i'l_i' = R_i'b_i'$ and thus

$$w'+w_i' \geq A'f(k_{fb})+k_{fb}(1-\delta)-Rb' \geq A'f(k_{fb})+(1-\theta)k_{fb}(1-\delta) > A'f(k^*)+(1-\theta)k^*(1-\delta)$$

whereas $w'^* + w_i'^* = \bar{A}'f(k^*) + (1-\theta)k^*(1-\delta)$. For $A' > \bar{A}'$, $w' + w_i' > w'^* + w_i'^*$, and either the intermediary or the firm (or both) must pay a dividend, a contradiction.

Part (ii): If $\lambda_i(s') = 0$, $\forall s' \in S$, then $(\beta\mu'/\mu)^{-1} = R'_i = (\beta_i\mu'_i/\mu_i)^{-1}$ where the first equality uses (14) and the second equality uses (19) and we use the fact that parts (iii) and (iv) of Lemma 1 hold for an economy that is deterministic from time 1 onward.

Part (iii): Since $\lambda(\hat{s}') = 0$, $\lambda_i(\hat{s}') = 0$ by part (ii) of Lemma 1 and $R_i(\hat{s}') = R$. From (13), $\mu(\hat{s}') = \mu(\check{s}') + \lambda(\check{s}') > \mu(\check{s}')$. Using (19), $(\beta_i\mu_i(\hat{s}')/\mu_i)^{-1} = R \leq R_i(\check{s}') = (\beta_i\mu_i(\check{s}')/\mu_i)^{-1}$ and thus $\mu_i(\hat{s}') \geq \mu_i(\check{s}')$. $\square$

Appendix B: Equivalence of limited enforcement and collateral constraints

We start by stating the representative firm's and the representative intermediary's problem in the stochastic economy with limited enforcement and limited participation, allowing for long-term contracts. To facilitate this we first develop some notation for states and the stochastic process for productivity, as well as state prices, stochastic discount factors, and state-dependent interest rates for both the morning and the afternoon.

Define a state at date t by $s^t = \{s_0, s_1, \ldots, s_t\}$ which includes the history of realizations of the stochastic process s_τ for dates $\tau = 0, 1, \ldots, t$, and the set of states at date t by S^t. Let the transition function from state s^t to s^{t+1} be $\Pi(s^t, s^{t+1})$ and let $\Pi(s^\tau, s^t)$ be the probability of state s^t occurring at date t given state s^τ at date $\tau < t$. Each date has two subperiods, morning and afternoon, and states are realized in the morning.

Each afternoon there are markets in Arrow securities for all subsequent mornings and afternoons. We define the (endogenous) state prices in these markets as follows: $Q^m_{\tau t} \equiv Q_m(s^\tau, s^t)$ is the price in the afternoon in state s^τ of one unit of goods in the morning in state s^t, $t > \tau$, and $Q^a_{\tau t} \equiv Q_a(s^\tau, s^t)$ the price in the afternoon in state s^τ of one unit of goods in the afternoon in state s^t, $t \geq \tau$. These prices are determined in equilibrium but are taken as given by firms and intermediaries. Note that $Q^a_{\tau\tau} = 1$, that is, the price of a unit of goods in the afternoon today is just one unit. We also define stochastic discount factors $q^m_{\tau t} \equiv q_m(s^\tau, s^t)$ and $q^a_{\tau t} \equiv q_a(s^\tau, s^t)$ as follows:

$$\begin{aligned} Q_m(s^\tau, s^t) &\equiv \Pi(s^\tau, s^t) q_m(s^\tau, s^t) \\ Q_a(s^\tau, s^t) &\equiv \Pi(s^\tau, s^t) q_a(s^\tau, s^t). \end{aligned}$$

Moreover, we define state-dependent interest rates $R_{mt+1} \equiv R_m(s^t, s^{t+1}) \equiv (q_m(s^t, s^{t+1}))^{-1}$ for the morning and $R_{at+1} \equiv R_a(s^t, s^{t+1}) \equiv (q_a(s^t, s^{t+1}))^{-1}$ for the afternoon, respectively. Finally, to simplify notation, for a stochastic random variable $y_t \equiv y(s^t)$, we define the short hand $\mathbf{y}_\tau \equiv \{y_t\}_{t=\tau}^\infty$. Similarly we define the short hand $\mathbf{Q}^a_{\tau\tau} \equiv \{\{Q^a_{ut}\}_{t=u}^\infty\}_{u=\tau}^\infty$ to be all the current and future state prices for all future dates from time τ onward; $\mathbf{q}^a_{\tau\tau}$ is defined analogously. Analogous definitions apply for the morning.

The *firm's problem* ($\mathbf{P}_\tau^{LE}$) at date τ in history s^τ in the afternoon is to choose $\mathbf{x}_\tau^{LE}$ where $x_t^{LE} = (d_t, k_t, p_t, p_{mt}, p_{at})$, that is, dividends d_t, capital k_t, net payments to households p_t, and net payments to intermediaries in the morning p_{mt} and in the afternoon p_{at}, given net worth w_τ, to solve

$$\max_{\mathbf{x}_\tau^{LE}} E_\tau \left[\sum_{t=\tau}^{\infty} \beta^{t-\tau} d_t \right] \tag{B.1}$$

subject to the budget constraints for the current and all subsequent dates and states,

$$\begin{aligned} w_\tau &\geq d_\tau + k_\tau + p_\tau + p_{a\tau}, && \text{(B.2)} \\ A_t f(k_{t-1}) + k_{t-1}(1-\delta) &\geq d_t + k_t + p_t + p_{mt} + p_{at}, \qquad \forall t > \tau, && \text{(B.3)} \end{aligned}$$

the participation constraints for the intermediary and the household,

$$\begin{aligned} E_\tau \left[\sum_{t=\tau+1}^{\infty} q_{\tau t}^m p_{mt} + \sum_{t=\tau}^{\infty} q_{\tau t}^a p_{at} \right] &\geq 0, && \text{(B.4)} \\ E_\tau \left[\sum_{t=\tau}^{\infty} R^{-(t-\tau)} p_t \right] &\geq 0, && \text{(B.5)} \end{aligned}$$

the limited enforcement constraints

$$E_{\hat{\tau}} \left[\sum_{t=\hat{\tau}}^{\infty} \beta^{t-\tau} d_t \right] \geq E_{\hat{\tau}} \left[\sum_{t=\hat{\tau}}^{\infty} \beta^{t-\tau} \hat{d}_t \right], \qquad \forall \hat{\tau} > \tau, \tag{B.6}$$

and the non-negativity constraints

$$d_t, p_{mt} \geq 0, \qquad \forall t \geq \tau, \tag{B.7}$$

where $\hat{\mathbf{d}}_{\hat{\tau}}$ together with the associated investment and financial policy $\hat{\mathbf{x}}_{\hat{\tau}}^{LE}$ solve problem $\mathbf{P}_{\hat{\tau}}^{LE}$ given net worth $\hat{w}_{\hat{\tau}} \equiv A_{\hat{\tau}} f(k_{\hat{\tau}-1}) + (1-\theta_i) k_{\hat{\tau}-1}(1-\delta)$ if the firm defaults in the morning and $\hat{w}_{\hat{\tau}} \equiv A_{\hat{\tau}} f(k_{\hat{\tau}-1}) + (1-\theta) k_{\hat{\tau}-1}(1-\delta) - p_{m\hat{\tau}}$ if the firm defaults in the afternoon. There are therefore two sets of limited enforcement constraints, one for the morning, where the firm can abscond with cash flows and $1-\theta_i$ of depreciated capital, and one for the afternoon, where the firm can abscond with cash flows minus any payments made in the morning and $1-\theta$ of capital. There is however only one budget constraint for every date, rather than separate budget constraints for the two subperiods, because the firm merely carries over funds from the morning to the afternoon. Note that net payments to intermediaries in the morning are restricted to be non-negative, as intermediaries have no other funds in the morning, but there are no restrictions on p_t and p_{at}.

The *intermediary's problem* ($\mathbf{P}_{i\tau}^{LE}$) at date τ in history s^τ in the afternoon is to choose $\mathbf{x}_{i\tau}^{LE}$ where $x_{it}^{LE} = (d_{it}, p_{ht}, \bar{p}_{mt}, \bar{p}_{at})$, that is, dividends d_{it}, net payments *from* households p_{ht}, and net payments *from* firms in the morning $\bar{p}_{m\tau}$ and in the afternoon $\bar{p}_{a\tau}$, given net worth $w_{i\tau}$, to solve

$$\max_{\mathbf{x}_{i\tau}^{LE}} E_\tau \left[\sum_{t=\tau}^{\infty} \beta^{t-\tau} d_{it} \right] \tag{B.8}$$

subject to the budget constraints for the current and all subsequent dates and states,

$$\begin{aligned} w_{i\tau} &\geq d_{i\tau} - p_{h\tau} - \bar{p}_{a\tau} && \text{(B.9)} \\ 0 &\geq d_{it} - p_{ht} - \bar{p}_{mt} - \bar{p}_{at}, \quad \forall t > \tau, && \text{(B.10)} \end{aligned}$$

the participation constraint for the firm and the household,

$$\begin{aligned} -E_\tau \left[\sum_{t=\tau+1}^{\infty} q_{\tau t}^m \bar{p}_{mt} + \sum_{t=\tau}^{\infty} q_{\tau t}^a \bar{p}_{at} \right] &\geq 0, && \text{(B.11)} \\ -E_\tau \left[\sum_{t=\tau}^{\infty} R^{-(t-\tau)} p_{ht} \right] &\geq 0, && \text{(B.12)} \end{aligned}$$

the limited enforcement constraints

$$E_{\hat{\tau}} \left[\sum_{t=\hat{\tau}}^{\infty} \beta_i^{t-\hat{\tau}} d_{it} \right] \geq E_{\hat{\tau}} \left[\sum_{t=\hat{\tau}}^{\infty} \beta_i^{t-\hat{\tau}} \hat{d}_{it} \right], \qquad \forall \hat{\tau} > \tau, \tag{B.13}$$

and the non-negativity constraints

$$d_{it}, \bar{p}_{mt} \geq 0, \qquad \forall t \geq \tau, \tag{B.14}$$

where $\hat{\mathbf{d}}_{i\hat{\tau}}$ together with associated lending policy $\hat{\mathbf{x}}_{i\hat{\tau}}^{LE}$ solve problem $\mathbf{P}_{i\hat{\tau}}^{LE}$ given wealth $\hat{w}_{i\hat{\tau}} = \bar{p}_{m\hat{\tau}}$ if the intermediary absconds in the afternoon. Since the intermediary at best receives payments in the morning, the limited enforcement constraint for the morning is redundant and we hence drop it. As in the case of the firm, there is only one budget constraint for each date, rather than separate budget constraints for each subperiod. Note that we again restrict the morning payments from the firm to the intermediary to be non-negative, but there are no restrictions on $\bar{p}_{at}$ and p_{ht}, $\forall t \geq \tau$.

We define an equilibrium for the economy with limited enforcement as follows:

Definition B.1 (Equilibrium with limited enforcement)**.** *An equilibrium with limited enforcement is an allocation $\mathbf{x}_0^{LE}$ for the representative firm and $\mathbf{x}_{i0}^{LE}$ for the representative intermediary and stochastic discount factors $\mathbf{q}_{00}^m$ and $\mathbf{q}_{00}^a$ such that: (i) $\mathbf{x}_0^{LE}$ and $\mathbf{x}_{i0}^{LE}$ solve the firm's problem $\mathbf{P}_0^{LE}$ and the intermediary's problem $\mathbf{P}_{i0}^{LE}$, respectively; (ii) the household participation constraints (B.5) and (B.12) hold; and (iii) markets clear, that is, the promises made by firms to intermediaries equal the promises received by intermediaries from firms, $\mathbf{p}_{m0} = \bar{\mathbf{p}}_{m0}$ and $\mathbf{p}_{a0} = \bar{\mathbf{p}}_{a0}$.*

For this environment we now state the equivalent problem with collateral constraints in sequence form. The *firm's problem* ($\mathbf{P}_0^{CC}$) at time 0 in the afternoon is to choose a sequence $\mathbf{x}_0^{CC} \equiv \{x_t^{CC}\}_{t=0}^{\infty}$ where $x_t^{CC} = (d_t, k_t, b_t, b_{it}, b_{at})$, that is, dividends d_t, capital k_t, state-contingent loans from households b_t, and state-contingent loans from intermediaries to be repaid in the morning b_{it} and in the afternoon b_{at}, given net worth w_0 and given the sequence of stochastic interest rates $\mathbf{R}_{i0} \equiv \{R_{it}\}_{t=0}^{\infty}$, to solve

$$\max_{\mathbf{x}_0^{CC}} E_0 \left[\sum_{t=0}^{\infty} \beta^t d_t \right] \tag{B.15}$$

subject to the budget constraints for the current and all subsequent dates and states, that is, $\forall t \geq 1$,

$$\begin{aligned} w_0 &\geq d_0 + k_0 - (E_0[b_1 + b_{i1} + b_{a1}]) && \text{(B.16)} \\ A_t f(k_{t-1}) + k_{t-1}(1-\delta) &\geq d_t + k_t + R(b_t + b_{at}) + R_{it} b_{it} - (E_t[b_{t+1} + b_{it+1} + b_{at+1}]), && \text{(B.17)} \end{aligned}$$

the collateral constraints for loans to be repaid in the morning and afternoon, for all dates and states,

$$\begin{aligned} (\theta_i - \theta) k_t (1-\delta) &\geq R_{it+1} b_{it+1}, && \text{(B.18)} \\ \theta k_t (1-\delta) &\geq R(b_{t+1} + b_{at+1}), && \text{(B.19)} \end{aligned}$$

and the non-negativity constraints for all dates and states

$$d_t, k_t, b_{it} \geq 0. \tag{B.20}$$

Note that there are no non-negativity constraints on (b_t, b_{at}). We emphasize that there are two types of collateral constraints restricting loans to be repaid in the morning and afternoon separately. Given our definition of the stochastic discount factor and the state-contingent interest rates, it is the expected value of the claims issued against the next period that enters the budget constraint in the current period. As we show below, the morning loans are provided by intermediaries at the equilibrium state-contingent interest rate R_{it}, and afternoon loans by both households and intermediaries are provided at interest rate R in equilibrium.[22]

