A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets

Ravi Bansal
Ivan Shaliastovich *

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*Bansal (email: ravi.bansal@duke.edu) is affiliated with the Fuqua School of Business, Duke University, and NBER, and Shaliastovich (email: iks5@duke.edu) is at the Department of Economics, Duke University. We would like to thank Tim Bollerslev, Riccardo Colacito, Bjorn Eraker, David Hsieh, Nikolai Roussanov, George Tauchen, Adrien Verdelhan, the participants of the 2007 Financial Research Association meeting and 2008 UBC Winter Conference for their helpful comments and suggestions. The usual disclaimer applies. The paper was previously circulated under the title "Risk and Return in Bond, Currency and Equity Markets."
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Abstract

We present a long-run risks based equilibrium model that can quantitatively explain the violations of expectations hypotheses in bond and currency markets. The key ingredients of the model include a low-frequency predictable component in consumption, time-varying consumption volatility and investor’s preferences for early resolution of uncertainty. In this model, varying consumption volatility in the presence of the predictable consumption component leads to appropriate variation in bond yields and the risk premia to provide an explanation for the puzzling violations of the expectations hypothesis. Using domestic and foreign consumption and asset markets data we provide direct empirical support for the volatility-related channels highlighted in the paper.
1 Introduction

A prominent puzzle in financial economics is the violation of the expectations hypotheses and the ensuing predictability of returns in bond and currency markets. In this paper we show that a long-run risks framework (see Bansal and Yaron, 2004), which features low-frequency movements in consumption and time-varying consumption volatility, leads to significant time-variation in risk premia that can quantitatively explain the violations of expectations hypotheses in bond and currency markets. We also provide a direct empirical evidence that the covariation of measured consumption volatility in the data with the bond and currency prices across countries agrees with the theoretical predictions from our model – this evidence supports the economic channels highlighted in this paper.

The violations of the expectations hypotheses and the implied predictability of returns are one of the prominent features of bond and currency markets data. Empirical evidence in US and foreign countries typically shows a significant drop in long rates following periods of high long-short yield differential, which stands in sharp contrast to the implications of the expectations hypothesis of the yield curve. Additional evidence on the predictability of bond returns is presented more recently in Cochrane and Piazzesi (2005), who show that a single bond factor constructed from a linear combination of three to five forward rates can sharply forecast future bond returns. In currency markets, Backus, Foresi, and Telmer (2001), Bansal (1997) and Fama (1984) show that the interest rate differential across countries forecasts future exchange rate changes — in particular, a rise in the domestic nominal rate forecasts an appreciation of the domestic currency. What economic mechanisms can account for all these bond and currency market puzzles? We argue that the economic channels of the long-run risks model can successfully account for the risk and returns in financial markets and explain the puzzling features of bond and currency markets.

The key ingredients of the long-run risks model include a low frequency component in consumption, time-varying aggregate volatility and Kreps and Porteus (1978) recursive utility of Epstein and Zin (1989). Time-varying consumption uncertainty provides a primary economic channel which generates predictability and time-variation of expected excess returns, while the expected growth component magnifies risk compensations which enables us to explain the violations of the expectations hypotheses in the
data. In earlier work, Bansal and Yaron (2004) highlight the implications of this model for the slope of the real yield curve. Eraker (2006), Piazzesi and Schneider (2005) consider the long-run risks model as well, and show that the inflation risk induces an upward sloping nominal yield curve in this model. These papers do not attempt to explain the violations of the expectations hypothesis as in their specification, consumption volatility and hence all the risk premia are constant. A similar two-country version of the model considered in Colacito and Croce (2005) also has constant risk premia and consequently cannot explain the currency market violations of the expectations hypothesis.

In terms of the model intuition, when agents prefer early resolution of uncertainty (i.e., when the risk aversion exceeds the reciprocal of the IES) a positive shock to consumption volatility moves the expected excess bond returns and the long-short yield spread in the same direction. Therefore, the slope of the term structure forecasts positively future excess returns on bonds, which can quantitatively account for the violations of expectations hypothesis in the data. Similarly, in foreign exchange markets, in response to a positive shock to domestic consumption uncertainty the agents demand higher expected excess returns in foreign bonds, forecast appreciation of the foreign currency and at the same time push the yield on domestic risk-free assets down. This can quantitatively account for the violations of the expectations hypothesis in currency markets.

There is considerable support for time-varying consumption volatility in the data—Kandel and Stambaugh (1990) present strong evidence of varying consumption volatility, while Stock and Watson (2002) highlight a time-variation and a decline in the volatility of several macro variables, such as real GDP and aggregate consumption. Further, in the data, the volatility of domestic consumption growth correlates negatively with the dollar price of foreign currencies and forward premia, and positively with the expected returns on foreign bonds for all the countries in our analysis. This evidence is consistent with the predictions of the model, and provides empirical support for the economic channels highlighted in the paper.


The rest of the paper is organized as follows. In the next section we document the violations of the expectations hypothesis in bond and currency markets. In Section 3 we setup the long-run risks model. We present the solution to the model and discuss its theoretical implications for financial markets in Section 4. Section 5 describes the data and calibration of the real and nominal economy and preference parameters. Model implications for bond and currency markets values are addressed in Section 6. In Section 7 we discuss an extension of the model to the two-volatility case which sharpens its quantitative results. Conclusion follows.

2 Predictability Puzzles and Evidence

A standard benchmark for the analysis of returns on bonds is provided by the expectations hypothesis. It states that in domestic bond markets, a high long-short yield spread today is offset by an anticipated loss on long maturity bonds in the future, and therefore should forecast an increase in the long rates. In foreign exchange context, low risk-free rates at home are compensated by the future appreciation of dollar and therefore should predict expected depreciation of the foreign currency. These conclusions formally obtain in structural models when the expected excess returns are constant, e.g., when investors are risk-neutral or economic uncertainty is constant.
As discussed below, none of these implications of the expectations hypothesis are supported by the data; in fact, the signs in predictability regressions are exactly the opposite. Reduced-form empirical projections imply that a high yield spread forecasts a drop in future long rates, and the regression coefficients become more negative with maturity. Likewise, low forward premium predicts appreciation of the foreign currency, though, the violations are less severe in longer horizon. Therefore, the forecasts of the change in future bond and currency prices based on expectations hypothesis or empirical projections will be radically different, both in terms of their magnitude and sign. The violations of these predictions in the data pose a challenge to the economic understanding of the asset markets and seriously question the constant (zero) expected excess return assumptions used to justify the expectations hypothesis model.

The economic principle of no-arbitrage across bond, currency and equity markets implies that the expected return in all these markets should be explained by common economic risk channels. In the context of long-run risks model, we show that these channels can successfully account for the predictability puzzles in bond and currency markets.

In the next two sub-sections we establish the notations and document the key empirical findings on predictability of domestic and foreign bond returns.

### 2.1 Bond Market Puzzles

Denote $y_{t,n}$ the yield on the real discount bond with $n$ months to maturity. Then, we can write the excess log return on buying an $n$ months bond at time $t$ and selling it at time $t + m$ as an $n - m$ period bond as

$$ rx_{t+m,n} = ny_{t,n} - (n - m)y_{t+m,n-m} - my_{t,m}. $$

(2.1)

Variables with a dollar superscript will refer to nominal quantities, such as nominal risk-free rate $y_{t,1}$. To avoid clustering of superscripts, we lay out the discussion using real variables; same arguments apply for the nominal economy as well.

Under the expectations hypothesis, the expected excess bond returns are constant.
This implies that the slope coefficient $\beta_{n,m}$ in bond regressions

$$y_{t+m,n-m} - y_{t,n} = const + \beta_{n,m} \frac{m}{n-m} (y_{t,n} - y_{t,m}) + error$$ (2.2)

should be equal to one at all maturities $n$ and time steps $m$. Indeed, with rational expectations, the population value for the slope coefficient is given by

$$\beta_{n,m} = 1 - \frac{Cov(E_{t}r_{x_{t+m,n}}; y_{t,n} - y_{t,m})}{mVar(y_{t,n} - y_{t,m})}.$$ (2.3)

If term-spread $y_{t,n} - y_{t,m}$ contains no information about expected excess bond returns $E_{t}r_{x_{t+m,n}}$, e.g. expected excess returns are constant under the expectations hypothesis, then the slope is equal to unity. Alternatively, high long-short spread should predict a proportional decline in future bond prices, which eliminates the yield advantage to long-term bonds by expected capital loss.

In the data, however, the regression coefficients in bond projections (2.2) are negative and increasing in absolute value with horizon (see Campbell and Shiller, 1991). In the second panel of Table 2, we tabulate the projection coefficients for nominal bond yields in US and UK. Consistent with previous studies, all of the slope coefficients are negative, and they are increasing in absolute value with maturity for US. The standard errors of the estimates, however, are quite large. Similar evidence obtains for inflation-adjusted projections when we subtract the measure of expected inflation from nominal yields.

This empirical evidence suggests that, contrary to the expectations hypothesis, high long-short yield spread forecasts an increase in future prices. That is, expected excess bond returns are time-varying and predictable by the term-spread with a positive sign. Dai and Singleton (2002) provide further discussion of the violations of the expectations hypothesis and bond return risk premia in context of affine models of the term structure.

Forward rate projections provide additional evidence for the time-variation in bond risk premia. We follow Cochrane and Piazzesi (2005) and regress the average of $m$ - period excess returns on bonds of different maturities on the forward rates over equally spaced short, middle and long horizon. The fitted values $\hat{r}_{x_{t,m}}$ from these regressions are then used as a single bond factor in projections

$$r_{x_{t+m,n}} = const + b_{m,n}\hat{r}_{x_{t,m}} + error.$$ (2.4)
Cochrane and Piazzesi (2005) show that the estimates $b_{m,n}$ are positive and increasing with horizon, and a single factor projection captures $20 - 30\%$ of the variation in bond returns. We demonstrate these results for US and UK bond markets in the last panel of Table 2.

### 2.2 Currency Market Puzzles

Let $s_t$ stand for a real spot exchange rate, in logs, per unit of foreign currency (dollars spot price of one pound), and denote by $f_{t}^{FX}$ the logarithm of the foreign exchange forward rate, i.e. current dollar price of a contract to deliver one pound tomorrow. Superscript $^*$ will denote the corresponding variable in the second country, e.g. $y_{t,1}^*$ stands for the foreign risk-free rate. To avoid clustering of superscripts, we present the discussion in real terms.

A one-period excess dollar return in foreign bonds is given by

$$rx_{t+1}^{FX} = s_{t+1} - s_t + y_{t,1}^* - y_{t,1}.$$  \hspace{1cm} (2.5)

This corresponds to an excess return on buying foreign currency today, investing the money into the foreign risk-free asset and converting the proceeds back using the spot rate next period.

