CHAPTER 5

Long-Run Risks and Risk Compensation in Equity Markets

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Abstract

What drives the compensation in equity markets? This article shows that long-run growth and economic uncertainty in the economy play an important role in determining the risk in equity markets. The size of the market risk premium, the level of the risk-free rate, the volatility of asset prices, and differences in the risk compensation across assets are shown to be related to risks pertaining to the long-run growth and uncertainty in the economy.

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1. INTRODUCTION

Several aspects of asset markets are puzzling. The work of Mehra and Prescott (1985) shows that the magnitude of risk compensation in equity markets is a puzzle—Treasury bills offer a return of about 1 percent per annum while the equity market portfolio offers 7.5 percent. What risks justify such a sizable compensation for holding equity? Another equally puzzling, and related, dimension is the large difference in the average returns across equity portfolios. For example, the return to value firms exceeds that of growth firms by about 7 percent per annum. In addition to these return puzzles, Shiller (1981) and Leroy and Porter (1981) document the volatility puzzle—it is hard to explain the high volatility of equity prices. This article highlights the ideas developed in Bansal and Lundblad (2002), Bansal and Yaron (2004), Bansal, Khatchatrian, and Yaron (2005), Bansal, Dittmar, and Lundblad (2002, 2005), and Bansal, Gallant, and Tauchen (2007) pertaining to various financial market anomalies. This research argues that the magnitudes of asset returns and volatility are a natural outcome of risks associated with the long-run growth prospects and changing economic uncertainty in the economy.

Bansal and Yaron (2004) argue that investors care about the long-run growth prospects and the level of economic uncertainty. Changes in these fundamentals drive the risks and volatility in asset prices. They document that consumption and dividend growth rates contain a small long-run component. That is, current shocks to expected growth alter expectations about future economic growth not only for short horizons but also for the very long run. Agents care a lot about these long-run components as small revisions in them lead to large changes in asset prices. Any adverse movements in the long-run growth components lower asset prices and concomitantly the wealth and consumption of investors. This makes holding equity very risky for investors, making them demand a high equity risk compensation.

Bansal and Yaron further argue that time variation in expected excess returns is due to variation in economic uncertainty. They model this uncertainty by incorporating time-varying consumption volatility in the consumption process. Empirical motivation for this channel is provided by Bansal, Khatchatrian, and Yaron (2005)—their robust empirical finding is that current consumption volatility predicts future asset valuations and that current asset valuations predict future consumption volatility. Both projection coefficients are significantly negative. A rise in economic uncertainty lowers asset prices—that is, asset markets dislike economic uncertainty. Bansal and Yaron derive the result that volatility shocks, in equilibrium, carry a separate risk premium; this is a novel feature of their model.

Epstein and Zin (1989) preferences play an important role in the Bansal and Yaron model. These preferences allow for separation between risk aversion and the intertemporal elasticity of substitution (IES) of investors. An IES larger than one is required for the wealth to consumption ratio to rise with expected consumption growth; when the IES is smaller than one, high expected growth lowers the wealth to consumption ratio. The equity price to dividend ratio mirrors the behavior of the wealth to consumption ratio. This ensures that equity payoffs are high when consumption and corporate profits rise, leading to a positive equity risk premium. An IES larger than one is also required to capture the data feature that asset markets dislike economic uncertainty.

The magnitude of the IES is a key empirical issue. Hansen and Singleton (1982), Attanasio and Weber (1989), and Attanasio and Vissing-Jorgensen (2003) estimate the IES to be well in excess of one. Hall (1988) and Campbell (1999), on the other hand, estimate its value to be well below one. Bansal and Yaron argue that the estimates for the IES in Hall and Campbell are not a robust guide for the magnitude of the IES parameter. They show that even if the population value of the IES is larger than one, the estimation methods used by Hall would measure the IES to be close to zero. That is, there is a severe downward bias in the point estimates of the IES. Bansal and Yaron as well as Bansal, Khatchatrian, and Yaron (2005) further argue that the economic implications when the IES is less than one—a rise in consumption volatility and/or a drop is expected growth raises asset valuations, are counterfactual, making the low magnitude of the IES suspect.