The *intermediary's problem* ($\mathbf{P}_{i0}^{CC}$) at time 0 in the afternoon is to choose $\mathbf{x}_{i0}^{CC} \equiv \{x_{it}^{CC}\}_{t=0}^{\infty}$ where $x_{it}^{CC} = (d_{it}, l_t, l_{it}, l_{at})$, that is, dividends d_{it}, state-contingent loans to

[22] Importantly, morning loans need to be repaid in the morning and postponing payment to the afternoon is not feasible. Morning loans can therefore not be simply rolled over but are extended every afternoon and repaid every morning. Our model thus provides a novel notion of short-term financing.

households l_t, and state-contingent loans to firms to be repaid in the morning l_{it} and in the afternoon l_{at}, given net worth w_{i0} and given the stochastic interest rates $\mathbf{R}_{i0}$, to solve

$$\max_{\mathbf{x}_{i0}^{CC}} E_0 \left[\sum_{t=0}^{\infty} \beta^t d_{it} \right] \tag{B.21}$$

subject to the budget constraints for the current and all subsequent dates and states,

$$\begin{aligned} w_{i0} &\geq d_{i0} + E_0[l_1 + l_{i1} + l_{a1}], && \text{(B.22)} \\ 0 &\geq d_{it} - Rl_t - R_{it}l_{it} - Rl_{at} + E_t[l_{t+1} + l_{it+1} + l_{at+1}], \qquad \forall t \geq 1, && \text{(B.23)} \end{aligned}$$

the collateral constraints for all dates and states

$$l_t + l_{at} \geq 0, \tag{B.24}$$

and the non-negativity constraints for all dates and states

$$d_{it}, l_{it} \geq 0. \tag{B.25}$$

Note that there are no non-negativity constraints on (l_t, l_{at}). Critically, the collateral constraints imply that intermediaries can borrow from households only to the extent that they have corporate loans that pay off (in the afternoon) in that state. Intermediaries cannot borrow against corporate loans that pay off in the morning. Again, the reason is intermediaries themselves could abscond with such payments.

We define an equilibrium for the economy with collateral constraints as follows:

Definition B.2 (Equilibrium with collateral constraints)**.** *An equilibrium with collateral constraint is an allocation $\mathbf{x}_0^{CC}$ for the representative firm and $\mathbf{x}_{i0}^{CC}$ for the representative intermediary and interest rates $\mathbf{R}_{i0}$ such that: (i) $\mathbf{x}_0^{CC}$ and $\mathbf{x}_{i0}^{CC}$ solve the firm's problem $\mathbf{P}_0^{CC}$ and the intermediary's problem $\mathbf{P}_{i0}^{CC}$, respectively; and (ii) markets for intermediated debt clear in each date and state, that is, $\mathbf{b}_{i0} = \mathbf{l}_{i0}$ and $\mathbf{b}_{a0} = \mathbf{l}_{a0}$.*

Observe that the firm's problem $\mathbf{P}_0^{CC}$ and the intermediary's problem $\mathbf{P}_{i0}^{CC}$ only determine the sum of $b_t + b_{at}$ and $l_t + l_{at}$, respectively, for all t. Thus, we can focus on the direct implementation and set $b_{at} = l_{at} = 0$ without loss of generality, that is, we can assume that all afternoon loans are extended by households only. With this assumption and defining the state variables, net worth, for the firm and intermediary as $w_t \equiv A_t f(k_{t-1}) + k_{t-1}(1-\delta) - Rb_t - R_{it}b_{it}$ and $w_{it} \equiv Rl_t + R_{it}l_{it}$, respectively, we obtain the recursive formulation of the firm's and intermediary's problem in equations (1)-(6) and (7)-(10), respectively.

We now show that the economy with limited enforcement and the economy with collateral constraints are equivalent. We proceed in two steps. First, limited enforcement implies that the present value of the sequence of promises, that is, long-term contracts, issued by the firm and the intermediary can never exceed the amount of collateral that can be seized. Second, after several intermediate steps, we show that any such sequence can be implemented with the two types of one-period ahead Arrow securities subject to the collateral constraints (B.18) and (B.19) in $\mathbf{P}_0^{CC}$ and (B.24) in $\mathbf{P}_{i0}^{CC}$ in the main text.[23] Therefore, long-term contracts are irrelevant and the restriction to one-period ahead contracts is without loss of generality.

Theorem B.1 establishes that limited enforcement implies present value collateral constraints (and vice versa).

Theorem B.1 (Enforcement constraints imply present value collateral constraints)**.** *The firm's limited enforcement constraints (B.6) for the morning and the afternoon for all dates and states are equivalent to present value collateral constraints for all dates and states for the morning*

$$\theta_i k_{\tau-1}(1-\delta) \geq p_\tau + p_{m\tau} + p_{a\tau} + \mathrm{E}_\tau \left[\sum_{t=\tau+1}^{\infty} \left(R^{-(t-\tau)} p_t + q_{\tau t}^m p_{mt} + q_{\tau t}^a p_{at} \right) \right], \qquad \text{(B.26)}$$

and for the afternoon

$$\theta k_{\tau-1}(1-\delta) \geq p_\tau + p_{a\tau} + \mathrm{E}_\tau \left[\sum_{t=\tau+1}^{\infty} \left(R^{-(t-\tau)} p_t + q_{\tau t}^m p_{mt} + q_{\tau t}^a p_{at} \right) \right]. \qquad \text{(B.27)}$$

Similarly, the intermediary's limited enforcement constraints (B.13) for the afternoon in all dates and states are equivalent to present value collateral constraints for the afternoon for all dates and states

$$p_{h\tau} + \bar{p}_{a\tau} + \mathrm{E}_\tau \left[\sum_{t=\tau+1}^{\infty} \left(R^{-(t-\tau)} p_{ht} + q_{\tau t}^m \bar{p}_{mt} + q_{\tau t}^a \bar{p}_{at} \right) \right] \geq 0. \qquad \text{(B.28)}$$

Proof of Theorem B.1. We first consider the firm and show that the limited enforcement implies present value collateral constraints. The proof is by contraposition, that is, we show that if the present value collateral constraint is violated, then so is the limited enforcement constraint.

Define, for $\tau' \geq \tau$,

$$\mathrm{PV}_\tau(\mathbf{p}_{\tau'}) = E_\tau \left[\sum_{t=\tau'}^{\infty} R^{-(t-\tau)} p_t \right]$$

[23] An important subtlety in establishing the equivalence is the determination of the present values of morning and afternoon promises. The fact that afternoon promises are discounted to the previous afternoon by both households and intermediaries at the interest rate charged by households obtains by no arbitrage. Morning promises are discounted to the previous afternoon at the interest rate on intermediated finance, and, if necessary, discounted further back at the interest rate charged by households.

and similarly

$$\mathrm{PV}_{m\tau}(\mathbf{p}_{m\tau'}) = E_\tau\left[\sum_{t=\tau'}^{\infty} q_{\tau t}^m p_{mt}\right]; \qquad \mathrm{PV}_{a\tau}(\mathbf{p}_{a\tau'}) = E_\tau\left[\sum_{t=\tau'}^{\infty} q_{\tau t}^a p_{at}\right].$$

Suppose that the present value collateral constraint (B.27) does not hold in the afternoon at time τ. The firm can default and issue new promises at date τ

$$\hat{p}_\tau = -\mathrm{PV}_\tau(\mathbf{p}_{\tau+1}) \quad \text{and} \quad \hat{p}_{a\tau} = -\mathrm{PV}_{m\tau}(\mathbf{p}_{m\tau+1}) - \mathrm{PV}_{a\tau}(\mathbf{p}_{a\tau+1})$$

while keeping the promises, investments, and dividends in all future dates and states the same. The firm can make the same investment k_τ and must be able to pay a higher dividend, since, if (B.27) is violated,

$$\theta k_{\tau-1}(1-\delta) - [p_\tau + p_{a\tau}] < \mathrm{PV}_\tau(\mathbf{p}_{\tau+1}) + \mathrm{PV}_{m\tau}(\mathbf{p}_{m\tau+1}) + \mathrm{PV}_{a\tau}(\mathbf{p}_{a\tau+1}) = -\hat{p}_\tau - \hat{p}_{a\tau},$$

and therefore

$$\begin{aligned} \hat{d}_\tau &= A_\tau f(k_{\tau-1}) + (1-\theta)k_{\tau-1}(1-\delta) - k_\tau - \hat{p}_\tau - p_{m\tau} - \hat{p}_{a\tau} \\ &> A_\tau f(k_{\tau-1}) + k_{\tau-1}(1-\delta) - k_\tau - p_\tau - p_{m\tau} - p_{a\tau} = d_\tau, \end{aligned}$$

which completes the proof in one direction.

Suppose now that the present value collateral constraint (B.27) holds in the afternoon at time τ. If the firm were to default in the afternoon, the firm's net worth $\hat{w}_\tau$ would be $\hat{w}_\tau = A_\tau f(k_{\tau-1}) + (1-\theta)k_{\tau-1}(1-\delta) - p_{m\tau}$. Consider an optimal plan given $\hat{w}_\tau$ say $\hat{\mathbf{x}}_\tau$. One can instead implement a plan $\check{\mathbf{x}}_\tau$ without defaulting that has $\check{\mathbf{x}}_{\tau+1} = \hat{\mathbf{x}}_{\tau+1}$, $\check{k}_\tau = \hat{k}_\tau$, and choose promises today given by

$$\begin{aligned} \check{p}_\tau &= \hat{p}_\tau + p_\tau + \mathrm{PV}_\tau(\mathbf{p}_{\tau+1}) \\ \check{p}_{a\tau} &= \hat{p}_{a\tau} + p_{a\tau} + \mathrm{PV}_{m\tau}(\mathbf{p}_{m\tau+1}) + \mathrm{PV}_{a\tau}(\mathbf{p}_{a\tau+1}). \end{aligned}$$

Hence, the firm could pay the present value of its current promises and make the same promises as under $\hat{\mathbf{x}}_\tau$. Using (B.4) and (B.5) at equality for $\hat{\mathbf{x}}_\tau$ and (B.27) for $\mathbf{x}_\tau$ we have

$$\theta k_{\tau-1}(1-\delta) \geq \check{p}_\tau + \check{p}_{a\tau} + \mathrm{PV}_\tau(\hat{\mathbf{p}}_{\tau+1}) + \mathrm{PV}_{m\tau}(\hat{\mathbf{p}}_{m\tau+1}) + \mathrm{PV}_{a\tau}(\hat{\mathbf{p}}_{a\tau+1}),$$

so (B.27) is satisfied for $\check{\mathbf{p}}_\tau$, $\check{\mathbf{p}}_{m\tau+1}$, and $\check{\mathbf{p}}_{a\tau}$. Moreover, using (B.27) for $\mathbf{x}_\tau$, the dividend $\hat{d}_\tau$ must satisfy

$$\begin{aligned} \hat{d}_\tau &= A_\tau f(k_{\tau-1}) + (1-\theta)k_{\tau-1}(1-\delta) - \hat{k}_\tau - \hat{p}_\tau - p_{m\tau} - \hat{p}_{a\tau} \\ &\leq A_\tau f(k_{\tau-1}) + k_{\tau-1}(1-\delta) - \hat{k}_\tau - \hat{p}_\tau - p_{m\tau} - \hat{p}_{a\tau} \\ &\qquad - p_\tau - p_{a\tau} - \mathrm{PV}_\tau(\mathbf{p}_{\tau+1}) - \mathrm{PV}_{m\tau}(\mathbf{p}_{m\tau+1}) - \mathrm{PV}_{a\tau}(\mathbf{p}_{a\tau+1}) \\ &= A_\tau f(k_{\tau-1}) + k_{\tau-1}(1-\delta) - \check{k}_\tau - \check{p}_\tau - \check{p}_{a\tau} = \check{d}_\tau. \end{aligned}$$

Thus, a feasible strategy without default yields a weakly higher payoff, implying that default cannot be optimal.

Next we show that if the present value collateral constraint (B.26) does not hold in the morning at time τ, then the limited enforcement constraint is violated, too. If (B.26) is violated, the firm could default in the morning, and in the afternoon set $\hat{\mathbf{x}}_{\tau+1} = \mathbf{x}_{\tau+1}$, $\hat{k}_\tau = k_\tau$, and issue new promises today such that (B.4) and (B.5) hold with equality and

$$0 = \hat{p}_\tau + \hat{p}_{a\tau} + \mathrm{PV}_\tau(\mathbf{p}_{\tau+1}) + \mathrm{PV}_{m\tau}(\mathbf{p}_{m\tau+1}) + \mathrm{PV}_{a\tau}(\mathbf{p}_{a\tau+1}).$$

Since the present value collateral constraint is violated, we have

$$\theta_i k_{\tau-1}(1-\delta) - [p_\tau + p_{m\tau} + p_{a\tau}] < \mathrm{PV}_\tau(\mathbf{p}_{\tau+1}) + \mathrm{PV}_{m\tau}(\mathbf{p}_{m\tau+1}) + \mathrm{PV}_{a\tau}(\mathbf{p}_{a\tau+1}) = -\hat{p}_\tau - \hat{p}_{a\tau},$$

and the firm can set $\hat{d}_\tau$ to

$$\begin{aligned} \hat{d}_\tau &= A_\tau f(k_{\tau-1}) + (1-\theta_i)k_{\tau-1}(1-\delta) - k_\tau - \hat{p}_\tau - \hat{p}_{a\tau} \\ &> A_\tau f(k_{\tau-1}) + k_{\tau-1}(1-\delta) - k_\tau - p_\tau - p_{m\tau} - p_{a\tau} = d_\tau, \end{aligned}$$

an improvement. Therefore, the limited enforcement constraint in the morning implies the corresponding present value collateral constraint.