Under the expectations hypothesis in currency markets, the excess returns are constant. Therefore, the slope coefficient in the projection

$$s_{t+1} - s_t = \text{const} + \beta^{UIP}(y_{t,1} - y_{t,1}^*) + \text{error.}$$  \hspace{1cm} (2.6)

should be equal to one. Indeed, with rational expectations, the population value for the regression coefficient can be written as,

$$\beta^{UIP} = 1 + \frac{\text{Cov}(E_trx_{t+1}^{FX}, y_{t,1} - y_{t,1}^*)}{\text{Var}(y_{t,1} - y_{t,1}^*)}.$$  \hspace{1cm} (2.7)

Therefore, if the forward premium $y_{t,1} - y_{t,1}^*$ contains no information about the foreign bond risk premium $E_trx_{t+1}^{FX}$, e.g. the latter is constant under the expectations hypothesis, the projection coefficient is unity. Alternatively, if the uncovered interest rate
parity condition holds, high interest rate bearing countries are expected to experience a proportional depreciation of their currency.

Fama (1984), Hodrick (1987), Backus et al. (2001) and many other studies show that at short maturities, the regression coefficient in foreign exchange projection (2.6) is negative and statistically significant. In the second panel in Table 3, we document these findings for UK, Germany and Japan for an investment horizon of 1 month. To focus on the return dimension, we subtract the forward premium from the both sides of (2.6). As investment horizon gets larger, the violations of the UIP condition in the data are less severe. As shown by Chinn and Meredith (2004) and Alexius (2001), at long maturities the slope coefficient turns positive but remains below one. We confirm these findings for nominal bonds in US and UK for 2 to 5 years to maturity: in monthly regressions from January 1988 to December 2005 (not shown), the slope coefficient is positive at 2 year horizon and is equal to 0.12 (0.98), and is very close to one for longer investment horizons.

To get additional insight into the violations of expectations hypothesis in currency markets, Fama (1984) decomposes the difference between the forward and spot price of the currency into the risk premium part and the expected depreciation of the exchange rate:

\[
f_t^{FX} - s_t = (f_t^{FX} - E_t s_{t+1}) + (E_t s_{t+1} - s_t) \\
≡ (-E_t r_{FX,t+1}) + (E_t s_{t+1} - s_t).
\]

As can be seen from the expression (2.7), to explain a negative slope in uncovered interest parity regressions, an asset-pricing model should deliver a negative covariance between the (negative of) foreign bond risk premium, \(-E_t r_{FX,t+1}\), and expected depreciation of the currency, \(E_t s_{t+1} - s_t\), and also a greater variance of the risk premium than that of the expected depreciation.
3 Long-Run Risks Model

3.1 Preferences and Real Economy

We consider a discrete-time real endowment economy developed in Bansal and Yaron (2004). The investors preferences over the uncertain consumption stream $C_t$ can be described by the Kreps-Porteus, Epstein-Zin recursive utility function, (see Epstein and Zin, 1989; Kreps and Porteus, 1978):

$$U_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\psi}} + \delta (\mathbb{E}_t U_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \right]^{\frac{1}{1-\gamma}}. \quad (3.1)$$

The time discount factor is $\delta$, $\gamma \geq 0$ is the risk aversion parameter and $\psi \geq 0$ is the intertemporal elasticity of substitution (IES). Parameter $\theta$ is defined $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. Its sign is determined by the magnitudes of the risk aversion and the elasticity of substitution, so that if $\psi > 1$ and $\gamma > 1$, then $\theta$ will be negative. Note that when $\theta = 1$, that is, $\gamma = 1/\psi$, the above recursive preferences collapse to the standard case of expected utility. As is pointed out in Epstein and Zin (1989), in this case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk aversion exceeds (is less than) the reciprocal of IES the agent prefers early (late) resolution of uncertainty of consumption path. Hence, these preferences allow for agent’s preference for the timing of the resolution of uncertainty. In the long-run risk model agents prefer early resolution of uncertainty of the consumption path.

As shown in Epstein and Zin (1989), the logarithm of the Intertemporal Marginal Rate of Substitution (IMRS) for these preferences is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1}, \quad (3.2)$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the log growth rate of aggregate consumption and $r_{c,t+1}$ is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends each time period. This return is not observable in the data. It is different from the observed return on the market portfolio as the levels of market dividends and consumption are not equal: aggregate consumption is much larger than aggregate dividends. Therefore, we assume exogenous process for consumption growth.
and use a standard asset-pricing restriction

$$E_t[\exp(m_{t+1} + r_{t+1})] = 1$$  \hspace{1cm} (3.3)$$

which holds for any continuous return $r_{t+1} = \log(R_{t+1})$, including the one on the wealth portfolio, to solve for the unobserved wealth-to-consumption ratio in the model.

Following Bansal and Yaron (2004), we assume that the real consumption growth contains a small and persistent long-run expected growth component and the time-varying volatility (economic uncertainty). The consumption dynamics is thus the following:

$$\Delta c_{t+1} = \mu_g + x_t + \sigma_g \eta_{t+1},$$  \hspace{1cm} (3.4)

$$x_{t+1} = \rho x_t + \varphi_e \sigma_g \epsilon_{t+1},$$  \hspace{1cm} (3.5)

$$\sigma_{g,t+1}^2 = \sigma_g^2 + \nu_g (\sigma_g^2 - \sigma_g^2) + \sigma_{gw} w^*_g, t+1.$$  \hspace{1cm} (3.6)

The unconditional mean of the time-varying variance of consumption growth is $\sigma_g^2$. The variance of the long-run component in expected growth and consumption volatility is determined by $\varphi_e$ and $\sigma_{gw}$, respectively. The parameters $\rho$ and $\nu_g$ control the persistence of shocks to expected growth and consumption volatility. For analytical tractability, we assume that all the innovations are Gaussian and independent from each other.

3.2 Nominal Economy

The economic channels in the real economy suffice to explain the violations of the expectations hypotheses and predictability of returns in bond and currency markets in real terms. However, most of the asset markets data is in nominal terms, and data on real bonds is not observable. To make our model implications comparable to observed data, we model the inflation exogenously and derive asset prices in nominal terms and inflation adjusted terms. Our model implications for the violations of the expectations hypothesis are similar across these various measures–real or nominal.

Our approach to directly model inflation is similar to that pursued by Wachter (2006) and Piazzesi and Schneider (2005) in context of equilibrium model for bond yields. In particular, we assume that the inflation process follows

\[
\pi_{t+1} = \bar{\pi}_t + \varphi_{\pi g} \sigma_{gt} \eta_{t+1} + \varphi_{\pi x} \varphi_{e} \sigma_{et} e_{t+1} + \sigma_{\pi} \xi_{t+1}, \tag{3.7}
\]

where the expected inflation, \(\bar{\pi}_t \equiv E_t \pi_{t+1}\), is given by

\[
\bar{\pi}_{t+1} = \mu_\pi + \alpha_\pi (\bar{\pi}_t - \mu_\pi) + \alpha_x x_t + \varphi_{zg} \sigma_{gt} \eta_{t+1} + \varphi_{zx} \varphi_{e} \sigma_{et} e_{t+1} + \sigma_z \xi_{t+1}. \tag{3.8}
\]

To maintain parsimony, we assume that the inflation shock \(\xi\) is homoscedastic and affects both the inflation rate and its conditional mean; extensions to time-varying volatility and separate shock structure are straightforward. Parameters \(\varphi_{\pi g}, \varphi_{zg}\) and \(\varphi_{\pi x}, \varphi_{zx}\) measure the sensitivity ("beta") of realized and expected inflation innovations to short and long-run consumption news. Thus, the conditional variance of realized and expected inflation is time-varying and proportional to that of consumption growth, \(\sigma^2_{zt}\).

Our specification of the expected consumption and inflation growth rates is similar to that of Piazzesi and Schneider (2005). For parsimony, we assume that the inflation process has no effect on the real economy, while the real consumption growth, in particular, its expected growth, affects future expectations of inflation rate. We discuss the plausibility of this specification in the empirical section of the paper.

Given the real discount factor \(m_{t+1}\) and inflation process \(\pi_{t+1}\), the logarithm of nominal pricing kernel is given by

\[
m^S_{t+1} = m_{t+1} - \pi_{t+1}. \tag{3.9}
\]
3.3 Two-Country Setup

We extend our model to two-country setup—a similar specification, without the time-varying volatility in consumption, is also entertained in Colacito and Croce (2005). In particular, we assume that the endowments are country specific, and the agents derive utility only from consumption of domestic goods. Financial markets are frictionless and open for domestic and foreign investment and equilibrium real and nominal exchange rates adjust exactly to offset all the net payoffs and preclude arbitrage. This is similar to setup considered by Backus et al. (2001).

For simplicity, we impose complete symmetry and equate model and preference parameters across the two countries. Only the states and innovations are allowed to be country-specific; those for the foreign country are indexed by a superscript *. The correlation structure of the innovations is summarized by

\[ Cov(e, e^*) = \tau_e, \quad Corr(\eta, \eta^*) = \tau_\eta, \]
\[ Corr(w_g, w_g^*) = \tau_{wg}, \quad Corr(\xi, \xi^*) = \tau_\xi. \]

The discount factor used to price assets denominated in foreign currency is given by

\[ m^*_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c^*_{t+1} + (\theta - 1) r^*_{c,t+1}, \quad (3.10) \]

where \( g^*_{t+1} \) is the log growth rate of foreign endowment growth, \( r^*_{c,t+1} \) is the log return on foreign consumption portfolio, and \( \delta, \gamma \) and \( \psi \) are the preference parameters of the representative agents at home and abroad.

4 Asset Markets

4.1 Real Marginal Rate of Substitution

The key ideas of the model rely on solutions which are derived using the standard log-linearization of returns. In particular, the log-linearized return on consumption claim is given by

\[ r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \quad (4.1) \]
where \( z_t \equiv \log (P_t/C_t) \) is the log price-to-consumption ratio. Parameters \( \kappa_0 \) and \( \kappa_1 \) are approximating constants which are based on the endogenous average price-to-consumption ratio in the economy.

