Cointegration and Consumption Risks in Asset Returns

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Comments Welcome

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Abstract

We argue that long run movement in consumption is a key determinant of the risk-return relation in asset markets. Our goal is to explain cross-sectional differences in risk premia by horizon. We show that as the investment horizon increases, the return’s consumption beta is dominated by the long-run (cointegrating) relation between dividends and consumption. Further, as cointegration alters dividend growth and return predictability, it has important conceptual and empirical implications for the risk-return tradeoff at all investment horizons. We show that asset betas, derived from an Error-Correction VAR model of returns, can successfully account for the cross-sectional variation in equity mean returns at both short and long horizons; this is not the case when the cointegrating restriction is ignored. In all, our evidence underscores the economic importance of cointegration and risks related to long run movements in consumption for understanding the whole term structure of the risk-return tradeoff in the cross-section of assets.
1 Introduction

The concerns of a short-run and long-run investor can be quite different, and these differences can provide new insights about the sources of risks in asset markets. An asset’s risk derives from the exposure of its price fluctuations and cash flows to aggregate consumption. At short horizons, both of these components are important determinants of the asset’s risk-return tradeoff. However, as the investment horizon lengthens, price fluctuation risks become less important and risks in dividends (cash flows) dominate. In the limit, we show that the long-horizon investor cares almost exclusively about risks in cash flows which can be measured by the cointegrating relation between cash flows and consumption. Empirically, we document that this cointegrating relation has important implications for the risk-return tradeoff not only at long horizons but also contains very valuable information about risk premia at short horizons. Hence, we argue that the cointegrating relation between cash flows and consumption is a key ingredient for understanding risk and return at all investment horizons.

We focus on cointegration since the cointegration parameter is a measure of the long-run covariation in two series, and thus closely linked to long-run risks that Bansal and Yaron (2004), Bansal, Dittmar, and Lundblad (2001, 2005), Hansen, Heaton, and Li (2005), and Bansal, Gallant, and Tauchen (2005) find to be important in explaining risk premia. In addition, cointegration, as is made clear by the representation theorem of Granger and Engle (1987), has sharp implications for predictability. In the context of dividends and consumption, the representation theorem implies the existence of a predictable component in dividend growth rates and thus in returns. As we show, this predictability has important implications for conditional expectations and hence asset betas. Cointegration implies that the deviation of the level of dividends from consumption is an important state variable for predicting future returns. This error-correction state variable alters the information set used to predict future returns relative to other specifications of return dynamics, and hence significantly alters the return innovation and conditional betas.

We establish several interesting results. First, we show that, at long horizons, an asset’s consumption beta converges almost exactly to the cointegration parameter. Thus, the long-run (cointegrating) relation between dividends and consumption is
of paramount importance to a long-run investor. Restrictions on the cointegration parameter significantly impact possible risk premia on assets. In particular, the empirical literature frequently imposes the implicit assumption of no cointegration by modeling returns with a standard vector autoregression (VAR). We show that these restrictions lead to significant deterioration in the ability of consumption-based models to explain risk premia at both short and long horizons. Further, by considering the risk-return relation at the very long horizon we are able to isolate the market compensation for long-run consumption risks and evaluate their importance relative to short-run risks in consumption.

Second, we show that the error-correction term in the dividend-consumption cointegrating relation contains important information for predicting future dividend growth and returns. Allowing for cointegration, we are able to predict on average 11.5% of the variation in returns, compared to 7.5% when we do not impose cointegration. This difference is even starker at longer horizons: at the 10-year horizon, the EC-VAR specification results in an average 44.0% adjusted $R^2$, compared to 9.9% for the standard growth-rate VAR specification. That is, at longer horizons, we are able to predict far more variation in returns allowing for cointegration than when we do not impose cointegration on the return dynamics. This predictability evidence suggests that cointegration has important implications for measuring the conditional mean of returns and, consequently, conditional betas.

Third, we demonstrate that cointegration indeed has important effects on the calculation of risk measures and resulting prices of risk in the cross-section. Our cross-sectional evidence shows that the major source of compensation is long-run fluctuations in consumption. We show that the typical profile of mean returns on size- and book-to-market-sorted portfolios, based on the EC-VAR, is declining with horizon. Further, variability in the returns of small firms and high book-to-market firms are dominated by long-run components in consumption and cash flows. In contrast, these components of returns and consumption are less important for large and low book-to-market firms. We document that the cointegration parameter itself can account for about 80% of the cross-sectional variation in average returns. Using the EC-VAR specification, we construct the term structure of conditional betas and mean returns and show that, at short horizons, these consumption betas account for about 75% of the cross-sectional variation in risk premia. At long horizons, the betas are able to explain over 85% of the the cross-
section of mean returns. The estimated market prices of risks are always positive and significant. In contrast, the unconditional consumption CAPM fails to explain the cross-sectional variation in unconditional mean returns. Conditional versions that ignore the cointegrating relation are also unable to account for the differences in risk premia across assets.

Although related, our approach differs from Bansal and Yaron (2004), Bansal, Dittmar, and Lundblad (2001, 2005) and Hansen, Heaton, and Li (2005) in several ways. Bansal and Yaron develop a model of long-run risks that accounts for the equity premium and a wide range of other asset market puzzles. Bansal, Dittmar, and Lundblad measure long-run consumption risks in cash flow growth rates and show that cash-flows risk exposures contain important information about one-period risk premia in the cross-section of assets. Hansen, Heaton, and Li evaluate the contribution of long-run risks for the valuation of dividend strips (that is, dividend payments at specific dates in the future) and the long-run risk premia on these strips. Bansal, Dittmar, and Lundblad use a growth rate and a stochastic cointegration specifications for dividend dynamics; Hansen, Heaton and Li consider both of these, as well as a richer set of dividend growth dynamics. We consider the implications of all these specifications for the risk return tradeoff in the cross-section. The unique dimension of our paper is to directly model the dynamics of dividend growth rates and price-dividend ratios in the data, and use the estimated models to measure mean returns and asset betas at various investment horizons. Our approach allows us to analyze the magnitude of the contribution of very long-run risks versus transitory risks in accounting for the cross-sectional differences in risk premia. Our goal is to identify risk sources that can account for the cross-sectional differences in average returns at all investment horizons. In this sense, our work differs from earlier papers, such as Fama and French (1993), Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Bansal, Dittmar, and Lundblad (2005), Jagannathan and Wang (2005), Malloy, Moskowitz, and Vissing-Jorgensen (2005), and Parker and Julliard (2005) that focus only on one-period returns.

The rest of the paper is organized as follows. In Section 2, we discuss a brief and simple theoretical framework and present our econometric specification for return dynamics. Section 3 describes the data. Section 4 contains the results of our empirical analyses. Concluding remarks are provided in Section 5.
2 Cointegration and Risk Premia

In this section, we discuss a simple theoretical framework that we employ and implications of cointegration for risk measures and risk premia. Throughout our discussion, we utilize a Taylor series approximation to log returns (see Campbell and Shiller (1988)). Let $z_t = p_t - d_t$ represent the log price-dividend ratio. The return approximation is given by

$$r_{t+1} = \kappa_0 + d_{t+1} - d_t + \kappa_1 z_{t+1} - z_t.$$  

(1)

Using $\Delta z_{t+s} = z_{t+s} - z_t$, the above equation also implies

$$r_{t+1} = \kappa_0 + \Delta d_{t+1} + \Delta z_{t+1} + (\kappa_1 - 1) z_{t+1}.$$  

(2)

We further assume that the log price-dividend ratio, $z_t$, and dividend growth rates, $\Delta d_{t+1}$, are covariance stationary processes.

2.1 Risk Premia

We assert the existence of a strictly positive stochastic discount factor, $M_{t+1}$, which satisfies the usual pricing restriction

$$E_t [\exp(m_{t+1} + r_{t+1})] = 1,$$  

(3)

where lower case letters represent logs of their uppercase counterparts. We assume that the log stochastic discount factor and log returns are conditionally jointly normal and homoskedastic. This allows us to express the pricing restriction compactly as

$$E_t \left[ r_{t+1} + 0.5 \sigma^2_{\eta_r} - r^f_t \right] = \beta \lambda,$$  

(4)

where $\lambda$ is the risk premium as a function of the parameters of the stochastic discount factor. $\beta$ is the covariance of the innovation in the asset return, $\eta_r$, with the innovation in the stochastic discount factor, $\eta_m$, scaled by the variance of the innovation in the stochastic discount factor.
The pricing condition above is expressed in single-period terms. This is consistent with the majority of the literature that typically focuses on the pricing implications for out-period risk premia. However, investors have different holding periods and, thus, different horizons over which this restriction will hold. In particular, the restriction should hold equally well for a long-horizon buy and hold investor as it does for a single period investor. We express the horizon-dependent pricing condition as follows. Define the horizon \( s \) return on an asset as \( r_{t+1-t+s} \). The expected return at horizon \( s \) is given by

\[
\frac{1}{s}E_t \left[ r_{t+1-t+s} + 0.5\sigma^2_{\eta_{r,t+s}} - sr^f_{t,t+s} \right] = \lambda_s \frac{\text{Cov}_t[\eta_{m,t+s}, \eta_{r,t+s}]}{\sigma^2_{\eta_{m,t+s}}} = \lambda_s \beta_s
\]

We scale by \( s \) to ensure that moments exist and to provide the interpretation of the risk premium as per unit of time. The unconditional mean return is obtained by taking unconditional expectations of (5). Let \( \mu_r \) be the one-period-ahead geometric mean return,

\[
E \left\{ \frac{1}{s}E_t \left[ r_{t+1-t+s} + 0.5\sigma^2_{\eta_{r,t+s}} \right] \right\} = \mu_r + \frac{0.5\sigma^2_{\eta_{r,t+s}}}{s} = E[r^f_{t,t+s}] + \beta_s \lambda_s.
\]

As shown in expression (6), unconditional mean returns per unit of time depend on the horizon both through the market price of horizon risk, \( \lambda_s \), and through the horizon-dependent beta, \( \beta_s \). Expected returns for all horizons are identical only in the case when returns are i.i.d.. Expression (6) provides a term structure for the risk-return tradeoff. At all horizons, this term structure is determined by the horizon-dependent covariance of the asset’s return with the stochastic discount factor, \( \beta_s \).