The proof that the present value collateral constraint in the morning implies the limited enforcement constraint in the morning follows the argument for the afternoon closely and is hence omitted.

Next we turn to the intermediary's problem. Since the intermediary receives a non-negative payment in the morning, intermediary default is a concern only in the afternoon. Suppose that the present value collateral constraint (B.28) at date τ is violated, that is,

$$p_{h\tau} + \bar{p}_{a\tau} + \mathrm{PV}_\tau(\mathbf{p}_{h\tau+1}) + \mathrm{PV}_{m\tau}(\bar{\mathbf{p}}_{m\tau+1}) + \mathrm{PV}_{a\tau}(\bar{\mathbf{p}}_{a\tau+1}) < 0.$$

The intermediary could default with $\hat{w}_{i\tau} = \bar{p}_{m\tau}$ and issue new promises at date τ

$$\hat{p}_{h\tau} = -\mathrm{PV}_\tau(\mathbf{p}_{h\tau+1}) \quad \text{and} \quad \hat{\bar{p}}_{a\tau} = -\mathrm{PV}_{m\tau}(\bar{\mathbf{p}}_{m\tau+1}) - \mathrm{PV}_{a\tau}(\bar{\mathbf{p}}_{a\tau+1})$$

while keeping the promises and dividends in all future dates and states the same. Hence, $\hat{p}_{h\tau} + \hat{\bar{p}}_{a\tau} > p_{h\tau} + \bar{p}_{a\tau}$ and the dividend at date τ is higher since

$$\hat{d}_{i\tau} = w_{i\tau} + \hat{p}_{h\tau} + \hat{\bar{p}}_{a\tau} > w_{i\tau} + p_{h\tau} + \bar{p}_{a\tau} = d_{i\tau},$$

implying that the limited enforcement constraint would be violated, too.

To see that (B.28) implies (B.13) proceed analogously to the proof for the firm by assuming that the intermediary defaults and showing that the intermediary could do at least as well without defaulting. $\square$

Before proceeding, we define a notion of consistency of state prices. Essentially, the price of a state s^t contingent claim is the price of the state s^{t-1} contingent claim payable in the afternoon times the state s^{t-1} one-period ahead price for date t of the morning or afternoon claim, as the case may be.

Definition B.3 (Consistency of state prices). *Let history s^{t-1} be such that $(s^{t-1}, s_t) \equiv s^t$, that is, history s^{t-1} occurs as part of history s^t. Then, for $t > \tau$, state prices are consistent if $Q_m(s^\tau, s^t) = Q_a(s^\tau, s^{t-1})Q_m(s^{t-1}, s^t)$ and $Q_a(s^\tau, s^t) = Q_a(s^\tau, s^{t-1})Q_a(s^{t-1}, s^t)$.*

State prices in an equilibrium in the economy with limited enforcement are consistent.

Lemma B.1 (Consistency of state prices). *A limited enforcement equilibrium has consistent state prices.*

Proof of Lemma B.1. Consider first consistency of state prices for afternoon payments. Suppose consistency is violated at time τ for state $s^{\tau+2}$ and assume w.l.o.g. that $Q_a(s^\tau, s^{\tau+2}) > Q_a(s^\tau, s^{\tau+1})Q_a(s^{\tau+1}, s^{\tau+2})$. The intermediary could issue a claim against state $s^{\tau+2}$ and receive $Q_a(s^\tau, s^{\tau+2})$ at time τ and at the same time purchase $Q_a(s^{\tau+1}, s^{\tau+2})$ units of state $s^{\tau+1}$ claims at a per unit cost of $Q_a(s^\tau, s^{\tau+1})$, yielding a positive payout today. In state $s^{\tau+1}$, the promise the intermediary issued will be worth $Q_a(s^{\tau+1}, s^{\tau+2})$ which equals the payoff of the one-period claim. Thus the intermediary can repurchase the claim at that time, yielding a zero payout then. Note that the payoff to the intermediary is positive at time τ and that the present value of the future promises is zero, implying that the present value collateral constraint is satisfied throughout. This is an arbitrage and hence afternoon state prices have to be consistent for time $\tau + 2$, and, proceeding recursively, for all $t \geq \tau + 2$.

To prove the consistency claim for morning state prices, we have to maintain the constraints that p_{mt} and $\bar{p}_{mt}$ have to be non-negative. Suppose first that $Q_m(s^\tau, s^{\tau+2}) > Q_a(s^\tau, s^{\tau+1})Q_m(s^{\tau+1}, s^{\tau+2})$. The firm could issue a claim against state $s^{\tau+2}$ in the morning and receive $Q_m(s^\tau, s^{\tau+2})$ at time τ and at the same time purchase $Q_m(s^{\tau+1}, s^{\tau+2})$ units of state $s^{\tau+1}$ claims at a per unit cost of $Q_a(s^\tau, s^{\tau+1})$, yielding a positive payout today. In state $s^{\tau+1}$, the promise the firm issued will be worth $Q_m(s^{\tau+1}, s^{\tau+2})$ which equals the payoff of the one-period claims bought. Thus the firm can repurchase the claim at that time, yielding a zero payout then. Note that the payoff to the firm is positive at time τ, that is, this is an arbitrage, and that the present value of the future promises is zero, implying that the present value collateral constraint is satisfied throughout. Suppose instead that $Q_m(s^\tau, s^{\tau+2}) < Q_a(s^\tau, s^{\tau+1})Q_m(s^{\tau+1}, s^{\tau+2})$, then the intermediary could purchase a claim against state $s^{\tau+2}$ in the morning and proceed analogously to the firm above, reversing all the signs of the transactions. In either case, there is thus an arbitrage and hence morning state prices have to be consistent for time $\tau + 2$, and, proceeding recursively, for all $t \geq \tau + 2$. $\square$

It turns out that the state prices of afternoon claims can be determined by no arbitrage:

Lemma B.2 (One-period ahead afternoon interest rate equals R)**.** *The interest rate on one-period ahead afternoon claims must equal the household's interest rate, that is, $R_a(s^\tau, s^{\tau+1}) = (q_a(s^\tau, s^{\tau+1}))^{-1} = R$ for all dates and states.*

Proof of Lemma B.2. Suppose $R_a(s^\tau, s^{\tau+1}) > R$ for some $(s^\tau, s^{\tau+1})$. The intermediary could lend t the firm at $R_a(s^\tau, s^{\tau+1})$ and borrow from the household at R against state $s^{\tau+1}$. This transaction satisfies the present value collateral constraints at both s^τ and $s^{\tau+1}$ and yields a zero payoff at s^τ and a strictly positive payoff at $s^{\tau+1}$, an arbitrage. Therefore, $R_a(s^\tau, s^{\tau+1}) \leq R$ for all $(s^\tau, s^{\tau+1})$. Moreover, if $R_a(s^\tau, s^{\tau+1}) < R$ for some $(s^\tau, s^{\tau+1})$ then the reverse transaction presents an arbitrage for the intermediary. Thus, the interest rate on one-period ahead afternoon claims equals R in all dates and states. $\square$

Essentially, the afternoon state-contingent interest rates must equal the riskless rate R, as otherwise the intermediary could arbitrage by borrowing at R and lending at $R_a(s^\tau, s^{\tau+1})$ in a state-dependent way. Using Lemmas B.1 and B.2 recursively then yields

$$\begin{aligned} Q_m(s^\tau, s^t) &= \Pi(s^\tau, s^t) R^{-(t-\tau-1)} (R_m(s^{t-1}, s^t))^{-1} \\ Q_a(s^\tau, s^t) &= \Pi(s^\tau, s^t) R^{-(t-\tau)}. \end{aligned}$$

It is noteworthy that multi-period ahead morning claims are priced using the morning state-contingent interest rate only from the afternoon immediately preceding the morning and are priced at the afternoon interest rate before. This is because a multi-period ahead morning claim can be replicated using an afternoon claim paying off in the preceding period and a one-period morning claim from then on. Effectively, the ability to enforce morning claims is used only once. The interest rates for one-period ahead morning claims $R_m(s^{\tau-1}, s^\tau)$ are to be determined in equilibrium.

We are now ready for the second step, which establishes the equivalence of the economies with limited enforcement and with collateral constraints and one-period ahead complete markets for morning and afternoon claims. In the paper, we work with the equivalent economy with collateral constraints which turns out to be more tractable.

Theorem B.2 (Equivalence of limited enforcement and collateral constraints)**.** *An equilibrium with limited enforcement $(\mathbf{x}_0^{LE}, \mathbf{x}_{i0}^{LE}, \mathbf{q}_{00}, \mathbf{q}_{00}^a)$ is equivalent to an equilibrium with collateral constraints $(\mathbf{x}_0^{CC}, \mathbf{x}_{i0}^{CC}, \mathbf{R}_{i0})$. More specifically, in the economy with limited enforcement, $\mathbf{x}_0^{LE}$ and $\mathbf{x}_{i0}^{LE}$ solve $\mathbf{P}_0^{LE}$ and $\mathbf{P}_{i0}^{LE}$, respectively, and markets clear, $\mathbf{p}_{m0} = \bar{\mathbf{p}}_{m0}$*

and $\mathbf{p}_{a0} = \bar{\mathbf{p}}_{a0}$*, with stochastic discount factors* $\mathbf{q}_{00}$ *and* $\mathbf{q}^a_{00}$*, if and only if, in the economy with collateral constraints,* $\mathbf{x}^{CC}_0$ *and* $\mathbf{x}^{CC}_{i0}$ *solve* $\mathbf{P}^{CC}_0$ *and* $\mathbf{P}^{CC}_{i0}$*, respectively, and markets clear,* $\mathbf{b}_{i0} = \mathbf{l}_{i0}$ *and* $\mathbf{b}_{a0} = \bar{\mathbf{l}}_{a0}$ *where*

(i) *Equivalence of state prices: The market clearing interest rates on one-period ahead morning claims* $\mathbf{R}_{i0}$ *are given by* $R_{it+1} = (q^m_{tt+1})^{-1}$ *for all dates and states.*

(ii) *Equivalence of allocations: Dividends and investment* $(\mathbf{d}_0, \mathbf{k}_0, \mathbf{d}_{i0})$ *are identical in the equilibrium with limited enforcement and with collateral constraints.*

Proof of Theorem B.2. Using Lemmas B.1 and B.2, we can write the present value collateral constraints (B.26) and (B.27) for the morning and afternoon, respectively, as

$$\theta_i k_{\tau-1}(1-\delta) \geq p_\tau + p_{m\tau} + p_{a\tau} + E_\tau\left[\sum_{t=\tau+1}^{\infty}\left(R^{-(t-\tau)}p_t + R^{-(t-\tau-1)}(R_{mt})^{-1}p_{mt} + R^{-(t-\tau)}p_{at}\right)\right]$$

and

$$\theta k_{\tau-1}(1-\delta) \geq p_\tau + p_{a\tau} + E_\tau\left[\sum_{t=\tau+1}^{\infty}\left(R^{-(t-\tau)}p_t + R^{-(t-\tau-1)}(R_{mt})^{-1}p_{mt} + R^{-(t-\tau)}p_{at}\right)\right].$$

Now define $p^+_{at} \equiv p_{at} + E_t[(R_{mt+1})^{-1}p_{mt+1}]$ and rewrite the collateral constraints as

$$\theta_i k_{\tau-1}(1-\delta) \geq p_\tau + p_{m\tau} + p^+_{a\tau} + \mathrm{PV}_\tau(\mathbf{p}_{\tau+1} + \mathbf{p}^+_{a\tau+1}) \tag{B.29}$$

and

$$\theta k_{\tau-1}(1-\delta) \geq p_\tau + p^+_{a\tau} + \mathrm{PV}_\tau(\mathbf{p}_{\tau+1} + \mathbf{p}^+_{a\tau+1}). \tag{B.30}$$

Note that determining $(\mathbf{p}_0, \mathbf{p}_{m0}, \mathbf{p}_{a0})$ and $(\mathbf{p}_0, \mathbf{p}_{m0}, \mathbf{p}^+_{a0})$ are equivalent. Next we show that collateral constraints (B.29) and (B.30) are equivalent to (B.30) and the following collateral constraint

$$(\theta_i - \theta)k_{\tau-1}(1-\delta) \geq p_{m\tau}. \tag{B.31}$$

First, observe that adding (B.30) and (B.31) yields (B.29) which establishes the first direction. Second, to establish the other direction, suppose (B.31) is violated, that is, $(\theta_i - \theta)k_{\tau-1}(1-\delta) < p_{m\tau}$. Then it must be that (B.30) is slack, that is, the inequality must be strict, as otherwise adding (B.31) and (B.30) would imply that (B.29) is violated. For such a state s^τ the firm could raise the payment p_τ to households in the afternoon by $R\varepsilon$ and reduce the payment $p_{m\tau}$ to intermediaries in the morning by $R_{m\tau}\varepsilon$ (and correspondingly reduce the payment $p_{\tau-1}$ to households at time $\tau - 1$ by $\Pi(s^\tau, s^{\tau+1})\varepsilon$ while raising the payment $p^+_{a\tau-1}$ to intermediaries by the same amount). This would yield an additional payoff $(R_{m\tau} - R)\varepsilon$ in state s^τ which is non-negative and strictly positive if $R_{m\tau} > R$; but in the latter case we obtain a strict improvement, which is not possible, and in the former case we can shift the payment to the afternoon and (C.16) holds without loss of generality. This establishes the equivalence of the economy with limited enforcement and an economy with collateral constraints as in (B.31) and (B.30).