The arguments presented in Bansal and Yaron also have immediate implications for the cross-sectional differences in mean returns across assets. Firms whose expected cash-flow (profits) growth rates move with the economy are more exposed to long-run risks and hence should carry a higher risk compensation. In Breeden (1979), Lucas (1978), and Hansen and Singleton (1982), the riskiness of the asset is determined by the consumption beta of the asset. However, the consumption beta of an asset is not exogenous—in equilibrium, it is determined by the systematic risks in cash flows and the preference parameters of the representative agent. That is, cross-sectional differences in betas of assets reflect differences in the systematic risks in cash flows. Risks in cash flows should consequently contain information about differences in mean returns across assets. We review the work of Bansal, Dittmar, and Lundblad (2002, 2005), who show that systematic risks in cash flows can account for the cross-sectional differences in risk premia of assets. Specifically, their cash-flow betas can account for the puzzling value, size, and the momentum spread in the cross section of assets.

Bansal and Lundblad (2002) rely on long-run components and varying risk premia to address issues in international equity markets. Developed markets' asset prices and returns show a high degree of correlation; however, dividends and earnings growth across these economies are virtually uncorrelated. Bansal and Lundblad show that high asset price volatility and correlation across national equity markets are due to the long-run component in dividend growth rates and time-varying systematic risk.

The argument that long-term economic growth and uncertainty are the key drivers of risks in equity markets is distinct from the arguments presented in Campbell and Cochrane (1999). They argue that equity market risks are driven largely (even exclusively) by fluctuations in the ex-ante rate of discount (cost of capital) through external habit formation. Sorting out which of the channels is critical for explaining the risk compensation in equity markets, consequently, is largely an empirical issue. Using the Efficient Method of Moments (EMM) estimation technique, Bansal, Gallant, and Tauchen (2007) document the differences between the Bansal and Yaron and Campbell and Cochrane models. A unique dimension of the Bansal, Gallant, and Tauchen paper is that consumption and dividends in their model are cointegrated—this feature is typically missing in earlier work on asset market models.

The remainder of the article has three sections. Section 2 discusses the long-run risks model of Bansal and Yaron. Section 3 discusses the issue of cross-sectional differences in returns across asset portfolios. Section 4 presents concluding comments.

2. LONG-RUN RISKS MODEL

2.1. Preferences and the Environment

We consider a representative agent with the generalized preferences developed in Epstein and Zin (1989). The logarithm of the Intertemporal Marginal Rate of Substitution (IMRS), m_{t+1}, for these preferences, as shown in the Epstein and Zin paper, is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}. \tag{1}$$

If R_{t+1} is the gross return on the asset, then $\log(R_{t+1}) \equiv r_{t+1}$ is the continuous return on the asset. Using the standard asset pricing restriction for any continuous return $r_{i,t+1}$, it follows that[1]

$$E_t \left[\exp\left(\theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} + r_{i,t+1} \right) \right] = 1, \tag{2}$$

where g_{t+1} equals $\log(C_{t+1}/C_t)$—the log growth rate of aggregate consumption. The return, $r_{a,t+1}$, is the log of the return (i.e., continuous return) on an asset that delivers aggregate consumption as its dividends each time period. The time discount factor is δ and the parameter $\theta \equiv 1 - \gamma/(1 - 1/\psi)$, where $\gamma \geq 0$ is the risk aversion (sensitivity) parameter, and $\psi \geq 0$ is the intertemporal elasticity of substitution. The sign of θ is determined by the magnitudes of the risk aversion and the elasticity of substitution.[2] Note that when θ equals one, the above IMRS and the asset pricing implications collapse to the usual case of power utility considered in Mehra and Prescott (1985).

The return to the aggregate consumption claim, $r_{a,t+1}$, is not observed in the data while the return on the dividend claim corresponds to the observed return on the market portfolio $r_{m,t+1}$. The levels of market dividends and consumption are not equal; aggregate consumption is much larger than aggregate dividends. The difference is financed by labor income. In the model, aggregate consumption and aggregate dividends are treated as two separate processes and the difference between them implicitly defines the agent's labor income process.

[1]Note that the standard asset pricing condition in frictionless markets is

$$E_t[\exp(m_{t+1} + r_{t+1})] = 1,$$

where the intertemporal marginal rate of substitution is M_{t+1} and $\log(M_{t+1}) \equiv m_{t+1}$. The logarithm of the gross return for an asset equals r_{t+1}.

