A Monetary Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles

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This paper develops and estimates a monetary model that offers an explanation of some puzzling features of observed returns on equities and default-free bonds. The key feature of the model is that some assets other than money play a special role in facilitating transactions, which affects the rate of return that they offer. The model is capable of producing a low risk-free rate, a high equity premium, and an average positive relationship between maturity and term premium for default-free bonds. The model's implications for the joint distribution of asset returns, velocity, inflation, money growth, and consumption growth are also compared to the behavior of these variables in the U.S. economy.

I. Introduction

In the U.S. economy we see transactions being carried out in a wide variety of ways. At one extreme we pay for goods with fiat money, and at the other extreme we obtain goods with only a promise to pay out of our wealth by some specified date (such as when we use a credit or charge card). Between these two extremes, we pay for goods with a check drawn on a bank demand deposit or a check drawn on a money market mutual fund. Evidently many assets other than narrowly defined money play a special role in facilitating transactions. This feature has long ago been recognized in the literature; for exam-

ple, Patinkin (1965) discusses the role of bonds in facilitating transactions, Tobin (1963) refers to the liquidity value of short-term government debt for banks in managing their reserve position, and Friedman (1969) refers in general to a nonpecuniary return to bonds.

To motivate the importance of considering the role of a variety of assets in facilitating transactions, consider the transaction service return of fiat money. If both cash and checks can be used for transactions, the transaction service return of fiat money relative to interest-bearing checkable deposits should equal, at the margin, the nominal interest rate paid on these deposits. The interest rate on unregulated NOW accounts averaged roughly 6 percent from 1983 to 1987 in the U.S. economy, and in 1987 currency in circulation amounted to 76 percent of these deposits (Moore, Porter, and Small 1990; 1993 *Economic Report of the President*).[1] These results suggest that households place a substantial value (at least 6 percent per year during the 1980s) on the role of fiat money in facilitating transactions, which provides a strong motivation to consider the transaction service return of other assets as well.

To capture the transaction service return of a variety of assets, we develop a monetary economy that distinguishes payment by cash, checks, and credit. As with fiat money, in our model interest-bearing checkable deposits are held in part for their return from facilitating transactions. Securities that back checkable deposits thus provide a transaction service return in addition to their nominal return. An implication of our model is that short-term government bonds facilitate transactions by backing checkable deposits, and any transaction service return that short-term government bonds offer leads in equilibrium to a lower nominal return for these bonds. We use this framework to provide an explanation for a variety of asset market puzzles, as well as the joint behavior of observed asset returns and macroeconomic variables.

Mehra and Prescott (1985) show that the representative-agent economy with time-additive expected utility reasonably restricted cannot simultaneously account for the observed low average real risk-free rate and high equity premium. Backus, Gregory, and Zin (1989) show that the Mehra-Prescott economy also cannot explain the observed positive relationship between average bond returns and maturity (see also Donaldson, Johnsen, and Mehra 1990). Hansen and Jagannathan (1991) provide an alternative characterization of asset

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[1] Note that the return on demand deposits was regulated, and therefore banks often provided additional services to the business and households that held these accounts; this makes it difficult to measure the return offered on these deposits. Also, we recognize that a large fraction of currency may not circulate in the U.S. economy.

return puzzles in terms of nonparametric variance bounds. In this paper we show that our model can simultaneously explain the low average risk-free rate, the high equity premium, and the positive relationship between average bond returns and maturity. We also explore the implications of the Hansen-Jagannathan bounds for our model.

Several authors have modified the basic Mehra-Prescott framework to resolve the risk-free rate and equity premium puzzles. Cecchetti and Mark (1990) and Kandel and Stambaugh (1990) use larger risk aversion and accommodate a consumption process different from the one used in Mehra and Prescott to explain these puzzles. Kocherlakota (1988) allows for negative time preference, Epstein and Zin (1990) use preferences that accommodate "first-order risk aversion," and Constantinides (1990) uses habit formation in preferences to explain these puzzles.[2] In contrast to these papers, we explain the low average risk-free rate and high equity premium puzzles with preference parameters that are similar to the ones used in Mehra and Prescott.

As we construct a monetary model, we are able to evaluate the ability of the model to provide some insight concerning the relationship between velocity, inflation, and asset returns. In particular, we focus on the behavior of velocity and its relationship to the nominal interest rate as well as the relationship between real equity returns and inflation. These issues have also been examined in Hodrick, Kocherlakota, and Lucas (1991) and Marshall (1992).

We begin this paper by developing a transaction cost model that distinguishes among payment with cash, checks, and credit. In terms of cash and credit goods, we make the same distinction that Lucas and Stokey (1987) make, except here the choice of purchasing goods on credit or with cash depends solely on the technology and not on preferences. By explicitly developing a monetary model that involves cash, checks, and credit, this paper extends the transaction cost literature, which includes Baumol (1952), Tobin (1956), Barro (1976), and McCallum (1983).[3] For a wide class of monetary policies, we prove

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[2] For a more complete set of references, see Donaldson and Willard (1993).

[3] See Feenstra (1986) for a survey of this literature and a connection of the transaction cost literature to both the money-in-utility and the cash-in-advance literatures. Our analysis of the role of riskless assets in facilitating transactions is in some ways similar to the analysis by Fried and Howitt (1983), who study the steady state in a deterministic model in which both money and bonds directly enter a transaction cost function. Our analysis is also related to the one used by King and Plosser (1984), who consider the role of privately produced accounting services in facilitating transactions. Other related models are those of Prescott (1987), who considers a fixed cost for purchasing goods with checks, and Poterba and Rotemberg (1987), who model assets as providing utility directly.

the existence and uniqueness of an equilibrium to this economy and show how to compute this equilibrium.

The remainder of this paper is structured as follows. In Section II we develop the model with one-period bonds and equities, and in Section III we use the model to price multiperiod bonds. In Section IV we estimate the model's parameters using the generalized method of moments estimation technique of Hansen (1982) and Hansen and Singleton (1982). In Section V we compare some of the model's predictions to corresponding features of U.S. data. In Section VI we provide some concluding remarks.

II. The Model

A. The Aggregate Supply of Money and Bonds

Denote by s the economy's exogenous state variables, which evolve according to a stationary first-order Markov process. Given a value of the current period's state variables equal to s, denote by E_s the conditional expectation over next period's value. Denote by A the aggregate amount of currency in circulation at the beginning of a period, which includes government bonds that are redeemed for cash and any lump-sum monetary transfer by the monetary authority. Denote by $q(s)$ the price of a one-period, pure-discount government bond, denote by $r(s)$ the one-period interest rate (so $r = [1/q] - 1$), and denote by B the amount of one-period bonds issued this period. These bonds are purchased with currency, so the amount of currency in circulation after such an open-market operation is $M = A - qB$. The aggregate stock of fiat money M evolves according to some state-dependent function h: $M' = h(s, s')M$. To attain this monetary growth rate, the monetary authority relies in part on lump-sum monetary transfers; denote by $x(s, s')M$ the lump-sum monetary transfer issued at the beginning of the next period. Suppose also that the government chooses to issue a value of bonds, qB, according to some state-dependent function g: $qB = g(s)M$. The government is free to choose h and g independently since it has two independent instruments: open-market operations and lump-sum monetary transfers.

Given a choice for $g > 0$, we assume that the monetary authority chooses h to attain state-contingent targets for the interest rate and inflation rate. We assume that the interest rate is always strictly positive, so $q < 1$. Denote the gross inflation rate from this period to the next by $\pi' = \pi(s, s')$, which we also assume is strictly positive. The model will provide a relationship between the interest rate and the inflation rate that must be satisfied, and we assume that the monetary authority's choices for the interest rate and inflation rate are consis-

tent with this relationship. This technique was also used in Coleman (1996).

B. The Household's Problem

Each household receives an endowment $y = y(s)$ of a perishable consumption good. Households cannot consume their own endowment, but rather must purchase consumption goods from other households. In terms of money, the price of the endowment is given by $P = p(s)M$. For the remainder of this paper, we measure all nominal aggregates relative to the aggregate stock of money M.

Consider a representative household that ranks stochastic consumption streams $\{c_t\}$ according to the utility function

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right].$$

Assume that $0 < \beta < 1$; that the period utility function u is bounded, strictly increasing, and strictly concave; and that $\lim_{c \to 0} u_1(c) = \infty$. The household begins each period with money balances, denoted by a, which includes the current lump-sum monetary transfer by the government (as above, next period's monetary transfer is given by x'). During the period the household purchases consumption goods c using cash, checks, and credit and one-period, pure-discount bonds b using cash.

Denote goods that are purchased with cash by c_1, goods purchased with checks by c_2, and goods purchased on credit by c_3. Goods and bonds purchased using cash must satisfy the currency constraint

$$pc_1 + qb \le a. \tag{1}$$

Denote the household's cash holdings after it purchases bonds by $m = a - qb$. Note that in equilibrium $m = 1$ (recall that M is measured after an open-market operation). Households deposit their bond holdings qb with a financial intermediary, which allows them to write checks on these deposits. These checks come due at the beginning of the next period; we assume that checks must clear before interest is paid on checkable deposits. A household's purchase of goods with checks must not exceed its checking account balance, so its checking constraint is

$$pc_2 \le qb. \tag{2}$$

[4] In Sec. III we allow for checkable deposits to be backed by risky securities. In that section we also bring out the special role of short-term government debt in backing checkable deposits. The constraint in eq. (2) is a special case of the more elaborate setup discussed in that section. The simplification here captures, in essence, the special role of short-term government debt in backing checkable deposits.