The return approximation, (2), shows that returns can be decomposed into three components: growth in dividends, growth in price-dividend ratios, and the level of the price-dividend ratio. The covariance of these components with the stochastic discount factor will determine the overall risk compensation. Specifically, note that the asset beta, \( \beta_s \), can be expressed as the sum of three covariances:

\[
\beta_s = \left[ \frac{\text{Cov}_t(\eta_{m,t+s}, \eta_{d,t+s})}{\sigma^2_{\eta_{m,t+s}}} + \frac{\text{Cov}_t(\eta_{m,t+s}, \eta_{z,t+s})}{\sigma^2_{\eta_{m,t+s}}} + (\kappa_1 - 1) \frac{\text{Cov}_t(\eta_{m,t+s}, \eta_{z,t+s})}{\sigma^2_{\eta_{m,t+s}}} \right]
\]
which we express compactly as

\[ \beta_s = [\beta_{d,s} + \beta_{\Delta z,s} + (\kappa_1 - 1)\beta_{z,s}] \quad \text{(8)} \]

Thus, the asset beta can be considered the sum of three betas, related to cash flow growth and price-dividend ratios. This distinction is important because it suggests that the joint dynamics of the stochastic discount factor, dividend growth, and price-dividend ratios contribute to the determination of assets’ risk. We discuss the implications of modeling these dynamics in the next section.

The expression for risk exposures, equation (8), also suggests that the risks faced by a long-run buy and hold investor will be different than those faced by a single-period investor. Note that, in the limit, price-dividend ratio variation will contribute little to the asset’s risk. That is, as \( s \to \infty \), the term \( \beta_{\Delta z,s} \to 0 \), as the growth in the price-dividend ratio represents the change in a stationary variable. Further, as the approximation parameter \( \kappa_1 \) is close to one, the final term in the expression, \((\kappa_1 - 1)\beta_{z,s}\), is close to zero. For a long-run investor, this limit implies that an asset’s risk is determined almost exclusively by the exposure of its dividends to aggregate consumption. Thus, a long-term investor will be concerned predominantly with the cash flow exposure of an asset, and much less with the exposure of prices to aggregate consumption. We will show in the next section that, under the assumption of cointegration between dividends and consumption, the consumption beta will essentially converge to the cointegration parameter, \( \delta \). Thus, for an investor with a long horizon, an asset’s risk is determined almost entirely by its cash flow’s parameter of cointegration with consumption. We also show that restricting this parameter will have important effects on the model’s ability to describe the risk-return tradeoff in asset markets.

2.2 Cointegration: Specification and Implications

We now assume for simplicity that the log stochastic discount factor is a linear function of consumption growth, \( \Delta c_{t+1} \), as in the C-CAPM. A number of recent papers (e.g. Bansal, Dittmar, and Lundblad (2001) and Hansen, Heaton, and Li (2005)) suggest that consumption and dividends are stochastically cointegrated (see Campbell and Perron (1991)). In this section, we examine the implications of cointegration for the calculation
of the risk measures outlined in the preceding section.

We focus on the per-share dividends series, traditionally used in the literature. Dividends per share are constructed as follows. The total return per dollar invested is $R_{t+1} = H_{t+1} + Y_{t+1}$, where $H_{t+1}$ is the price appreciation and $Y_{t+1}$ is the dividend yield. The level of dividends per share can then be computed as $D_{t+1} = Y_{t+1}V_t$, where $V_{t+1} = H_{t+1}V_t$ and $V_0 = 1$. Note that the present value of dividends per share is price per share. Further, as we discuss below, these dividend series are very different from aggregate dividends (or payouts) as they ignore the evolution of the change in scale (or size) of the firm (for more details on this issue see Bansal and Yaron (2006)). Using these dividends we discuss the EC-VAR specification that characterizes the dynamics of returns.

If two non-stationary variables are cointegrated, a linear combination of the variables is stationary. We express the relation between two such variables, dividends per share and consumption, as

$$d_t = \tau_0 + \tau_1 t + \delta c_t + \epsilon_{d,t},$$

(9)

where the level of dividends and consumption in logs is $d_t$ and $c_t$ respectively and $E[\epsilon_{d,t}] = 0$. From equation (9) it also follows that $\tau_1 = \mu_d - \delta \mu_c$, where $\mu_c$ and $\mu_d$ are the average growth of consumption and dividends respectively. Substituting for $\tau_1$, the above equation can equivalently be stated as

$$d_t - \mu_d t = \tau_0 + \delta (c_t - \mu_c t) + \epsilon_{d,t}$$

(10)

That is, the cointegration parameter $\delta$ can be estimated via the projection of deterministically de-trended dividends on de-trended consumption (as in Bansal, Dittmar, and Lundblad (2001)). The Granger and Engle (1987) representation theorem states that the error-correction variable, $\epsilon_{d,t}$, will forecast future dividend growth rates, and consequently, perhaps returns. As we show below, this predictability has important implications for the calculations of risk measures in standard models.

Several features of our cointegration specification are worthy of further discussion. In particular, the specification in (9) includes a time trend and does not restrict the cointegration coefficient, $\delta_i$, across assets. Hansen, Heaton, and Li (2005) consider alternative specifications with and without a time trend and with and without restricting
the cointegration parameter to unity, and Menzly, Santos, and Veronesi (2004) consider solely a specification with no time trend and unit cointegration parameter. We argue that including the time trend in estimating the cointegrating relation and allowing for heterogeneity in the cointegration parameter are important for the following reasons. First, without the time trend in equation (9), the cointegration parameter simply equals the ratio of average dividend growth to average growth in aggregate consumption (see Hamilton (1994)). In this case, cross-sectional differences in \( \delta_i \)'s will tautologically reflect differences in average dividend growth (and average capital gains) and, therefore, average ex-post returns. Including the time trend purges the effect of mean growth rates in dividends on the cointegration parameter, and ensures that long-run risk measures do not mechanically replicate cross-sectional differences in ex-post average returns.

Additionally, it may be economically appealing to omit the time trend and restrict the cointegration parameter of aggregate dividends on the market or a sector of the economy to one, as this implies that the average growth rate in these dividends will match that of aggregate consumption. However, there is no economic rationale for this restriction on the cointegration parameter for dividends per share, which we and other empirical studies employ. In order to reinforce this important point, we plot the log ratio of aggregate dividends to consumption in Figure 1(a) and the log ratio of market dividends per share to consumption in Figure 1(b). Aggregate dividends at date \( t \) for the market are computed by multiplying the dividend yield at date \( t \) by the lagged market capitalization \( K \),

\[
D_{t, agg} = Y_t K_{t-1}.
\] (11)

As shown in Figure 1(a), the ratio of aggregate dividends to consumption is quite stationary. Further, the average growth rate of aggregate dividends is 3.2%, which is comparable to that of aggregate consumption. Thus, restricting this series to be cointegrated with consumption with a parameter of one and omitting a time trend appears to be quite reasonable. In contrast, the ratio of market per-share dividends to consumption displays a dramatic decline over time, as shown in Figure 1(b). The average growth rate in the dividend per share series is 0.9%, which is considerably lower than that in consumption. This difference arises as the growth in aggregate dividend series reflects the growth in market capital (capitalization), while the growth in dividends per share reflects the growth in price per share. The average growth of market capital (capitalization) is 4.3%, while that in price per share is only about 2.0%. Conceptually,
the asset pricing literature usually focuses on the per-share dividends as their present value corresponds to the price of the asset, which is not true for aggregate dividend series. We follow this tradition and use dividends constructed on the per-share basis.

2.3 Deriving the Term Structure of Mean Returns and Betas

This section provides the details for estimating assets’ consumption betas for different investment horizons. We first estimate the cointegrating relation (9) between dividends and consumption via OLS, that is by regressing the de-trended portfolio’s cash flows onto the stochastic trend in consumption. Using the resulting cointegrating residual, $\epsilon_{d,t}$, we model its dynamics jointly with the portfolio’s price-dividend ratio, $z_t$, and consumption growth, $\Delta c_t$, in the first-order error-correction VAR (EC-VAR) structure:

$$X_t = AX_{t-1} + Gu_t,$$

where $X_t' = (\Delta c_t \quad \epsilon_{d,t} \quad z_t \quad \Delta d_t \quad \Delta z_t)$, $A$ is a $5 \times 5$ matrix of coefficients, $G$ is a $5 \times 3$ matrix, and $u$ is a three by one matrix of shocks, $u_t' = (\eta_t \quad \eta_{c,t} \quad \eta_{z,t})$. Details of the EC-VAR specification are provided in the appendix; all growth rates are de-meaned throughout our discussion. Notice that this specification reduces to the standard growth-rates based VAR if the error-correction term, $\epsilon_{d,t}$, is excluded from the set of variables that predict future growth rates and returns.\(^1\)

Using the recursive structure of the EC-VAR, we can compute the long-run variance of consumption growth and the various pieces of long-run risks in equation (8). Specifically, let $B_j = B_{j-1} + A^{j-1}$, $j = 1, 2, ..., $ and $B_0 = 0$. The horizon-$s$ covariance matrix of the above variables satisfies the recursion

$$\Sigma^*_s = B_s \Sigma_c B'_s + \Sigma^*_{s-1},$$

where $\Sigma_c = G\Sigma_u G'$, and $\Sigma^*_0 = 0$. As $s$ increases, $\Sigma^*_s$ grows without a bound; hence we consider $\Sigma_s \equiv \frac{\Sigma^*_s}{s}$. In the long-run limit, this covariance matrix becomes:

$$\Sigma_{lr} = [I - A]^{-1} \Sigma_c [I - A]^{-1}'.$$

\(^1\)We have also considered specifications that include more lags in the vector error-correction model. As this does not materially change the results, we only report evidence based on the first-order EC-VAR.
Details of the derivation of the covariance matrix are provided in the Appendix.