[2]In particular, if $\psi > 1$ and $\gamma > 1$, then θ will be negative. Note that when $\theta = 1$, that is, $\gamma = (1/\psi)$, the above recursive preferences collapse to the standard case of expected utility. Further, when $\theta = 1$ and in addition $\gamma = 1$, we get the standard case of log utility.

The key ideas of the model are developed and the intuition is provided via approximate analytical solutions. However, all the quantitative results reported in the paper are based on numerical solutions of the model. To derive the approximate analytical solutions for the model, we use the standard first-order Taylor series approximation developed in Campbell and Shiller (1988),[3]

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{3}$$

where lowercase letters refer to variables in logs, in particular, $r_{a,t+1} = \log(R_{a,t+1})$ is the continuous return on the consumption claim, and $z_t \equiv \log(P_t/C_t)$ is the log price to consumption ratio. Analogously, $r_{m,t+1}$ and $z_{m,t}$ correspond to the continuous return on the dividend claim and its log price-dividend ratio. As $P_t + C_t/C_t$ is the agent's wealth to consumption ratio, fluctuations in the price to consumption ratio, consequently, also correspond to movements in the wealth to consumption ratio. Parameters κ_0 and κ_1 are approximating constants that both depend only on the average level of z.[4]

From Eq. (1) it follows that the innovation in the IMRS, m_{t+1}, is driven by the innovations in g_{t+1} and $r_{a,t+1}$. Covariation with the innovation in m_{t+1} determines the risk premium for any asset. The simpler model specification, with only long-run growth rate risks, is discussed first. The full model that incorporates long-run growth rate and economic uncertainty risks is presented after that.

2.2. Long-Run Growth Rate Risks

The agents' IMRS depends on the endogenous consumption return, $r_{a,t+1}$. The risk compensation on all assets depends on this return, which itself is determined by the process for consumption growth. The dividend process is needed for determining the return on the market portfolio. To capture long-run risks, consumption and dividend growth rates, g_{t+1} and $g_{d,t+1}$, are modeled to contain a small persistent predictable component x_t,

$$\begin{aligned} x_{t+1} &= \rho x_t + \varphi_e \sigma e_{t+1}, \\ g_{t+1} &= \mu + x_t + \sigma \eta_{t+1}, \\ g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma u_{t+1}, \\ & e_{t+1}, u_{t+1}, \eta_{t+1} \sim \text{N.i.i.d.}(0,1), \end{aligned} \tag{4}$$

[3] Any equity return can be written as

$$R_{t+1} = \frac{1 + \frac{P_{t+1}}{D_{t+1}}}{\frac{P_t}{D_t}} \frac{D_{t+1}}{D_t}.$$

The approximate return expression follows from taking the log of the gross return and then taking a first-order Taylor series approximation of $\log\left(1 + \frac{P_{t+1}}{D_{t+1}}\right)$ around the average value of the log of the price-dividend ratio, which is referenced as z in Eq. (3). The approximating constants κ_0 and κ_1 are solely determined by the average value of z.

[4] The Campbell–Shiller approximate return follows from using a first-order Taylor series expansion of the continuous return. Note that $\kappa_1 = \exp(\bar{z})/(1+\exp(\bar{z}))$. The value of κ_1 is set at 0.997, which is consistent with the magnitude of $\bar{z}$ in our sample and with the magnitudes used in Campbell and Shiller (1988).

with the three shocks, e_{t+1}, u_{t+1}, and η_{t+1}, assumed to be mutually independent. The volatility of the consumption growth rate innovation is σ. Similarly, the volatility of the innovation in x_{t+1} and $g_{d,t+1}$ is $\varphi_e\sigma$ and $\varphi_d\sigma$, respectively.

The parameter ρ determines the persistence of the expected growth rate process. First, note that when $\varphi_e = 0$, the processes g_t and $g_{d,t+1}$ are i.i.d. Second, if $e_{t+1} = \eta_{t+1}$, the process for consumption is the ARMA(1,1) used in Campbell (1999), Cecchetti, Lam, and Mark (1993), and Bansal and Lundblad (2002). If, in addition, $\varphi_e = \rho$, then consumption growth corresponds to an AR(1) process used in Mehra and Prescott (1985).