Finally, to capture a feature of credit and charge card usage in actual economies, goods purchased on credit must be nonnegative,

$$pc_3 \geq 0, \tag{3}$$

and must satisfy the household's budget constraint.

Purchasing goods, whether they are purchased with cash, with checks, or on credit, incurs a transaction cost in terms of forgone output (or in terms of time devoted to the production of consumption goods). Purchasing a larger fraction of goods with cash reduces this transaction cost, and purchasing a larger fraction of goods with checks or on credit increases this transaction cost. Denote this transaction cost by $\psi(c_1 + c_2 + c_3, c_1, c_2)$, where ψ is homogeneous of degree one in all three arguments. Assume $\psi_1 > 0$, $\psi_2 < 0$, and $\psi_3 < 0$ (ψ_i is the derivative of ψ with respect to its ith argument). To ensure that some goods are always purchased with cash and checks, assume $\psi_2(c, 0, c_2) = -\infty$ and $\psi_3(c, c_1, 0) = -\infty$. We shall impose some additional assumptions on ψ later in this section, at which point it will be clear what role these additional restrictions play.

The traditional approach taken in the transaction cost literature, such as in Feenstra (1986), Marshall (1992), and Bansal et al. (1995), enters real money balances m/p instead of c_1 directly as the second argument of ψ. Hence, the traditional approach does not impose the nonnegativity restriction on credit purchases and does not distinguish purchases of goods with checks, that is, $c_2 = 0$. With this specification, all the money in the economy indeed circulates each period, but only because households can directly *sell* goods on credit (in those economies, $m/p > y - \psi(c, m/p, 0)$ is possible). In this event, one should define transaction costs on gross transactions $|c_1| + |c_3|$ instead of net transactions c; alternatively, one should prohibit households from directly selling goods on credit. We take the latter approach. With this approach, however, it is possible that not all the money in the economy circulates each period. To avoid attributing liquidity services to money that is not used for goods transactions, we enter only goods purchased with money, c_1, as the second argument in the transaction cost function ψ.

Goods purchased on credit this period must be paid for in cash at the beginning of the next period, so next period's cash balances before any government monetary transfer must equal

$$\hat{a} = py - \psi(pc, pc_1, pc_2) + (a - pc_1 - qb) + (b - pc_2) - pc_3 \tag{4}$$

($\hat{a}$ denotes next period's pretransfer currency holdings relative to this period's aggregate money supply M). Note that receipts $py - \psi(pc, pc_1, pc_2)$ are all in the form of cash, either because the goods were sold for cash or because the goods were sold in exchange for checks or on credit, which was later settled in cash.

Let $J(a, s)$ denote the value of the optimal discounted stream of utility for a household starting a given period with money balances a, while the economy is in state s. The value function J satisfies the Bellman equation

$$J(a, s) = \sup_{c_1, c_2, c_3, b, \hat{a}} E_s \left[u(c) + \beta J\left(\frac{\hat{a} + x'}{h'}, s'\right) \right],$$

where choices for c_1, c_2, c_3, b, and $\hat{a}$ are subject to equations (1)–(4).[5] Denote multipliers for the feasibility constraints (1)–(4) by, respectively, φ, ζ, μ, and λ. Standard arguments, such as those in Stokey, Lucas, and Prescott (1989), prove the existence of a unique solution to the household's dynamic programming problem.

C.   The Equilibrium

When we derive first-order and envelope conditions, it is straightforward to characterize the equilibrium as satisfying the following equations:

$$c_1 + c_2 + c_3 = c, \tag{5}$$

$$pc + \psi(pc, pc_1, pc_2) = py, \tag{6}$$

$$pc_1 \leq 1 \quad \text{with equality if } \varphi > 0, \tag{7}$$

$$pc_2 \leq g \quad \text{with equality if } \zeta > 0, \tag{8}$$

$$pc_3 \geq 0 \quad \text{with equality if } \mu > 0, \tag{9}$$

$$u_1(c) = \lambda p[1 + \psi_1(pc, pc_1, pc_2) + \psi_2(pc, pc_1, pc_2)] + \varphi p, \tag{10}$$

$$u_1(c) = \lambda p[1 + \psi_1(pc, pc_1, pc_2) + \psi_3(pc, pc_1, pc_2)] + \zeta p, \tag{11}$$

$$u_1(c) = \lambda p[1 + \psi_1(pc, pc_1, pc_2)] - \mu p, \tag{12}$$

$$(\lambda + \varphi)q = \lambda + \zeta q, \tag{13}$$

and

$$\lambda = \beta E_s \left[\frac{\lambda' + \varphi'}{h'} \right]. \tag{14}$$

Given y, h, and g, these 10 equations determine the 10 functions c, c_1, c_2, c_3, p, q, λ, φ, ζ, and μ; all functions depend on s. Alternatively, as we describe here, given y, g, q, and a function $\pi' = p'h'/p$ that satisfies equation (14), the nine equations (5)–(13) determine the nine functions c, c_1, c_2, c_3, p, λ, φ, ζ, and μ; h is determined as $h' =$

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[5] To rule out Ponzi schemes, we assume that there exists some upper limit on the selling of debt.

$\pi' p/p'$. We search for an equilibrium in which all three modes of payment are used; the general case is handled in Appendix A.

The first result to be established is that if some goods are purchased on credit, then both the currency and checking constraints hold with equality. If $pc_3 > 0$, then it follows from equation (9) that $\mu = 0$. From equation (12) it thus follows that

$$u_1(c) = \lambda p[1 + \psi_1(pc, pc_1, pc_2)]. \tag{15}$$

Use this result in equations (10)–(11) to show that

$$\varphi = -\lambda \psi_2(pc, pc_1, pc_2) \tag{16}$$

and

$$\zeta = -\lambda \psi_3(pc, pc_1, pc_2). \tag{17}$$

It follows that $\varphi > 0$ and $\zeta > 0$, and thus equations (7)–(8) hold with equality. Hence, $pc_1 = 1$, $pc_2 = g$, and $pc_3 = pc - 1 - g$. Note that pc is the consumption velocity of cash; to simplify the notation, we sometimes denote this as v.

The next result to be established is that there exists a function pc, which is unique, that is consistent with an equilibrium. First, to simplify the notation, denote $\psi_0 = \psi$ and define ξ_i, for $i = 0, 1, 2,$ or 3, as

$$\xi_i(v; g) = \psi_i\left(1, \frac{1}{v}, \frac{g}{v}\right).$$

Also, we shall often write $\xi_i(v; g)$ simply as $\xi_i(v)$. To solve for pc, substitute equations (16)–(17) into equation (13) to derive

$$r = \xi_3(pc) - \xi_2(pc). \tag{18}$$

Given r from the monetary authority, equation (18) determines pc. With the following assumptions on ψ, there exists a pc, which is unique, that solves this equation. First, assume that $\xi_3(v) - \xi_2(v)$ is a strictly increasing function of v for any $v > 1$. Second, assume that

$$\lim_{v \to \infty} \xi_3(v) - \xi_2(v) > r$$

and also that

$$\lim_{v \to 1+g} \xi_3(v) - \xi_2(v) < r.$$

These assumptions ensure that $pc > 1 + g$, which is consistent with our search for an equilibrium in which all three modes of payment are used.

To interpret these assumptions, note that as v rises, relatively fewer goods are purchased with cash and checks, although the proportion

of goods purchased with cash to goods purchased with checks remains constant (at $1/g$). If checkable deposits offer no transaction service return (so $\xi_3 = 0$), then the assumption that $\xi_3(v) - \xi_2(v)$ is rising in v simply means that the transaction service return to using cash, $\xi_2(v)$, rises as relatively fewer goods are purchased with cash. If checkable deposits offer a transaction service return, then this assumption means that the change in the transaction cost differential $\xi_3(v) - \xi_2(v)$ is dominated by how transaction costs change as relatively more or fewer goods are purchased with cash.

With pc determined, equation (6) determines c as

$$c = \frac{y}{1 + \xi_0(pc)}.$$

(With both pc and c determined, so is p.) The term λp is then determined from equation (15) as

$$\lambda p = \frac{u_1(c)}{1 + \xi_1(pc)}. \tag{19}$$

Given this function for λp, the functions φp and ζp are determined by equations (16)–(17). Write equation (14) as

$$\lambda p = \beta E_s \left[\frac{\lambda' p' + \varphi' p'}{\pi'} \right], \tag{20}$$

where $\pi' = p'h'/p$ is the (gross) inflation rate. Given the monetary authority's choice for q and g, a choice for inflation π' that is consistent with this choice need satisfy only equation (20). Under the stated assumptions, there thus exists a unique equilibrium to this economy.

D. The Transaction Service Return to Monetary Assets

Equation (18) summarizes how the nominal interest rate determines the transaction service return to currency and riskless assets. In equilibrium, the transaction service return to currency, $-\xi_2(pc)$, equals the nominal interest rate plus the transaction service return to interest-bearing riskless assets, $-\xi_3(pc)$. To explain this relationship, note that in equilibrium, households must be indifferent to purchasing dx less nominal consumption with cash, thereby increasing their bond holdings by dx and raising their purchases of nominal consumption with checks by dx. This shift leaves current utility unaffected, since total consumption is unchanged, but increases future interest income by rdx. Hence, in equilibrium, this interest income must be exactly offset by the higher transaction cost of purchasing fewer goods with cash and more goods with checks. In this model the transaction cost

increases by $[\xi_3(pc) - \xi_2(pc)]\,dx$. Equating these two returns delivers equation (18).