The $s$-period covariance matrix, $\Sigma_s$, allows us to calculate the $s$-period beta of an asset using the appropriate covariance and variance terms in the matrix. For a given horizon $s$, the covariance risk in the asset is

$$\beta_s = [\Sigma_s(1, 4) + \Sigma_s(1, 5) + (\kappa_1 - 1)\Sigma_s(1, 3)]\Sigma_s(1, 1)^{-1},$$  \hfill (15)

where $\Sigma_s(i, j)$ is the $(i, j)$-element of the covariance matrix $\Sigma_s$. There are several implications of this expression that are worthy of note. First, the cointegration parameter will be an important determinant of the asset’s risk at any investment horizon. To illustrate this point, consider the beta for one-period investment. Given (15),

$$\beta_1 = \delta + [\Sigma_1(1, 2) + \Sigma_1(1, 5) + (\kappa_1 - 1)\Sigma_1(1, 3)]\Sigma_1(1, 1)^{-1}.$$  \hfill (16)

Notice that the cointegration parameter is one of the four components of the consumption beta at the one-period horizon. Further, the presence of cointegration not only alters betas directly (via the first term), it also alters the beta through the other terms (via its impact on the transition matrices $A$ and $G$). Thus, the parameter of cointegration that reflects long-run consumption risks in the asset’s cash flows is an important determinant of the risk-return relation at all investment horizons.

For a long-horizon buy and hold investor, these cointegration risks are especially important. As noted above, variation in transitory components in returns die out in the long run. Thus, variation in the error-correction term and the growth in the price-dividend ratio will vanish in the limit. This means that, in the long run, $lr$, the beta will become

$$\beta_{lr} = \delta + [0 + 0 + (\kappa_1 - 1)\Sigma_1(1, 3)]\Sigma_1(1, 1)^{-1}.$$  \hfill (17)

That is, the beta becomes the cointegration parameter plus a term related to the risk in the price-dividend ratio. However, since $(\kappa_1 - 1)$ in practice is close to zero, the cointegration term dominates. Thus, a long-horizon buy and hold investor cares almost exclusively about long-run consumption risks in asset’s cash flows, measured by the cointegration parameter.

Expression (15) characterizes the entire term structure of betas and highlights important implications of the cointegrating relation for risk measures. For example,
when $\delta$ is assumed to be one, as in Menzly, Santos, and Veronesi (2004), the cross-sectional dispersion in betas will be significantly reduced at both short and long horizons. Further, when cointegration is not imposed, $\epsilon_{d,t}$ is not a predictive variable for dividend growth rates and returns, and the permanent term $\delta$ is absent from the term structure of betas. This predictive power is an empirical consideration which we investigate below.

The profile of expected returns by horizon can be read from the above EC-VAR specification by adding half the variance on return innovation for a given horizon to the mean log return. We use the term structure of betas and mean returns, implied by the EC-VAR dynamics, to analyze the cross-sectional relation in risk and return at various investment horizons. Specifically, we investigate the relation by considering cross-sectional regressions for different horizons $s$,

$$E[r + 0.5\sigma_u^2] = \lambda_0,s + \lambda_1,s\beta_i,s.$$  \hspace{1cm} (18)

Evidence on the sources of betas at various horizons and the explanatory power of these betas for the cross-section of mean returns is presented in Section 4.

3 Data

The portfolios employed in our empirical tests sort firms on dimensions that lead to cross-sectional dispersion in measured risk premia. The particular characteristics that we consider are firms’ market value and book-to-market ratio. Our rationale for examining portfolios sorted on these characteristics is that size and book-to-market based sorts are the basis for factor models used in Fama and French (1993) to explain the risk premia on other assets. Consequently, understanding the risk premia on these assets is an economically important step toward understanding the risk compensation of a wider array of assets.

We construct the set of portfolios formed on the basis of market capitalization by ranking all firms covered by CRSP on the basis of their market capitalization at the end of June of each year using NYSE capitalization breakpoints. We form annual returns on these portfolios over the period 1929 through 2002. In Table I, we present means and standard deviations of market value-weighted returns for size decile portfolios. The
data evidence a substantial size premium over the sample period; the mean real annual return on the lowest decile firms is 13.45%, contrasted with a return of 7.58% for the highest decile.

Book-to-market portfolios are formed by ranking firms on their book-to-market ratios as of the end of June of each year using NYSE book-to-market breakpoints. Book values are computed using Moody's data prior to 1955 and Compustat data in the post-1955 period. The book-to-market ratio at year \( t \) is computed as the ratio of book value at fiscal year end \( t - 1 \) to CRSP market value of equity at calendar year \( t - 1 \). Average value-weighted portfolio returns are also presented in Table I. The data evidence a book-to-market spread of similar magnitude to the size spread; the highest book-to-market firms earn average real annual returns of 13.37%, whereas the lowest book-to-market firms average 7.01%.

We utilize the dividends paid on these value-weighted portfolios to explore the relations between portfolio cash flows and consumption. Our construction of the dividend series is standard; details of the construction can be found in Campbell and Shiller (1988) and Bansal, Dittmar, and Lundblad (2005). We construct the level of cash dividends per share, \( D_t \), for the size and book-to-market portfolios on a monthly basis as described above. From this series, we construct the annual levels of dividends by summing the cash flows within the year. These series are converted to real by the personal consumption deflator. Log growth rates are constructed by taking the log first difference of the cash-flow series. Summary statistics for the cash dividend growth rates of the portfolios under consideration are presented in Table I. Earlier work shows that alternative measures of dividends, such as including repurchases, do not affect the results.\(^2\)

4 Empirical Results

In this section, we investigate the implications of the preceding framework for the measurement of assets' risks. We first analyze the cointegration of assets' dividends

\(^2\)Bansal, Dittmar, and Lundblad (2005)) show that alternative dividend measures, which include share repurchases do not make a big difference to their cash-flow risk measures. We find the same is true for the empirical evidence in this paper.
with consumption and investigate the implications of the cointegrating relation for
the predictability of assets’ growth rates and returns. We then compute the profile
of consumption betas and expected returns for different investment horizons, implied by
our EC-VAR framework, and analyze the cross-sectional implications of the model.

4.1 Cointegration Evidence

In Table II, we present point estimates of the cointegration parameters between
portfolios’ cash flows and consumption, the sample autocorrelation functions (ACF)
of the cointegrating residuals, and unit root tests of the stationarity of the cointegrating
residuals. As discussed earlier, we estimate cointegration parameters via OLS by
regressing the deterministically de-trended dividends on de-trended consumption. We
first note that, for the majority of portfolios analyzed, the sample autocorrelations of
the resulting cointegrating residuals exhibit a relatively rapid decline. This supports
our assumption that the long-run dynamics of portfolios’ dividends and aggregate
consumption are governed by the same permanent component that can be eliminated
by the appropriate linear combination of the levels. In addition, large cross-sectional
variation in the estimated cointegration parameters, presented in the first column,
suggests that assets’ cash flows differ in their exposures to this low-frequency component.
The unit root tests suggest cointegration in 10 of the 20 portfolios, and a number of
portfolios’ test statistics are close to the Mackinnon critical value of -3.59. Because of
the low power of this test in samples of the size that we analyze (74 observations), we
conclude that the results in the table provide reasonable support for the cointegration
specification.

We now examine the point estimates of the cointegration parameters more closely.
Note that for the size portfolios, the parameters exhibit a near-monotonic decreasing
pattern across the market capitalization deciles. The estimate for the small size portfolio
is 9.62 compared to 0.82 for the large size portfolio, mirroring the pattern in observed
risk premia. For the book-to-market portfolios, the cointegration parameters exhibit
an increasing relation in the book-to-market decile. The point estimate for the highest
book-to-market portfolio is 10.25, compared to -0.27 for the growth portfolio. Again,
this result is broadly consistent with the pattern of observed risk premia. Indeed, we
find that the cointegration parameter by itself explains about 81% of the cross-sectional
variation in average one-period returns on size and book-to-market sorted assets. The price of risk of 0.486 (SE=0.053) is positive and statistically significant. This evidence suggests that long-run risks embodied in assets’ cash flows are able to account for a significant portion of the differences in risk premia. The fact that small and high book-to-market stocks have large exposures to permanent risks in consumption implies that the performance of these firms is linked to the permanent risks in the economy, while that of large and low book-to-market portfolio firms is not. As consumption is largely dominated by permanent shocks, risks in large market-capitalization and low book-to-market stocks are largely unrelated to the long-run evolution of the economy. This, as we document further below, is exactly why large and low book-to-market portfolios should bear a low ex-ante risk premium and already highlights the importance of long-run risks.