We now show how to recover the collateral constraints in the one-period debt form (B.18), (B.19), and (B.24). First, let $R_{i\tau} \equiv R_{m\tau}$, for all (τ, s^τ). Second, for $\tau \geq 1$ define

$$Rb_\tau \equiv \mathrm{PV}_\tau(\mathbf{p}_\tau), \quad b_{i\tau} = (R_{i\tau})^{-1} p_{m\tau}, \quad Rb_{a\tau} \equiv \mathrm{PV}_\tau(\mathbf{p}_{a\tau}^+),$$

and rewrite (B.31) and (B.30) as

$$\begin{aligned} (\theta_i - \theta)k_{\tau-1}(1-\delta) &\geq R_{i\tau}b_{i\tau} \\ \theta k_{\tau-1}(1-\delta) &\geq R(b_\tau + b_{a\tau}), \end{aligned}$$

that is, as in (B.18) and (B.19). Similarly, given one-period borrowing $(\mathbf{b}_0, \mathbf{b}_{i0}, \mathbf{b}_{a0})$ we can recover payments $(\mathbf{p}_0, \mathbf{p}_{m0}, \mathbf{p}_{a0}^+)$ by constructing, for all $\tau \geq 1$,

$$p_\tau = Rb_\tau - E_\tau[b_{\tau+1}], \quad p_{m\tau} = R_{i\tau}b_{i\tau}, \quad p_{a\tau}^+ = Rb_{a\tau} - E_\tau[b_{a\tau+1}],$$

which can be seen for p_τ, for example, as follows:

$$Rb_\tau \equiv \mathrm{PV}_\tau(\mathbf{p}_\tau) = p_\tau + R^{-1}E_\tau[\mathrm{PV}_{\tau+1}(\mathbf{p}_{\tau+1})] = p_\tau + R^{-1}E_\tau[Rb_{\tau+1}] = p_\tau + E_\tau[b_{\tau+1}],$$

and analogously for $p_{a\tau}^+$. For date 0, using the household's participation constraint (B.5) at equality we have

$$0 = \mathrm{PV}_0(\mathbf{p}_0) = p_0 + R^{-1}E_0[PV_1(\mathbf{p}_1)] = p_0 + R^{-1}E_0[Rb_1] = p_0 + E_0[b_1],$$

that is, $p_0 = -E_0[b_1]$, and similarly using the intermediary's participation constraint at equality we have

$$\begin{aligned} 0 = \mathrm{PV}_0(\mathbf{p}_{a0}^+) &= p_{a0}^+ + R^{-1}E_0[PV_1(\mathbf{p}_{a1}^+)] \\ &= p_{a0} + E_0[(R_{m1})^{-1}p_{m1}] + R^{-1}E_0[Rb_{a1}] = p_{a0} + E_0[b_{i1}] + E_0[b_{a1}], \end{aligned}$$

so $p_{a0} = -E_0[b_{i1}] - E_0[b_{a1}]$, and thus (B.2) and (B.16) are equivalent. Hence, given w_0, we have shown how to translate an allocation $\mathbf{x}_0^{LE}$ (with $p_{m0} = 0$) into an allocation $\mathbf{x}_0^{CC}$ for the firm and vice versa.

Consider next the intermediary's present value collateral constraint (B.28) which using Lemmas (B.1) and (B.2) can be written as

$$p_{h\tau} + \bar{p}_{a\tau} + E_\tau\left[\sum_{t=\tau+1}^{\infty}\left(R^{-(t-\tau)}p_{ht} + R^{-(t-\tau)-1}(R_{mt})^{-1}\bar{p}_{mt} + R^{-(t-\tau)}\bar{p}_{at}\right)\right] \geq 0,$$

and further simplified by proceeding as before and defining $\bar{p}^+_{at} \equiv \bar{p}_{at} + E_t[(R_{mt+1})^{-1}\bar{p}_{mt+1}]$ and for $\tau \geq 1$,

$$Rl_\tau \equiv \text{PV}_\tau(\mathbf{p}_{h\tau}), \quad l_{i\tau} = (R_{i\tau})^{-1}\bar{p}_{m\tau}, \quad Rl_{a\tau} \equiv \text{PV}_\tau(\bar{\mathbf{p}}^+_{a\tau}),$$

reducing the collateral constraint for the intermediary to

$$l_t + l_{at} \geq 0,$$

which is (B.24), the collateral constraint with one-period loans. Moreover, by mimicking the proof for the firm above, we can show that for $\tau \geq 1$

$$p_{h\tau} = Rl_\tau - E_\tau[l_{\tau+1}], \quad \bar{p}_{m\tau} = R_{i\tau}l_{i\tau}, \quad \bar{p}^+_{a\tau} = Rl_{a\tau} - E_\tau[l_{a\tau+1}],$$

and, using the participation constraints for the firm and household at equality, that $p_{h0} = -E_0[l_1]$ and $\bar{p}_{a0} = -E_0[l_{i1}] - E_0[l_{a1}]$. Hence, given w_{i0}, we have shown how to translate an allocation $\mathbf{x}^{LE}_{i0}$ (with $\bar{p}_{m0} = 0$) into an allocation $\mathbf{x}^{CC}_{i0}$ for the intermediary and vice versa.

Consider now a given equilibrium with limited commitment. Using the interest rates $\mathbf{R}_{m0}$ implied by the state prices for morning payments, in the equivalent collateral constraint problem with $\mathbf{R}_{i0} = \mathbf{R}_{m0}$ our construction ensures that the same dividends and investment and the one-period borrowing defined above are optimal for the firm. Similarly, the same dividends and the one-period loans defined above are optimal for the intermediary. Hence, given one-period interest rates $\mathbf{R}_{i0}$ for intermediary loans repaid in the morning and the interest rate R for loans repaid in the afternoon, the market clears and we have an equilibrium with collateral constraints. The converse argument obviously obtains as well. Therefore, we have shown the equivalence of the economy with limited enforcement, which allows for long-term contracts, and the economy with collateral constraints and one-period ahead contracts. $\square$

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Figure 1: Capital, Collateral Value, and Financing

This figure shows, at the top, the extent to which one unit of capital can be collateralized by households (fraction θ, interpreted as structures) and intermediaries (fraction θ_i, interpreted to include equipment), in the middle, the collateral value next period after depreciation, and at the bottom, the maximal amount that households and intermediaries can finance, as well as the minimum amount of internal funds required.

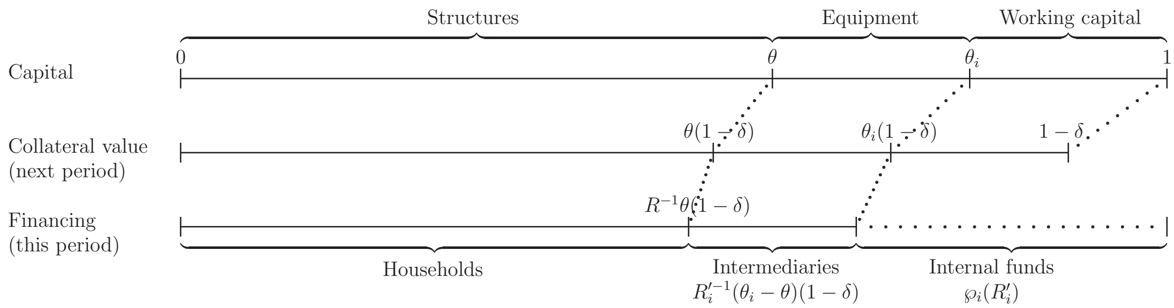


Figure 2: Time Line of Firm's Problem

This figure shows the time line of the firm's problem in the afternoon of the current period including the repayment decisions in the morning (on loans from intermediaries b_i' due then) and in the afternoon of the next period (on loans from intermediaries b_a' and from households b' due then).

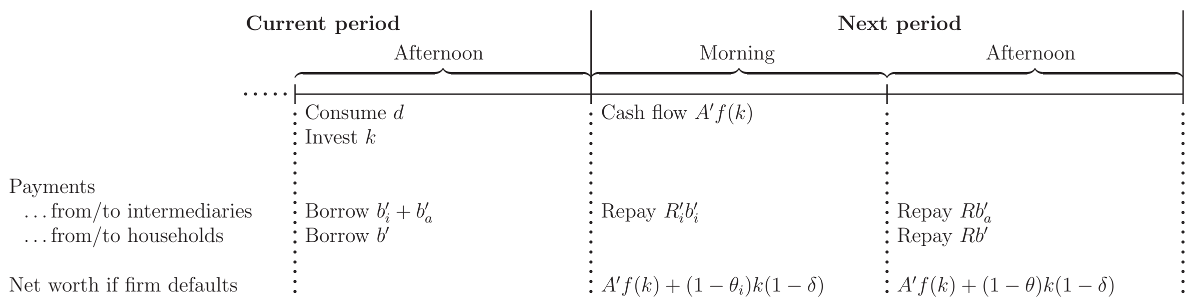


Figure 3: Dynamics of Firm and Financial Intermediary Net Worth

Contours of the regions describing the dynamics of firm and financial intermediary net worth (see Proposition 5). Region ND, in which firms pay no dividends, is to the left of the solid line and Region D, in which firms pay positive dividends, is to the right of the solid line. The point where the solid line reaches the dotted line is the steady state (w^*, w_i^*). The kink in the solid line is the point $(\bar{w}, \bar{w}_i)$ where $R_i' = \beta^{-1}$ and the collateral constraint just binds. The solid line segment between these two points is $\bar{w}(w_i) = \wp k(w_i) - w_i$ (with $R_i' \in (\beta_i^{-1}, \beta^{-1})$). The solid line segment sloping down is $\bar{w}(w_i) = \wp \bar{k} - w_i$ (with $R_i' = \beta^{-1}$). Region ND is dividend by two dash dotted lines: below the dash dotted line through $(\bar{w}, \bar{w}_i)$ $R_i' > \beta^{-1}$; between the two dash dotted lines $R_i' \in (\beta_i^{-1}, \beta^{-1})$; and above the dash dotted line through (w^*, w_i^*) $R_i' < \beta_i^{-1}$. The parameter values are: $\beta = 0.90$, $R = 1.05$, $\beta_i = 0.94$, $\delta = 0.10$, $\theta = 0.60$, $\theta_i = 0.80$, $A' = 0.20$, and $f(k) = k^\alpha$ with $\alpha = 0.80$.

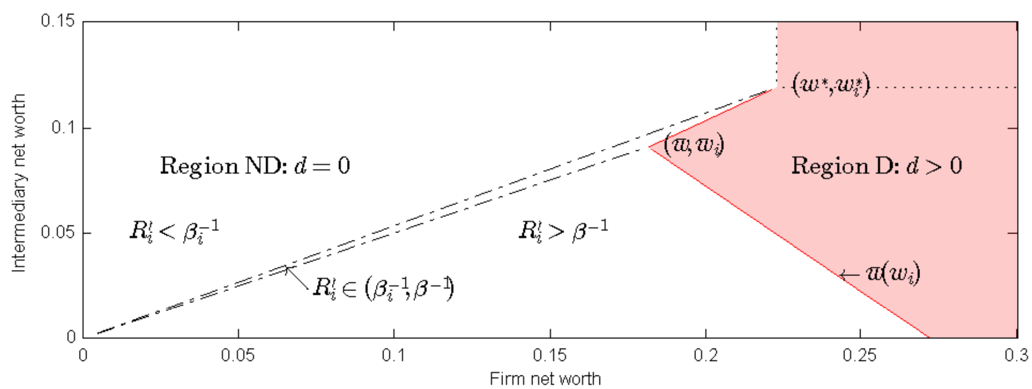
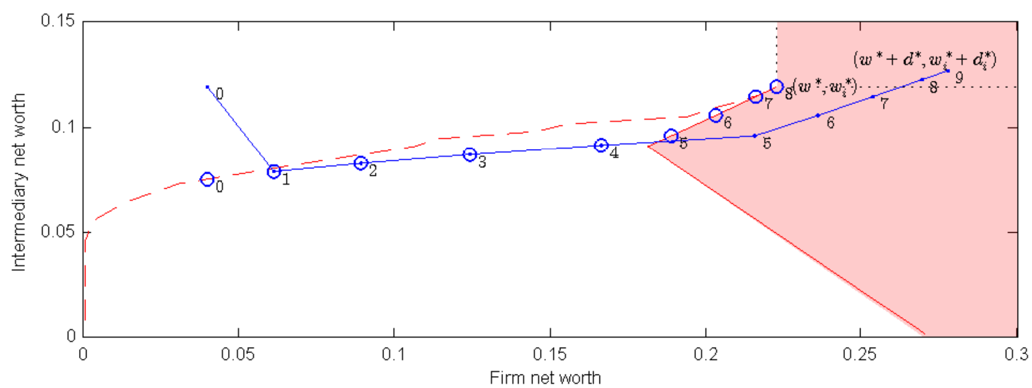


Figure 4: Dynamics of a Downturn without a Credit Crunch

This figure illustrates the dynamics after a downturn in corporate net worth starting from initial values of net worth $w = 0.04$ and $w_i = w_i^*$. Panel A traces out the path of firm and intermediary net worth in w vs. w_i space with the contours as in Figure 3. Panel B shows the evolution of the interest rate on intermediated finance (Panel B1), firm net worth (dashed) and intermediary net worth (solid) (cum dividend (higher) and ex dividend (lower)) (Panel B2), intermediated lending to firms (solid) and households (dashed) (Panel B3), and investment (Panel B4). The parameter values are as in Figure 3.