The approximate solution for the price-consumption ratio is linear in states, \( z_t = A_0 + A_x x_t + A_{gs} (\sigma^2_{gt} - \sigma^2_g) \). From the Euler condition (3.2) and the assumed dynamics of consumption growth, the solutions for \( A_x \) and \( A_{gs} \) satisfy

\[
A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \quad (4.2)
\]

\[
A_{gs} = \frac{1}{2} \frac{(1 - \gamma)(1 - \frac{1}{\psi})}{1 - \kappa_1 \nu_g} \left( 1 + \left[ \frac{\varphi \kappa_1}{1 - \kappa_1 \rho} \right]^2 \right). \quad (4.3)
\]

It follows that \( A_x \) is positive if the IES, \( \psi \), is greater than one. In this case the intertemporal substitution effect dominates the wealth effect. In response to higher expected growth, agents buy more assets, and consequently the wealth to consumption ratio rises. In the standard power utility model with risk aversion larger than one, the IES is less than one, and hence \( A_x \) is negative — a rise in expected growth potentially lowers asset valuations. That is, the wealth effect dominates the substitution effect.

Coefficient \( A_{gs} \) measures the sensitivity of price-consumption ratio to volatility fluctuations. If the IES and risk aversion are larger than one, then \( A_{gs} \) is negative. In this case a rise in consumption volatility lowers asset valuations and increases the risk premia on all assets. An increase in the permanence of volatility shocks, that is \( \nu_g \), magnifies the effects of volatility shocks on valuation ratios as changes in economic uncertainty are perceived by investors as being long lasting. Similarly, \( A_{gs} \) increases in absolute value with the persistence of expected consumption growth \( \rho \), as the effects of volatility shocks are magnified as they feed in through the expected growth channel \( x_t \).

Using the approximate solutions for the price-consumption ratio, we can provide an analytical expression for the marginal rate of substitution in (3.2). The log of the IMRS \( m_{t+1} \) can always be decomposed to its conditional mean and innovation. The former is affine in expected mean and variance of consumption growth and can be expressed as

\[
E_t m_{t+1} = \mu_m - \frac{1}{\psi} x_t + \frac{(1 - \gamma)(1 - \frac{1}{\psi})}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi c}{1 - \kappa_1 \rho} \right)^2 \right] (\sigma^2_{gt} - \sigma^2_g) \quad (4.4)
\]
for some constant $\mu_m$.

The innovation in the IMRS is very important for thinking about risk compensation (risk premia) in various markets. Specifically, it is equal to

$$m_{t+1} - E_t m_{t+1} = -\lambda_\eta \sigma_g \eta_{t+1} - \lambda_e \varphi_e \sigma_g e_{t+1} - \lambda_g \sigma_g w_{g,t+1}. \quad (4.5)$$

Parameters $\lambda_\eta$, $\lambda_e$, and $\lambda_g$ are the market price of risks for the short-run, long-run, and volatility risks. The market prices of systematic risks, including the compensation for stochastic volatility risk in consumption, can be expressed in terms of the underlying preferences and parameters that govern the evolution of consumption growth:

$$\lambda_\eta = \gamma$$
$$\lambda_e = (\gamma - \frac{1}{\psi}) \left( \frac{\kappa_1}{1 - \kappa_1 \rho} \right)$$
$$\lambda_g = \frac{1}{2} (\gamma - \frac{1}{\psi}) (1 - \gamma) \frac{\kappa_1}{1 - \kappa_1 \nu_g} \left( 1 + \left[ \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right]^2 \right) \quad (4.6)$$

In the special case of power utility, $\theta = 1$ or more specifically, $\gamma = \frac{1}{\psi}$, the risk compensation parameters $\lambda_e$ and $\lambda_g$ are zero, and the IMRS collapses to the standard power utility specification

$$m_{t+1}^{CRRA} = \log \delta - \gamma \Delta c_{t+1}. \quad (4.7)$$

With power utility there is no separate risk compensation for long-run growth rate risks and volatility risks — with generalized preferences both risks are priced. The pricing of long-run and volatility risks is an important feature of the long-run risks model. Specifically, at the calibrated parameter values, consumption volatility shocks, $w_{g,t+1}$, together with the innovations into the expected consumption, $e_{t+1}$, are the most important sources of risks in the economy, as measured by their contribution to the maximal Sharpe ratio in the economy (conditional variance of the discount factor).

The logarithm of nominal pricing kernel can be obtained from the condition (3.9). In particular, nominal prices of immediate and long-run consumption risks depend on the inflation betas to short and long-run consumption news. Real and nominal discount factors in foreign country have analogous solutions.
4.2 Bond Returns

The equilibrium real and nominal yields are affine in the state variables:

\[
y_{t,n} = \frac{1}{n} \left[ B_{0,n} \ B_{x,n} \ B_{gs,n} \right] \left[ 1 \ x_t \ \sigma^2_{gt} - \sigma^2_g \right]', \tag{4.8}
\]

\[
y_{t,n}^s = \frac{1}{n} \left[ B_{0,n}^s \ B_{x,n}^s \ B_{gs,n}^s \ B_{\pi,n}^s \right] \left[ 1 \ x_t \ \sigma^2_{gt} - \sigma^2_g \ \bar{\pi}_t - \mu_{\pi} \right]', \tag{4.9}
\]

The bond coefficients, which measure the sensitivity ("beta") of bonds to the fundamental risks in the economy, are pinned down by the preference and consumption dynamics parameters – the expressions for the loadings are presented in Appendix B.

For typical parameter values, real yields respond positively to expected growth shocks, \( B_x > 0 \), and negatively to consumption volatility \( B_{gs} < 0 \), – while this is also true with power utility (\( \gamma = \frac{1}{\psi} \)), the full model with recursive preferences separates the risk aversion from the IES which allows for greater flexibility in modeling and explaining the yield curve. The solutions for nominal bond yields are more complicated as they take into account inflation risks in the economy. For example, in our calibrations nominal yields load negatively on the expected growth factor, which allows us to match an upward sloping term-structure of nominal bonds.

As discussed in Section 2.1, the violations of expectations hypothesis in the data imply a positive covariation between the expected excess return on bonds and long-short yield spread. In the model, up to Jensen’s adjustment term the one period expected excess return on real bond with \( n \) months to maturity can be rewritten in the following form:

\[
E_t(r_{x_{t+1},n}) + \frac{1}{2} \text{Var}_t(r_{x_{t+1},n}) = -\text{Cov}_t(m_{t+1}, r_{x_{t+1},n}) \]
\[
= -B_{gs,n-1} \lambda_{gw} \sigma^2_{gw} - B_{x,n-1} \lambda_e \bar{e} \sigma^2_{gt},
\tag{4.10}
\]

while the real term-spread is given by,

\[
y_{t,n} - y_{t,1} = \text{const} + \left( \frac{1}{n} B_{x,n} - B_{x,1} \right)x_t + \left( \frac{1}{n} B_{gs,n} - B_{gs,1} \right) \sigma^2_{gt}.
\tag{4.11}
\]

The model implies that in period of high consumption uncertainty, the expected excess return on real bonds is low. Additionally, both short and long real yields fall,
and the long-short term-spread declines as well, as in calibrations long yields are more sensitive to consumption variance, that is, \( \frac{1}{n} |B_{gs,n}| > |B_{gs,1}| \). This leads to a positive correlation of the real term-spread and bond risk premia, as required to explain the violations of the expectations hypothesis in the data. The actual magnitudes of the slopes coefficients in bond regressions (2.2) depend on the amount of persistence and variation in bond risk premium generated by the model.

Notably, all the three ingredients of the long-run risks model—preference for early resolution of uncertainty and time-variation in expected growth and volatility of consumption—are critical to explain the predictability of bond returns and violations of expectations hypothesis in the data. Indeed, as can be seen from expression (4.10), the risk premium on holding period bond returns reflects the compensation for long-run and volatility risks. With power utility, these two sources of risks are not priced, as \( \lambda_{gw} \) and \( \lambda_e \) are all zero, so up to Jensen’s adjustment term, the risk premium is zero. Further, all the time-variation in bond risk premium comes from the uncertainty about expected growth rate, so that if the expected consumption growth is constant (\( \varphi_e = 0 \)) or the aggregate volatility is constant (\( \sigma^2_{gt} = \sigma^2_g \)), the bond risk premium is constant as well, and expectation hypothesis holds.

A similar discussion of the bond risk premia holds in nominal terms. Market prices of risks and bond return sensitivities to sources of uncertainty should now account for inflation risks, so the responses of bond yields and excess bond returns to consumption volatility depend on the calibration of consumption and inflation rates. For example, in our calibration, both nominal bond risk premium and nominal long-short yield spread actually increase with the variance of consumption growth. Still, same as for the real bonds, as the expected bond returns and the term-spread move in the same direction in response to consumption volatility shock, the slope coefficient in expectations hypothesis regressions for nominal bonds is also below one and negative.

### 4.3 Expected Depreciation and the Forward premium

As discussed in Backus et al. (2001), the Euler equation implies that the change in exchange rate is equal to the difference between the logarithms of the discount factors in the two countries:

\[
s_{t+1} - s_t = m^*_{t+1} - m_{t+1},
\]

(4.12)
and similar expression holds for nominal exchange rates.

Therefore, given the equilibrium solution to the pricing kernel, one-period expected depreciation rate of the domestic currency can be calculated in the following way:

\[ E_t s_{t+1} - s_t = const + \frac{1}{\psi} (x_t - x_t^*) + \frac{1}{2} \left( \frac{\gamma - 1}{\gamma} \right) \left( 1 + \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right) (1 - \psi) (\gamma - 1) \left( 1 + \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right) \left( \sigma_{gt}^2 - \sigma_{gt}^{*2} \right). \] (4.13)

while the expression for the forward premium satisfies

\[ y_{t,1} - y_{t,1}^* = const + \frac{1}{\psi} (x_t - x_t^*) \]

\[ - \frac{1}{2} \left( \lambda_\eta^2 + \varphi_e^2 \lambda_e^2 + \frac{1}{\psi} - \gamma \right) \left( 1 + \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right) \left( \sigma_{gt}^2 - \sigma_{gt}^{*2} \right). \] (4.14)

Therefore, the solution to expected excess return in foreign bond is linear in consumption volatilities at home and abroad:

\[ E_t r_{FX, t+1} = E_t \left( s_{t+1} - s_t + y_{t,1}^* - y_{t,1} \right) \]

\[ = \frac{1}{2} \left( \lambda_\eta^2 + \varphi_e^2 \lambda_e^2 \right) (\sigma_{gt}^2 - \sigma_{gt}^{*2}). \] (4.15)

As can be seen from the above expressions, in the full long-run risks specification with time-varying economic uncertainty and preference for early resolution of uncertainty, the expected excess return on foreign bonds unambiguously increases when the consumption uncertainty at home is high. Indeed, with a positive shock to domestic uncertainty, the equilibrium dollar price of the foreign currency drops immediately, so that relative to the new level today, the foreign currency is expected to appreciate tomorrow (expression (4.13)), and the dollar return on investments abroad is expected to be high (expression (4.15)). At the same time, when \( \gamma > 1 \) and \( \psi > 1 \), increase in domestic consumption uncertainty lowers the yields on domestic bonds and the yield spread across countries (equation (4.14)). All together, in response to a positive shock to consumption uncertainty, the agents demand higher expected excess returns in foreign bonds, forecast appreciation of the foreign currency and at the same time push the yield on domestic risk-free assets down. This can qualitatively account for the violations of the uncovered interest rate parity condition in the data. The actual magnitude of the model-implied
slope coefficients in foreign exchange projections depend on the calibration of preference
and consumption growth parameters.