The transaction service return to riskless assets alters the relationship among the nominal interest rate and the inflation rate and thereby affects the real interest rate. To see this, substitute equations (16) and (17) into equation (20) to obtain

$$\frac{u_1(c)}{1 + \xi_1(pc)} = \beta E_s \left[\frac{u_1(c')}{1 + \xi_1(p'c')} \frac{1 - \xi_2(p'c')}{\pi'} \right]. \tag{21}$$

Given y, a choice of r determines both pc and c, and equation (21) determines the possible inflation rates that are consistent with this interest rate. To derive a more revealing form of this equation, write equation (18) as $q[1 - \xi_2(pc) + \xi_3(pc)] = 1$, and use this result in equation (21) to obtain

$$\frac{1 - \xi_2(pc) + \xi_3(pc)}{1 - \xi_2(pc)} \frac{u_1(c)}{1 + \xi_1(pc)} [1 - \xi_2(pc)]q$$

$$= \beta E_s \left[\frac{u_1(c')}{1 + \xi_1(p'c')} \frac{1 - \xi_2(p'c')}{\pi'} \right]. \tag{22}$$

We shall examine special cases of the equation just derived in a subsequent section. At this point, though, it is clear that the average real interest rate will be affected by the term

$$\frac{1 - \xi_2(pc) + \xi_3(pc)}{1 - \xi_2(pc)},$$

which falls below one if checkable deposits begin to offer a transaction service return $(-\xi_3(pc) > 0)$. A higher transaction service return offered on checkable deposits will lower both sides of equation (22) for a given nominal interest rate, which is consistent with higher inflation and hence a lower real interest rate.

E.   The Return to Risky Assets

To consider the effect of a transaction service return to riskless assets on the equity premium, we must first derive the price of a risky asset. Suppose that households trade claims to a series of uncertain cash payments: Holding a claim z at the beginning of a period entitles its bearer to the realization of zpd each period (relative to that period's money stock M), where the uncertain real value of the payment (in terms of goods) is captured by $d = d(s)$. In equilibrium, $z = 0$. Assume that the payment zpd is made in cash at the beginning of the period,

which is one period before households collect payment from the sale of their endowment. Denote the price of this claim by $\Theta = \theta(s)M$. At the beginning of the period, these claims can be traded, and the cash from this transaction can be used to purchase goods this period. The currency constraint thus becomes

$$pc_1 + qb + \theta(z' - z) \leq a + zpd,$$

and the budget constraint becomes

$$\hat{a} = py - \psi(pc, pc_1, pc_2)$$
$$+ [a + zpd - pc_1 - qb - \theta(z' - z)] + (b - pc_2) - pc_3.$$

It is straightforward to derive the equilibrium condition

$$(\lambda + \varphi)\theta = \beta E_s[(\lambda' + \varphi')(\theta' + p'd')],$$

which is in addition to equations (5)–(14). From the results obtained above, if all three modes of payment are used to purchase consumption goods, then this equation can be written as

$$\frac{u_1(c)}{1 + \xi_1(pc)} [1 - \xi_2(pc)] \frac{\theta}{p}$$
$$= \beta E_s \left[\frac{u_1(c')}{1 + \xi_1(p'c')} [1 - \xi_2(p'c')] \left(\frac{\theta'}{p'} + d' \right) \right]. \quad (23)$$

Note that the nominal return from buying the claim z' at the beginning of this period and selling it at the beginning of the next period, that is, the holding period return, is equal to

$$\frac{\theta' + p'd'}{\theta} h'.$$

The ex post real holding period return is thus equal to

$$\frac{(\theta'/p') + d'}{\theta/p}.$$

Note also that in a comparison of equations (22) and (23), the term $[1 - \xi_2(pc) + \xi_3(pc)]/[1 - \xi_2(pc)]$ is absent from equation (23). This simply reflects the fact that equities offer no transaction service return.

F. Two Examples

Consider the following two examples to illustrate some key properties of this model. In both examples, suppose that households exhibit

constant relative risk aversion utility,

$$u(c) = \frac{c^{1-\tau}}{1-\tau}, \quad \tau > 0,$$

and suppose that the transaction cost function ψ is given as

$$\psi(c, c_1, c_2) = \overline{\psi} c^\alpha (c_1^\omega + \kappa c_2^\omega)^{(1-\alpha)/\omega}, \tag{24}$$

where $\overline{\psi} > 0$, $\alpha > 1$, $\kappa > 0$, and $0 < \omega < 1$. Transaction services are "produced" according to the constant elasticity of substitution production function $l = (c_1^\omega + \kappa c_2^\omega)^{1/\omega}$. This measure of transaction services enters a Cobb-Douglas transaction cost function $\overline{\psi} c^\alpha l^{1-\alpha}$. With this specification, the solution $pc > 1 + g$ to $r = \psi_3(pc, 1, g) - \psi_2(pc, 1, g)$ is

$$pc = \left[\frac{r}{\overline{\psi}(\alpha - 1)(1 + \kappa g^\omega)^{(1-\alpha-\omega)/\omega}(1 - \kappa g^{\omega-1})} \right]^{1/\alpha} \tag{25}$$

(the right side must always exceed $1 + g$). Note that the elasticity of the consumption velocity of cash (pc) with respect to the nominal interest rate (with g held constant) equals $1/\alpha$.

Example 1

Suppose that q and g are constant, the growth rate of the endowment $\gamma' = y'/y$ and the growth rate of dividends $\chi' = d'/d$ are independently and identically distributed, and inflation is constant at $\overline{\pi}$. Note that pc is then also constant. Write equation (22) as

$$\frac{1 - \xi_2(pc) + \xi_3(pc)}{1 - \xi_2(pc)} q = \frac{\beta}{\overline{\pi}} E[\gamma^{-\tau}].$$

The constant term $[1 - \xi_2(pc) + \xi_3(pc)]/[1 - \xi_2(pc)]$ thus captures the transaction service return that affects interest rates. The average (gross) real interest rate equals

$$\frac{1}{\beta E[\gamma^{-\tau}]} \frac{1 - \xi_2(pc) + \xi_3(pc)}{1 - \xi_2(pc)}.$$

The (gross) real interest rate in standard models (such as ours with $\kappa = 0$) is $1/\beta E[\gamma^{-\tau}]$. If $\kappa > 0$, then $\xi_3 < 0$, so a positive transaction service return to checkable deposits lowers the real interest rate.

The transaction service return that lowers the real interest rate also raises the equity premium. To see this, note that the solution for θ/pd is

$$\frac{\theta}{pd} = \frac{\beta E[\gamma^{-\tau}] E[\chi]}{1 - \beta E[\gamma^{-\tau}\chi]}.$$

The expected gross nominal return to equities is

$$\frac{\overline{\pi}}{\beta} \frac{E[\chi]}{E[\gamma^{-\tau}\chi]}.$$

The ratio of the expected gross return to equities over the gross return to bonds is given by

$$\frac{E[\gamma^{-\tau}]E[\chi]}{E[\gamma^{-\tau}\chi]} \frac{1 - \xi_2(pc)}{1 - \xi_2(pc) + \xi_3(pc)}.$$

The equity premium depends in the usual way on the covariance of the growth in marginal utility $\gamma^{-\tau}$ with the growth in dividends χ, but here this premium rises proportionately with $[1 - \xi_2(pc)]/[1 - \xi_2(pc) + \xi_3(pc)]$. If $\kappa > 0$, then this term is strictly greater than one; hence a positive transaction service return to checkable deposits raises the equity premium.

Example 2

Some additional insight can be gained by comparing the following special case of our model with the cash-credit model of Lucas and Stokey (1987). Suppose, in the transaction cost function ψ as parameterized in (24), that $\kappa = 0$. In this case, checkable deposits do not facilitate transactions. It is straightforward to verify that the solution pc is

$$pc = \left[\frac{r}{\overline{\psi}(\alpha - 1)}\right]^{1/\alpha}.$$

If the nominal interest rate is held constant, then pc is constant as well, and the equation corresponding to equation (20) can be written as

$$q = \beta E_s\left[\frac{(\gamma')^{-\tau}}{\pi'}\right].$$

For this example, the model thus reproduces exactly the Fisher interest rate equation.

This example can be seen as an alternative conceptualization of Lucas and Stokey in the following way.[6] Suppose that checkable de-

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[6] We wish to thank a referee for pointing out the particular way in which our model provides an alternative conceptualization of Lucas and Stokey. Suppose that $c_2 = 0$, and define the two goods, $c_A = c_1$ and $c_B = c_3 + \psi(c_1 + c_3, c_1, 0)$. Note that $c = c_1 + c_3$, c_A, and c_B are related by $c = c_A + c_B - \psi(c, c_A, 0)$, which can be used to write c as a function of c_A and c_B, say $c = G(c_A, c_B)$. One can then define utility directly over c_A and c_B as $v(c_A, c_B) = u(G(c_A, c_B))$ and write the sequence of constraints as $m \le a - qb$, $pc_A \le m$, and $\hat{a} \le py + m - pc_A - pc_B + rqb$. This is the setup of Lucas and Stokey.

posits do not facilitate transactions, so equation (18) becomes

$$q = \frac{1}{1 - \xi_2(pc)}.$$

This equation is formally identical to one in Lucas and Stokey (eq. 5.13), which is motivated by equating the marginal rate of transformation between cash and credit goods (q) to the marginal rate of substitution between these two types of goods ($1/[1 - \xi_2(pc)]$). In our model, the term $\xi_2(pc)$ is part of the technology, and not preferences, so our interpretation of this relationship is somewhat different. Here, the interest rate equals the transaction service return to using money.