Panel A: Joint evolution of firm and intermediary net worth



Panel B: Interest rates, net worth, lending, and investment over time

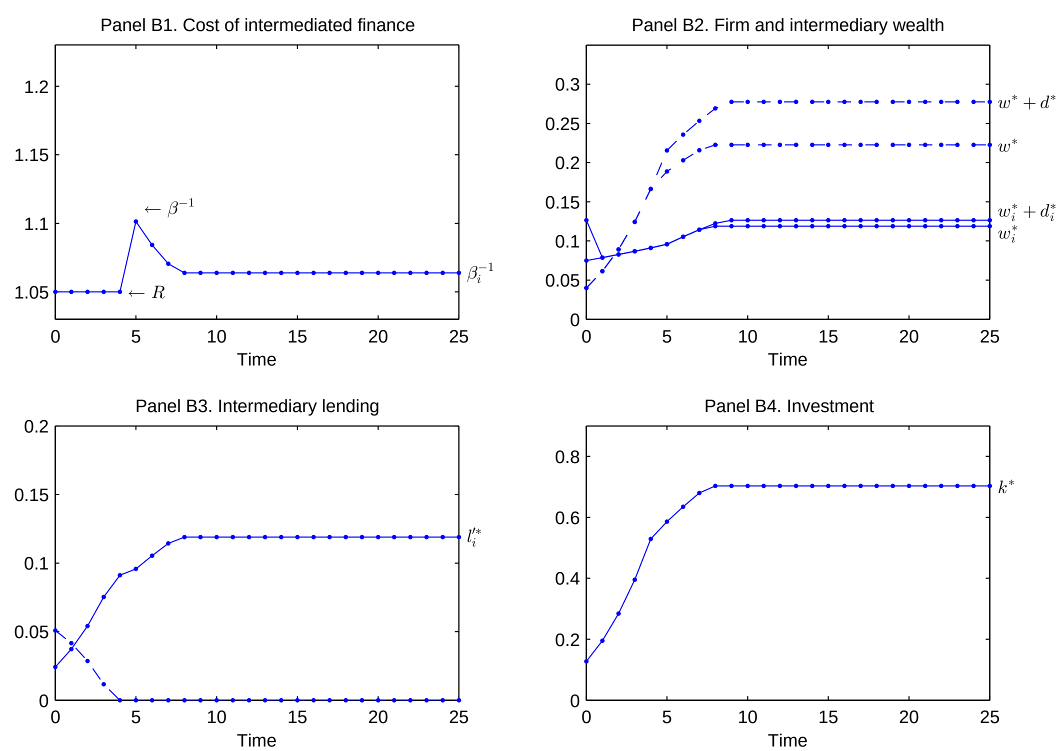
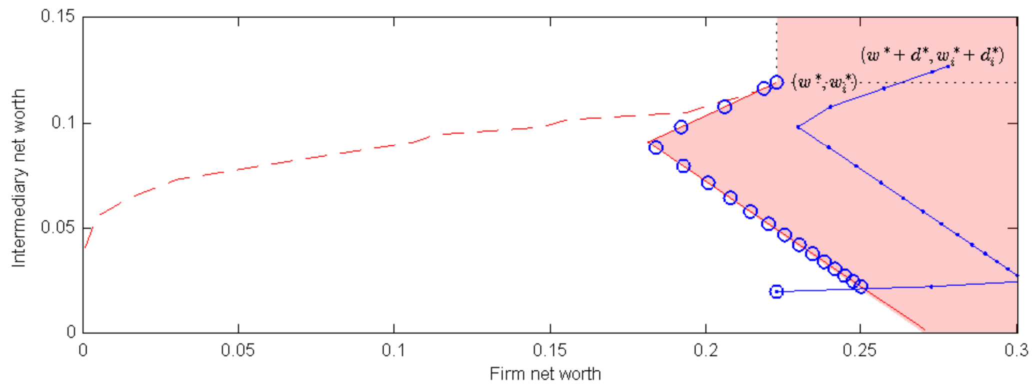


Figure 5: Dynamics of a Credit Crunch

This figure illustrates the dynamics after a credit crunch starting from initial values of net worth $w = w^*$ and $w_i = 0.02$. Panel A traces out the path of firm and intermediary net worth in w vs. w_i space with the contours as in Figure 3. Panel B shows the evolution of the interest rate on intermediated finance (Panel B1), firm net worth (dashed) and intermediary net worth (solid) (cum dividend (higher) and ex dividend (lower)) (Panel B2), intermediated lending to firms (solid) and households (dashed) (Panel B3), and investment (Panel B4). The parameter values are as in Figure 3.

Panel A: Joint evolution of firm and intermediary net worth



Panel B: Interest rates, net worth, lending, and investment over time

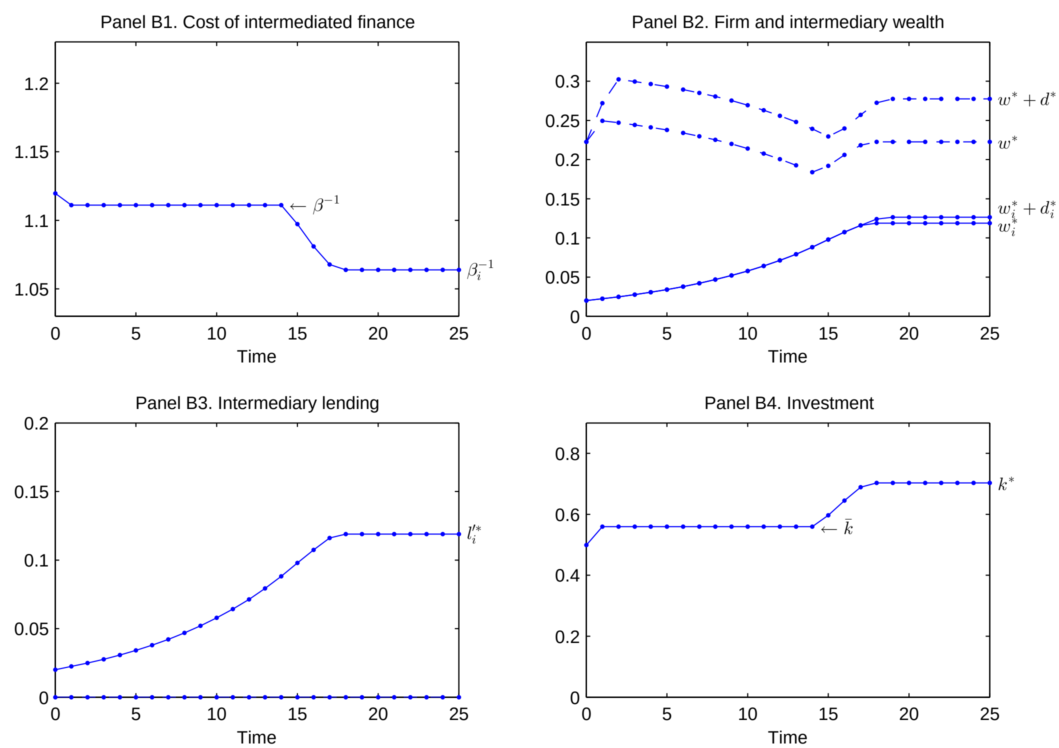
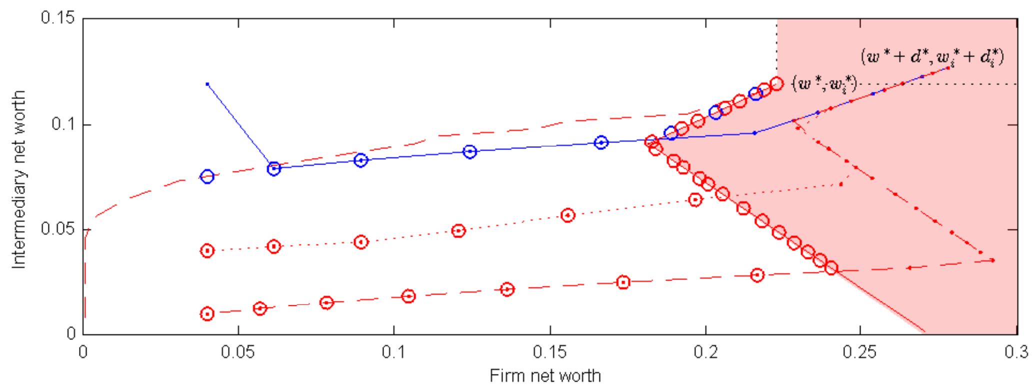


Figure 6: Dynamics of a Downturn Associated with a Credit Crunch

This figure illustrates the dynamics after a downturn in corporate net worth associated with a credit crunch starting from initial values of net worth $w = 0.04$ and three values of intermediary net worth w_i: w_i^* (solid; see also Figure 4), 0.04 (dotted), and 0.01 (dashed). Panel A traces out the path of firm and intermediary net worth in w vs. w_i space with the contours as in Figure 3. Panel B shows the evolution of the interest rate on intermediated finance (Panel B1), firm net worth (higher lines) and intermediary net worth (lower lines) (ex dividend) with dividend initiations marked with triangles pointing up (firms) and down (intermediaries) (Panel B2), intermediated lending to firms (increasing lines) and households (decreasing lines) (Panel B3), and investment (Panel B4). The parameter values are as in Figure 3.

Panel A: Joint evolution of firm and intermediary net worth



Panel B: Interest rates, net worth, lending, and investment over time

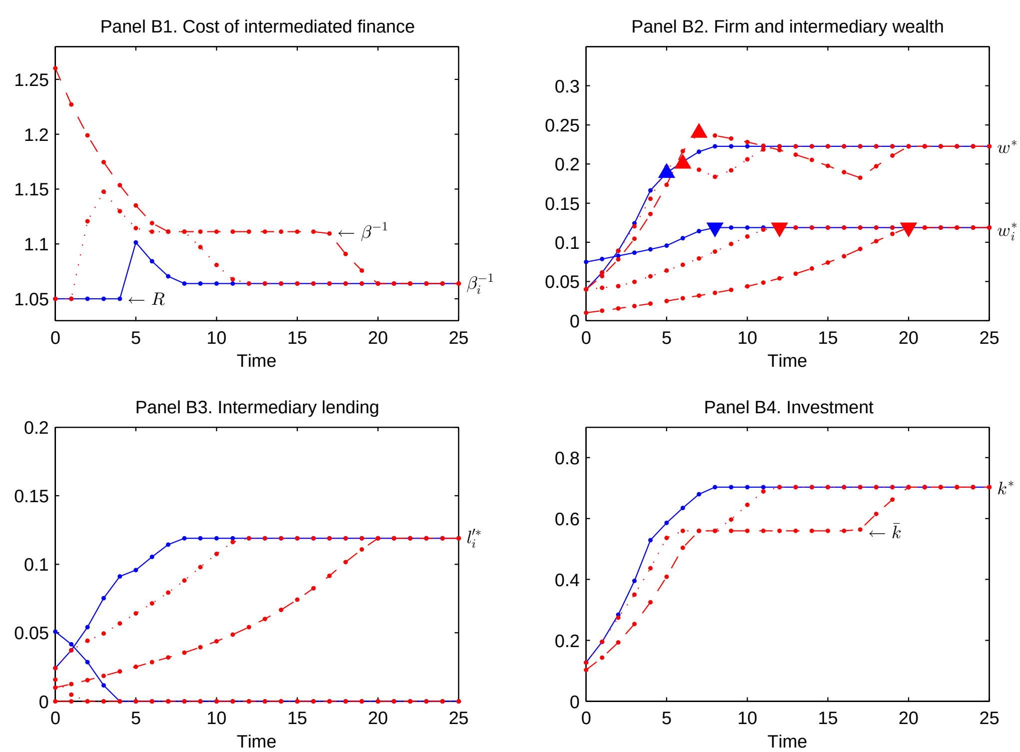
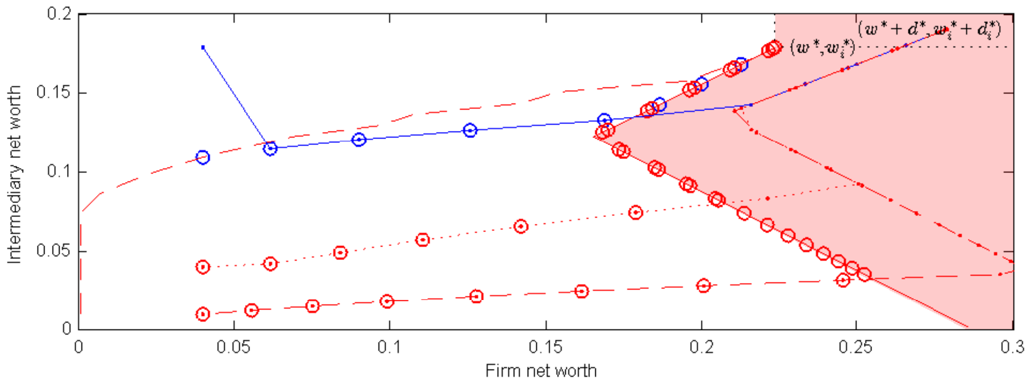