4.2 Predictability Evidence

As stated above, cointegration implies that dividend growth rates are predicted by the cointegrating residuals. That is, the current deviations of an asset’s cash flows from their long-run relation with consumption should forecast the dynamics of dividend growth rates while dividends are moving back towards the equilibrium. For example, if dividends are unusually high today, dividend growth is expected to fall in order for cash flows to adjust to the stochastic trend in consumption. Given the approximation for the log return in equation (2), the predictability of dividend growth rates potentially translates into return predictability. The variation in the cointegrating residuals, therefore, may also be able to account for the variation in expected future returns.

We explore the ability of our EC-VAR specification to predict future dividend growth rates and returns at various horizons. To emphasize the importance of the cointegrating relation, we compare the adjusted-$R^2$’s for dividend growth and return projections implied by the EC-VAR model outlined above with the corresponding $R^2$’s from the growth-rates VAR specification. The latter simply excludes the cointegrating residuals from a set of VAR variables. Results for dividend growth are presented in Table III. We examine results at horizons of 1, 5, and 10 years. As shown in the table, at the one-year horizon, the differences between the EC-VAR specification and the simple growth-rates VAR are quite stark. The mean (median) adjusted $R^2$ for the EC-VAR specification is 0.19 (0.16), compared to 0.15 (0.11) for the growth rates specification. At longer
horizons, the inclusion of the error-correction term becomes less important; at the ten-year horizon the mean (median) adjusted $R^2$ for the EC-VAR and VAR specification are 0.20 (0.17) and 0.26 (0.24) respectively. This is not surprising, as the error-correction term captures transitory variation in dividend growth rates.

Further, asset return predictability is also altered by the cointegration between dividends and consumption. Return projections’ $\bar{R}^2$’s for horizons 1, 5 and 10 years implied by the EC-VAR model as well as the alternative, growth-rates specification are reported in Table IV.\(^3\) The EC-VAR specification, on average, is able to explain an average (median) of 11.5% (10.5%) of return variation at one year horizon, and 44.0% (43.5%) of the variation in ten-year returns. Excluding the cointegrating residual significantly lowers the predictability of asset returns and alters the conditional mean of returns. We illustrate this point in Figure 2 by plotting one and ten-year returns predicted by the EC-VAR specification along with the forecasts implied by the alternative VAR model. Predicted conditional means are displayed for the top and bottom market capitalization and book-to-market portfolios. It can be seen that the two specifications produce quite different predictions of future expected returns, especially at longer horizons. That is, the cointegrating residual, included in the error-correction specification, contains distinct information about future returns beyond that in the growth rates model. Return innovations, therefore, also differ across the two specifications, and most importantly so do the consumption betas measured from the two alternative models.

The results of this section underscore the importance of the cointegration specification in the measurement of risk and return. The specification has important implications for returns in that the cointegration parameter governs the long-run relation between risk and return, and is an important determinant of the short-run relation as well. Further, as emphasized in this section, temporary deviations of cash flows from the permanent component of consumption contain valuable information for predicting dividend growth rates and returns, and thus represent an important component in the calculation of the term structure of expected returns and betas.\(^4\) We turn to this

\(^3\)We have also examined predictability regressions based on compounded returns without relying on the Campbell and Shiller (1988) return approximation. These results are materially unchanged from those presented in this section.

\(^4\)Our evidence on predictability of cash-flows is consistent with that in Lettau and Ludvigson (2005), Ang and Belaert (2003), and Bansal, Khatchatrian, and Yaron (2005).
point in the following section, and analyze how the risk-return relation changes with the investment horizon.

4.3 The Term Structure of Betas and Expected Returns

Mean returns for the portfolios at the one-, five-, and ten-year horizons, implied by the EC-VAR, are presented in Table V. As shown in the table, the general pattern observed in unconditional means is preserved across the various horizons. Small firm portfolios tend to earn higher conditional mean returns than do large firm portfolios, and low book-to-market portfolios earn lower conditional mean returns than high book-to-market portfolios. Further, at all horizons, the mean returns exhibit considerable cross-sectional variation; the dispersion in mean returns at the ten-year horizon is as high as that at the one-year horizon.

We now explore the implications of cointegration for the determination of assets’ consumption risks. Table VI presents betas at various horizons for each of the portfolios; Newey and West (1987) standard errors are given in parentheses. Similarly to mean return, risk measures implied by the EC-VAR exhibit substantial cross-sectional variation. At the 1-year horizon, the small-firm portfolio beta (4.12) exceeds the large-firm portfolio beta (1.54), and the high book-to-market beta (3.89) exceeds the low book-to-market beta (1.81). Further, there is a generally declining pattern in the size dimension and increasing pattern in the book-to-market dimension that is consistent with conditional and unconditional mean returns. As the horizon increases, the cross-sectional patterns in these risk measures generally remains the same, although the precision of the estimates suffers. At the 10-year horizon, the small-firm beta (6.54) continues to exceed the large-firm beta (0.34) and the high book-to-market beta (4.33) exceeds the low book-to-market beta (-0.83).

As argued in Section 2, the cointegration parameter is one of the components of the overall consumption beta at each horizon, along with risks arising from transient fluctuations in dividends and prices. The relative importance of price risks and short-run dividend risks, however, decreases over time, and in the long run, systematic risks in returns should be dominated by permanent risks in assets’ cash flows. We find that this theoretical proposition is strongly supported in the data. While at the one-year horizon, the correlation between assets’ betas and cointegration parameters is about 0.87, by
the five-year horizon it already exceeds 0.90, reaching virtually 1 in the limit. Thus, the contribution of the cash-flow component of risks to the beta dominates in the long run. Moreover, even at short horizons, long-run consumption risks in dividends are an important determinant of assets’ betas.

The magnitude of mean returns for the growth-rates VAR specification, reported in Table V, is somewhat higher relative to the EC-VAR specification but their cross-sectional pattern by horizon is very comparable to that implied by the EC-VAR. However, the betas in the VAR specification, reported in Table VI, significantly differ from the EC-VAR based betas. For comparison, in Table VI we also present the unconditional C-CAPM betas. Both the unconditional betas and those based on the VAR do not reflect the cross-sectional differences in mean returns on size and book-to-market sorted portfolios. This evidence underscores the importance of conditioning information subsumed in the cointegrating residual in computing assets’ exposures to consumption risks.

In the next section, we more formally analyze the relation between mean returns and risk measures in the cross-section. Additionally, we examine the implications of restrictions on the cointegration parameter and time trend for capturing cross-sectional relations between mean returns and risk measures.

### 4.4 Cross-Sectional Risk and Return

A wide variety of consumption risk measures have been proposed in the asset pricing literature that are able, to some extent, to account for the observed heterogeneity in risk premia. Some papers rely on constant beta models (for example, Parker and Julliard (2005), Malloy, Moskowitz, and Vissing-Jorgensen (2005), Jagannathan and Wang (2005)), others exploit the cross-sectional implications of conditional versions of the C-CAPM (as in Jagannathan and Wang (1996), Lettau and Ludvigson (2001)). The aforementioned papers focus merely on short-horizon, one-period returns, as is typical in empirical studies. However, as we emphasize, the risk and return relation varies considerably across investment horizons. Thus, it is important to explore the cross-sectional implications of an asset pricing model not only in the short but also in the long run. We therefore investigate the cross-sectional risk-return relation at various investment horizons, and explore the ability of our consumption betas to explain
the cross-sectional dispersion in risk premia at both short, as well as long ends of the investment horizon.

Table VII presents results of estimating the cross-sectional regression (18) for different investment horizons. The market prices of risk are estimated jointly with the time-series parameters via one-step GMM. The reported standard errors of the cross-sectional parameters, therefore, are robust to the estimation error in betas. As shown in the table, at the one-year horizon, betas impled by our EC-VAR specification explain 75% of the cross-sectional variation in mean returns with a positive price of risk of 1.19 (SE=0.41). This explanatory power is maintained at the 5- and 10-year horizons, with adjusted $R^2$ of 0.73 and 0.84 respectively, and prices of risk of 0.73 (SE=0.32) and 0.65 (SE=0.24) respectively. At the very long horizon, i.e., $s = \infty$, the estimate of the market price of risk remains strongly significant of 0.72 (SE=0.25), and the cross-section of long-run consumption betas accounts for a sizeable portion of the variation in long-run risk premia ($\bar{R}^2=87\%$). The fit for the one-period and the very long horizon is plotted in Figure 3. Thus, at all horizons, the EC-VAR specification explains most of the cross-sectional variation in mean returns across assets. This evidence manifests the empirical importance of the cointegrating relation between dividends and consumption in determining assets’ risk premia not only at long but also at short horizons.

Table VII also provides the cross-sectional risk-return tradeoff comparison of the EC-VAR relative to the VAR-based specification and the unconditional C-CAPM. As we might expect, given the estimates of the standard C-CAPM beta in the preceding section, the standard risk measures have no power in explaining the cross-sectional variation in mean returns. While the estimated price of risk is positive, the betas fail to capture cross-sectional differences in mean returns, as indicated by the adjusted $R^2$ of -0.04. The VAR specification provides marginal improvement at the very short horizon. For example, at the one-year horizon, is explains about 22% of the cross-sectional variation in mean returns and implies a positive, although still insignificant, price of risk of 1.278 (SE=1.57). As the horizon extends, however, the ability of the VAR specification deteriorates substantially. At both five and ten-year horizons, it produces negative prices of risks and fails to account for the differences in risk premia across assets, as indicated by adjusted $R^2$’s of -0.02 and -0.05 respectively.

This evidence emphasizes that the cointegration-based specification is critical for understanding the risk-return tradeoff at all investment horizons. The EC-VAR relative
to alternative specifications, such as the growth-rates based VAR, incorporates the error-correction term $\epsilon_{d,t}$ as a predictive variable. This error-correction state variable alters the information set used to predict future returns relative to other specifications of return dynamics, and hence significantly alters the return innovation and conditional betas.