Our model with cash, checks, and credit is also formally identical to one that distinguishes among three types of goods in household preferences, where the marginal rates of substitution depend on the transaction cost function ψ. Here, though, interpreting the relationship between the interest rate and marginal rates of substitution is not so clear, since purchasing fewer goods with cash and more on credit also relaxes the checking constraint.

III. An Extension to Multiperiod Bonds

In the data, term premia and maturity are positively related. However, standard models produce an essentially flat relationship between maturity and average term premia. In this section we explore the ability of our model to resolve this term premium puzzle.

Addressing this puzzle requires pricing multiperiod bonds in our model, which is not as straightforward as pricing multiperiod bonds in standard models. At issue is the capital risk inherent in multiperiod bonds other than one-period bonds and how this risk interacts with the ability of risky bonds to facilitate transactions. In particular, one would wish to ensure that checks do not bounce because of a drop in the value of risky assets backing checkable deposits. In what follows we extend our model to allow checkable deposits to be backed by multiperiod government bonds, and we ensure that checks do not bounce.

Suppose that households consider using longer-term bonds to back up their checking deposits. In particular, suppose that households can purchase two-period government bonds, b_2, at a price of q_2 and three-period government bonds, b_3, at a price of q_3. Households initially choose a portfolio of one-, two-, and three-period bonds, which has a current value of $qb + q_2 b_2 + q_3 b_3$. They write checks in the amount of pc_2, which clear at the beginning of the next period. The value of the bond portfolio when checks clear is $qb + q' b_2 + q'_2 b_3$, where q' and q'_2 are the market values of two- and three-period bonds

that have matured by one period (for simplicity, we still assume that checks clear before interest is paid on one-period bonds). Households do not know this value when they write checks, so to ensure that checks always clear, they write checks only up to the minimum value that this bond portfolio can take. The checking constraint is thus

$$pc_2 \leq qb + \min\{q'b_2 + q_2'b_3\}. \tag{26}$$

At this point we conjecture (we discuss this issue later) that q' and q_2' can realize their minimum values at the same time. Denote the minimum gross return on holding a two-period bond for one period by $\delta_2 = \min\{q'\}/q_2$ and that on a three-period bond by $\delta_3 = \min\{q_2'\}/q_3$. With δ_2 and δ_3, the checking constraint can be written as

$$pc_2 \leq qb + \delta_2 q_2 b_2 + \delta_3 q_3 b_3. \tag{27}$$

This approach can be extended to include any maturity of government bonds.[7]

We feel that this extension of our model can resolve some empirical puzzles concerning the term structure. The bond prices q, q_2, and q_3 must satisfy the Euler equations

$$\frac{1 - \xi_2 + \xi_3}{1 - \xi_2} \frac{u_1}{1 + \xi_1} (1 - \xi_2)q = \beta E_s\left[\frac{u_1'}{1 + \xi_1'} \frac{1 - \xi_2'}{\pi'}\right],$$

$$\frac{1 - \xi_2 + \delta_2\xi_3}{1 - \xi_2} \frac{u_1}{1 + \xi_1} (1 - \xi_2)q_2 = \beta E_s\left[\frac{u_1'}{1 + \xi_1'} \frac{1 - \xi_2'}{\pi'} q'\right],$$

and

$$\frac{1 - \xi_2 + \delta_3\xi_3}{1 - \xi_2} \frac{u_1}{1 + \xi_1} (1 - \xi_2)q_3 = \beta E_s\left[\frac{u_1'}{1 + \xi_1'} \frac{1 - \xi_2'}{\pi'} q_2'\right].$$

Note that the equilibrium we considered in Section II is unaltered by these equations if one assumes that all government bonds of maturity greater than one are in zero net supply.[8] Intuitively, the values of δ_2

[7] Note that, because of the possibility of default, conceptually, privately issued assets should have a δ of zero. Hence, primarily government bonds would be used to back checkable deposits. A useful extension of our model would be to consider a fixed but finite cost to violating the checking constraint in eq. (27). This would allow for the possibility of privately issued securities, in addition to government-issued securities, to back checkable deposits.

[8] This assumption avoids an explosion of the state space. Without this assumption, the state space would include the supplies of each maturity of government debt. Note, however, the following equivalence result. Consider two economies, one that issues only one-period bonds and another that issues bonds of various maturities. For the first economy denote the bond/money ratio by g_1, and for the second economy denote the n-period bond/money ratio by $\hat{g}_n$. As is evident from eqq. (8) and (27), if $g_1 = \Sigma \delta_n \hat{g}_n$, then these two economies are otherwise identical.

and δ_3 would crucially depend on the volatility of the asset (δ would fall as volatility rises), and on average the values of δ_2 and δ_3 should be less than one. Since $1 - \xi_2 + \delta_2\xi_3$, for example, is a decreasing function of δ_2, the average term structure should slope upward.

Because of the dependence of the δ's on volatility, neither long-term bonds nor equities play a role in facilitating transactions; hence primarily short-term bonds (and money) facilitate transactions. This result of our model seems consistent with certain observed features of money market mutual funds and bank checkable deposits. Money market mutual funds that offer a check writing feature have their funds invested in short-term government bonds. Bank checkable deposits also offer a fixed interest rate and are not subject to any capital risk. Evidently one buys the convenience of check writing at the expense of investing in nominally riskless assets.[9] Moreover, many accounts require that they be collateralized by short-term government securities, such as a margin account on organized securities exchanges across the United States (e.g., the Chicago Board of Trade and the New York Stock Exchange [NYSE]). These examples corroborate our result that there is a special role for short-term government bonds that is intimately related to managing accounts that are used for transaction purposes.

IV. Estimation of the Structural Parameters

A. The Data

The monthly data set we use runs from January 1959 to June 1991, which gives us 390 observations. The macroeconomic time series are taken from Citibase (using seasonally adjusted data). We identify the real consumption series with the consumption of nondurables (GMCN82) plus services (GMCS82), and we identify the implicit deflator on this series as the price series. Fiat currency in the model is identified with the series FMSCU, which consists of currency in circulation. We equate the quantity of riskless assets with the series FML (net of currency in circulation), which consists of M3 plus the nonbank public holdings (net of money market mutual fund holdings) of U.S. savings bonds, short-term (less than 12 months to maturity) Treasury securities, commercial paper, and banker's acceptances. Since the series FML contains securities with a maturity in excess of 1 month, this series is not strictly nominally riskless at a 1-month horizon. We nevertheless use FML since the monthly capital

[9] There are also legal restrictions on what types of deposits can be checkable. Indeed, one interpretation of our model is that it explains why legal restrictions require that money market mutual funds be invested in bonds of maturity less than a year.

risk for these securities is relatively small. Since currency and riskless assets are used to purchase more than just nondurables and services, we scaled down our measure of currency and riskless assets by the average ratio of nondurables and services to gross national product over this time period, which equals .55. As in the model, all nominal aggregates are deflated by the stock of fiat currency. Value-weighted equity returns for the NYSE and monthly holding period returns for bonds with 1 and 6 months left to maturity are taken from data distributed by the Center for Research in Security Prices.

B.   Estimation Procedure

With this data set we exploit the restrictions imposed by the model to estimate the structural parameters using the generalized method of moments (GMM) procedure developed by Hansen (1982) and Hansen and Singleton (1982). We estimate the five structural parameters β, τ, $\overline{\psi}$, α, and κ. The two parameters β and τ describe preferences: β is the subjective discount factor, and τ is the degree of risk aversion in a constant relative risk aversion utility function $u(c) = c^{1-\tau}/(1-\tau)$. The three parameters $\overline{\psi}$, α, and κ describe the transaction cost function [10]

$$\psi(c, c_1, c_2) = \overline{\psi} c^{\alpha} (\sqrt{c_1} + \kappa \sqrt{c_2})^{2(1-\alpha)}.$$

With this functional form for the transaction cost function, note that

$$\xi_1(v; g) = \overline{\psi} \alpha v^{(\alpha-1)} (1 + \kappa \sqrt{g})^{2(1-\alpha)}$$

and

$$\xi_2(v; g) = \overline{\psi}(1 - \alpha) v^{\alpha} (1 + \kappa \sqrt{g})^{2(1-\alpha)-1}.$$

We use equations (21) and (23) to construct stationary forecast errors in the following way. Define k' as

$$k' = \beta \left(\frac{c'}{c}\right)^{-\tau} \frac{1}{\pi'} \left[\frac{1 + \xi_1(v; g)}{1 + \xi_1(v'; g')}\right] \left[\frac{1 - \xi_2(v'; g')}{1 - \xi_2(v; g)}\right]. \tag{28}$$

With k', the forecast errors for equations (21) and (23) are, respectively,

$$e_1' = 1 - k'[1 - \xi_2(v; g)]$$

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[10] We did not estimate the parameter ω in the transaction cost function specified in eq. (24) since it seemed to be nearly collinear with κ. We set $\omega = .5$. It was necessary to choose $0 < \omega < 1$ to satisfy the condition $1 - \kappa g^{\omega-1} > 0$ for a wide range of values for κ.

and

$$e'_2 = 1 - k' R'_e,$$

where R_e denotes the gross return to equity. As long as real consumption growth, v, and g are stationary, the forecast errors will be stationary as well. In the model we assume that consumption growth and g are stationary, and the stationarity of v follows from equation (18) and our assumption that nominal interest rates are stationary.