Two additional parameters, $\phi > 1$ and $\varphi_d > 1$, calibrate the overall volatility of dividends and its correlation with consumption. The parameter ϕ can be interpreted, as in Abel (1999), as the leverage ratio on expected consumption growth. Alternately, this says that corporate profits, relative to consumption, are more sensitive to changing expected economic growth conditions. That is, any fluctuation in x_t leads to larger changes in expected dividend growth relative to expected consumption growth. The maintained assumption is that the three innovations are uncorrelated. It is straightforward to allow the three shocks to be correlated; however, to maintain parsimony in the number of parameters, they are assumed to be independent. Note that consumption and dividends are not cointegrated in the above specification—Bansal, Gallant, and Tauchen (2007) develop a specification that does allow for cointegration between consumption and dividends.

Asset prices reflect expectations of future growth rates. To develop some intuition about long-run risks, consider the quantity

$$E_t\left[\sum_{j=1}^{\infty}\kappa_1^j g_{t+j}\right],$$

with κ_1 less than one, this expectation equals $\kappa_1 x_t/(1-\kappa_1\rho)$. Even if the variance of x is tiny, but ρ fairly high, then shocks to x can alter growth rate expectations for the long run, leading to volatile asset prices. Bansal and Lundblad (2002) and Bansal and Yaron (2004) provide empirical support for the existence of this long-run component in observed growth rates.

2.2.1. Equilibrium and Asset Prices

The consumption and dividend growth rates processes are exogenous in this endowment economy. Further, the IMRS depends on an endogenous return $r_{a,t+1}$. To characterize the IMRS and the behavior of asset returns, a solution for the log price to consumption ratio z_t and the log price-dividend ratio $z_{m,t}$ is needed. The relevant state variable for deriving the solution for z_t and $z_{m,t}$ is the expected growth rate of consumption x_t.

Exploiting the Euler equation (2), the approximate solution for the log price-consumption z_t has the form $z_t = A_0 + A_1x_t$. An analogous expression holds for the log price-dividend ratio $z_{m,t}$.[5] Bansal and Yaron (2004) show that the solution coefficients are

$$A_1 = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}, \quad A_{1,m} = \frac{\phi-\frac{1}{\psi}}{1-\kappa_{1,m}\rho}. \tag{5}$$

It follows that A_1 is positive if the IES, ψ, is greater than one. In this case the intertemporal substitution effect dominates the wealth effect. In response to higher expected growth, agents buy more assets, and consequently the wealth to consumption ratio rises. The level of consumption rises due to a rise in expected growth; however, wealth rises more than consumption. In the standard power utility model with risk aversion larger than one, the IES is less than one, and hence A_1 is negative—a rise in expected growth potentially lowers asset valuations. That is, the wealth effect dominates the substitution effect.[6] Corporate payouts (i.e., dividends), with $\phi > 1$, are more sensitive to long-run risks (i.e., $A_{1,m} > A_1$), and changes in expected growth rate lead to a larger reaction in the price of the dividend claim than in the price of the consumption claim.

Equation (3), the solution in (5), and the dynamics for the consumption and dividend growth rates provide a complete characterization for the endogenous returns on the consumption and the dividend asset.

2.2.2. Pricing of Long-Run Growth Risks

Substituting the equilibrium return for $r_{a,t+1}$ into the IMRS, Bansal and Yaron show that the innovation in m_{t+1} is

$$\begin{aligned} m_{t+1} - E_t(m_{t+1}) &= -\left[\frac{\theta}{\psi} - \theta + 1\right]\sigma\eta_{t+1} - (1-\theta)\left[\kappa_1\left(1-\frac{1}{\psi}\right)\frac{\varphi_e}{1-\kappa_1\rho}\right]\sigma e_{t+1} \\ &= -\lambda_{m,\eta}\sigma\eta_{t+1} - \lambda_{m,e}\sigma e_{t+1}. \end{aligned} \tag{6}$$

The parameters $\lambda_{m,e}$ and $\lambda_{m,\eta}$ determine the risk compensation for expected growth rate shock and the independent consumption shock η_{t+1}.