For example, if the consumption uncertainty $\sigma^2_{gt}$ is constant, the expected excess returns on foreign bonds are constant as well, so that the expectations hypothesis holds and the projection coefficient in foreign exchange regression should be centered at one. At the other extreme, when agent has power utility, the slope coefficient is given by

$$\beta_{UIP}^{CRRA} = \frac{Var(x_t - x^*_t)}{\gamma^2 Var(\sigma^2_{gt} - \sigma^*_2) + Var(x_t - x^*_t)},$$

so that it less than one, but bigger than zero.

In terms of Fama (1984) conditions, while the power utility model can generate a negative covariation of the expected excess returns in foreign bonds and forward premium, so that the slope coefficient is below one, it fails to produce enough variation in the risk premium $E_{t+1} x_{t+1}^{FX}$. In the long-run risks model, the risk premium is magnified by the compensation for the uncertainty in expected consumption growth, see expression (4.15). In particular, for the right persistence and variance of the long-run and volatility risks, we can match the empirical findings that the projection coefficients are negative at short horizon but become positive and closer to one at longer maturities.

The discussion for nominal variables is completely analogous, and is omitted for the interests of space.

5 Data and Calibration

5.1 Real Economy

We choose four countries for our empirical analysis, such as United Kingdom, Germany, Japan and United States (domestic country). The financial data for the foreign countries is taken from Datastream. This includes spot and forward rates, Euro-Currency Middle Rates of 1 month to maturity and returns on Morgan Stanley International Index for the period of January 1976 (July 1978 for Japan) to November 2005. Additional data on 1 to 5 year nominal discount bonds in US and UK come from CRSP and Bank of England, respectively. Market returns in US are calculated for a broad value-weighted portfolio
from CRSP. The consumption and CPI measures for foreign countries are taken from the IMF’s International Financial Statistics,\textsuperscript{2} while the US consumption data come from BEA tables of real expenditures on non-durable goods and services.

The second column of Table 5 shows summary statistics for the US quarterly consumption series for the period from 1976Q2 to 2005Q2. The real consumption growth rate is mildly persistent, has an annualized volatility just below 1\% and correlates positively with household expenditure rates in UK. To view the long-horizon properties of the series we compute autoregressive coefficients at different lags as well as the variance ratios which are themselves determined by the autocorrelations. In the data the variance ratios first rise significantly and at about 3 years start to decline, while the autocorrelations fall uniformly with number of lags. The standard errors on these statistics, not surprisingly, are quite substantial. As the constructed series in other countries are more noisy proxies for the true consumption process of the agent, they are more volatile and less persistent than the US series. For the interests of space, we do not report their statistics in the paper.

The first panel in Table 1 tabulates summary statistics for excess market returns and inflation-adjusted rates across the four countries\textsuperscript{3}. Notably, one-month interest rates are very persistent and vary from 1.9\% in Japan to 3.61\% in UK, while their standard deviations range between 2.1\% and 3.1\% for the sample period. For a post-war sample in US, the interest rate is equal to 0.8\% with a standard deviation of 2.8\%. Realized excess equity returns are several times more volatile and average 4.9\% – 5.5\% for UK, Germany and US (7.4\% for a post-war US sample). High mean and variance of the market return relative to the interest rate are well-known puzzles in financial literature (see Mehra and Prescott, 1985).

The first panel of Table 3 reports summary statistics for the nominal and real foreign exchange rates across the countries. Consistent with previous findings, foreign exchange rates have a zero autoregressive coefficient (not reported), and the range for their volatilities is 10.7\% – 12.1\%.

\textsuperscript{2} We thank John Campbell for providing us the dataset.
\textsuperscript{3} Inflation adjustment of interest rates is based on AR(2) filtered inflation, while realized inflation is used to adjust changes in foreign exchange rates.
5.2 Nominal Economy

The second panel of Table 1 contains summary statistics for seasonally-adjusted monthly inflation rates and 1 month nominal yields across countries, while a more detailed description of the quarterly inflation in US is given in the second column of Table 6. The inflation rates are fairly persistent, as evidenced by the first and tenth-order autoregressive coefficients of 0.64 and 0.36, respectively, in the US sample. In fact, the variance ratios rise substantially and start to decline only at 8.5 year horizon. The series co-move positively across countries, with a correlation of US and UK inflation rates of 0.73, and negatively with consumption growth rates, −0.21 being the correlation coefficient in US sample. The output for the other countries is similar, safe for a low predictability of inflation rates in Japan, and is omitted for the interest of space.

To capture the sensitivity of inflation news to consumption uncertainty, we set up and estimate a bivariate VAR(1) for these two series. We use the estimated model to compute analytically a $k$-period inflation beta defined as

\[
\hat{Cov}_t(\frac{1}{k} \sum_{i=1}^{k} \pi_{t+i}, \frac{1}{k} \sum_{i=1}^{k} \Delta c_{t+i}) / \hat{Var}_t(\frac{1}{k} \sum_{i=1}^{k} \Delta c_{t+i}).
\]

A solid line in Figure 1 draws the inflation beta as a function of horizon. Consistent with negative correlation of consumption and inflation rates in the data, the inflation beta is negative and stabilizes at −1.3 at long horizons. Notably, if the conditional expectations of consumption growth and inflation rates were constant, the inflation beta computed using the unconditional moments in Tables 5 and 6 would amount to −0.44 at all maturities. This evidence suggests that the expected consumption and inflation rates are time-varying and negatively correlated.

5.3 Calibration of Consumption and Inflation

We calibrate the model outlined in (3.4) - (3.6) and (3.7) - (3.8) at monthly frequency and time-aggregate the output from monthly simulations to match the key aspects of the 1976Q2 - 2005Q2 sample of quarterly consumption growth and inflation rate in US. We use the solutions provided in the theoretical section of the paper to derive our model implications for the asset prices. In particular, we use the numerical method discussed in Bansal et al. (2007). They develop a procedure to solve for the endogenous constants $\kappa_0$ and $\kappa_1$ in equation (4.1) associated with each return and document that the numerical
solution to the model is accurate. We provide some details on this method in Appendix A.

The baseline calibration parameter values are reported in Table 4. Specifically, we set the persistence in the expected consumption growth $\rho$ at 0.991. Our choice of $\varphi_e$ and $\sigma$ ensures that the model matches the moments of consumption growth in the data. In particular, the annualized volatility of monthly consumption growth is set to 1.1%, while the long-run risks volatility parameter is $\varphi_e = 0.055$. The persistence of the variance shocks is set at $\nu_g = 0.996$.

To capture the international dimensions of the data, similar to Colacito and Croce (2005) we set the correlation of long-run news $\tau_e$ to be nearly perfect, 0.999, and additionally impose a high correlation of the volatility news across the countries, $\tau_{gw} = 0.99$. In the extension of the model discussed subsequently in Section 7, we allow the volatilities of short-run and long-run consumption news to be different from each other, and calibrate the correlation between the short-run volatilities across countries to be zero, while the correlation of the long-run volatilities is set to one. This captures the intuition that in the long run, the distribution of consumption processes across all the countries are nearly identical (the long-run means and volatilities are the same), while in the short-run, they can be quite different due to uncorrelated immediate consumption and short-run volatility news at home and abroad.

In Table 5 we report the calibration output of our model, which is based on 1,000 simulations of 360 months of consumption series aggregated to quarterly horizon. The model implications for the volatility, persistence and multi-horizon properties of consumption growth rates are close to their empirical counterparts. Additionally, the model also delivers low correlation coefficient of consumption growth rates across countries, which matches well the historical estimates (0.24 in the model versus 0.25 in the US and UK data).

The inflation rate process is calibrated as follows. To maintain parsimony, we zero out inflation and expected inflation betas to immediate consumption news, $\varphi_{\pi g} = \varphi_{z g} = 0$, and set their sensitivity to long-run risks to be negative, $\varphi_{\pi x} = -2$ and $\varphi_{\pi z} = -1$. This is consistent with Piazzesi and Schneider (2005), who show that the negative correlation of inflation innovations with future long-horizon consumption growth helps explain the term structure of nominal bonds. We calibrate the parameters of the expected inflation
to match the key properties of the data. In particular, the expected inflation loads negatively on the expected consumption growth, $\alpha_x = -0.35$, and its own autoregressive coefficient is $\alpha_{\pi}$ is 0.83. For simplicity, we set to zero the correlation of independent inflation shocks across countries and capture the co-movements in the series through covariation of long-run risks shocks.

Table 6 shows the calibration output for the inflation process, while Figure 1 depicts the model-implied inflation beta to consumption news based on the bivariate VAR(1) specification. The model quite successfully matches the univariate properties of the inflation series, as well as the joint behavior of inflation and consumption growth rate and the correlation of the inflation rates across countries.

5.4 Preference Parameters

We calibrate the subjective discount factor $\delta = 0.9987$. The risk-aversion coefficient is set at $\gamma = 8$. Mehra and Prescott (1985) and Bansal and Yaron (2004) do not entertain risk aversion values larger than 10.

There is a debate in the literature about the magnitude of the IES. As in Bansal and Yaron (2004), we focus on an IES of 1.5 — an IES value larger than one is important for our quantitative results. Bansal et al. (2005) document that the asset valuations fall when consumption volatility is high; this is consistent only with $\psi > 1$. Further, as we show in the next section of the paper, in the data domestic consumption volatility co-moves negatively with dollar prices of foreign currency and forward premia and positively with expected returns on foreign bonds. This evidence is consistent with model predictions only when $\gamma > \frac{1}{\psi}$ and $\psi > 1$, which further supports our calibration of preference parameters.