As is evident from Table VI, risk measures (betas) are measured somewhat imprecisely. Bansal, Dittmar, and Lundblad (2001) and Hansen, Heaton, and Li (2005) also find that long-run dividend-based risk measures are imprecisely estimated. To a large extent, this is not surprising as measuring any long-run dependencies with 74 observations is challenging (as highlighted in Campbell and Mankiw (1987)). What is economically interesting is that these measures contain valuable information about the cross-sectional differences in risk premia that is the key focus of this paper. Note that the market prices of risks are precisely estimated for the EC-VAR model reflecting the very high cross-sectional correlation between mean returns and asset betas. To confirm this intuition, we also conduct a series of block bootstraps and Monte Carlo experiments. The main message of these experiments is that while consumption betas are imprecisely measured in the time series, the cross-sectional evidence is very robust to this measurement error. For brevity we do not include detailed tables, which are available on request.

4.5 Long-Run Risks Compensation

Since consumption growth is a covariance-stationary process, its level satisfies a Beveridge and Nelson (1981) decomposition. That is, the consumption process can be presented as a sum of a deterministic trend, a random walk component or stochastic trend, and a transitory component. The covariance of the return with consumption (i.e., the beta) can, therefore, be broken into two parts: the covariance with trend shocks and the covariance with transitory shocks in consumption. Consequently, at each horizon, the market price of risk, $\lambda_{1,s}$, reflects the premium for both very long risks and short-run fluctuations in consumption. As the horizon increases, transitory consumption shocks die out, and the transitory risk compensation shrinks to zero. Thus, the total risk compensation in the long-run limit (when $s = \infty$) provides a measure of the compensation solely for long-run consumption risks. We isolate and estimate this long-run compensation by considering the long-run risk-return relation in the cross-section
of assets. Subtracting it from the overall risk compensation for a given horizon \( s \) allows us to construct the time-horizon profile of risk premia for transient consumption risks.\(^5\)

We find that the compensation for long-run risks in consumption is about 60% of the overall compensation at the one-year horizon. Specifically, the estimate of the long-run risk compensation is 0.72 per annum compared to 1.19 at the one-period horizon. Figure 4 plots the compensation for short-run consumption risks along with the total market price of risks for investment horizons up to 15 years. The compensation for transitory fluctuations is small relative to the premium for long-run risks and exhibits a rapid decline as the time horizon grows, starting at about 50 basis points at the first horizon and falling to zero by the 5th year. That is, long-run fluctuations in consumption are a dominant source of the premium for consumption risks in asset markets.

### 4.6 Alternative Cointegration Specifications

In this section, we discuss the implications of various restrictions on the cointegration specification discussed above. In particular, we focus on two restrictions: omitting the time trend in the cointegration specification and restricting the cointegration parameter to one in the cross-section.

Results for cross-sectional regressions incorporating these restrictions are presented in Table VIII. The first set of columns present results restricting the cointegration parameter to unity across all assets (while still allowing for differences in time trends). The results at the one-period horizon are somewhat weaker, but similar to those for the

\[^5\text{Specifically, the stochastic part of log consumption can be decomposed into a trend, } T_t, \text{ and a stationary component, } S_t, \text{ as follows: } c_t = \mu_s t + T_t + S_t. \text{ The asset betas with respect to trend shocks, } \beta_{T,s}, \text{ and transitory shocks, } \beta_{S,s}, \text{ are defined as:} \]

\[
\beta_{T,s} = \frac{\text{Cov}(r_{t+1-t+s}, T_{t+1-t+s})}{\sigma_{T,s}^2}, \quad \beta_{S,s} = \frac{\text{Cov}(r_{t+1-t+s}, S_{t+1-t+s})}{\sigma_{S,s}^2}.
\]

The overall beta is, therefore, a weighted combination of the two risk measures: \( \beta_s \equiv \beta_{T,s} \frac{\sigma_{T,s}^2}{\sigma_s^2} + \beta_{S,s} \frac{\sigma_{S,s}^2}{\sigma_s^2} \). Consequently, the risk premium comprises the premium for very long risks plus the premium for short-run uncertainty in consumption, that is,

\[
\frac{1}{s} \sum_{t=1}^{s} \left[ r_{t+1-t+s} + 0.5 \sigma_{s,t+1-s}^2 - sr_{t,t+1} \right] \equiv \lambda_s \beta_s = \lambda_{T,s} \beta_{T,s} + \lambda_{S,s} \beta_{S,s}
\]

As \( s \rightarrow \infty \), short-run consumption risks are washed out, so that \( \lambda_s \rightarrow 0 \). Thus, at the very long horizon, the market price of risks corresponds to the price of long-run risks in consumption: \( \lambda_{\infty} = \lambda_{T,s} \).
unrestricted case; the price of risk is 1.58 (S.E. = 0.39) and the betas explain 45% of the cross-sectional variation in average returns, as indicated by the $R^2$. However, as the horizon increases, the explanatory power of the specification deteriorates rapidly. At the two- and five-year horizons, the prices of risk are no longer statistically significant and the explanatory power of the regression is near zero. These results indicate that allowing for heterogeneity in the long-run risk in dividends is important for capturing variation in risk premia not only in the long run, but also the short term.

In the second set of columns, we omit the time trend, but allow the cointegration parameter to vary across assets. These results represent an improvement over the case in which the cointegration parameter is restricted. Across all horizons, the estimated price of risk is positive and statistically significant, and the explanatory power of the regression varies from an $R^2$ of 35% for the two-year horizon to 64% for the ten-year horizon. However, as mentioned previously, these results may occur because, in this case, the cointegration parameter equals the ratio of mean growth rate in the portfolio dividends to the mean growth of aggregate consumption (see Hamilton (1994)), and hence tautologically reflects average ex-post returns. We have also considered the specification with $\tau = 0$ and $\delta = 1$ for all assets as in Menzly, Santos, and Veronesi (2004). This specification is sharply rejected, and for brevity we do not report the detailed evidence.

In all, our empirical results suggest that the heterogeneity in the cointegration coefficient is important for accounting for the cross-sectional differences in risk premia at all investment horizons.

5 Conclusion

We argue that the concerns of a long and short-horizon investor are very different. A buy and hold investor with a long horizon will be concerned almost exclusively with an asset’s cash flow exposure to long-run consumption risks (measured by the cointegration parameter). In contrast, a single-period investor will care about both the cash flow exposure and the exposure of price fluctuations to consumption. We analyze these concerns and their implications for risk premia at different return horizons. This focus on multiple investment horizons is a novel feature of our paper.
We show that allowing for cointegration between dividends and consumption is important for understanding the dynamics of asset returns and their risk compensations. Cointegration has sharp implications for both the predictability of returns and for the measurement of assets’ betas. We demonstrate that allowing for cointegration vastly improves the consumption-based model’s ability to explain risk premia on size and book-to-market-sorted assets. In particular, at the one-year horizon, the cross-sectional explanatory power rises from an adjusted $R^2$ of less than zero for the unconditional C-CAPM and 22% for a growth-rates based conditional C-CAPM, to 75% for a C-CAPM based on a cointegration specification. Importantly, specifying a cointegrating relation between dividends and consumption allows us to interpret the whole term-structure of the risk-return tradeoff. We show that consumption betas, implied by the EC-VAR, can successfully account for the cross-sectional differences in mean returns at all investment horizons whereas risk measures based on the standard, growth-rates VAR specification fail to do so.

Our empirical evidence underscores the importance of the cointegration specification in measuring expected returns and assets’ betas. Deviations of dividends from the long-run equilibrium with consumption contain valuable information about future evolution of dividend growth rates and returns. We show that incorporating this information via an EC-VAR framework is crucial for extracting return innovations and computing conditional betas. The growth-rate VAR specification excludes the error-correction term from a set of variables that predict future returns and, therefore, is not able to explain the cross-sectional differences in risk premia.

Finally, our approach allows us to assess the importance of long-run consumption risks relative to the overall consumption uncertainty. At all but the very long horizon, the cross-sectional slope coefficient from projecting mean returns on consumption betas reflects the risk compensation for both short- and long-run fluctuations in aggregate consumption. In the limit, on the other hand, it provides a market measure of risk compensation solely for long-run consumption risks. We find it to be a dominant source of the total risk premium, which, once again, points strongly towards the importance of long-run consumption risks in asset markets.
Appendix

In this appendix, we provide the details of the EC-VAR structure employed in the paper and the calculation of the horizon-dependent covariance matrix. Given estimates of the parameters and residuals in the cointegrating relation (9) between dividends and consumption, we model the dynamics of the resulting cointegrating residual, \( \epsilon_{d,t} \), jointly with the portfolio’s price-dividend ratio, \( z_t \), and consumption growth, \( \Delta c_t \), by the following EC-VAR structure:

\[
\begin{pmatrix}
\Delta c_t \\
\epsilon_{d,t} \\
z_t \\
\Delta d_t \\
\Delta z_t
\end{pmatrix} =
\begin{pmatrix}
\rho_c & 0 & 0 & 0 & 0 \\
a_{cc} & \rho_c & a_{cz} & a_{cd} & 0 \\
a_{zc} & a_{ze} & \rho_z & a_{zd} & 0 \\
a_{zc} & a_{ze} & (\rho_z - 1) & a_{zd} & 0 \\
a_{zc} & a_{ze} & (\rho_z - 1) & a_{zd} & 0
\end{pmatrix}
\begin{pmatrix}
\Delta c_{t-1} \\
\epsilon_{d,t-1} \\
z_{t-1} \\
\Delta d_{t-1} \\
\Delta z_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
\eta_t \\
\eta_{e,t} \\
\eta_{z,t} \\
\eta_{e,t} + \delta \eta_t \\
\eta_{z,t}
\end{pmatrix}.
\]

The first three variables form the basis of the EC-VAR process. The last two variables provide no additional information; they are included into the EC-VAR to simplify the computation of long-run covariances. The dynamics of \( \Delta d_t \) and \( \Delta z_t \) are derived from the dynamics of the first three variables by exploiting \( \Delta d_t = \Delta \epsilon_{d,t} + \delta \Delta c_t \), and \( \Delta z_t = z_t - z_{t-1} \).