We construct 10 orthogonality conditions by multiplying each forecast error by five instruments. The five instruments consist of a constant and one-period lagged values of real consumption growth, inflation, the gross nominal risk-free return, and the gross nominal monthly holding period return on a 6-month bond.

C. Estimation Results

Table 1 contains the GMM parameter estimates along with their standard errors. The GMM criterion function, which is distributed as a χ^2 with five degrees of freedom, has a value of 10.79. This implies a p-value of 5.57 percent. The discount factor β and the risk aversion parameter τ are estimated reasonably well at .998 and 1.49, respectively. The parameters in the transaction cost technology are estimated somewhat imprecisely as $\overline{\psi} = .00569$, $\alpha = 5.19$, and $\kappa = 1.23$. The overall transaction cost ξ_0 is fairly small; the average ξ_0 at the

TABLE 1

A. GMM PARAMETER ESTIMATES

Parameter	Estimate	Standard Error
β	.99818	.0024
τ	1.48808	.4199
κ	1.23112	.9468
α	5.18807	8.1028
ψ	.00569	.0234

B. STATISTICS OF TRANSACTION COSTS*

Variable	Average	Standard Deviation
ξ_0	.0140	.0018
ξ_1	.0033	.0004
ξ_2	−.0096	.0013
ξ_3	−.0028	.0005

NOTE.—GMM criterion function is 10.7866; p-value is .0557.
* Average and standard deviation refer to the sample statistics of the transaction cost function evaluated at the estimated parameter values.

estimated parameters is .014, implying that 1.4 percent of the endowment is lost in transaction costs. The average value of ξ_3, which influences the average equity premium, is $-.0028$.

Note that the estimate of the risk aversion parameter is similar to the value used by Mehra and Prescott as well as the value estimated by Hansen and Singleton. Moreover, we estimate β to be less than one, whereas many studies that estimate a relatively low risk aversion parameter also estimate β to be greater than one (e.g., Dunn and Singleton 1986; Kocherlakota 1988).

We also evaluate the hypotheses that $\kappa = 0$ or $\kappa = 2$. Under the null that $\kappa = 0$, only cash provides a transaction service. This hypothesis is tested by reestimating the structural parameters at the null of $\kappa = 0$ using the optimal weighting matrix of the unrestricted model. The difference in the value of the restricted and unrestricted GMM criterion function is then distributed as a χ^2 with one degree of freedom. In our case this difference was 19.45, implying a strong rejection of the null that $\kappa = 0$. In this sense, this test rejects the Lucas and Stokey special case of our model. On the other hand, the model could not reject the hypothesis that $\kappa = 2$: the difference in the GMM criterion function was .09. This suggests that the transaction service return to riskless assets is an important feature of the data.

It is important to note that equation (18) implies a stochastic singularity among r, v, and g in the model. Clearly one should expect this exact relationship to be violated in the data. We would like to make two observations regarding this issue. First, to provide a sense of the degree to which this relationship is violated in the data, at the estimated parameter values the mean and standard deviation of $r - \xi_3(v; g) + \xi_2(v; g)$ are $-.0018$ and $.0021$, respectively, in the data (these values are zero in the model).[11] Also, at the estimated parameter values the correlation between r and $\xi_3(v; g) - \xi_2(v; g)$ is .38 in the data (it is one in the model). Second, it is possible to break the stochastic singularity in the model by introducing shocks to the transaction cost function. Equation (18) could then be used to identify these shocks in the data. As these shocks should not substantially affect average transaction service returns, we suspect that the results of such an exercise would not alter the main conclusions of this paper.

V. A Comparison of the Model's Predictions to the Data

In this section we compare the model's predicted distribution for asset returns, velocity, inflation, money growth, riskless assets, and

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[11] The variables r, v, and g are measured on a monthly basis (not annualized).

consumption growth to the corresponding distribution estimated from U.S. data.

A. Procedure for Solving the Model

To solve the model we require the parameter estimates from the previous section, a specification of the stochastic process for the exogenous variables, and an inflation process that satisfies equation (21). The process for the log interest rate, log riskless assets, and endowment growth is modeled as a first-order vector autoregression (VAR(1)). The gross inflation process is modeled as

$$\log[\pi(s, s')] = \log[\bar{\pi}(s)] + \eta(s'),$$

where the forecast error η is normally distributed, has mean zero, and is correlated with the innovations in the other exogenous variables. Hence, the forecast errors for inflation are exogenous (and chosen by us) and the model determines the conditional mean $\log(\bar{\pi})$ to satisfy equation (21). Given this specification of our model, the details of the algorithm for solving the model are described in Appendix B.

Table 2 reports summary statistics for a restricted VAR(1) fitted to U.S. data on the log of the interest rate, the log of riskless assets, consumption growth, and inflation.[12] The restriction we impose is that lagged inflation does not enter the first three equations. In the model, the VAR coefficients on the log interest rate, log riskless assets, and endowment growth are those reported in table 2. Since the correlations among the forecast errors for the interest rate, riskless assets, and consumption growth are quite small, for the model we simply impose a diagonal covariance matrix for these innovations. The diagonal entries are chosen so that the variances of these innovations match those reported in table 2. We choose the forecast error η to have the same variance as the forecast error of inflation and the same covariance with the remaining three forecast errors as reported in table 2.

B. The Risk-Free and the Equity Premium

Mehra and Prescott (1985) documented that standard representative-agent models, reasonably restricted, are unable to simultaneously ex-

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[12] We use consumption as our proxy for the endowment since at the estimated parameter values these two variables differ on average only by 1.4 percent. Also, by consumption growth we mean the first difference of the log of consumption, and by inflation we mean the first difference of the log of the price level.

TABLE 2

VAR(1) FITTED TO U.S. DATA, JANUARY 1959 TO JUNE 1991

	$\log(r)$	g	c	p
Constant	$-.9394$	.0258	$-.0079$	.0163
	(.2708)	(.0078)	(.0089)	(.0055)
$\log(r)$	.9182	.0005	$-.0017$	.0022
	(.0209)	(.0006)	(.0007)	(.0005)
g	.1717	.9922	$-.0006$	$-.0007$
	(.0612)	(.0017)	(.0020)	(.0011)
c	3.1094	.0475	$-.2834$	.0681
	(1.4891)	(.0425)	(.0491)	(.0303)
p				.3727
				(.0516)
R^2	.9279	.9995	.0942	.3384
Standard deviation residual	.1267	.0036	.0042	.0025
	Residual Correlations			
$\log(r)$	1.0000	.0188	$-.0656$	.1881
g	.0188	1.0000	.0047	$-.0005$
c	$-.0656$	.0047	1.0000	$-.3149$
p	.1881	$-.0005$	.3149	1.0000

NOTE.—Standard errors are in parentheses. Definition of variables: r is the net nominal interest rate (1-month Treasury bill); g is the log of riskless assets (M3 plus nonbank public holdings of U.S. savings bonds, short-term Treasury securities, commercial paper, and banker's acceptances; c is the consumption growth rate (nondurables plus services); p is the inflation rate (implicit consumption deflator); residual refers to the one-step-ahead forecast errors. All variables are measured on a monthly basis (not annualized).

plain the low average risk-free rate and the high equity premium observed in the data. Table 3 reports the behavior of the real interest rate and the equity premium in the data and in simulations of our model with the parameter estimates in tables 1 and 2. Equity in our model corresponds to a claim on aggregate consumption (i.e., $d = c$). In the data the average ex post real interest rate is 1.12 percent at an annual rate, and the average equity premium is 5.02 percent at an annual rate. In the estimated model these two rates are 4.00 percent and 2.42 percent, respectively, so at the point estimate of the model's parameters, the model captures about 50 percent of the equity premium. This is a substantial improvement over previous models (as documented, e.g., by Mehra and Prescott [1985]).

A key parameter that determines the average risk-free rate and equity premium is κ. Recall that κ = 2 could not be rejected whereas κ = 0 was strongly rejected. Table 4 reports the real interest rate and the equity premium as κ ranges from zero to two. At κ = 2, the average real interest rate becomes 1.46 percent and the average equity premium becomes 5.25 percent. Although not reported here, the remaining statistics for the model with κ = 2 (and $\overline{\psi}$ = .350 to

TABLE 3

RISK-FREE RATE AND EQUITY PREMIUM

Statistic	Data	Model
Nominal interest rate (r):		
Average	5.93	5.63
Standard deviation	2.76	1.85
Inflation rate (p):		
Average	4.81	1.36
Standard deviation	3.61	4.89
Nominal equity return:*		
Average	10.95	8.05
Standard deviation	52.42	6.25
Ex post real interest rate:		
Average	1.12	4.00
Standard deviation	3.27	3.95
Equity premium:†		
Average	5.02	2.42
Standard deviation	52.68	5.69

NOTE.—All variables are annualized percentage rates (multiplied by 1,200). Data refers to the data described in table 2; model refers to the model at the estimated parameter values of table 2.
    * Value-weighted NYSE.
    † Nominal equity return minus the nominal interest rate.

TABLE 4

RISK-FREE RATE AND EQUITY PREMIUM IN THE MODEL:
DEPENDENCE ON κ

κ	Ex Post Real Interest Rate	Equity Premium
.00	6.61	.06
.25	6.26	.42
.50	5.85	.83
.75	5.39	1.30
1.00	4.85	1.84
1.25	4.22	2.47
1.50	3.47	3.22
1.75	2.57	4.13
2.00	1.46	5.25

NOTE.—All variables are anualized. Model refers to the model at the estimated parameter values in table 1 and the indicated value of κ. For this range of κ, the standard deviation of the equity premium is always close to 5.7.

match average velocity)[13] are not much affected by the higher choice for κ. Hence this model can generate the low average real risk-free interest rate and high equity premium that are observed in the data. Moreover, this model generates these values without predicting a standard deviation of the real interest rate that is higher than that observed in the data.