6 Model Implications

6.1 Bond Markets

As shown in the first panel of Table 7, at the calibrated parameter values the model-implied term structure of nominal bond yields is upward sloping. The one-year nominal
yield is 5.44%, and it increases to 6.22% at 5 years. The volatilities of the yields fall uniformly from 2.25% at 1 year to 1.91% at 5 year horizon. The population values for levels and volatilities of nominal yields are thus consistent with US historical estimates reported in Table 2. The term-structure of real rates is downward sloping: the model-implied real rate is 1.3% at one year horizon and 0.52% at 5 years. At the calibrated parameter values, the inflation risk premium is increasing with the maturity, which enables us to match the upward slope of the nominal term structure.

The second panel of Table 7 shows model-implied slope coefficients in bond projections (2.2). These regressions are done using the annual time step and bond maturities of 2 to 5 years, so they are directly comparable to the projections in the data reported in Table 2. The theoretical slope coefficients are all negative and decreasing with horizon: the slopes fall from −0.17 to −0.34. These values are consistent with the estimates based on historical data in US and UK bond markets in Table 2, taking into account the standard error of the estimates. As in the data, these violations are more severe at longer horizons, as model-implied slope coefficients in bond regressions increase in absolute value with horizon. In the model we also verify that the slope coefficients for real bond regressions are also negative and increasing in absolute value with horizon, and their values are similar to the ones based on nominal regressions. As we do not observe the counterparts for real yields in the data, we do not report the model-based regression results in the paper.

Additional evidence on the time-variation of bond risk premia comes from forward-rate projections considered in Cochrane and Piazzesi (2005). The preferred regression model of Cochrane and Piazzesi (2005) includes five forward rates, but we have to limit ourselves to three regressors to avoid perfect multicollinearity in the model. Indeed, with three states — expected consumption growth, expected inflation rate and consumption volatility — the three forward rates summarize all the information in the economy. Last panel of Table 7 and second panel of Table 2 compare the magnitudes of the coefficients and $R^2$ in common bond factor regressions (2.4) in the data and in the model. The loadings on a single bond factor are very similar across the countries: for US, they increase from 0.44 at 2 year horizon to 1.45 at 5 years. These values are well captured by the model: the slope coefficients increase from 0.39 to 1.58 for 2 and 5 year maturities, respectively. The $R^2$ in the data are in 20% – 30% range. The population $R^2$’s are about 10 – 12%, however, the estimates in small samples (not reported) often reach magnitudes
found by Cochrane and Piazzesi (2005).

6.2 Currency Markets

Table 8 shows that the nominal slope coefficient in foreign exchange projections is equal to $-1.23$ at one month horizon, and it increases to $-0.71$ at 1 year and to $3.19$ at 5 year horizon. The value of the nominal projection coefficient at short horizon matches well the empirical estimates shown in Table 3. The model-implied nominal slope becomes positive at maturities above 2 years, so that the violations of the expectations hypothesis are less pronounced at longer maturities, which is consistent with the evidence reported in Chinn and Meredith (2004) and Alexius (2001). Further, consistent with the data, our model delivers that the foreign exchange rates are virtually unpredictable – the $R^2$s in foreign exchange projections (not shown) and the persistence in changes in spot prices of foreign currencies are all very close to zero. In the model we also verify that the slope coefficients in real regressions are negative as well – as there is no direct counterpart for real yields in the data, we do not report the model output in the paper. These findings are broadly consistent with Hollifield and Yaron (2003), who argue that risks from the real side of the economy are potentially important to capture the violations of the uncovered interest rate parity condition.

As shown in Table 3, the model-implied volatility of the foreign exchange rates is 19.86%, both in real and nominal terms, which is somewhat higher than the usual estimates of 11 – 12% in the data.

We can provide a direct evidence for the volatility channel in the model using consumption and asset prices data across countries. In Figures 2 - 4 we plot inflation-adjusted spot prices, forward premia and expected foreign bond returns against the difference in consumption volatility across countries. We follow Bansal et al. (2005) and construct consumption volatility measures non-parametrically as a 4.5 year sum of absolute residuals from AR(3) projections of consumption growth rates. Inflation adjustment for the interest rates is based on the fitted values from AR(2) model for the inflation rate. Consistent with the theoretical predictions in Section 4.3 for $\gamma > \frac{1}{\psi}$ and $\psi > 1$, consumption volatility differential co-moves negatively with dollar prices of foreign currency and forward premia, and positively with the expected dollar returns in foreign bonds. Indeed, the correlation coefficients for the spot exchange prices range
between -0.1 and -0.5, and are equal to about -0.3 and 0.3 for the forward premia and expected excess returns, respectively, for all the countries in the sample.

The magnitudes of the preference parameters and in particular, the value of the IES relative to one, have the first-order implications for the dynamics of asset markets in the model. Indeed, if $\psi > 1$, a rise in consumption volatility lowers asset valuations (see equation (4.3)), while the opposite happens if the IES is less than one. This has a direct effect on the co-movements of the consumption variance with asset and currency valuations and forward premium in the economy. Ultimately, it determines the ability of the model to explain the violations of the expectations hypothesis. In Figure 5 we plot the model-implied nominal slope coefficient against the IES when all the other parameters are fixed at their benchmark values. When IES is less than one, the theoretical slope coefficient is positive, so calibrating the IES the at the value above one is important to explain the foreign exchange puzzle in the data.

### 6.3 Equity Return

We note that the model, at the calibrated values for preferences and consumption above, also matches the equity data. This is not surprising, given the earlier work on equity markets in Bansal and Yaron (2004). Following this work, we consider a dividend process of the form,

$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_g \epsilon_{d,t+1}. \tag{6.1}$$

and calibrate it similar to that in Bansal and Yaron (2004). The third panel of Table 7 computes summary statistics for the real excess return on market portfolio and the real one-month interest rate. The model generates a sizable equity premium of 6.6%, which matches well the historical estimates in US and foreign countries reported in Table 1. The model-implied population value of the volatility of market returns is about 11.11%, which is broadly comparable to the historical estimates of 14% – 15% in US data (see Table 1). Model-implied real interest rate is 1.5%.

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$^4$ We calibrate the dividend process and set the exposure of the corporate sector to long-run risks to $\phi = 1.5$, and choose $\varphi_d = 6$ to match a high volatility of dividend stream relative to consumption. The correlation of consumption and dividend news is set to $\tau_{gd} = 0.1$. The model-implied dynamics of the dividend growth series matches the data very well, and is omitted for the interest of space.
7 Robustness and Model Extension: Two Volatility Model

One interpretation of the evidence regarding consumption volatility in Stock and Watson (2002) on one hand and Kandel and Stambaugh (1990) on the other is that there are two volatility processes that drive macroeconomic volatility. Indeed, the evidence in Stock and Watson (2002) highlights a long-run decline in aggregate uncertainty, which can be modeled as a very slow mean-reverting process, and another that is featured in Kandel and Stambaugh (1990) is a shorter run volatility which mean-reverts more quickly after rising during recessions. This two volatility process view is consistent with the evidence on macroeconomic volatility, and at the same time, when incorporated in the long-run risks model, has sharp implications for the violations of the expectations hypothesis – in particular, it brings many of the quantitative model implications closer to the magnitudes in the data. We feature this model in this section and also point out the quantitative magnitudes upon which this two-volatility model improves.

Specifically, we extend the benchmark model specification and allow the volatilities of long-run and short-run consumption news to be driven by individual shocks—the structure is,

\[ \Delta c_{t+1} = \mu_g + x_t + \sigma_g \eta_{t+1}, \]
\[ x_{t+1} = \rho x_t + \sigma_x e_{t+1}, \]
\[ \sigma_{g,t+1}^2 = \sigma_g^2 + \nu_g (\sigma_g^2 - \sigma_g^2) + \sigma_{gw} w_{g,t+1}, \]
\[ \sigma_{x,t+1}^2 = \sigma_x^2 + \nu_x (\sigma_x^2 - \sigma_x^2) + \sigma_{wx} w_{x,t+1}. \]

The immediate news in consumption growth have a stochastic volatility \( \sigma_{g,t}^2 \), while \( \sigma_{x,t}^2 \) is equal to the conditional variance of the low-run component \( x_t \). We view the volatility of \( x_t \) as corresponding to the long-run volatility, and the volatility for the short-run shock to be associated with the rapidly mean reverting short-run volatility. We show that with a minimal change in the calibration of the model, we can provide a sharper explanation of the predictability of bond and currency returns while being consistent with the consumption and inflation data. Indeed, as we show in Tables in Appendix C, the coefficients in the expectations hypothesis regressions in bond markets become more negative and are quite close to those the data. We also obtain higher population
$R^2$ in the single factor projections. Relative to the benchmark case with one volatility, it increases from from 9\% to 11 – 13\% (see Tables 7 and C.2).

In currency markets, a relatively high mean-reverting short-run volatility which is uncorrelated across countries help better match the slope coefficients in foreign exchange projections at long maturities and the volatility of foreign exchange rate. Indeed, nominal slope coefficient in foreign exchange regressions is -1.2 at one month horizon, turns positive within 2 to 3 years and remains below one at the considered maturities (it is equal to 0.86 at 5 year horizon). The volatility of the exchange rate is about 15\%, which is quite close to its data counterpart.

This augmented long-run risks model provides a sharper match to the data, however, the main economic insights are quite similar to the main model presented in the paper. For this reason, the details of the solution to the this model and its quantitative implications are discussed in Appendix C.

**Conclusion**

We show that the long-run risks model (see Bansal and Yaron, 2004) can explain the violations of expectation hypothesis and predictability puzzles in bond and currency markets. The key ingredients of the model include long-run growth fluctuations, time-varying consumption uncertainty and preference for early resolution of uncertainty. These channels generate a significant variation in risk premia, driven by the consumption volatility, which can quantitatively account for the negative coefficients in the tests of expectations hypothesis and for the level of return predictability in bond and currency markets. Using consumption and asset markets data, we provide direct empirical evidence to support the key economic channels highlighted in the paper.

The model captures the intuition that a positive shock to consumption volatility moves the expected excess bond returns and the yield-spread in the same direction, which explains negative slope coefficients in the expectations hypothesis regressions in bond markets. At the same time, forward premium decreases and the domestic currency is expected to depreciate, which accounts for the violations of the uncovered interest rate parity condition in currency markets. In numerical calibrations, we show that our model can match the key dimensions of bond and currency markets in the data.
A Solution to $\kappa_1$

From the log-linearization of returns, we obtain that the mean price-to-consumption ratio $E(z_t) = A_0$ and a constant $\kappa_0$ are equal to

$$A_0 = \log \frac{\kappa_1}{1 - \kappa_1}, \quad k_0 = -\log \left[ (1 - k_1)_2^{-k_1 k_1} \right]. \quad (A.1)$$

We can substitute the above expressions for $A_0$ and $\kappa_0$ into the solution for $A_0$ from the Euler equation, and after some algebra obtain

$$\log \kappa_1 = \log \delta + (1 - \frac{1}{\psi}) \mu_g + A_{gs} (1 - \kappa_1 \nu_g) \sigma_g^2 + \frac{1}{2} \theta \kappa_1^2 A_{gs} \sigma_g^2. \quad (A.2)$$

To find the solution for $\kappa_1$ at the calibrated parameter values, we iterate on the equation above from an initial value $\delta$. Similar expression can be derived for the approximating coefficients in the log-linearization of the return on the stock market portfolio. This approach closely follows Bansal et al. (2005).