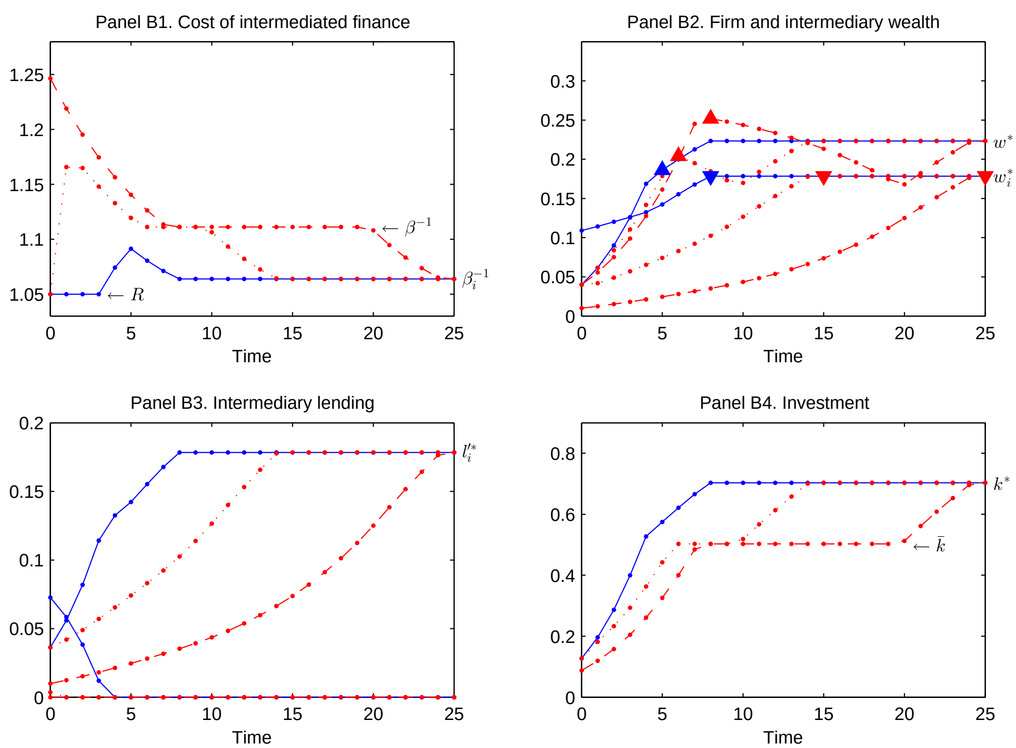
Figure 7: Downturn with a Credit Crunch in Bank-Oriented Economy

This figure illustrates the dynamics after a downturn in corporate net worth associated with a credit crunch in a bank-oriented economy starting from initial values of net worth $w = 0.04$ and three values of intermediary net worth w_i: w_i^* (solid), 0.04 (dotted), and 0.01 (dashed). Panel A traces out the path of firm and intermediary net worth in w vs. w_i space with the contours analogous to Figure 3. Panel B shows the evolution of the interest rate on intermediated finance (Panel B1), firm net worth (higher lines) and intermediary net worth (lower lines) (ex dividend) with dividend initiations marked with triangles pointing up (firms) and down (intermediaries) (Panel B2), intermediated lending to firms (increasing lines) and households (decreasing line) (Panel B3), and investment (Panel B4). The parameter values are as in Figure 3 except that $\theta = 0.50$ and that $A' = 0.2014$ is adjusted to keep k^* and thus $w^* + w_i^*$ constant.

Panel A: Joint evolution of firm and intermediary net worth



Panel B: Interest rates, net worth, lending, and investment over time



Appendix C: Static model of intermediary capital

In this appendix we study how the choice between intermediated and direct finance varies with firm and intermediary net worth in a static (one-period) version of our model with a representative firm. We consider a deterministic economy, although the results in this appendix do not depend on this assumption.[24] For this case, we first show the equivalence of limited enforcement and collateral constraints, then characterize the effect of intermediary net worth on spreads, and finally, analyze the choice between intermediated and direct finance in the cross section of firms with different net worths.

Appendix C.1: Equivalence of limited enforcement and collateral constraints

To show the equivalence of the economy with limited enforcement and limited participation and with collateral constraints, consider the firm's and the intermediary's problem with limited enforcement first. The *firm's problem* is to choose dividends $\{d, d'\}$, investment k, and payments to the household $\{p, p'\}$ and intermediary $\{p_a, p'_m, p'_a\}$ to maximize

$$d + \beta d' \tag{C.1}$$

subject to the budget constraints for time 0 and time 1

$$\begin{aligned} w &\geq d + k + p + p_a, &\text{(C.2)}\\ A'f(k) + k(1-\delta) &\geq d' + p' + p'_m + p'_a, &\text{(C.3)}\end{aligned}$$

the participation constraints for the intermediary and household

$$\begin{aligned} p_a + q'_m p'_m + q'_a p'_a &\geq 0, &\text{(C.4)}\\ p + R^{-1}p' &\geq 0, &\text{(C.5)}\end{aligned}$$

the limited enforcement constraints for the morning and afternoon

$$\begin{aligned} d' &\geq A'f(k) + (1-\theta_i)k(1-\delta), &\text{(C.6)}\\ d' &\geq A'f(k) + (1-\theta)k(1-\delta) - p'_m, &\text{(C.7)}\end{aligned}$$

and the non-negativity constraints

$$d, d', p'_m \geq 0. \tag{C.8}$$

[24] With one period only, the interest rate on intermediated finance is independent of the state s', as the marginal value of net worth next period for financial intermediaries and firms equals 1 for all states, that is, $\mu' = \mu'_i = 1$, rendering the model effectively deterministic.

The firm can abscond with a fraction $1-\theta_i$ of capital in the morning, whereas in the afternoon it can abscond with fraction $1-\theta$ but not payments p'_m already made.

The *intermediary's problem* is to choose dividends $\{d_i, d'_i\}$ and payments *from* the household $\{p_h, p'_h\}$ and *from* the firm $\{\bar{p}_a, \bar{p}'_m, \bar{p}'_a\}$ to maximize

$$d_i + \beta_i d'_i \tag{C.9}$$

subject to the budget constraints for time 0 and time 1

$$w_i \geq d_i - p_h - \bar{p}_a, \tag{C.10}$$
$$0 \geq d'_i - p'_h - \bar{p}'_m - \bar{p}'_a, \tag{C.11}$$

the participation constraints for the firm and household

$$-(\bar{p}_a + q'_m \bar{p}'_m + q'_a \bar{p}'_a) \geq 0, \tag{C.12}$$
$$-(p_h + R^{-1} p'_h) \geq 0, \tag{C.13}$$

the limited enforcement constraint for the afternoon

$$d'_i \geq \bar{p}'_m, \tag{C.14}$$

and the non-negativity constraints

$$d_i, d'_i, \bar{p}'_m \geq 0. \tag{C.15}$$

The intermediary can abscond in the afternoon with payments received in the morning. We emphasize that the intermediary's limited enforcement constraint in the morning is redundant, because the intermediary would abscond without anything in the morning.

Using the firm's time 1 budget constraint (C.3) which holds with equality and the limited enforcement constraint for the morning (C.6), we have

$$A' f(k) + k(1-\delta) - (p' + p'_m + p'_a) = d' \geq A' f(k) + (1-\theta_i) k(1-\delta)$$

which is equivalent to the following collateral constraint

$$\theta_i k(1-\delta) \geq p' + p'_m + p'_a, \tag{C.16}$$

and similarly the limited enforcement constraint for the afternoon is equivalent to the following collateral constraint

$$\theta k(1-\delta) \geq p' + p'_a. \tag{C.17}$$

We will further simplify the collateral constraint for the morning below, after a few intermediate steps.

Using the intermediary's time 1 budget constraint (C.11) which holds with equality and the limited enforcement constraint for the afternoon (C.14), we have

$$p'_h + \bar{p}'_m + \bar{p}'_a = d'_i \geq \bar{p}'_m$$

which is equivalent to the following collateral constraint

$$\bar{p}'_a \geq -p'_h. \tag{C.18}$$

The intermediary can borrow from the household only against claims it has on the firm.

Next we show that the interest rate on intermediated loans repaid in the afternoon must equal R and that the interest rate on intermediated loans repaid in the morning must (weakly) exceed R. The intuition is that if the afternoon interest rate would differ from the interest rates charged by the household, the intermediary could arbitrage this spread. Moreover, since the intermediary can always lend at R to the household, loans repaid in the morning must yield at least R.

Lemma C.1. *Equilibrium state prices satisfy (i) $q'_a = R^{-1}$ and (ii) $q' \leq R^{-1}$ without loss of generality.*

Proof of Lemma C.1. Part (i): Suppose not and assume $q'_a < R^{-1}$ without loss of generality. Take $\varepsilon > 0$ and consider the alternative payments $\hat{p}_h = p_h + \varepsilon$ and $\hat{\bar{p}}_a = \bar{p}_a - \varepsilon$ at time 0 and $\hat{p}'_h = p'_h - R\varepsilon$ and $\hat{\bar{p}}'_a = \bar{p}'_a + (q'_a)^{-1}\varepsilon$ in the afternoon. These payments satisfy the intermediary's time 0 budget constraint (C.10) and the firm's and household's participation constraints, (C.12) and (C.13), by construction. Moreover, the intermediary's afternoon collateral constraint (C.18) is satisfied as

$$\hat{p}'_h + \hat{\bar{p}}'_a = p'_h + \bar{p}'_a + ((q'_a)^{-1} - R)\varepsilon > p'_h + \bar{p}'_a \geq 0.$$

and using the intermediary's time 1 budget constraint (C.11) we can choose $\hat{d}'_i = d'_i + ((q'_a)^{-1} - R)\varepsilon > d'_i$, which is an improvement, contradicting the optimality of the original solution.

Part (ii): Suppose not, i.e., $q' > R^{-1}$ and $\bar{p}'_i > 0$. (If $\bar{p}'_i = 0$, then we can set $q' = R^{-1}$ without loss of generality.) Then consider the alternative choice $\hat{\bar{p}}'_i = \bar{p}_i - \varepsilon$ and $\hat{\bar{p}}'_a = \bar{p}'_a + q'/R^{-1}\varepsilon$, where $\varepsilon > 0$, which satisfies the firm's participation constraint (C.12) by construction. Moreover, we can choose $\hat{d}'_i = d'_i + (q'/R^{-1} - 1)\varepsilon > d'_i$, which is an improvement and hence again impossible. □

Observe that the firm's problem only determines the sum of $p + p_a$ and $p' + p'_a$. Similarly, the intermediary's problem only determines the sum of $p_h + \bar{p}_a$ and $p'_h + \bar{p}'_a$.

We can now show that the firm's limited enforcement constraints are equivalent to the following collateral constraints

$$(\theta_i - \theta)k(1-\delta) \geq p'_m \tag{C.19}$$
$$\theta k(1-\delta) \geq p' + p'_a. \tag{C.20}$$

We need to show that (C.19) and (C.20) are equivalent to (C.16) and (C.17). First, note that (C.20) and (C.17) are identical. Second, observe that adding (C.19) and (C.20) yields (C.16) which establishes the first direction. To establish the other direction, suppose that (C.19) does not hold, i.e., $(\theta_i - \theta)k(1-\delta) < p'_m$. Then (C.20) must be slack as otherwise adding (C.19) and (C.20) would imply that (C.16) is violated. Thus, $\theta k(1-\delta) > p' + p'_a$. Consider the alternative payments $\hat{p}'_m = p'_m - \varepsilon$ and $\hat{p}'_a = p'_a + q'/R^{-1}\varepsilon$ which satisfy (C.4), (C.19), and (C.20) by construction, and $\hat{d}' = d' + (1 - q'/R^{-1})\varepsilon \geq d'$, which is a (weak) improvement (and a strict improvement and hence contradiction whenever $q' < R^{-1}$). Therefore, (C.19) holds without loss of generality. This establishes the equivalence of the economies with limited enforcement and with collateral constraints.

To recover the formulation in Section 4 of the paper set $p'_a = \bar{p}'_a = 0$, that is, the firm makes no payments to the intermediary in the afternoon, and change notation by letting $R'_i \equiv (q')^{-1}$, $b \equiv -p$, $b_i \equiv (R'_i)^{-1}p'_m$, $l \equiv -p_h$, and $l_i \equiv (R'_i)^{-1}\bar{p}'_m$, where $\{b, b_i\}$ are the amounts the firm borrows from the household and intermediary and $\{l, l_i\}$ are the amounts the intermediary lends to the household and firm. Using the fact that the participation constraints for the intermediary and the household, (C.4) and (C.5), bind, we can rewrite the firm's problem as maximizing (C.1) by choosing $\{d, d', k, b, b_i\}$ subject to the constraints

$$w \geq d + k - b - b_i \tag{C.21}$$
$$A'f(k) + k(1-\delta) \geq d' + Rb + R'_i b_i \tag{C.22}$$
$$(\theta_i - \theta)k(1-\delta) \geq R'_i b_i \tag{C.23}$$
$$\theta k(1-\delta) \geq Rb \tag{C.24}$$
$$d, d', b_i \geq 0. \tag{C.25}$$

Similarly, using the fact that the participation constraints for the firm and the household, (C.12) and (C.13), bind, we can rewrite the intermediary's problem as maximizing (C.9) by choosing $\{d_i, d'_i, l, l_i\}$ subject to the constraints

$$w_i \geq d_i + l + l_i \tag{C.26}$$
$$0 \geq d'_i - Rl - R'_i l_i \tag{C.27}$$
$$l \geq 0 \tag{C.28}$$
$$d_i, d'_i, l_i \geq 0. \tag{C.29}$$

We refer to this implementation as the *direct implementation* as all afternoon loans to the firm are extended by the household directly. The intermediary has hence no income from collateralized loans in the afternoon and thus cannot make pledges to the household and can lend to but not borrow from the household. This can be seen from the collateral constraint (C.28) which reduces to a non-negativity constraint on lending to the household.