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[13] Changing $\bar{\psi}$ essentially changes only average velocity.

C. Inflation, Real Equity Returns, and Real Interest Rates

A variety of papers have attempted to explain the observed negative relationship between inflation and real equity returns and between inflation and real interest rates. For example, Marshall (1992) is able to explain these relationships using a related transaction cost model. Table 5 documents the relationship between inflation, real equity returns, and the real interest rate observed in the data and in simulations of our estimated model with $\kappa = 2$. The correlation between inflation and real equity returns is $-.17$ in the data and $-.36$ in the estimated model. Moreover, the correlation between forecast errors for inflation and real equity returns is $-.12$ in the data and $-.29$ in the estimated model. This model evidently does quite well in explaining the negative relationship between inflation and real equity returns. Note also that the estimated model matches reasonably well the negative correlation between inflation and the ex post real interest rate.

D. Term Structure Implications

Backus et al. (1989) documented that the model considered by Mehra and Prescott is unable to generate an upward-sloping term structure that quantitatively matches the one observed in the data. This implies that in these models the relationship between average holding period returns and maturity is essentially flat. In the data (see table 6) there is a positive relationship between average holding period returns and maturity. Qualitatively, as we argued above, it seems clear that our model can generate an upward-sloping term structure and a positive relationship between average bond holding period returns and maturity. To explore whether or not this model can quantitatively match this feature of the observed term structure, we simulated prices of multiperiod bonds under the following assumptions. First, we assume that all bonds but one-period bonds are in zero net supply. Second, we use the data to estimate the δ_j's that we use to generate simulation of our model. We assume a parametric relationship among δ_j's such that $\delta_j = \bar{\delta}^{j-1}/q_j$. A value of $\bar{\delta} = .98$ is consistent with the minimum prices for all bonds of maturity from 1 to 11 months observed in the data, which are reported in table 6.[14]

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[14] Under the assumption of normally distributed VAR innovations, the values of δ_j should be much lower in the model, which would lead to a steeper term structure. Alternative approaches would be to estimate the value of $\bar{\delta}$ along with the other structural parameters or to use the δ_j's that are consistent with the model at the estimates of the parameters in table 1 and the exogenous processes.

TABLE 5

INFLATION, REAL INTEREST RATES, AND REAL EQUITY RETURNS

Statistic	Data	Model
Autocorrelation inflation rate	.50	.41
Correlation:		
Inflation rate and ex post real interest rate	−.68	−.96
Inflation rate and real equity return	−.17	−.36
Inflation rate residual and real interest rate residual	−.96	−.97
Inflation rate residual and real equity return residual	−.12	−.29

NOTE.—Residual refers to the forecast errors from a first-order vector autoregression with the ex post real interest rate, log riskless assets, consumption growth, inflation, log velocity, real equity return, and currency growth. Data refers to the data described in table 2. Model refers to the model at the estimated parameter values in table 1, except with $\kappa = 2$ and $\psi = .35$.

Table 6 reports average simulated monthly holding period returns (annualized) for 1–11-month bonds for two versions of the model, one with $\kappa = 0$ and one with $\kappa = 2$, as well as for U.S. data. Note that, for both the model with $\kappa = 2$ and the data, average holding period returns rise with maturity, whereas they do not vary with maturity for the model with $\kappa = 0$. In particular, for the data there is a 90-basis-point spread between the average 1- and 11-month holding period returns, for the model with $\kappa = 2$ there is a 74-basis-point spread, and for the model with $\kappa = 0$ there is a four-basis-point spread. Note also that for all three cases the standard deviation of holding period returns rises with maturity. These results show that our model is capable of explaining some important features of the term structure that standard models (such as ours with $\kappa = 0$) have had considerable difficulty with.[15] One thing that our model does not explain, however, is why the term structure is so steeply sloped at very short maturities (2–6 months). Figure 1 displays the ability of our model to match some features of the observed average term premium, as well as the average real interest rate and equity premium.

Given the high positive correlation among bond yields that is observed for various maturities, the assumption that allows us to use equation (27) is very attractive. We numerically verified that this assumption is valid for the model with the just mentioned values of δ_j.

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[15] There are a variety of other term structure related puzzles that we do not explore and that this model may explain, such as the different behavior of the short and long ends of the term structure, and the relationship between spot and forward interest rates for all maturities (see, e.g., Backus et al. 1989; Mishkin 1990). An in-depth analysis of these issues is a full-length project in itself.

TABLE 6
MATURITY AND EXCESS HOLDING PERIOD RETURNS

| | DATA | | | | MODEL | | | |
| | 1/59–6/91 | | 7/63–6/91* | | $\kappa = 2$ | | $\kappa = 0$ | |
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
h2	.57	.81	.36	.71	.08	.74	.03	.74
h3	.78	1.29	.67	1.37	.16	1.43	.03	1.43
h4	.72	1.96	.68	2.03	.24	2.06	.03	2.05
h5	1.02	2.56	.85	2.68	.31	2.64	.04	2.62
h6	1.05	3.10	.89	3.24	.39	3.17	.04	3.13
h7			.84	3.84	.46	3.67	.04	3.60
h8			1.06	4.44	.54	4.13	.04	4.03
h9			1.12	5.14	.61	4.55	.04	4.42
h10			.84	5.87	.68	4.95	.04	4.78
h11			.90	6.42	.74	5.32	.04	5.10

NOTE.—$h2$–$h11$ are (annualized) monthly holding period returns for maturities 1–11 in excess of the one-period interest rate. Both data sets are taken from the Center for Research in Security Prices. Data refers to the data described in table 2. Model refers to the model at the estimated parameter values in tables 1 and 2 and the indicated value of κ. The model with $\kappa = 2$ has $\bar{\delta} = .98$.

* The minimum prices for discount bonds for maturities 1–11 months for the sample period 7/63–6/91 are .9865, .9735, .9609, .9480, .9477, .9212, .9105, .9877, .8852, .8531, and .8613.
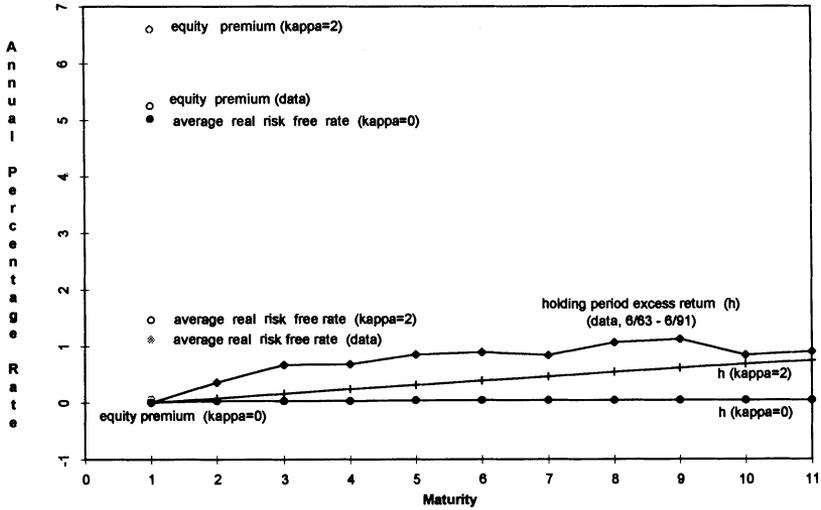
Fig. 1.—The real risk-free rate, equity premium, and term premium

E. Hansen-Jagannathan Bounds

Hansen and Jagannathan (1991) develop nonparametric variance bounds that provide an alternative characterization of the excess return puzzles.[16] Using these bounds, Hansen and Jagannathan and Cochrane and Hansen (1992) document the failure of a variety of models in explaining asset market puzzles. In this subsection we explore the implications of these bounds for our model.

As shown in Hansen and Jagannathan (1991), the ratio of average asset returns in excess of the risk-free interest rate to the standard deviation of this excess return (the Sharpe ratio) must satisfy the following bound:

$$\frac{\sigma(k)}{E(k)} \geq \frac{E(\zeta)}{\sigma(\zeta)},$$ (29)

where ζ is any excess return and k is the intertemporal marginal rate of substitution of a given model. The excess return puzzle associated with equity and bond returns implies that the ratio $\sigma(k)/E(k)$ in many models is much lower than observed Sharpe ratios for equities and bonds. Note that this way of confronting asset return puzzles leads one to consider the mean relative to the standard deviation of the asset's excess return.

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[16] Bansal and Lehmann (1996) provide an additional nonparametric bound to characterize asset market puzzles.

From the perspective of our model, with regard to equities the bound in expression (29) imposes restrictions on the excess return

$$\zeta_e' = R_e' - \frac{1 - \xi_2}{1 - \xi_2 + \xi_3}(1 + r).$$

Notice that this excess return is $R_e' - (1 + r)$ in our model with $\kappa = 0$; this is the case in which no asset other than money offers a transaction service return. For values of κ that exceed zero, the average excess return, $E(\zeta_e')$, is smaller than the observed equity premium, $E[R_e' - (1 + r)]$, by an amount equal to the transaction service return offered by risk-free assets. This feature tends to lower the Sharpe ratio on ζ_e' and hence makes it easier for a given model to satisfy expression (29). A similar argument also applies to bonds. The appropriate excess returns for bonds also include any transaction service return that they may offer, which will also lower the Sharpe ratios of their excess returns.