B Solution to Bond Yields

The log prices of real and nominal discount bonds $q_{t,n}, q_{t,n}^g$ satisfy the Euler conditions

$$e^{q_{t,n}} = E_t e^{m_{t+1} + q_{t+1,n-1}}, \quad e^{q_{t,n}^g} = E_t e^{m_{t+1} + q_{t+1,n-1}}, \quad (B.1)$$

for $q_{t,0} = q_{t,0}^g = 0$. Using the solutions to the real and nominal pricing kernels in (4.4) and (3.9), we obtain that the bond prices and therefore, bond yields, are affine in the states, as shown in expression (C.7) and (C.8), and the bond yield loadings satisfy

$$B_{x,n} = \rho B_{x,n-1} + \frac{1}{\psi},$$

$$B_{gs,n} = \nu_g B_{gs,n-1} - \left( \frac{1}{\psi} - \gamma (\gamma - 1) \right) \left[ 1 + \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right] - \frac{1}{2} \left( \lambda_g^2 + \varphi_e^2 [\lambda_e + B_{x,n-1}]^2 \right),$$

$$B_{0,n} = B_{0,n-1} - \mu_m - \frac{1}{2} \left( \sigma_g^2 [\lambda_g^2 + \varphi_e^2 (\lambda_e + B_{x,n-1})^2] + \sigma_g^2 [\lambda_g w + B_{gs,n-1}]^2 \right)$$
for real bonds, and

\[ B_{x,n}^s = \rho B_{x,n-1}^s + \alpha_x B_{\pi,n-1}^s + \frac{1}{\psi}, \]
\[ B_{\pi,n}^s = \alpha_\pi B_{\pi,n-1}^s + 1, \]
\[ B_{gs,n}^s = \nu_g B_{gs,n-1}^s - \frac{(\frac{1}{\psi} - \gamma)(\gamma - 1)}{2} \left[ 1 + \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right] \]
\[ \frac{1}{2} \left[ \lambda_e + \varphi_{\pi g} + \varphi_z B_{\pi,n-1}^s \right]^2 + \varphi_e^2 \left[ \lambda_e + \varphi_{\pi x} + \varphi_{xx} B_{\pi,n-1}^s + B_{x,n-1}^s \right]^2, \]
\[ B_{0,n}^s = B_{0,n-1}^s - \mu_m + \mu_\pi - \frac{1}{2} \left[ (\sigma_{\pi} + B_{\pi,n-1}^s \sigma_z)^2 + \sigma_{gw}^2 \lambda_g + B_{gs,n-1}^s \right]^2 \]
\[ + \sigma_g^2 \left[ (\varphi_{\pi g} + \lambda_g + B_{\pi,n-1}^s \right)^2 + \varphi_e^2 (\varphi_{\pi x} + \lambda_e + B_{x,n-1}^s + \varphi_{xx} B_{\pi,n-1}^s)^2) \]

for nominal ones.

C Two Volatilities Model

C.1 Model Solution

The equilibrium price-to-consumption ratio is affine in the expected consumption and inflation rates and the two volatilities:

\[ v_t = \mu_v + A_x x_t + A_{gs} (\sigma_{gt}^2 - \sigma_g^2) + A_{xs} (\sigma_{xt}^2 - \sigma_x^2), \]  

(C.1)

where the loadings satisfy

\[ A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \quad A_{gs} = \frac{1}{2} \frac{(1 - \gamma)(1 - \frac{1}{\psi})}{1 - \kappa_1 \nu_g}, \quad A_{xs} = \frac{1}{2} \kappa_1^2 \frac{(1 - \gamma)(1 - \frac{1}{\psi})}{(1 - \kappa_1 \nu_x)(1 - \kappa_1 \rho)^2}. \]  

(C.2)

The log-linearization parameter \( \kappa_1 \) is given implicitly by

\[ \log \kappa_1 = \log \delta + (1 - \frac{1}{\psi}) \mu_g + A_{gs} (1 - \kappa_1 \nu_g) \sigma_g^2 + A_{xs} (1 - \kappa_1 \nu_x) \sigma_x^2 + \frac{1}{2} \theta \kappa_1^2 \left( A_{gs}^2 \sigma_{gw}^2 + A_{xs}^2 \sigma_{xw}^2 \right). \]  

(C.3)
The real discount rate now takes the following form:

\[
\begin{align*}
m_{t+1} &= \mu_m + m_x x_t + m_{gs}(\sigma_{gt}^2 - \sigma_g^2) + m_{xs}(\sigma_{xt}^2 - \sigma_x^2) \\
&\quad - \lambda_\eta \sigma_{gt} \eta_{t+1} - \lambda_e \sigma_{xt} e_{t+1} - \lambda_{gw} \sigma_{gw} w_{g,t+1} - \lambda_{xw} \sigma_{xw} w_{x,t+1},
\end{align*}
\]  

(C.4)

for

\[
\begin{align*}
\mu_m &= \log \delta - (\theta - 1) \log \kappa_1 - \gamma \mu_g, \quad m_x = - \frac{1}{\psi}, \quad m_{gs} = \frac{1}{2} (\gamma - 1) \left( \frac{1}{\psi} - \gamma \right), \\
m_{xs} &= \frac{1}{2} (\gamma - 1) \left( \frac{1}{\psi} - \gamma \right) \left( \frac{\kappa_1}{1 - \kappa_1 \rho} \right)^2,
\end{align*}
\]  

(C.5)

and

\[
\begin{align*}
\lambda_\eta &= \gamma, \quad \lambda_e = (\gamma - 1) \left( \frac{1}{\psi} \right) \frac{\kappa_1}{1 - \kappa_1 \rho}, \quad \lambda_{gw} = \frac{1}{2} (\gamma - 1) \left( \frac{1}{\psi} - \gamma \right) \frac{\kappa_1}{1 - \kappa_1 \nu_g}, \\
\lambda_{xw} &= \frac{1}{2} (\gamma - 1) \left( \frac{1}{\psi} - \gamma \right) \frac{\kappa_1}{1 - \kappa_1 \nu_x} \left( \frac{\kappa_1}{1 - \kappa_1 \rho} \right)^2.
\end{align*}
\]  

(C.6)

The equilibrium real and nominal bond prices are affine in the state variables:

\[
\begin{align*}
q_{t,n} &= -B_{0,n} - B_{x,n} x_t - B_{gs,n} (\sigma_{gt}^2 - \sigma_g^2) - B_{xs,n} (\sigma_{xt}^2 - \sigma_x^2), \\
q_{t,n}^s &= -B_{0,n}^s - B_{x,n}^s x_t - B_{gs,n}^s (\sigma_{gt}^2 - \sigma_g^2) - B_{xs,n}^s (\sigma_{xt}^2 - \sigma_x^2) - B_{\bar{\pi},n}^s (\bar{\pi}_t - \mu_\pi),
\end{align*}
\]  

(C.7)

where the loadings satisfy the recursions

\[
\begin{align*}
B_{x,n} &= \rho B_{x,n-1} - m_x, \\
B_{gs,n} &= \nu_g B_{gs,n-1} - (m_{gs} + \frac{1}{2} \lambda_\eta^2), \\
B_{xs,n} &= \nu_x B_{xs,n-1} - m_{xs} - \frac{1}{2} (\lambda_e + B_{x,n-1})^2, \\
B_{0,n} &= B_{0,n-1} - \mu_m - \frac{1}{2} (\lambda_\eta^2 \sigma_g^2 + (\lambda_e + B_{x,n-1})^2 \sigma_x^2) \\
&\quad + (\lambda_{gw} + B_{gs,n-1})^2 \sigma_{gw}^2 + (\lambda_{xw} + B_{xs,n-1})^2 \sigma_{xw}^2).
\end{align*}
\]
and

\begin{align*}
B^s_{x,n} & = \rho B^s_{x,n-1} + \alpha_x B^s_{\pi,n-1} - m_x, \\
B^s_{\pi,n} & = \alpha_\pi B^s_{\pi,n-1} + 1, \\
B^s_{gs,n} & = \nu_{gs} B^s_{gs,n-1} - m_{gs} - \frac{1}{2} \left( \varphi_{\pi g} + \lambda_{\eta} + \varphi_{zg} B^s_{\pi,n-1} \right)^2, \\
B^s_{xs,n} & = \nu_x B^s_{xs,n-1} - m_{xs} - \frac{1}{2} \left( \varphi_{\pi x} + \lambda_{e} + B^s_{x,n-1} + \varphi_{zx} B^s_{\pi,n-1} \right)^2, \\
B^s_{0,n} & = B^s_{0,n-1} - \mu_m + \mu_\pi - \frac{1}{2} \left( (\sigma_{\pi} + B^s_{\pi,n-1}\sigma_{z})^2 + (\lambda_{gw} + B^s_{gs,n-1})^2 \sigma_{gw}^2 + (\lambda_{xw} + B^s_{xs,n-1})^2 \sigma_{xw}^2 \\
& \quad + (\varphi_{\pi g} + \lambda_{\eta} + \varphi_{zg} B^s_{\pi,n-1})^2 \sigma_{g}^2 + (\varphi_{\pi x} + \lambda_{e} + B^s_{x,n-1} + \varphi_{zx} B^s_{\pi,n-1})^2 \sigma_{x}^2 \right)
\end{align*}

C.2 Model Implications

Table C.2 reports the adjustments to the baseline calibration values for the variance parameters and correlations of news across the countries. We keep the persistence of \( \sigma_{xt}^2 \) at 0.996, and decrease the short-run volatility autoregressive coefficient to 0.85. While we increase the variance of consumption growth and the volatility of the long-run variance, the variance long-run risks was decreased to keep the predictability of consumption growth low. In international dimension, we increase the correlation of long-run risks shocks across the countries and shift all the co-movements of volatilities to low frequency. For the interest of space, we do not report the calibration output for consumption, inflation and dividend processes as they are virtually the same as the in baseline case.

