Practice Problem: Financing with Costly State Verification

This is a suggested practice problem. Solving this problem should help you prepare for the course.

Problem. This problem studies the optimal contract between a risk neutral entrepreneur and a risk neutral (representative) lender when there is private information about the project outcome but the lender has access to a costly state verification (CSV) technology as in Townsend (1979). We assume that the lender can commit to stochastic monitoring as in Mookherjee and Png (1989).

There are two dates, 0 and 1, and $n$ possible outcomes of the project, i.e., cash flows $y_i$, $i = 1, ..., n$, where $0 \leq y_1 < y_2 < \cdots < y_n$ and probability of cash flow $i$ is $\pi_i$ ($\pi_i > 0, \forall i$). The project requires an investment $I > 0$ and the entrepreneur has assets (or internal funds) $A \geq 0$, where $I > A$ so that there is a financing need. Assume w.l.o.g. that the entrepreneur contributes all his assets to the project. The entrepreneur consumes at date 1 only and is subject to limited liability, i.e., the entrepreneur's consumption has to be non-negative. The entrepreneur observes the cash flow $y_i$, i.e., state $i$, but the lender observes it only at a cost of $\kappa > 0$. By the revelation principle, we can restrict attention to direct truth-telling mechanisms, in which the entrepreneur announces the state, say $j$. We can assume that only announcements of the type $j \leq i$ are feasible in state $i$, e.g., because the entrepreneur has to show the cash to the investor and can only hide cash (which turns out to be w.l.o.g.).

Consider the optimal incentive-compatible contract which maximizes the utility of the entrepreneur. Notation: Let $c_i$ be the entrepreneur's consumption when he announces $i$ (truthfully) and is not monitored. Let $c_i^m$ be his consumption when he announces $i$ (truthfully) and is monitored. When the entrepreneur is monitored and found to be lying his consumption is 0 (why?). Let the probability that the agent is monitored in state $i$ be $p_i$. Finally, assume the interest rate is $r$ so that the lender needs to be repaid $(1 + r)(I - A)$.

[Remark: To prepare for the course, solve the questions below for $n = 2$. Of course, if you would like, you should feel free to (and be able to) solve the questions below for general $n$ as well.]

(a) Write down the problem of maximizing the entrepreneur’s expected utility, subject to (i) the participation constraint for the lender (with Kuhn-Tucker multiplier $\mu$), (ii) incentive compatibility constraints (with multipliers $\lambda_{ij}$, $j < i$), (iii) non-negativity constraints on the entrepreneur’s consumption in all states (with multipliers $\nu_i$ and $\nu_i^m$, respectively), and subject to (iv) the constraints that $p_i$ are between zero and one (with multipliers $\eta_i^0$ and $\eta_i^1$, respectively). Write down the first order conditions.

(b) Prove that the optimal contract minimizes expected auditing costs subject to (i)-(iv).
(c) Prove that expected auditing costs under the optimal contract are nonincreasing in the entrepreneur’s assets $A$.

(d) Note that the multiplier on the lender’s participation constraint $\mu \geq 1$. (The multiplier $\mu$ can be interpreted as the shadow-value of internal funds. Why?) Thus, there are two cases either $\mu = 1$ or $\mu > 1$. Prove that the optimal contract involves no auditing if and only if the lender’s required return is less than the value of the worst possible outcome of the project, i.e., $p_i = 0, \forall i, \iff y_1 \geq (1 + r)(I - A)$.

(e) For this part and all following parts, consider the case where $(1 + r)(I - A) > y_1$ and hence $\mu > 1$. Prove that in any state in which there is a positive probability of monitoring, the entrepreneur receives positive consumption only if he is audited, i.e., $p_i > 0 \Rightarrow c_i = 0$.

(f) Prove that the entrepreneur receives no consumption in the worst state $c_1^m = c_1 = 0$.

(g) Let $\tilde{c}_i = p_i c_i^m + (1 - p_i) c_i$ be the entrepreneur’s expected consumption in state $i$. Prove that $\tilde{c}_i$ is nondecreasing in $i$, i.e., the entrepreneur does better in better states.

(h) Prove that there is never any auditing in the highest state $p_n = 0$.

(k) Prove that the probability of auditing is nonincreasing in the announced state, i.e., $p_i$ is nonincreasing in $i$. 