Using the data and the point estimates reported in table 1, we construct a time series for k as defined by equation (28). On the basis of this time series, the ratio $\sigma(k)/E(k)$ equals .01. Without adjustment for transaction service returns, the Sharpe ratio in the data for the market return is .10 and for a 6-month bond is .33. When we adjust for transaction service returns, at the point estimates reported in table 1 the Sharpe ratio in the data for the market return is .03 and for a 6-month bond is .25. Hence, the Sharpe ratios on the transaction service adjusted returns are considerably smaller. Even though the bounds above are still violated, from the perspective of our model the high Sharpe ratios observed in the data are less puzzling.

An alternative characterization of the failure of a variety of models is that they generate Sharpe ratios (constructed in the usual way) that are too low relative to those observed in the data. Sharpe ratios in our model can easily be computed from simulations of the model. With $\kappa = 0$,[17] the model generates a Sharpe ratio for equities equal to .01 and for 6-month bonds equal to .01. Indeed, the Sharpe ratios for 2–11-month bonds are all roughly equal to .01. These values are very low compared to those observed in the data. As κ is increased from zero to two, the Sharpe ratio for equities rises from .01 to .92 and for 2–11-month bonds rises from roughly .01 to roughly .12.

As mentioned above, the equity premium at $\kappa = 2$ in the model is close to that observed in the data. However, at this value of κ, the Sharpe ratio for equities is much higher than that observed in the

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[17] In terms of real variables, such a version of our model is essentially the same as models simulated by Mehra and Prescott (1985) and Backus et al. (1989).

data. This is an outcome of the rather low variability of the equity return in the model. At $\kappa = 2$, the Sharpe ratio on 2–11-month bonds in our model equals roughly .12, which is the value observed for 11-month bonds in the data but is about a third of the value observed for 2-month bonds. In terms of the term structure, these results reflect a substantial improvement over standard models. These results suggest that increasing the variability of equity returns without affecting average equity and bond returns or variability of bond returns is required to explain observed risk premia and Sharpe ratios.

As just mentioned, our model, as well as the model of Mehra and Prescott (see Cecchetti, Lam, and Mark 1993), generates a standard deviation of the equity return that is much lower than that observed in the data. This failure of these models presents a substantial challenge. A potential explanation for this low-variance puzzle may be related to the appropriate measure of payoffs to equity. Recent results of Ackert and Smith (1993) show that the variance bounds tests of LeRoy and Porter (1981) and Shiller (1981) are satisfied if one identifies the price process with the market value of firms and the dividend process with the sum of ordinary dividends, share repurchases, and takeover distributions. This suggests that a model that matches the properties of such a dividend process may also match the standard deviation of equity returns. As in Mehra and Prescott, we identify the dividend process on equity with aggregate consumption, which may have properties quite different from those of the dividend process constructed by Ackert and Smith.

F. Velocity

As emphasized by Hodrick et al. (1991), an important prediction of monetary models concerns the behavior of the velocity of money. They show that, for a wide range of parameter configurations, the cash-credit model of Lucas and Stokey (1987) predicts too low a standard deviation of velocity and too high a correlation between velocity and the nominal interest rate.

For the transaction velocity of currency, table 7 reports summary statistics from the data and from simulations of the model with the parameter estimates in tables 1 and 2, but with $\kappa = 2$ and $\overline{\psi} = .35$. The average value of log velocity in the data is 3.10, and the average value of log velocity in the model is 3.05. The standard deviation of velocity in the data is .11, which is somewhat higher than the value of .07 predicted by the model. The correlation between velocity and nominal interest rates is .74 in the data, which is somewhat lower than .94 observed in the model. An examination of forecast errors,

TABLE 7

VELOCITY

Statistic	Data	Model
Log velocity:		
Average	3.10	3.05
Standard deviation	.11	.07
Autocorrelation	.99	.94
Residual standard deviation	.0044	.0245
Correlation:		
Log velocity and nominal interest rate	.74	.93
Log velocity residual and log nominal interest rate		
residual	−.0133	.9944

NOTE.—Log velocity refers to the consumption velocity of fiat currency. Residual refers to the forecast errors from a first-order vector autoregression with the log interest rate, log riskless assets, consumption growth, inflation, log velocity, equity return, and currency growth. Data refers to the data described in table 2. Model refers to the model at the estimated parameter values of table 1, but with $\kappa = 2$ and $\bar{\psi} = .35$.

however, reveals more significant mismatches between the data and the model. In particular, the model predicts a standard deviation of the forecast error of velocity equal to .0245, which is substantially higher than .0044 observed in the data. The model predicts a correlation between the forecast errors of velocity and the nominal interest rate equal to .9944, which is much higher than −.0133 observed in the data. The high correlation in the model is due to the contemporaneous relationship between the interest rate and velocity as a consequence of equation (18). These failures of the model should not be surprising since both the model in Lucas and Stokey (1987) and this model are designed so that velocity reacts to the interest rate in roughly the same way. In these models velocity reacts immediately to changes in the nominal interest rate, but evidently a more delayed response is required to match the behavior of velocity observed in the data.

G. Money Growth

In solving the model, we determined endogenously that money growth was consistent with the behavior of interest rates and inflation. It seems particularly important, then, to compare the behavior of money growth in the data to that in the model. For both the data and the model, table 8 documents the relationship between money growth and interest rates, consumption growth, inflation, and velocity. In the data, the forecast error correlation between money growth and interest rates is .11 and the forecast error correlation between money growth and velocity is −.41. In the model with $\kappa = 2$, these correlations are −.98 and −.99, respectively.

TABLE 8

MONEY GROWTH

Statistic: Correlation	Data	Model
Money growth rate residual and log of nominal interest rate residual	.11	−.98
Money growth rate residual and consumption growth rate residual	.10	.16
Money growth rate residual and inflation rate residual	.14	−.11
Money growth rate residual and log velocity residual	−.41	−.99
Inflation rate residual and consumption growth rate residual	−.31	−.30

NOTE.—Residual refers to the forecast errors from a first-order vector autoregression with the log interest rate, log riskless assets, consumption growth, inflation, log velocity, equity return, and currency growth. Model refers to the model at the estimated parameter values in table 1, but with $\kappa = 2$ and $\bar{\psi} = .35$.

As discussed in the previous section, in the model the forecast error correlation between the interest rate and velocity is almost one; hence the forecast errors for velocity and interest rates are almost identical. This explains why, in the model, the forecast error correlations between money growth and interest rates are almost the same as the forecast error correlations between money growth and velocity. To explain the large discrepancy in the forecast error correlations between money growth and interest rates, note that in the model money growth is determined by

$$\ln h' = \ln \pi' - \ln\left(\frac{v'}{v}\right) + \ln\left(\frac{c'}{c}\right).$$

Recall also that the forecast error correlations between the interest rate and inflation and consumption growth were chosen to match those in table 2. These correlations are fairly small. Hence the forecast error correlation for money growth and interest rates, to satisfy this functional relationship, is negative. As it turns out, in the model the variance of the forecast error for velocity far exceeds that for inflation or consumption growth, which explains the high magnitude of the correlation. To summarize, as in the previous section, the anomalous behavior of money in the model seems due to the tight contemporaneous relationship between interest rates and velocity.

VI. Concluding Remarks

In this paper we extended a well-understood monetary model (such as Lucas and Stokey [1987]) to include payment of goods with cash, checks, and credit. This model allowed us to consider the usefulness

of assets other than narrowly defined money in facilitating transactions. We evaluated the ability of this model to explain the risk-free rate, equity premium, and term premium puzzles. We also explored the model's implications for the joint behavior of asset returns, velocity, and inflation.

We found that the model could quantitatively match up with the observed average risk-free rate and equity premium. The model could also quantitatively account for much of the positive relationship between term premia on bonds and maturity. These results are presented in figure 1. Our model had difficulties in satisfying the Hansen-Jagannathan (1991) bounds, but our modification of the standard representative-agent model made it easier to satisfy these bounds. The model matched up well with the observed correlations among inflation, real equity returns, and real interest rates. The model, however, had difficulties in matching the observed behavior of velocity and its correlation with interest rates.

The model developed in this paper primarily focused on the role of government debt in facilitating transactions. One can also interpret this role of government debt from the perspective of liability management, which is similar in spirit to Tobin (1963, p. 156). Faced with a known value of some future liability and costs of not meeting this liability, the economic agent must decide what portfolio best allows him to meet this liability in every future state. From this perspective, short-term government debt offers an advantage over risky securities. This in turn affects the observed market value of government debt. This paper showed that incorporating this feature is important to understanding relative rates of return across equity and bond markets. More generally, this paper emphasizes the need to understand the special role some securities play in a representative-agent economy with frictions to explain observed returns on various financial assets.