In Table C.2 we show the output for the nominal term structure and the violations of expectations hypothesis in the two-volatility setup. The first panel documents that the levels and variances of yields are very similar to the baseline calibration and are consistent with historical evidence. On the other hand, the violations of the expectations hypothesis in bond markets are more pronounced than in the baseline setup: for example, the slope coefficients in nominal bond projection are now equal to -0.26 and -0.45 at 2 and 5 year frequencies, respectively. The predictability of bond returns in single bond factor regressions also increases relative to the benchmark case: the population \( R^2 \) increase to 11 – 13%, while the loadings on the single bond factor remain the same.

Table C.2 shows that the nominal slope coefficients in foreign exchange regressions
is \(-1.2\) at 1 month horizon. It turn positive within 1 year and remains below one at the considered maturities (nominal one is equal to 0.86 at 5 year horizon), which is consistent with historical evidence. The decrease in persistence of short-run consumption volatility also helps to match the volatility of foreign exchange rate, which now becomes 15.78\% and is closer to the historical estimates of 11 – 12\% in the data.

Finally, the third panel of Table C.2 shows that model generates the equity risk premium of 7.5\% and the standard deviation of market returns of 12.11\%, which is again closer to the estimates in the data. The mean of the real risk-free rate (1.32\%) and its volatility (1.21\%) are quite similar to the benchmark values. Therefore, the extension of the model to the two volatilities case can improve the quantitative predictions of the benchmark model.
Table C.1: **Two-Volatility Model Parameter Values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption Growth Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Short-run volatility level</td>
<td>$\sigma_g$ 0.004</td>
</tr>
<tr>
<td>Short-run volatility persistence</td>
<td>$\nu_g$ 0.85</td>
</tr>
<tr>
<td>Short-run volatility of volatility</td>
<td>$\sigma_{gw}$ 1.15e-06</td>
</tr>
<tr>
<td>Long-run volatility level</td>
<td>$\sigma_x$ 0.04 $\times \sigma_g$</td>
</tr>
<tr>
<td>Long-run volatility persistence</td>
<td>$\nu_x$ 0.996</td>
</tr>
<tr>
<td>Long-run volatility of volatility</td>
<td>$\sigma_{xw}$ 0.06$^2 \times \sigma_{gw}$</td>
</tr>
<tr>
<td><strong>Cross-Country Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Correlation of long-run news</td>
<td>$\tau_e$ 0.99995</td>
</tr>
<tr>
<td>Correlation of short-run news</td>
<td>$\tau_{\eta}$ 0.0</td>
</tr>
<tr>
<td>Correlation of short-run volatility news</td>
<td>$\tau_{gw}$ 0.00</td>
</tr>
<tr>
<td>Correlation of long-run volatility news</td>
<td>$\tau_{gw}$ 1.00</td>
</tr>
</tbody>
</table>

Calibrated parameter values in the two-volatility model. The model is calibrated at monthly frequency.

Table C.2: **Two-Volatility Model: Bond Markets**

<table>
<thead>
<tr>
<th></th>
<th>1y</th>
<th>2y</th>
<th>3y</th>
<th>4y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Term Structure:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(y^t)$</td>
<td>5.23</td>
<td>5.35</td>
<td>5.52</td>
<td>5.73</td>
<td>5.98</td>
</tr>
<tr>
<td>$\sigma(y^t)$</td>
<td>2.04</td>
<td>1.92</td>
<td>1.84</td>
<td>1.82</td>
<td>1.84</td>
</tr>
<tr>
<td><strong>EH Projection:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>-0.26</td>
<td>-0.39</td>
<td>-0.43</td>
<td>-0.45</td>
<td></td>
</tr>
<tr>
<td><strong>Single Factor Projection:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>0.39</td>
<td>0.82</td>
<td>1.21</td>
<td>1.58</td>
<td></td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.11</td>
<td>0.12</td>
<td>0.12</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td><strong>Excess Market Return:</strong></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>AR(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.51</td>
<td>12.11</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Real Risk Free Rate:</strong></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>AR(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.32</td>
<td>1.21</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Model-implied nominal term structure, tests of expectations hypothesis and return predictability, and model output for equity markets in the two-volatility model.
Table C.3: Two-Volatility Model: Currency Markets

<table>
<thead>
<tr>
<th></th>
<th>1m</th>
<th>3m</th>
<th>1y</th>
<th>2y</th>
<th>3y</th>
<th>4y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UIP Projection:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>-1.18</td>
<td>-0.81</td>
<td>0.25</td>
<td>0.66</td>
<td>0.81</td>
<td>0.82</td>
<td>0.86</td>
</tr>
<tr>
<td><strong>Std. Dev. AR(1)</strong></td>
<td>15.77</td>
<td>0.00</td>
<td>15.68</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Model-implied foreign exchange rate and tests of expectations hypothesis in currency markets in the two-volatility model.
References


Colacito, Riccardo, and Mariano M. Croce, 2005, Risks for the long run and the real exchange rate, working paper.


Farhi, Emmanuel, and Xavier Gabaix, 2008, Rare disasters and exchange rates, Working paper.


# Tables and Figures

Table 1: **Descriptive Statistics Across Countries**

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>Germany</th>
<th>Japan</th>
<th>US</th>
<th>US47</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation-Adjusted</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest Rate:</td>
<td>Mean</td>
<td>3.61</td>
<td>2.73</td>
<td>1.91</td>
<td>2.59</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>3.06</td>
<td>2.06</td>
<td>2.80</td>
<td>3.07</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.74</td>
<td>0.90</td>
<td>0.91</td>
<td>0.84</td>
</tr>
<tr>
<td><strong>Excess Market Return:</strong></td>
<td>Mean</td>
<td>5.46</td>
<td>4.86</td>
<td>2.83</td>
<td>5.50</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>17.30</td>
<td>20.78</td>
<td>18.56</td>
<td>14.74</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Sharpe Ratio</td>
<td>0.32</td>
<td>0.23</td>
<td>0.15</td>
<td>0.37</td>
</tr>
<tr>
<td><strong>Nominal</strong></td>
<td>Mean</td>
<td>9.15</td>
<td>5.27</td>
<td>3.57</td>
<td>6.82</td>
</tr>
<tr>
<td>Interest Rate:</td>
<td>Std. Dev.</td>
<td>3.91</td>
<td>2.53</td>
<td>3.26</td>
<td>3.76</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.97</td>
<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td><strong>Inflation Rate:</strong></td>
<td>Mean</td>
<td>5.52</td>
<td>2.55</td>
<td>1.84</td>
<td>4.24</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>1.58</td>
<td>0.72</td>
<td>1.24</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.63</td>
<td>0.44</td>
<td>0.21</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Descriptive statistics for interest rates and equity returns across countries. Inflation-adjusted interest rate corresponds to 1 month nominal interest rate adjusted for expected inflation. Excess market return is the return on Morgan Stanley International Index (CRSP portfolio for US) over the one month interest rate. Nominal interest rate is Euro-Currency Middle Rate of 1 month to maturity (CRSP risk-free rate for US47). Monthly observations from Jan 1976 (July 1978 for Japanese interest rate) to Nov 2005, and Feb 1947 to Nov 2005 for US47. Means and standard deviations are annualized.
<table>
<thead>
<tr>
<th></th>
<th>1y</th>
<th>2y</th>
<th>3y</th>
<th>4y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Yield:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Mean</td>
<td>5.56</td>
<td>5.77</td>
<td>5.94</td>
<td>6.07</td>
<td>6.16</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.91</td>
<td>2.86</td>
<td>2.79</td>
<td>2.75</td>
<td>2.72</td>
</tr>
<tr>
<td>UK Mean</td>
<td>7.33</td>
<td>7.35</td>
<td>7.39</td>
<td>7.43</td>
<td>7.46</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.82</td>
<td>2.57</td>
<td>2.45</td>
<td>2.40</td>
<td>2.37</td>
</tr>
</tbody>
</table>

|                |       |       |       |       |       |
| **EH Projection:** |       |       |       |       |       |
| US Slope       | -0.70 | -1.03 | -1.41 | -1.39 |       |
|                | (0.43)| (0.51)| (0.57)| (0.64)|       |
| UK Slope       | -0.14 | -0.12 | -0.10 | -0.09 |       |
|                | (0.57)| (0.64)| (0.72)| (0.82)|       |

|                |       |       |       |       |       |
| **Single Latent Factor Projection:** |       |       |       |       |       |
| US Slope       | 0.44  | 0.87  | 1.24  | 1.45  |       |
| $R^2$          | 0.20  | 0.23  | 0.24  | 0.22  |       |
| UK Slope       | 0.47  | 0.88  | 1.20  | 1.45  |       |
| $R^2$          | 0.28  | 0.30  | 0.30  | 0.29  |       |

Nominal term structure and tests of expectations hypothesis and return predictability in US and UK bond markets. Monthly observations of 1-5 year yields on US and UK discount bonds for June 1952 to Dec 2005 and Jan 1985 to Dec 2005, respectively. EH Projection reports slope coefficient $\beta_{n,m}$ in regression $y_{t+m-n-m} - y_{t,n} = const + \beta_{n,m} m - m (y_{t,n} - y_{t,m}) + error$, where time step $m$ is set at 12 months and bond maturities $n$ run from 2 to 5 years. Standard errors are Newey-West adjusted with 10 lags. Single Latent Factor Projection report the slope coefficient $b_{m,n}$ and $R^2$ in single latent factor regression $r_{x_{t+m,n}} = const + b_{m,n} x_{t,m} + error$, where $r_{x_{t+m,n}}$ is an $m$-months excess return on $n$—period nominal bond, and $r_{x_{t,m}}$ corresponds to a single bond factor obtained from a first-stage projection of average bond returns on three forward rates.
Table 3: Currency Market Data