Denoting \( X_t' = (\Delta c_t \ \epsilon_{d,t} \ z_t \ \Delta d_t \ \Delta z_t) \), we can rewrite the EC-VAR compactly as

\[ X_t = AX_{t-1} + Gu_t, \]

where the matrix \( A \) is defined above, \( G \) is a 5 \times 3 matrix, and \( u \) is a three by one matrix of shocks, \( u_t' = (\eta_t \ \eta_{e,t} \ \eta_{z,t}) \), that is

\[
X_t - AX_{t-1} = Gu_t \equiv \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\eta_t \\
\eta_{e,t} \\
\eta_{z,t}
\end{pmatrix}.
\]

Given this structure and a horizon \( s \geq 1 \), the innovation in the sum of \( s \) consecutive \( X \)'s can be extracted as follows,

\[
\sum_{j=1}^{s} X_{t+j} - E_t[\sum_{j=1}^{s} X_{t+j}] \equiv \zeta_{t,t+s},
\]

23
where $\zeta_{t,t+s}$ is

$$\zeta_{t,t+s} = \sum_{j=1}^{s} B_j e_{t+1+s-j},$$

with $e_t \equiv Gw_t$, $B_j = B_{j-1} + A^{j-1}$ and $B_0 = 0$, for $j = 1, ..., s$.

Exploiting the fact that the errors are identically distributed and uncorrelated, the covariance matrix of $\zeta_{t,t+s}$ for any given horizon $s$ is

$$\Sigma^*_s = B_s \Sigma_e B'_s + \Sigma^*_{s-1},$$  \hspace{1cm} (A-3)

where $\Sigma_e = G\Sigma_u G'$, and $\Sigma^*_0 = B_0 \Sigma_e B'_0 = 0$. As $s$ increases, $\Sigma^*_s$ grows without bound; hence we consider $\Sigma_s \equiv \Sigma^*_s$, that is the covariance matrix of $\zeta_{t,t+s}$ scaled by the horizon. Given $\Sigma_e$ and $B_s$, the evolution of $\Sigma_s$ is given by

$$\Sigma_s = \frac{1}{s} B_s \Sigma_e B'_s + (1 - \frac{1}{s}) \Sigma_{s-1},$$  \hspace{1cm} (A-4)

Equation (A-4) provides a direct recursive algorithm for the construction of the covariance matrix of interest. For large $s$, the long-run matrix is determined by the limit of $B_s$, which is $[I - A]^{-1}$, i.e.,

$$\Sigma_{lr} = [I - A]^{-1} \Sigma_e [I - A]^{-1}'. $$  \hspace{1cm} (A-5)
References


Table I presents descriptive statistics for the returns and cash flow growth rates on the 20 characteristic-sorted portfolios used in estimation. The portfolios examined are portfolios formed on market capitalization (S1-S10), and book-to-market ratio (B1-B10). Capitalization portfolios are formed by sorting NYSE, AMEX, and NASDAQ firms by their market capitalization as of June of each year (using NYSE breakpoints), and holding the capitalization decile constant for one year. Book-to-market portfolios are formed by sorting NYSE, AMEX, and NASDAQ firms based on their market capitalization as of June of each year divided by their book value as of the most recent fiscal year end available. Returns are value-weighted. The cash flow growth rates are constructed by taking the first difference of the logarithm of dividend series. The data are converted to real using the PCE deflator. The data are sampled at the annual frequency, and cover the period 1929 through 2002, for a total of 74 observations.
Table II
Cointegration Parameters and ACF

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\hat{\delta}$</th>
<th>ACF(1)</th>
<th>ACF(5)</th>
<th>ACF(10)</th>
<th>Unit Root</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>9.62 (6.07)</td>
<td>0.90</td>
<td>0.34</td>
<td>-0.02</td>
<td>-2.09</td>
</tr>
<tr>
<td>S2</td>
<td>9.71 (5.06)</td>
<td>0.64</td>
<td>0.41</td>
<td>-0.01</td>
<td>-5.86*</td>
</tr>
<tr>
<td>S3</td>
<td>6.69 (2.86)</td>
<td>0.75</td>
<td>0.17</td>
<td>0.05</td>
<td>-3.65*</td>
</tr>
<tr>
<td>S4</td>
<td>6.21 (2.37)</td>
<td>0.81</td>
<td>0.30</td>
<td>0.11</td>
<td>-2.75</td>
</tr>
<tr>
<td>S5</td>
<td>4.41 (1.70)</td>
<td>0.70</td>
<td>0.17</td>
<td>0.05</td>
<td>-3.32</td>
</tr>
<tr>
<td>S6</td>
<td>4.38 (1.88)</td>
<td>0.84</td>
<td>0.28</td>
<td>0.20</td>
<td>-2.41</td>
</tr>
<tr>
<td>S7</td>
<td>2.42 (1.12)</td>
<td>0.72</td>
<td>0.24</td>
<td>0.14</td>
<td>-3.41</td>
</tr>
<tr>
<td>S8</td>
<td>2.38 (0.77)</td>
<td>0.65</td>
<td>-0.03</td>
<td>-0.16</td>
<td>-3.47</td>
</tr>
<tr>
<td>S9</td>
<td>2.38 (0.96)</td>
<td>0.81</td>
<td>0.22</td>
<td>0.03</td>
<td>-3.45</td>
</tr>
<tr>
<td>S10</td>
<td>0.82 (0.89)</td>
<td>0.79</td>
<td>0.24</td>
<td>-0.04</td>
<td>-3.71*</td>
</tr>
<tr>
<td>B1</td>
<td>-0.27 (1.28)</td>
<td>0.73</td>
<td>0.16</td>
<td>-0.13</td>
<td>-3.98*</td>
</tr>
<tr>
<td>B2</td>
<td>-2.59 (1.41)</td>
<td>0.72</td>
<td>-0.05</td>
<td>0.17</td>
<td>-3.72*</td>
</tr>
<tr>
<td>B3</td>
<td>-0.11 (0.79)</td>
<td>0.61</td>
<td>0.07</td>
<td>-0.02</td>
<td>-3.87*</td>
</tr>
<tr>
<td>B4</td>
<td>0.82 (1.91)</td>
<td>0.64</td>
<td>0.17</td>
<td>0.13</td>
<td>-3.77*</td>
</tr>
<tr>
<td>B5</td>
<td>2.79 (1.59)</td>
<td>0.91</td>
<td>0.61</td>
<td>0.17</td>
<td>-1.83</td>
</tr>
<tr>
<td>B6</td>
<td>4.83 (1.75)</td>
<td>0.68</td>
<td>-0.01</td>
<td>0.05</td>
<td>-5.07*</td>
</tr>
<tr>
<td>B7</td>
<td>6.36 (2.20)</td>
<td>0.76</td>
<td>0.20</td>
<td>0.07</td>
<td>-3.12</td>
</tr>
<tr>
<td>B8</td>
<td>9.70 (3.07)</td>
<td>0.79</td>
<td>0.21</td>
<td>0.16</td>
<td>-3.80*</td>
</tr>
<tr>
<td>B9</td>
<td>12.54 (5.95)</td>
<td>0.67</td>
<td>0.17</td>
<td>0.03</td>
<td>-4.02*</td>
</tr>
<tr>
<td>B10</td>
<td>10.25 (4.55)</td>
<td>0.77</td>
<td>0.42</td>
<td>0.09</td>
<td>-1.88</td>
</tr>
</tbody>
</table>

* indicates significance at the 10% level (MacKinnon critical value = -3.59)

Table II presents the estimates of cointegration (CI) parameters for the 20 portfolios sorted by market capitalization (S1-S10) and book-to-market ratio (B1-B10). CI parameter estimates are obtained by regressing the deterministically de-trended log level of portfolio’s dividends on the de-trended log level of aggregate consumption. Aggregate consumption is defined as seasonally adjusted real consumption of nondurables plus services. Consumption data are taken from the NIPA tables available from the Bureau of Economic Analysis. The three columns labeled “ACF” report sample autocorrelation function of the cointegrating residuals for lags 1, 5 and 10. The final column presents a unit root test for null that the error correction residuals contain a unit root. Test statistics that exceed the 10% critical threshold are indicated with an asterisk.
Table III
Predictability Evidence: Dividend Growth