Appendix A

Here we study the equilibrium without assuming that all three modes of payment are used. As in the text, denote $\psi_0 = \psi$, but now define ξ_i, for $i = 0$, 1, 2, or 3, as

$$\xi_i(v; g) = \psi_i\left(1, \frac{1}{v}, \min\left\{1 - \frac{1}{v}, \frac{g}{v}\right\}\right).$$

To economize on notation, write $\xi_i(v; g)$ simply as $\xi_i(v)$. For $0 < q < 1$ and for a given λ, we shall first show that if the eight functions c, c_1, c_2, c_3, p, φ, μ, and ζ satisfy the following eight equations, then they satisfy the eight

equations (5)–(12):

$$p = \max \begin{cases} \min\left\{ \dfrac{1+g}{c}, \dfrac{u_1(c)}{\lambda[1 + \xi_1(pc) + \xi_3(pc)]} \right\} \\[2ex] \dfrac{u_1(c)}{\lambda[1 + \xi_1(pc)]}, \end{cases} \tag{A1}$$

$$\varphi = \min \begin{cases} \max\left\{ \dfrac{cu_1(c)}{1+g} - \lambda[1 + \xi_1(pc) + \xi_2(pc)], \ -\lambda[\xi_2(pc) - \xi_3(pc)] \right\} \\[2ex] -\lambda\xi_2(pc), \end{cases} \tag{A2}$$

$$\mu = \max \begin{cases} \min\left\{ \lambda[1 + \xi_1(pc)] - \dfrac{cu_1(c)}{1+g}, \ -\lambda\xi_3(pc) \right\} \\[2ex] 0, \end{cases} \tag{A3}$$

$$\zeta = \min \begin{cases} \max\left\{ \dfrac{cu_1(c)}{1+g} - \lambda[1 + \xi_1(pc) + \xi_3(pc)], 0 \right\} \\[2ex] -\lambda\xi_3(pc), \end{cases} \tag{A4}$$

$$c = \frac{y}{1 + \xi_0(pc)}, \tag{A5}$$

$$pc_1 = 1, \tag{A6}$$

$$pc_2 = \min\{pc - 1, g\}, \tag{A7}$$

$$pc_3 = \max\{0, pc - 1 - g\}. \tag{A8}$$

PROPOSITION 1. For $0 < q < 1$ and for a given λ, if the eight functions c, c_1, c_2, c_3, p, φ, μ, and ζ satisfy equations (A1)–(A8), then they satisfy the eight equations (5)–(12).

Proof. Clearly, if equations (A6)–(A8) hold, then so does equation (5). Because of the homogeneity of ψ, equations (A6) and (A7) imply that

$$\xi_0(pc) = \frac{\psi(pc, pc_1, pc_2)}{pc},$$

$$\xi_i(pc) = \psi_i(pc, pc_1, pc_2), \quad i = 1, 2, 3.$$

Hence, if equation (A5) holds, then so does equation (6). It remains, then, to show that equations (7)–(12) hold. The proof is similar to that in the previous economy. There are three terms that could be selected for p in equation (A1); number these terms sequentially, starting with the upper left-most term $(1 + g)/c$. What will be proved is that if the ith term is selected for p, then the ith term must also be selected for φ, α, and ζ. This defines all the combinations that are possible, from which it is a straightforward matter to verify that equations (10)–(12) are satisfied for each combination. These results will then be used to show that equations (7)–(9) are satisfied.

Term 1.—If $p = (1 + g)/c$ is selected, then this implies the two inequalities

$$\frac{u_1(c)}{\lambda[1 + \xi_1(pc)]} \le \frac{1 + g}{c} \le \frac{u_1(c)}{\lambda[1 + \xi_1(pc) + \xi_3(pc)]}.$$

With these results, it is straightforward to verify that the first terms are selected for φ, α, and ζ.

Term 2.—If

$$p = \frac{u_1(c)}{\lambda[1 + \xi_1(pc) + \xi_3(pc)]}$$

is selected, then this implies the inequality

$$\frac{u_1(c)}{\lambda[1 + \xi_1(pc) + \xi_3(pc)]} \le \frac{1 + g}{c}.$$

With these results, it is straightforward to verify that the second terms are selected for φ, α, and ζ.

Term 3.—If

$$p = \frac{u_1(c)}{\lambda[1 + \xi_1(pc)]}$$

is selected, then this implies the inequality

$$\frac{1 + g}{c} \le \frac{u_1(c)}{\lambda[1 + \xi_1(pc)]}.$$

With this result, it is straightforward to verify that the third terms are selected for φ, α, and ζ.

As mentioned above, it is straightforward then to verify that for each of these three cases, equations (10)–(12) hold. We also leave it for the reader to verify that equations (7)–(9) are satisfied. For each of these equations, this simply involves searching the three terms above for when the corresponding multiplier is strictly positive and then using the implied inequalities to show that the equation holds with equality. Q.E.D.

Using equations (14) and (13), note that

$$(\lambda + \varphi)q = \lambda + \zeta q.$$

Use this result in equation (10) to show that

$$(\lambda + \zeta q)p = \{u_1(c) - \lambda p[\xi_1(pc) + \xi_2(pc)]\}q.$$

Rewrite equation (11) as

$$(\lambda q + \zeta q)p = \{u_1(c) - \lambda p[\xi_1(pc) + \xi_3(pc)]\}q.$$

Subtract these two equations and divide by λp to arrive at

$$r = \xi_3(pc) - \xi_2(pc). \tag{A9}$$

Given r, equation (A9) determines pc. There exists a unique pc that solves this equation under the same assumptions on ψ that were mentioned in the text.

With pc determined, equation (A5) determines c. Divide both sides of equation (A1) by p to derive an equation in pc, c, and λp:

$$1 = \max\begin{cases} \min\left\{\dfrac{1 + g}{pc}, \dfrac{u_1(c)}{\lambda p[1 + \xi_1(pc) + \xi_3(pc)]}\right\} \\[2ex] \dfrac{u_1(c)}{\lambda p[1 + \xi_1(pc)]}. \end{cases} \tag{A10}$$

With pc and c already determined, this is one equation in the unknown function λp. Note that the right side of equation (A10) is a decreasing (although not strictly decreasing) function of λp. If $pc \neq 1 + g$, then there exists a unique λp that solves this equation. If $pc = 1 + g$, then any λp such that

$$\frac{u_1(c)}{1 + \xi_1(pc)} \leq \lambda p \leq \frac{u_1(c)}{1 + \xi_1(pc) + \xi_3(pc)}$$

solves this equation. To be definite, in this event choose the left end point of this interval for the value of λp. Given this function for λp, the function φp is determined by equation (A2), and π' is determined by equation (20). Under the stated assumptions, there thus exists a unique equilibrium to this economy.

Appendix B

This Appendix describes the algorithm we used to compute an equilibrium and simulate time series. This algorithm is essentially the one described in Coleman (1990) and used in Coleman (1996), but applied to an economy without an endogenous state variable (in this paper, all state variables are exogenous).

The exogenous state variables consist of the nominal interest rate, r, the ratio of the supply of risk-free assets to the supply of currency, g, and the gross growth rate of the endowment, γ. Denote the vector of the logs of these variables by $\mathbf{x}$, which is assumed to follow a first-order vector autoregression: $\mathbf{x}' = A\mathbf{x} + B\epsilon$. The innovations ϵ_i, $i = 1, \ldots, 3$, are independent of each other and over time and are drawn from a discrete distribution that approximates a mean-zero, unit-variance normal distribution. This is done by choosing five values and associated probabilities for each ϵ from a five-point Hermite-Gauss quadrature rule (normalized appropriately so that they each have a unit variance).

The solution consists of velocity, v, gross expected inflation, $\bar{\pi}$, and the price of equity, θ, as functions of the state vector $\mathbf{x}$. Each function is approximated as a multilinear function over a rectangular grid of values of $\mathbf{x}$. The grid consists of 15 uniformly spaced values for each of the state variables (a total of 15^3 values). The grid points for $\log(r)$ range from -6.64 to -4.03, the grid points for $\log(g)$ range from 2.74 to 2.96, and the grid points for $\log(\gamma)$ range from $-.0166$ to $.0225$.

Given $\mathbf{x}$, the solution for v is given by equation (25).

The solution for $\bar{\pi}$ is constructed as follows. Recall that $\pi' = \bar{\pi} \exp(\eta')$,

where $\bar{\pi}$ is known in the current period. Use this relation in equation (21) to write

$$\bar{\pi} = \beta E_s\left[\frac{(c'/c)^{-\tau}}{1 + \xi_1(v')}\frac{1 - \xi_2(v')}{\exp(\eta')}\right][1 + \xi_1(v)],$$

where c'/c is given by

$$\frac{c'}{c} = \gamma'\frac{1 + \xi_0(v)}{1 + \xi_0(v')}.$$

The innovation η is represented as a linear combination of the innovations ϵ_i, $i = 1, \ldots, 3$, in addition to its own innovation ϵ_4. The innovation ϵ_4 is also drawn from the discrete distribution generated by a five-point Hermite-Gauss quadrature rule. The function $\bar{\pi}$ is computed such that the equation above holds exactly at the grid points. Note that the expectation is computed as the sum over the 15^4 possible values that ϵ_i, $i = 1, \ldots, 4$, can take on.

The solution θ is constructed as follows. Equation (23) can be used to define a contraction mapping in

$$\zeta = \frac{1 - \xi_2(v)}{1 + \xi_1(v)}[1 + \xi_0(v)]^{\tau-1}\frac{\theta}{v}$$

in the usual way. We simply iterate on the operator underlying the contraction mapping until equation (23) (with ζ) holds almost exactly at the grid points. The expectation is computed as in the case for $\bar{\pi}$, except that values for ζ' off the grid points need to be obtained by interpolation. The function θ at the grid points is then computed from ζ at the grid points.

All statistics for the model are based on simulations of 100,000 time periods. To simulate time series we first simulate a time series for the exogenous state variables $\mathbf{x}$, as well as η. Here we draw the innovations ϵ_i, $i = 1, \ldots, 4$, according to a normal(0, 1) distribution (and not the discrete approximation). The time series for v is given by equation (25), and the time series for $\bar{\pi}$ and θ are obtained by multilinear interpolation of the functions representing these variables. These variables are used to obtain all remaining variables (such as the return to equity).

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