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>Germany</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Foreign Exchange Rate:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal Mean</td>
<td>-0.53</td>
<td>1.47</td>
<td>3.10</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>10.69</td>
<td>11.18</td>
<td>12.09</td>
</tr>
<tr>
<td>Real Mean</td>
<td>0.72</td>
<td>-0.25</td>
<td>0.65</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>10.80</td>
<td>11.17</td>
<td>12.18</td>
</tr>
<tr>
<td><strong>UIP Projection:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>-1.72</td>
<td>-0.85</td>
<td>-2.83</td>
</tr>
<tr>
<td>(Nominal) R²</td>
<td>(0.95)</td>
<td>(0.77)</td>
<td>(0.66)</td>
</tr>
<tr>
<td>(Real) R²</td>
<td>0.04</td>
<td>0.02</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Foreign exchange rate and tests of expectations hypothesis in currency markets. Monthly observations of changes in log spot foreign exchange rates from Jan 1976 to Nov 2005 for Germany and UK and from July 1978 to Nov 2005 for Japan. UIP Projection reports the slope coefficient $\beta_{UIP}^s$ and $R^2$ in regression

$$s_{t+1}^s - s_t^s - y_{t,1}^s + y_{t,1}^{s*} = const + (\beta_{UIP}^s - 1)(y_{t,1}^s - y_{t,1}^{s*}) + error,$$

where $y_{t,1}^s$ and $y_{t,1}^{s*}$ are US and foreign nominal interest rate, respectively, and $s_t^s$ is the nominal exchange rate. Standard errors are Newey-West adjusted with 10 lags.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preference Parameters:</strong></td>
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</tr>
<tr>
<td>Subjective discount factor</td>
<td>$\delta$ 0.9987</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\psi$ 1.5</td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>$\gamma$ 8</td>
</tr>
<tr>
<td><strong>Consumption Growth Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Mean of consumption growth</td>
<td>$\mu_c$ 0.0016</td>
</tr>
<tr>
<td>Long-run risks persistence</td>
<td>$\rho$ 0.991</td>
</tr>
<tr>
<td>Long-run risks volatility</td>
<td>$\varphi_c$ 0.055</td>
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<tr>
<td>Volatility level</td>
<td>$\sigma_g$ 0.0032</td>
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<tr>
<td>Volatility persistence</td>
<td>$\nu_g$ 0.996</td>
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<tr>
<td>Volatility of volatility</td>
<td>$\sigma_{gw}$ 1.15e-06</td>
</tr>
<tr>
<td><strong>Dividend Growth Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Dividend leverage</td>
<td>$\phi$ 1.5</td>
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<tr>
<td>Volatility loading of dividend growths</td>
<td>$\varphi_d$ 6.0</td>
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<tr>
<td>Correlation of consumption and dividend news</td>
<td>$\tau_{gd}$ 0.1</td>
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<tr>
<td><strong>Inflation Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Mean of inflation rate</td>
<td>$\mu_\pi$ 0.0032</td>
</tr>
<tr>
<td>Inflation leverage on long-run news</td>
<td>$\varphi_{\pi x}$ -2.0</td>
</tr>
<tr>
<td>Inflation shock volatility</td>
<td>$\sigma_\pi$ 0.0035</td>
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<tr>
<td>Expected inflation AR coefficient</td>
<td>$\alpha_\pi$ 0.83</td>
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<tr>
<td>Expected inflation loading on long-run risks</td>
<td>$\alpha_\pi$ -0.35</td>
</tr>
<tr>
<td>Expected inflation leverage on long-run news</td>
<td>$\varphi_{\pi x}$ -1.0</td>
</tr>
<tr>
<td>Expected inflation shock volatility</td>
<td>$\sigma_\pi$ 4.0e-06</td>
</tr>
<tr>
<td><strong>Cross-Country Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Correlation of long-run news</td>
<td>$\tau_e$ 0.999</td>
</tr>
<tr>
<td>Correlation of short-run news</td>
<td>$\tau_\eta$ 0.0</td>
</tr>
<tr>
<td>Correlation of volatility news</td>
<td>$\tau_{gw}$ 0.99</td>
</tr>
<tr>
<td>Correlation of inflation news</td>
<td>$\tau_\xi$ 0.0</td>
</tr>
</tbody>
</table>

Calibrated parameter values for the baseline model. The model is calibrated at monthly frequency.
### Table 5: Consumption Growth Dynamics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Median</th>
<th>95%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>0.78</td>
<td>(0.09)</td>
<td>1.38</td>
<td>2.03</td>
<td>0.96</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.36</td>
<td>(0.07)</td>
<td>0.38</td>
<td>0.61</td>
<td>0.18</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.20</td>
<td>(0.08)</td>
<td>0.21</td>
<td>0.47</td>
<td>-0.03</td>
</tr>
<tr>
<td>AR(5)</td>
<td>0.08</td>
<td>(0.08)</td>
<td>0.17</td>
<td>0.45</td>
<td>-0.08</td>
</tr>
<tr>
<td>VR(2)</td>
<td>1.36</td>
<td>(0.07)</td>
<td>1.38</td>
<td>1.59</td>
<td>1.18</td>
</tr>
<tr>
<td>VR(5)</td>
<td>2.15</td>
<td>(0.27)</td>
<td>2.03</td>
<td>2.97</td>
<td>1.32</td>
</tr>
<tr>
<td>VR(10)</td>
<td>3.01</td>
<td>(0.65)</td>
<td>2.78</td>
<td>5.06</td>
<td>1.31</td>
</tr>
<tr>
<td>$Corr(\Delta c, \Delta c^*)$</td>
<td>0.25</td>
<td>(0.09)</td>
<td>0.24</td>
<td>0.49</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Calibration of consumption growth. Quarterly observations of US real consumption growth from 1976Q2 to 2005Q2. Cross-country correlation is computed for US and UK series. Standard errors are Newey-West corrected using 10 lags. Model output is based on 1000 simulations of 360 months aggregated to quarterly horizon.

### Table 6: Inflation Dynamics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Median</th>
<th>95%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\pi)$</td>
<td>1.62</td>
<td>(0.32)</td>
<td>1.68</td>
<td>2.70</td>
<td>1.19</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.64</td>
<td>(0.16)</td>
<td>0.70</td>
<td>0.88</td>
<td>0.40</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.67</td>
<td>(0.14)</td>
<td>0.60</td>
<td>0.84</td>
<td>0.22</td>
</tr>
<tr>
<td>AR(5)</td>
<td>0.57</td>
<td>(0.17)</td>
<td>0.48</td>
<td>0.79</td>
<td>0.13</td>
</tr>
<tr>
<td>AR(10)</td>
<td>0.36</td>
<td>(0.17)</td>
<td>0.32</td>
<td>0.69</td>
<td>-0.04</td>
</tr>
<tr>
<td>VR(2)</td>
<td>1.61</td>
<td>(0.17)</td>
<td>1.69</td>
<td>1.87</td>
<td>1.40</td>
</tr>
<tr>
<td>VR(5)</td>
<td>3.56</td>
<td>(0.60)</td>
<td>3.41</td>
<td>4.34</td>
<td>2.10</td>
</tr>
<tr>
<td>VR(10)</td>
<td>6.44</td>
<td>(1.81)</td>
<td>5.69</td>
<td>8.25</td>
<td>2.75</td>
</tr>
<tr>
<td>$Corr(\pi, \Delta c)$</td>
<td>-0.21</td>
<td>(0.14)</td>
<td>-0.39</td>
<td>-0.13</td>
<td>-0.63</td>
</tr>
<tr>
<td>$Corr(\pi, \pi^*)$</td>
<td>0.73</td>
<td>0.07</td>
<td>0.63</td>
<td>0.86</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Calibration of inflation rate. Quarterly observations of US inflation rate from 1976Q2 to 2005Q2. Cross-country correlation is computed for US and UK. Standard errors are Newey-West corrected using 10 lags. Model output is based on 1000 simulations of 360 months aggregated to quarterly horizon.
<table>
<thead>
<tr>
<th></th>
<th>1m</th>
<th>1y</th>
<th>2y</th>
<th>3y</th>
<th>4y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Term Structure:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>5.39</td>
<td>5.44</td>
<td>5.59</td>
<td>5.77</td>
<td>5.98</td>
<td>6.22</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.33</td>
<td>2.25</td>
<td>2.12</td>
<td>2.01</td>
<td>1.94</td>
<td>1.91</td>
</tr>
<tr>
<td><strong>EH Projection:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>-0.17</td>
<td>-0.30</td>
<td>-0.33</td>
<td>-0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Single Latent Factor Projection:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>0.39</td>
<td>0.82</td>
<td>1.21</td>
<td>1.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Excess Market Return:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>6.64</td>
<td>11.11</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.50</td>
<td>1.30</td>
<td>0.99</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Model-implied nominal term structure, tests of expectations hypothesis and return predictability, and model output for equity markets. EH Projection reports the slope coefficient $\beta_{n,m}^s$ in regression $y_{t+n-m}^s - y_{t,n}^s = const + \beta_{n,m}^s \frac{m}{n-m} (y_{t,n}^s - y_{t,m}^s) + error$, where time step $m$ is set at 12 months and bond maturities $n$ run from 2 to 5 years. Single Latent Factor Projection report the slope coefficient $b_{m,n}^s$ and $R^2$ in single latent factor regression $r_{x_{t+m,n}}^s = const + b_{m,n}^s \hat{r}_{x_{t,m}}^s + error$, where $r_{x_{t+m,n}}^s$ is an $m$-months excess return on $n$-period nominal bond, and $\hat{r}_{x_{t,m}}^s$ corresponds to a single bond factor obtained from a first-stage projection of average bond returns on three forward rates. Model output is based on long simulation from the model.
Table 8: Model Implications: Currency Markets

<table>
<thead>
<tr>
<th></th>
<th>1m</th>
<th>3m</th>
<th>1y</th>
<th>2y</th>
<th>3y</th>
<th>4y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>UIP Projection</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>-1.23</td>
<td>-1.18</td>
<td>-0.71</td>
<td>0.20</td>
<td>1.25</td>
<td>2.30</td>
<td>3.19</td>
</tr>
<tr>
<td>Std. Dev. AR(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal FX Rate:</td>
<td>19.86</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real FX Rate:</td>
<td>19.80</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Model-implied foreign exchange rate and tests of expectations hypothesis in currency markets. UIP Projection reports the slope coefficient $\beta^{UIP}$ and $R^2$ in regression $s_{t+1}^s - s_t^s - y_{t,1}^s + y_{t,1}^{s*} = const + (\beta^{UIP} - 1)(y_{t,1}^s - y_{t,1}^{s*}) + error$, where $y_{t,1}^s$ and $y_{t,1}^{s*}$ are US and foreign nominal interest rate, respectively, and $s_t^s$ is the nominal exchange rate. Model output is based on long simulation from the model.
Figure 1: Inflation beta. Computations are based on VAR(1) fit of consumption growth and inflation rate. Data are quarterly observations of US real consumption growth and inflation from 1976Q2 to 2005Q2. Model output is based on 1,000 simulations of 360 months aggregated to quarterly horizon.
Figure 2: Real exchange rate (solid line) and domestic minus foreign consumption volatility (dashed line).
Figure 3: Forward premium (solid line) and domestic minus foreign consumption volatility (dashed line).
Figure 4: Ex-ante excess return in foreign bonds (solid line) and domestic minus foreign consumption volatility (dashed line). The ex-ante excess returns are constructed based on the foreign exchange projections.
Figure 5: Model-implied slope coefficient in nominal foreign exchange projection for different values of the IES.