The risk compensation for the η_{t+1} shocks is very standard as $\lambda_{m,\eta}$ equals the risk aversion parameter γ.[7] In addition, with power utility, that is, when θ equals one, $\lambda_{m,e} = 0$. Long-run risks are priced only when θ differs from one, that is, when risk aversion is not the reciprocal of the IES—this highlights the importance of the generalized preferences of Epstein and Zin (1989). The market price of long-run risks is sensitive to the magnitude of the permanence parameter ρ. The risk compensation for

[5] The expression for the intercept terms A_0 for the valuation ratio for the consumption claim, and $A_{0,m}$ for the dividend claim are not important for our qualitative results.

[6] An alternative interpretation with the power utility model is that higher expected growth rates increase the risk-free rate to an extent that discounting dominates the effects of higher expected growth rates. This leads to a fall in asset prices.

[7] This follows by substituting the expression for θ and simplifying the expression $[\theta/\psi - \theta + 1]$.

long-run risks, $\lambda_{m,e}$, rises as the permanence parameter ρ rises. The conditional volatility of the pricing kernel is constant, as all risk sources have constant conditional variances.

As asset returns and the pricing kernel in this model economy are conditionally log-normal, the risk premium on any asset i is $E_t[r_{i,t+1} - r_{f,t}] = -\text{cov}_t(m_{t+1}, r_{i,t+1}) - 0.5\sigma^2_{r_i,t}$. Given the solutions for A_1 and $A_{1,m}$, it is straightforward to derive the equity premium on the market portfolio,

$$E(r_{m,t+1} - r_{f,t}) = \beta_{m,\eta}\lambda_{m,\eta}\sigma^2 + \beta_{m,e}\lambda_{m,e}\sigma^2 - 0.5\text{Var}(r_{m,t}). \tag{7}$$

The market portfolio's beta with respect to the long-run risk component is

$$\beta_{m,e} \equiv \left[\kappa_{1,m}\left(\phi - \frac{1}{\psi}\right)\frac{\varphi_e}{1-\kappa_{1,m}\rho}\right]. \tag{8}$$

The exposure of the market return to long-run risk is $\beta_{m,e}$, and the market price of the long-run risk is $\lambda_{m,e}$. The expressions for these variables reveal that a rise in ρ increases both $\beta_{m,e}$ and $\lambda_{m,e}$. Consequently, the risk premium on the asset also increases with ρ. The market portfolio's beta with respect to the short-run risk component η_{t+1} is determined by the exposure of the dividend's innovation to η_{t+1}. The assumption that the dividend innovation of the market portfolio is independent of the short-run shock in consumption η_{t+1} implies that $\beta_{m,\eta}$ will be zero in our calibration exercise.

As all shocks have a constant conditional variance, the conditional risk premium on the market portfolio and its conditional volatility are constant. The ratio of the two, namely the Sharpe ratio, is also constant. In order to address issues that pertain to time-varying risk premia and predictability of risk premia, Bansal and Yaron augment the above model by incorporating time-varying economic uncertainty.

2.3. Long-Run Growth and Uncertainty Risks

Bansal and Yaron model fluctuating economic uncertainty as time-varying volatility of consumption growth. The consumption and dividends dynamics that incorporate stochastic volatility are

$$\begin{aligned}
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1},\\
g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1},\\
g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1},\\
\sigma^2_{t+1} &= \sigma^2 + \nu_1(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1},\\
& e_{t+1}, u_{t+1}, \eta_{t+1}, w_{t+1} \sim \text{N.i.i.d.}(0,1),
\end{aligned} \tag{9}$$

where σ_{t+1}, the conditional volatility of consumption growth, represents the time-varying economic uncertainty incorporated in consumption growth rate. The unconditional mean of the time-varying variance of consumption growth is σ^2, and σ_w determines the volatility of shocks to consumption uncertainty. The parameter ν_1 determines the persistence of shocks to consumption variance. To maintain parsimony, it is assumed that the shocks are uncorrelated and only one source of time-varying economic uncertainty affects consumption and dividends.

The relevant state variables in solving for the equilibrium price-consumption (and price-dividend) ratio are now x_t and σ_t^2. Thus, the approximate solution for the price-consumption ratio is $z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$. The solution for A_1 is unchanged (Eq. (5)). The solution coefficient A_2 for measuring the sensitivity of the price-consumption ratio to volatility fluctuations is

$$A_2 = \frac{0.5\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta A_1 \kappa_1 \varphi_e)^2\right]}{\theta(1 - \kappa_1 \nu_1)} . \tag{10}$$

An analogous coefficient for the market price-dividend ratio, $A_{2,m}$, is provided in Bansal and Yaron (2004).