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Horizon — 1 —</th>
<th>Horizon — 5 —</th>
<th>Horizon — 10 —</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EC-VAR</td>
<td>VAR</td>
<td>EC-VAR</td>
</tr>
<tr>
<td>S1</td>
<td>0.00</td>
<td>0.04</td>
<td>0.14</td>
</tr>
<tr>
<td>S2</td>
<td>0.11</td>
<td>0.00</td>
<td>0.15</td>
</tr>
<tr>
<td>S3</td>
<td>0.44</td>
<td>0.52</td>
<td>0.10</td>
</tr>
<tr>
<td>S4</td>
<td>0.11</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>S5</td>
<td>0.11</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>S6</td>
<td>0.15</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>S7</td>
<td>0.12</td>
<td>0.08</td>
<td>0.21</td>
</tr>
<tr>
<td>S8</td>
<td>0.12</td>
<td>0.06</td>
<td>0.22</td>
</tr>
<tr>
<td>S9</td>
<td>0.17</td>
<td>0.15</td>
<td>0.24</td>
</tr>
<tr>
<td>S10</td>
<td>0.19</td>
<td>0.03</td>
<td>0.43</td>
</tr>
<tr>
<td>B1</td>
<td>0.27</td>
<td>0.06</td>
<td>0.55</td>
</tr>
<tr>
<td>B2</td>
<td>0.25</td>
<td>0.08</td>
<td>0.52</td>
</tr>
<tr>
<td>B3</td>
<td>0.24</td>
<td>0.12</td>
<td>0.54</td>
</tr>
<tr>
<td>B4</td>
<td>0.39</td>
<td>0.25</td>
<td>0.48</td>
</tr>
<tr>
<td>B5</td>
<td>0.14</td>
<td>0.19</td>
<td>0.24</td>
</tr>
<tr>
<td>B6</td>
<td>0.23</td>
<td>0.25</td>
<td>0.21</td>
</tr>
<tr>
<td>B7</td>
<td>0.34</td>
<td>0.37</td>
<td>0.15</td>
</tr>
<tr>
<td>B8</td>
<td>0.17</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>B9</td>
<td>0.13</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>B10</td>
<td>0.07</td>
<td>0.10</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table III presents the adjusted-$R^2$ for dividend projections implied by the EC-VAR specification and the growth-rates VAR model that does not assume the long-run relation between assets’ cash flows and consumption. The entries are reported for the 20 portfolios sorted by market capitalization (S1-S10) and book-to-market ratio (B1-B10). Data are sampled at the annual frequency, expressed in real terms, and cover the period 1929-2002.
Table IV
Predictability Evidence: Returns

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Horizon 1</th>
<th>Horizon 5</th>
<th>Horizon 10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EC-VAR</td>
<td>VAR</td>
<td>EC-VAR</td>
</tr>
<tr>
<td>S1</td>
<td>0.17</td>
<td>0.06</td>
<td>0.40</td>
</tr>
<tr>
<td>S2</td>
<td>0.13</td>
<td>0.11</td>
<td>0.53</td>
</tr>
<tr>
<td>S3</td>
<td>0.10</td>
<td>0.03</td>
<td>0.40</td>
</tr>
<tr>
<td>S4</td>
<td>0.15</td>
<td>0.10</td>
<td>0.44</td>
</tr>
<tr>
<td>S5</td>
<td>0.16</td>
<td>0.15</td>
<td>0.36</td>
</tr>
<tr>
<td>S6</td>
<td>0.11</td>
<td>0.07</td>
<td>0.37</td>
</tr>
<tr>
<td>S7</td>
<td>0.09</td>
<td>0.08</td>
<td>0.29</td>
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<tr>
<td>S8</td>
<td>0.08</td>
<td>0.05</td>
<td>0.32</td>
</tr>
<tr>
<td>S9</td>
<td>0.04</td>
<td>0.01</td>
<td>0.28</td>
</tr>
<tr>
<td>S10</td>
<td>0.03</td>
<td>0.02</td>
<td>0.23</td>
</tr>
<tr>
<td>B1</td>
<td>0.04</td>
<td>0.00</td>
<td>0.19</td>
</tr>
<tr>
<td>B2</td>
<td>0.07</td>
<td>0.00</td>
<td>0.23</td>
</tr>
<tr>
<td>B3</td>
<td>0.23</td>
<td>0.14</td>
<td>0.27</td>
</tr>
<tr>
<td>B4</td>
<td>0.06</td>
<td>-0.01</td>
<td>0.26</td>
</tr>
<tr>
<td>B5</td>
<td>0.04</td>
<td>0.02</td>
<td>0.24</td>
</tr>
<tr>
<td>B6</td>
<td>0.08</td>
<td>0.06</td>
<td>0.26</td>
</tr>
<tr>
<td>B7</td>
<td>0.15</td>
<td>0.06</td>
<td>0.44</td>
</tr>
<tr>
<td>B8</td>
<td>0.17</td>
<td>0.11</td>
<td>0.49</td>
</tr>
<tr>
<td>B9</td>
<td>0.20</td>
<td>0.06</td>
<td>0.42</td>
</tr>
<tr>
<td>B10</td>
<td>0.20</td>
<td>0.09</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table IV presents the adjusted-$R^2$ for return projections implied by the EC-VAR specification and the growth-rates VAR model that does not assume the long-run relation between assets’ cash flows and consumption. The entries are reported for the 20 portfolios sorted by market capitalization (S1-S10) and book-to-market ratio (B1-B10). Data are sampled at the annual frequency, expressed in real terms, and cover the period 1929-2002.
Table V
Conditional Mean Returns

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EC-VAR</th>
<th>VAR</th>
<th>EC-VAR</th>
<th>VAR</th>
<th>EC-VAR</th>
<th>VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>11.74</td>
<td>11.72</td>
<td>10.65</td>
<td>11.69</td>
<td>10.74</td>
<td>11.67</td>
</tr>
<tr>
<td>S2</td>
<td>11.09</td>
<td>10.98</td>
<td>10.02</td>
<td>10.14</td>
<td>9.97</td>
<td>9.78</td>
</tr>
<tr>
<td>S3</td>
<td>11.71</td>
<td>11.57</td>
<td>10.39</td>
<td>10.98</td>
<td>10.02</td>
<td>10.58</td>
</tr>
<tr>
<td>S4</td>
<td>11.40</td>
<td>11.32</td>
<td>10.22</td>
<td>10.50</td>
<td>10.07</td>
<td>10.16</td>
</tr>
<tr>
<td>S7</td>
<td>9.86</td>
<td>9.85</td>
<td>9.13</td>
<td>9.24</td>
<td>8.68</td>
<td>8.84</td>
</tr>
<tr>
<td>S8</td>
<td>9.19</td>
<td>9.20</td>
<td>8.58</td>
<td>8.76</td>
<td>8.14</td>
<td>8.44</td>
</tr>
<tr>
<td>S9</td>
<td>8.47</td>
<td>8.47</td>
<td>7.83</td>
<td>7.97</td>
<td>7.44</td>
<td>7.66</td>
</tr>
<tr>
<td>S10</td>
<td>7.19</td>
<td>7.21</td>
<td>6.74</td>
<td>6.77</td>
<td>6.40</td>
<td>6.51</td>
</tr>
<tr>
<td>B1</td>
<td>6.61</td>
<td>6.64</td>
<td>6.32</td>
<td>6.37</td>
<td>5.99</td>
<td>6.19</td>
</tr>
<tr>
<td>B2</td>
<td>7.92</td>
<td>7.94</td>
<td>7.85</td>
<td>7.72</td>
<td>7.58</td>
<td>7.55</td>
</tr>
<tr>
<td>B3</td>
<td>6.60</td>
<td>6.70</td>
<td>6.68</td>
<td>6.63</td>
<td>6.36</td>
<td>6.45</td>
</tr>
<tr>
<td>B4</td>
<td>7.22</td>
<td>7.30</td>
<td>6.90</td>
<td>7.09</td>
<td>6.40</td>
<td>7.02</td>
</tr>
<tr>
<td>B5</td>
<td>9.09</td>
<td>9.10</td>
<td>8.38</td>
<td>8.57</td>
<td>8.02</td>
<td>8.29</td>
</tr>
<tr>
<td>B6</td>
<td>9.18</td>
<td>9.20</td>
<td>8.73</td>
<td>9.01</td>
<td>8.48</td>
<td>8.82</td>
</tr>
<tr>
<td>B7</td>
<td>9.96</td>
<td>9.84</td>
<td>9.05</td>
<td>9.22</td>
<td>9.20</td>
<td>9.06</td>
</tr>
<tr>
<td>B8</td>
<td>11.76</td>
<td>11.53</td>
<td>11.28</td>
<td>10.97</td>
<td>11.79</td>
<td>10.81</td>
</tr>
<tr>
<td>B9</td>
<td>12.89</td>
<td>12.45</td>
<td>12.80</td>
<td>12.36</td>
<td>13.57</td>
<td>12.40</td>
</tr>
<tr>
<td>B10</td>
<td>11.83</td>
<td>11.63</td>
<td>10.76</td>
<td>10.95</td>
<td>11.05</td>
<td>10.98</td>
</tr>
</tbody>
</table>

Table V presents mean returns for investment horizons of 1, 5 and 10 years for each of the 20 portfolios sorted by market capitalization (S1-S10) and book-to-market ratio (B1-B10). Mean returns for a given horizon s are computed as \( \bar{r}_s + 0.5\sigma^2_{\bar{r}_s} \). Data are sampled at the annual frequency and cover the period 1929-2002.
Table VI
Consumption Betas by Horizon