Alternatively, let, as before, $R_i' \equiv (q')^{-1}$, $b_i \equiv (R_i')^{-1}p_m'$, and $l_i \equiv (R_i')^{-1}\bar{p}_m'$, but now let $b \equiv R^{-1}p_a'$, $l \equiv R^{-1}p_h'$, and $l_a' \equiv R^{-1}\bar{p}_a'$, and set $p = p' = 0$, that is, the firm does not borrow from the household directly. Then (C.4), which holds with equality, implies that $p_a = -(b_i + b)$, and substituting into (C.2), (C.3), (C.19), and (C.20), we obtain the constraints of the firm's problem which are identical to equations (C.21) through (C.25). However, now we interpret b as loans extended by the intermediary to be repaid in the afternoon. Similarly, for the intermediary, (C.13) at equality implies that $p_h = -l$ and (C.12) at equality implies that $\bar{p}_a = -(l_i + l_a)$ which yields the following set of constraints:

$$w_i \geq d_i + l + l_i + l_a \tag{C.30}$$

$$0 \geq d_i' - Rl - R_i'l_i - Rl_a \tag{C.31}$$

$$l_a \geq -l \tag{C.32}$$

and (C.29). This is the *indirect implementation* in which the intermediary extends morning and all afternoon loans to the firm and in turn borrows from the household against its collateralized loans. The afternoon collateral constraint (C.32), similar to equation (C.18), implies that the intermediary can borrow from the household up to the amount that the firm is due to repay in the afternoon. We emphasize that the firm needs to repay morning loans (b_i) in the morning.

Appendix C.2: Effect of intermediary capital on spreads

The equilibrium spread on intermediated finance depends on both firm and intermediary net worth. Given firm net worth, spreads are higher when the intermediary is less well capitalized. Importantly, the spread on intermediated finance depends on the relative capitalization of firms and intermediaries. Spreads are particularly high when firms are poorly capitalized and intermediaries are relatively poorly capitalized at the same time. Poor capitalization of the corporate sector does not per se imply high spreads, as firms' limited ability to pledge may result in a reduction in firms' loan demand which intermediaries with given net worth can more easily accommodate.[25]

[25] Note that in contrast to our model in Holmström and Tirole (1997) aggregate investment only depends on the sum of firm and intermediary capital.

The representative firm solves (C.1) by choosing $\{d, d', k, b, b_i\}$ subject to the constraints (C.21) through (C.25). The representative intermediary solves (C.9) by choosing $\{d_i, d'_i, l, l_i\}$ subject to the constraints (C.26) through (C.29). An equilibrium is defined in Definition 1 of the paper. In addition to the equilibrium allocation, the spread on intermediated finance, $R'_i - R$, is determined in equilibrium.

The following proposition summarizes the characterization of the equilibrium spread. Figure C.1 illustrates the results. The key insight is that the spread on intermediated finance depends on both the firm and intermediary net worth. Importantly, low capitalization of the corporate sector does not necessarily result in a high spread on intermediated finance. Indeed, it may reduce spreads. Similarly, while low capitalization of the intermediation sector raises spreads, spreads are substantial only when the corporate sector is poorly capitalized and intermediaries are poorly capitalized relative to the corporate sector at the same time.

Proposition C.1 (Firm and intermediary net worth)**.** ***(i)*** *For $w_i \geq w_i^*$, intermediaries are well capitalized and there is a minimum spread on intermediated finance $\beta_i^{-1} - R > 0$ for all levels of firm net worth.* ***(ii)*** *Otherwise, there is a threshold of firm net worth $\underline{w}(w_i)$ (which depends on w_i) such that intermediaries are well capitalized and the spread on intermediated finance is $\beta_i^{-1} - R > 0$ as long as $w \leq \underline{w}(w_i)$. For $w > \underline{w}(w_i)$, intermediated finance is scarce and spreads are higher. For $w_i \in [\bar{w}_i, w_i^*)$, spreads are increasing in w until w reaches $\hat{w}(w_i)$, at which point spreads stay constant at $\hat{R}'_i(w_i) - R \in (\beta_i^{-1} - R, \beta^{-1} - R]$. For $w_i \in (0, \bar{w}_i)$, spreads are increasing in w until w reaches $\hat{w}(w_i)$, then decreasing in w until $\bar{w}(w_i)$ is reached, at which point spreads stay constant at $\beta^{-1} - R$. As $w_i \to 0$, $\hat{w}(w_i) \to 0$.*

Proof of Proposition C.1. First, consider the intermediary's problem. The first-order conditions, which are necessary and sufficient, are

$$\mu_i = 1 + \eta_d, \tag{C.33}$$

$$\mu_i = R\beta_i\mu'_i + R\beta_i\eta', \tag{C.34}$$

$$\mu_i = R'_i\beta_i\mu'_i + R'_i\beta_i\eta'_i, \tag{C.35}$$

$$\mu'_i = 1 + \eta'_d, \tag{C.36}$$

where the multipliers on the constraints (C.26) and (C.27) are μ_i and $\beta_i\mu'_i$, and η_d, $\beta_i\eta'_d$, $R\beta_i\eta'$, and $R'_i\beta_i\eta'_i$ are the multipliers on the non-negativity constraints on dividends and direct and intermediated lending. Since (C.27) holds with equality, the non-negativity constraints on l' and l'_i render the non-negativity constraint on d'_i redundant and hence $\mu'_i = 1$. Using (C.34) we have $\eta' = (R\beta_i)^{-1}\mu_i - 1 \geq (R\beta_i)^{-1} - 1 > 0$ (and $l' = 0$) and similarly using (C.35) $\eta'_i > 0$ as long as $R'_i < \beta_i^{-1}$. Therefore, for $l'_i > 0$ it is necessary that $R'_i \geq \beta_i^{-1}$. If $R'_i > \beta_i^{-1}$, then $\mu'_i > 1$ (and $l'_i = w_i$) while if $R'_i = \beta_i^{-1}$, $0 \leq l'_i \leq w_i$.

Now consider the representative firm's problem. The first-order conditions, which are necessary and sufficient, are

$$\begin{aligned}
\mu &= 1+\nu_d, && \text{(C.37)}\\
\mu &= \beta\left(\mu'\left[A'f_k\left(k\right)+(1-\delta)\right]+\left[\lambda'\theta+\lambda_i'(\theta_i-\theta)\right](1-\delta)\right), && \text{(C.38)}\\
\mu &= R\beta\mu'+R\beta\lambda', && \text{(C.39)}\\
\mu &= R_i'\beta\mu'+R_i'\beta\lambda_i'-R_i'\beta\nu_i', && \text{(C.40)}\\
\mu' &= 1+\nu_d', && \text{(C.41)}
\end{aligned}$$

where the multipliers on the constraints (C.21) through (C.24) are μ, $\beta\mu'$, $\beta\lambda'$, and $\beta\lambda_i'$, and ν_d, $\beta\nu_d'$, and $R_i'\beta\nu_i'$ are the multipliers on the non-negativity constraints on dividends and intermediated borrowing. By the Inada condition, (C.38) implies that $k>0$ and using (C.22) at equality and (C.23) and (C.24) we have $d'\geq A'f(k)+k(1-\theta_i)(1-\delta)>0$ and $\mu'=1$. Suppose $\nu_i'>0$ (and hence $b_i'=0$). Since $k>0$, (C.23) is slack and $\lambda_i'=0$. Using (C.37) and (C.40) we have $1\leq\mu<R_i'\beta$ which implies that $R_i'>\beta^{-1}$. But at such an interest rate on intermediated finance $l_i'=w_i>0$, which is not an equilibrium as $b_i'=0$. Therefore, $\nu_i'=0$ and $R_i'\leq\beta^{-1}$. Moreover, if $R_i'<\beta^{-1}$, then $\lambda_i'=(R_i'\beta)^{-1}\mu-1>0$ and hence $b_i'=(R_i')^{-1}(\theta_i-\theta)k(1-\delta)>0$. Since $l_i'=0$ if $R_i'<\beta_i^{-1}$, we have $R_i'\in[\beta_i^{-1},\beta^{-1}]$ in equilibrium. Defining $\wp_i(R_i')=1-[R^{-1}\theta+(R_i')^{-1}(\theta_i-\theta)](1-\delta)$ and using equations (C.38) through (C.40) the firm's investment Euler equation is

$$1=\beta\frac{1}{\mu}\frac{A'f_k\left(k\right)+(1-\theta_i)(1-\delta)}{\wp_i(R_i')}. \tag{C.42}$$

Given the interest rate on intermediated finance, the firm's problem induces a concave value function and thus μ (weakly) decreases in w, implying that k (weakly) increases.

We first show that intermediaries are well capitalized and there is a minimum spread on intermediated finance $\beta_i^{-1}-R>0$ for all levels of firm net worth when $w_i\geq w_i^*$ and for levels of firm net worth $w\leq\underline{w}(w_i)$ when $w_i<w_i^*$. If $R_i'=\beta_i^{-1}$, a well capitalized firm invests k^* which solves (C.42) specialized to $1=\beta[A'f_k(k^*)+(1-\theta_i)(1-\delta)]/\wp_i(\beta_i^{-1})$, while less well capitalized firms invests $k\leq k^*$. The intermediary can meet the required demand for intermediated finance for any level of firm net worth w if $w_i\geq w_i^*\equiv\beta_i(\theta_i-\theta)k^*(1-\delta)$. Suppose instead that $w_i<w_i^*$. In this case the intermediary is able to meet the firm's loan demand at $R_i'=\beta_i^{-1}$ only if the firm is sufficiently constrained; the constrained firm invests $k=w/\wp_i(\beta_i^{-1})$ using (C.21), (C.23), and (C.24) at equality, and thus $b_i'=\beta_i(\theta_i-\theta)k(1-\delta)$; the intermediary can meet this demand as long as $w\leq\underline{w}(w_i)\equiv\wp_i(\beta_i^{-1})/[\beta_i(\theta_i-\theta)(1-\delta)]w_i$.

Suppose now that $w_i<w_i^*$ and $w>\underline{w}(w_i)$ as defined above. First, consider $w_i\in[\bar{w}_i,w_i^*)$ where $\bar{w}_i\equiv\beta(\theta_i-\theta)\bar{k}(1-\delta)$ and $1=\beta[A'f_k(\bar{k})+(1-\theta)(1-\delta)]/\wp$, that is, $\bar{w}_i$

is the loan demand of the well capitalized firm when the cost of intermediated finance is $R_i' = \beta^{-1}$. Note that $R_i' < \beta^{-1}$ on $(\bar{w}_i, w_i^*)$ since the intermediary has more than enough net worth to accommodate the loan demand of the well capitalized firm (and thus any constrained firm) at $R_i' = \beta^{-1}$. Thus, the firm's collateral constraint binds, that is, $w_i = (R_i')^{-1}(\theta_i - \theta)k(1-\delta)$. If the firm is poorly capitalized, $d = 0$ and (C.21) implies $w + w_i = \wp k$, and $R_i' = (\theta_i - \theta)(1-\delta)(w/w_i + 1)$. If the firm is well capitalized, $\mu = 1$ and $\bar{k}(w_i)$ solves $1 = \beta[A' f_k(\bar{k}(w_i)) + (1-\theta_i)(1-\delta)]/[\wp - w_i/\bar{k}(w_i)]$. Moreover, $\bar{w}(w_i) \equiv \wp\bar{k}(w_i) - w_i$ and for $w \geq \bar{w}(w_i)$ the cost of intermediated finance is constant at $\bar{R}_i'(w_i) = (\theta_i - \theta)\bar{k}(w_i)(1-\delta)/w_i$. Note that $\bar{R}_i'(w_i^*) = \beta_i^{-1}$ and $\bar{w}(w_i^*) = \wp k^* - w_i^* = \wp_i(\beta_i^{-1})k^* = \underline{w}(w_i^*)$, that is, the two boundaries coincide at w_i^*. In contrast, at $\bar{w}_i$ we have $\underline{w}(\bar{w}_i) = \wp_i(\beta_i^{-1})/[\beta_i(\theta_i - \theta)(1-\delta)]\bar{w}_i = \wp_i(\beta_i^{-1})\beta/\beta_i\bar{k} = \wp\bar{k}\beta/\beta_i - \bar{w}_i < \bar{w}(\bar{w}_i)$ and $\bar{R}_i'(\bar{w}_i) = \beta^{-1}$.