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Uncond.</th>
<th>EC-VAR</th>
<th>VAR</th>
<th>Uncond.</th>
<th>EC-VAR</th>
<th>VAR</th>
<th>Uncond.</th>
<th>EC-VAR</th>
<th>VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.71 (1.47)</td>
<td>4.12 (2.43)</td>
<td>1.77 (2.10)</td>
<td>4.51 (4.66)</td>
<td>-1.46 (3.94)</td>
<td>6.54 (4.26)</td>
<td>-1.55 (4.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>0.80 (1.38)</td>
<td>2.09 (1.12)</td>
<td>0.89 (1.09)</td>
<td>1.82 (2.65)</td>
<td>-1.24 (2.42)</td>
<td>4.21 (3.02)</td>
<td>0.13 (2.37)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>0.52 (1.36)</td>
<td>4.14 (1.29)</td>
<td>2.95 (1.98)</td>
<td>2.14 (2.38)</td>
<td>0.04 (2.44)</td>
<td>3.38 (2.29)</td>
<td>0.35 (2.86)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>0.77 (1.13)</td>
<td>3.52 (1.21)</td>
<td>2.66 (1.50)</td>
<td>2.09 (1.97)</td>
<td>0.79 (1.94)</td>
<td>3.56 (2.40)</td>
<td>1.59 (1.76)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>0.36 (1.21)</td>
<td>2.76 (0.99)</td>
<td>2.36 (1.22)</td>
<td>0.99 (1.72)</td>
<td>0.39 (1.52)</td>
<td>2.05 (2.12)</td>
<td>1.14 (1.37)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>0.64 (1.10)</td>
<td>3.07 (0.81)</td>
<td>2.58 (1.07)</td>
<td>1.42 (1.51)</td>
<td>0.71 (1.87)</td>
<td>2.50 (1.83)</td>
<td>1.41 (1.56)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>0.33 (1.10)</td>
<td>2.37 (0.86)</td>
<td>2.20 (0.82)</td>
<td>0.46 (1.37)</td>
<td>0.14 (1.28)</td>
<td>1.00 (1.67)</td>
<td>0.61 (1.18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>-0.31 (1.14)</td>
<td>1.62 (0.68)</td>
<td>1.43 (0.76)</td>
<td>-0.12 (1.35)</td>
<td>-0.44 (1.62)</td>
<td>0.35 (1.68)</td>
<td>-0.13 (1.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S9</td>
<td>0.13 (1.09)</td>
<td>1.58 (0.77)</td>
<td>1.38 (0.80)</td>
<td>0.60 (1.41)</td>
<td>0.27 (1.62)</td>
<td>1.02 (1.59)</td>
<td>0.62 (1.49)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S10</td>
<td>0.69 (0.83)</td>
<td>1.54 (0.56)</td>
<td>1.64 (0.46)</td>
<td>0.31 (1.09)</td>
<td>0.51 (1.09)</td>
<td>0.34 (1.07)</td>
<td>0.67 (1.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>0.82 (0.97)</td>
<td>1.81 (0.54)</td>
<td>2.16 (0.33)</td>
<td>-0.58 (1.40)</td>
<td>0.31 (1.48)</td>
<td>-0.83 (1.32)</td>
<td>0.14 (1.63)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>-0.18 (0.81)</td>
<td>0.16 (0.56)</td>
<td>0.65 (0.39)</td>
<td>-1.69 (0.91)</td>
<td>-0.76 (0.98)</td>
<td>-2.05 (0.86)</td>
<td>-0.86 (1.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>-0.33 (0.84)</td>
<td>-0.09 (0.37)</td>
<td>0.32 (0.37)</td>
<td>-1.79 (0.77)</td>
<td>-1.34 (0.85)</td>
<td>-1.70 (0.84)</td>
<td>-1.33 (0.98)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>0.29 (1.10)</td>
<td>1.48 (1.29)</td>
<td>1.94 (1.50)</td>
<td>-0.67 (1.99)</td>
<td>-0.07 (2.10)</td>
<td>-0.59 (2.06)</td>
<td>-0.28 (2.22)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B5</td>
<td>0.27 (1.11)</td>
<td>1.94 (0.86)</td>
<td>1.67 (0.96)</td>
<td>1.18 (1.58)</td>
<td>0.81 (1.79)</td>
<td>1.60 (1.69)</td>
<td>1.10 (1.57)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B6</td>
<td>2.24 (1.01)</td>
<td>3.18 (1.49)</td>
<td>2.64 (1.97)</td>
<td>2.75 (2.42)</td>
<td>1.91 (1.96)</td>
<td>3.27 (2.37)</td>
<td>2.17 (1.77)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B7</td>
<td>0.21 (1.21)</td>
<td>2.74 (0.98)</td>
<td>1.46 (1.68)</td>
<td>2.67 (1.53)</td>
<td>1.01 (1.59)</td>
<td>4.22 (1.63)</td>
<td>1.70 (1.69)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B8</td>
<td>0.84 (1.23)</td>
<td>4.34 (1.83)</td>
<td>2.18 (2.14)</td>
<td>4.39 (2.72)</td>
<td>0.37 (2.22)</td>
<td>6.36 (2.69)</td>
<td>0.98 (2.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B9</td>
<td>-0.39 (1.54)</td>
<td>5.47 (2.31)</td>
<td>2.16 (2.91)</td>
<td>6.11 (3.75)</td>
<td>-1.55 (2.72)</td>
<td>8.32 (3.43)</td>
<td>-2.03 (2.79)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B10</td>
<td>0.14 (1.63)</td>
<td>3.89 (1.06)</td>
<td>2.87 (1.03)</td>
<td>2.14 (2.60)</td>
<td>0.54 (3.15)</td>
<td>4.33 (3.38)</td>
<td>1.49 (3.54)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table VI presents consumption betas for investment horizons of 1, 5 and 10 years for each of the 20 portfolios sorted by market capitalization (S1-S10) and book-to-market ratio (B1-B10). In columns labeled “EC-VAR,” betas are measured using the error-correction specification for consumption and asset returns. Columns labeled “VAR” present betas measured using a growth rate VAR omitting the error-correction information. These consumption betas are estimated as in equation (15) using the covariance matrices implied by the relevant time series model. The column labeled “Uncond.” represents the standard consumption beta. Standard errors are reported in parentheses.
Table VII
Cross-Sectional Regressions by Horizon

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Uncond.</th>
<th>EC-VAR</th>
<th>VAR</th>
<th>Uncond.</th>
<th>EC-VAR</th>
<th>VAR</th>
<th>Uncond.</th>
<th>EC-VAR</th>
<th>VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Risk (λ₁ₘ)</td>
<td>0.51</td>
<td>1.19</td>
<td>1.28</td>
<td>0.73</td>
<td>-0.31</td>
<td>0.65</td>
<td>-0.03</td>
<td>0.84</td>
<td>-0.05</td>
</tr>
<tr>
<td>SE</td>
<td>(2.24)</td>
<td>(0.41)</td>
<td>(1.57)</td>
<td>(0.32)</td>
<td>(0.40)</td>
<td>(0.24)</td>
<td>(0.44)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ř²</td>
<td>-0.04</td>
<td>0.75</td>
<td>0.22</td>
<td>0.73</td>
<td>-0.03</td>
<td>0.84</td>
<td>-0.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table VII presents results for cross-sectional regressions for a set of 10 portfolios sorted by market capitalization and 10 portfolios sorted by book-to-market ratio. Consumption risk for different investment horizons is measured by the corresponding consumption beta. In columns labeled “EC-VAR,” betas are measured using the error-correction specification for consumption and asset returns. Columns labeled “VAR” presents betas measured using a growth rate VAR omitting the error correction information. These consumption betas are estimated as in equation (15) using the covariance matrices implied by the relevant time series model. The column labeled “Uncond.” represents the standard consumption beta. All risk prices are expressed in annual percentage terms. Robust standard errors, reported in parentheses, are computed by estimating time-series and cross-sectional parameters in one step via GMM. The number of lags used in Newey-West covariance estimator is 8.
Table VIII
Cross-Sectional Regressions by Horizon with Cointegration Restrictions

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\delta = 1$</th>
<th>$\tau_1 = 0$</th>
<th>$\delta = 1$</th>
<th>$\tau_1 = 0$</th>
<th>$\delta = 1$</th>
<th>$\tau_1 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Risk ($\lambda_{1,s}$)</td>
<td>1.58</td>
<td>1.55</td>
<td>0.79</td>
<td>1.20</td>
<td>1.12</td>
<td>1.12</td>
</tr>
<tr>
<td>SE</td>
<td>(0.39)</td>
<td>(0.28)</td>
<td>(0.54)</td>
<td>(0.31)</td>
<td>(0.41)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.45</td>
<td>0.62</td>
<td>0.06</td>
<td>0.43</td>
<td>0.26</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table VIII presents results for cross-sectional regressions for a set of 10 portfolios sorted by market capitalization and 10 portfolios sorted by book-to-market ratio. Consumption risk for different investment horizons is measured by the corresponding consumption beta. These betas are measured under two restrictions on the cointegrating relation between dividends and consumption:

$$d_t = \tau_0 + \tau_1 t + \delta c_t + \epsilon_{d,t}.$$  

In the left columns, we restrict the cointegration parameter, $\delta = 1$, for all assets while still allowing for different time trends. In the right columns, we suppress the time trend in the cointegration specification, $\tau_1 = 0$, but do not restrict the cointegration parameter, $\delta$. The beta for a given horizon is calculated from the error-correction VAR model as in equation (15). All risk prices are expressed in annual percentage terms. Robust standard errors, reported in parentheses, are computed by estimating time-series and cross-sectional parameters in one step via GMM. The number of lags used in Newey-West covariance estimator is 8.
Figure 1. Dividends to Consumption Ratio

Figure 1 plots the logarithm of the aggregate dividends to consumption ratio and the ratio of per-share dividends to consumption.
Figure 2. Expected Returns

Figure 2 plots one- and ten-year returns predicted by the EC-VAR model and the alternative, growth-rates VAR specification. The latter ignores the error-correction information in predicting returns. Expected returns are plotted for small (S1) and large (S10) market capitalization firms, and low (B1) and high (B10) book-to-market portfolios.
Figure 3 displays the fit from the cross-sectional regressions for the investment horizon of one year, as well as in the long-run limit. The figures plot fitted expected returns, implied by the model, against mean returns.

Figure 3. Cross-Sectional Fit
Figure 4. Consumption Risks Compensation by Horizon

Figure 4 plots the market price of risk along with the profile of implied compensations for transient consumption risk for investment horizons up to 15 years. The prices of risks are expressed in annual percentage terms.