Finally, consider $w_i \in (0, \bar{w}_i)$ and $w > \underline{w}(w_i)$ as defined above. If the firm is well capitalized (C.40) implies $\lambda_i' = (R_i'\beta)^{-1} - 1 \geq 0$. Moreover, since $w_i < \bar{w}_i$ the intermediary cannot meet the well capitalized firm's loan demand at $R_i' = \beta^{-1}$ and thus the cost of intermediated finance is in fact β^{-1} and $\lambda_i' = 0$, that is, the collateral constraint for intermediated finance does not bind. Thus, the firm's investment Euler equation (C.42) simplifies to $1 = \beta[A' f_k(\bar{k}) + (1-\theta_i)(1-\delta)]/\wp_i(\beta^{-1})$ which is solved by $\bar{k}$ as defined earlier in the proof. Define $\bar{w}(w_i) \equiv \wp\bar{k} - w_i$; the firm is well capitalized for $w \geq \bar{w}(w_i)$. Suppose $w < \bar{w}(w_i)$ and hence $\mu > 1$. If the collateral constraint for intermediated finance does not bind, then (C.40) implies $R_i' = \beta^{-1}\mu > \beta^{-1}$ and (C.42) implies $R_i' = [A' f_k(k) + (1-\theta)(1-\delta)]/\wp$, while (C.21) yields $w + w_i = \wp k$. Observe that $k < \bar{k}$ and R_i' decreases in w. If instead the collateral constraint binds, then $R_i' = (\theta_i - \theta)k(1-\delta)/w_i$ and $w + w_i = \wp k$ (so long as $w > \underline{w}(w_i)$). Note that k and R_i' increase in w in this range. The collateral constrain is just binding at $\hat{w}(w_i) \equiv \wp\hat{k}(w_i) - w_i$ where $[A' f_k(\hat{k}(w_i)) + (1-\theta)(1-\delta)]/\wp = (\theta_i - \theta)\hat{k}(w_i)(1-\delta)/w_i$.

We now show that if the collateral constraint for intermediated finance binds at some $w < \bar{w}(w_i)$ then it binds for all $w^- < w$. Note that $d = 0$ in this range and $w + w_i = \wp k$. At w^-, either $b_i'^- < w_i$ and $R_i' = \beta_i^{-1}$ and hence $\lambda_i'^- = (\beta_i^{-1}\beta)^{-1}\mu^- - 1 > 0$ or $b_i'^- = w_i$ and $w^- + w_i = \wp k^-$, implying $k^- < k$. Suppose the collateral constraint for intermediated finance is slack at w^-. Then $R_i'^- b_i'^- < (\theta_i - \theta)k^-(1-\delta) < (\theta_i - \theta)k(1-\delta) = R_i' b_i'$ and since $b_i'^- = w_i$ and $b_i' \leq w_i$ by above $R_i'^- w_i < R_i' b_i' \leq R_i' w_i$ which implies $R_i'^- < R_i'$. But

$$R_i'^-\beta = \mu^- = \beta\frac{A' f_k(k^-) + (1-\theta_i)(1-\delta)}{\wp - (R_i'^-)^{-1}(\theta_i - \theta)(1-\delta)} > \beta\frac{A' f_k(k) + (1-\theta_i)(1-\delta)}{\wp - (R_i')^{-1}(\theta_i - \theta)(1-\delta)} = \mu > R_i'\beta$$

or $R_i'^- > R_i'$, a contradiction.

Moreover, $\underline{w}(w_i) < \hat{w}(w_i) < \bar{w}(w_i)$ on $w_i \in (0, \bar{w}_i)$. Suppose, by contradiction, that $\hat{w}(w_i) \leq \underline{w}(w_i)$ and recall that $\underline{w}(w_i) + w_i = \wp k$ and $\hat{w}(w_i) + w_i = \wp\hat{k}(w_i)$, so $\hat{k}(w_i) \leq k$. But $\hat{R}_i'(w_i) = (\theta_i - \theta)\hat{k}(w_i)(1-\delta)/w_i \leq (\theta_i - \theta)k(1-\delta)/w_i = \beta_i^{-1}$. But if $\hat{R}_i'(w_i) \leq \beta_i^{-1}$, then at $\hat{w}(w_i)$ we have $\mu = \hat{R}_i'(w_i)\beta < 1$ (since the collateral constraint is slack), a contradiction. Thus, $\underline{w}(w_i) < \hat{w}(w_i)$. Suppose, again by contradiction, that $\bar{w}(w_i) \leq \hat{w}(w_i)$ and hence $\bar{k} \leq \hat{k}(w_i)$. Recall that $\hat{k}(w_i)$ solves $[A' f_k(\hat{k}(w_i)) + (1-\theta)(1-\delta)]/\wp = (\theta_i - \theta)\hat{k}(w_i)(1-\delta)/w_i$. At $\bar{w}_i$ this equation is solved by $\bar{k}$ (and $\hat{R}_i'(\bar{w}_i) = \beta^{-1}$), but since $w_i < \bar{w}_i$, $\hat{k}(w_i) < \bar{k}$, a contradiction. Moreover, as $w_i \to 0$, $\hat{k}(w_i) \to 0$ and $\hat{w}(w_i) = \wp\hat{k}(w_i) - w_i \to 0$. $\square$

Panel A of Figure C.1 displays the cost of intermediated finance as a function of firm net worth (w) and intermediary net worth (w_i). Panel B of Figure C.1 displays the contours of the various areas in Panel A. When intermediary capital is below w_i^* and the corporate sector is not too poorly capitalized ($w > \underline{w}(w_i)$), spreads on intermediated finance are higher. Indeed, when intermediary capital is in this range, higher firm net worth initially raises spreads as loan demand increases (until firm net worth reaches $\hat{w}(w_i)$). This effect can be substantial when $w_i < \bar{w}_i$; indeed, interest rates spike when financial intermediary net worth is very low. If firm net worth is still higher, spreads decline as the marginal product of capital and hence firms' willingness to borrow at high interest rates declines. When corporate net worth exceeds $\bar{w}(w_i)$, the cost on intermediated finance is constant at β^{-1}, which equals the shadow cost of internal funds of well capitalized firms.

To sum up, spreads are determined by firm and intermediary net worth jointly. Spreads are higher when intermediary net worth is lower. But firm net worth affects both the demand for intermediated loans and, via investment, the collateral available to back such loans. When collateral constraints bind, lower firm net worth reduces spreads.

Appendix C.3: Intermediated vs. direct finance in cross section

To show that our model has plausible implications for the choice between intermediated and direct finance in the cross section of firms, consider the static environment without uncertainty analyzed above, but now taking the spread on intermediated finance $R_i' - R$ as given. Each firm maximizes (C.1) subject to the constraints (C.21) through (C.25) given its net worth w. Assume that $R_i' > \beta^{-1}$.[26] Severely constrained firms borrow as much as possible from intermediaries while less constrained firms borrow less from intermediaries and dividend paying firms do not borrow from intermediaries at all, consistent with the cross sectional stylized facts. These cross-sectional results are similar to the ones

[26] We consider the case in which $R_i' > \beta^{-1}$ since one can show that $R_i' < \beta^{-1}$ would imply that $\lambda_i' > 0$ and thus the cross sectional financing implications would be trivial as all firms would borrow the maximal amount from intermediaries. When $R_i' = \beta^{-1}$, this would also be true without loss of generality.

in Holmström and Tirole (1997), although in their model all firms that borrow from intermediaries raise the same amount of intermediated finance.

Proposition C.2 (Intermediated vs. direct finance across firms)**.** *Suppose $R_i' > \beta^{-1}$. **(i)** Firms with net worth $w \leq \underline{w}_l$ borrow as much as possible from intermediaries, firms with net worth $\underline{w}_l < w < \underline{w}_u$ borrow a positive amount from intermediaries but less than the maximal amount, and firms with net worth exceeding $\underline{w}_u$ do not borrow from intermediaries, where $0 < \underline{w}_l < \underline{w}_u$. **(ii)** Only firms with net worth exceeding $\bar{w}$ pay dividends at time 0, where $\underline{w}_u < \bar{w} < \infty$. **(iii)** Investment is increasing in w and strictly increasing for $w \leq \underline{w}_l$ and $\underline{w}_u < w < \bar{w}$.*

Proof of Proposition C.2. The first-order conditions are (C.37)-(C.41). By the Inada condition, (C.38) implies that $k > 0$ and using (C.22) at equality and (C.23) and (C.24) we have $d' \geq A'f(k) + k(1-\theta_i)(1-\delta) > 0$ and $\mu' = 1$. But (C.37) and (C.39) imply $1 \leq \mu = R\beta + R\beta\lambda'$ and thus $\lambda' > 0$ since $R\beta < 1$ by assumption; that is, all firms raise as much financing as possible from households.

Suppose the firm pays dividends at time 0. Then $\mu = \mu' = 1$ and (C.40) implies $0 > 1 - R_i'\beta = R_i'\beta\lambda_i' - R_i'\beta\nu_i'$ and thus $\nu_i' = 1 - (R_i'\beta)^{-1} > 0$, $b_i' = 0$, and $\lambda_i' = 0$; thus, the firm does not use intermediated finance. Note that the problem of maximizing (C.1) subject to the constraints (C.21) through (C.25) has a (weakly) concave objective and a convex constraint set and hence induces a (weakly) concave value function. Thus, μ is (weakly) decreasing in w and let $\bar{w}$ be the lowest value of net worth for which $\mu = 1$; by the Inada condition, such a $\bar{w} < +\infty$ exists. At $\bar{w}$, $d = 0$, $\bar{w} = \bar{k}\wp$ (using (C.21)), and $\bar{k}$ solves $1 = \beta[A'f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$ (using (C.38)). For $w \geq \bar{w}$, $d = w - \bar{w}$ while the rest of the optimal policy is unchanged.

Suppose $\lambda_i' = 0$ and $\nu_i' = 0$. Then $\mu = R_i'\beta > 1$. Moreover, rearranging (C.38) we have $1 = \beta/(R_i'\beta)[A'f_k(\underline{k}) + (1-\theta)(1-\delta)]/\wp$ which defines $\underline{k} < \bar{k}$. Define $\underline{w}_u$ such that investment is $\underline{k}$ and $b_i' = 0$; then $\underline{w}_u = \underline{k}\wp$. Similarly, define $\underline{w}_l$ such that investment is $\underline{k}$ and $b_i' = (R_i')^{-1}(\theta_i - \theta)\underline{k}(1-\delta)$; then $\underline{w}_l = \underline{k}[\wp - (R_i')^{-1}(\theta_i - \theta)(1-\delta)]$. Note that $\underline{w}_l < \underline{w}_u < \bar{w}$. So firms below $\underline{w}_l$ raise as much financing as possible from intermediaries (since $\mu > R_i'\beta$ by concavity and hence $\lambda_i' > 0$). Firms with net worth between $\underline{w}_l$ and $\underline{w}_u$ pay down intermediary financing linearly. Firms with net worth above $\underline{w}_u$ do not borrow from intermediaries and scale up until $\bar{k}$ is reached at $\bar{w}$, at which point they initiate dividends. □

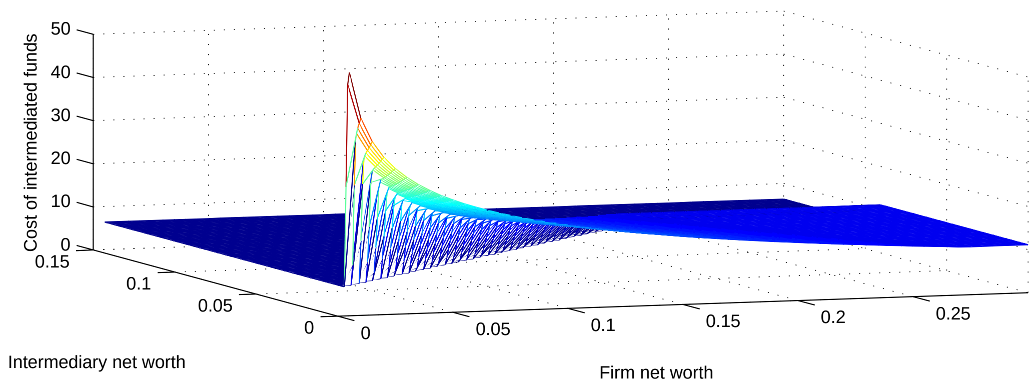
Intermediated finance is costlier than direct finance. Indeed, under the conditions of the proposition, intermediated finance is costlier than the shadow cost of internal finance of well capitalized firms. Thus, well capitalized firms, which pay dividends, do not borrow from financial intermediaries. In contrast, firms with net worth below $\underline{w}_u$ have a shadow cost of internal finance which is sufficiently high that they choose to borrow a positive amount from intermediaries. For severely constrained firms, with net worth below $\underline{w}_l$, the shadow cost of internal funds is so high that they borrow as much as they can from intermediaries, that is, their collateral constraint for intermediated finance binds. Moreover, more constrained firms have lower investment and are hence smaller.

The cross-sectional capital structure implications are plausible: smaller and more constrained firms borrow more from financial intermediaries and have higher costs of financing, while larger and less constrained firms borrow from households, for example in bond markets, and have lower financing costs.

Figure C.1: Role of Firm and Financial Intermediary Net Worth

This figure displays the equilibrium of the static economy. Panel A shows the interest rate on intermediated finance $R_i' - 1$ (percent) as a function of firm (w) and intermediary net worth (w_i). Panel B shows the contour of area where spread exceeds $\beta_i^{-1} - R$: w_i^* (dash dotted) and $\underline{w}(w_i)$ (dash dotted); $\hat{w}(w_i)$ (dashed); contour of area where spread equals $\beta^{-1} - R$: $\bar{w}_i$ (solid) and $\bar{w}(w_i)$ (solid). The parameter values are: $\beta = 0.90$, $R = 1.05$, $\beta_i = 0.94$, $\delta = 0.10$, $\theta = 0.60$, $\theta_i = 0.80$, $A' = 0.20$, and $f(k) = k^\alpha$ with $\alpha = 0.80$.

Panel A: Interest rate on intermediated finance $R_i' - 1$



Panel B: Contours of regions with different equilibrium interest rate