The expression for A_2 provides two valuable insights. First, if the IES and risk aversion are larger than one, then θ and consequently A_2 are negative. In this case, a rise in consumption volatility lowers asset valuations and increases the risk premia on all assets. To capture the intuition that a rise in economic uncertainty lowers asset valuations requires that the IES be larger than one. Bansal, Khatchatrian, and Yaron (2005) present robust empirical evidence that asset markets dislike economic uncertainty—that is, A_2 is negative. This empirical evidence, given the expression for A_2, has a direct bearing on the plausible magnitude for the IES. Second, an increase in the permanence of volatility shocks, that is ν_1, magnifies the effects of volatility shocks on valuation ratios as changes in economic uncertainty are perceived by investors as being long-lasting.

2.3.1. Pricing of Uncertainty Risks

As the wealth to consumption ratio is affected by consumption volatility shocks, so are the return $r_{a,t+1}$ and the IMRS. Specifically, the innovation in m_{t+1} is

$$m_{t+1} - E_t(m_{t+1}) = -\lambda_{m,\eta}\sigma_t\eta_{t+1} - \lambda_{m,e}\sigma_t e_{t+1} - \lambda_{m,w}\sigma_w w_{t+1}, \tag{11}$$

where $\lambda_{m,\eta}$, $\lambda_{m,e}$, and $\lambda_{m,w}$ are the market prices of risks for the short-run, long-run, and volatility risks. The market prices of systematic risks, including the compensation for stochastic volatility risk in consumption, can be expressed in terms of underlying preferences and parameters that govern the evolution of consumption growth as

$$\begin{aligned}
\lambda_{m,\eta} &= \gamma, \\
\lambda_{m,e} &= \left(\gamma - \frac{1}{\psi}\right)\left[\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho}\right], \\
\lambda_{m,w} &= \left(\gamma - \frac{1}{\psi}\right)(1 - \gamma)\left[\frac{\kappa_1\left(1 + \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho}\right)^2\right)}{2\,(1 - \kappa_1 \nu_1)}\right].
\end{aligned} \tag{12}$$

Expression (11) is similar to the earlier model (see Eq. (6)) save for the inclusion of w_{t+1}, the shocks to consumption volatility. In the special case of power utility, when

$\theta = 1$ or more specifically, where $\gamma = 1/\psi$, the risk compensation parameters $\lambda_{m,e}$ and $\lambda_{m,w}$ are zero. The long-run risks and volatility risks are not reflected in the innovation of the pricing kernel. With power utility there is no separate risk compensation for long-run growth rate risks and volatility risks—with Epstein and Zin preferences, both risks are priced. The pricing of long-run and volatility risks is an important and novel feature of the Bansal and Yaron model.

The equity premium in the presence of time-varying economic uncertainty is

$$E_t(r_{m,t+1} - r_{f,t}) = \beta_{m,\eta}\lambda_{m,\eta}\sigma_t^2 + \beta_{m,e}\lambda_{m,e}\sigma_t^2 + \beta_{m,w}\lambda_{m,w}\sigma_w^2 - 0.5\mathrm{Var}_t(r_{m,t+1}), \tag{13}$$

where $\beta_{m,w} \equiv \kappa_{1,m}A_{2,m}$. The first β corresponds to the exposure to short-run risks, and the second to long-run risks. The last beta, $\beta_{m,w}$, is the exposure of the asset to volatility risks.

The risk premium on the market portfolio is time-varying as σ_t fluctuates. The ratio of the conditional risk premium to the conditional volatility of the market portfolio fluctuates with σ_t, and hence the Sharpe ratio is time-varying. The maximal Sharpe ratio in this model economy, which approximately equals the conditional volatility of the log IMRS, also varies with σ_t.[8] This implies that during periods of high economic uncertainty all risk premia will rise.

The first-order effects on the level of the risk-free rate, as discussed in Bansal and Yaron (2006), are the rate of time preference and the average consumption growth rate, divided by the IES. Increasing the IES keeps the level low. The variance of the risk-free rate is determined by the volatility of the expected consumption growth rate and the IES. Increasing the IES lowers the volatility of the risk-free rate. In addition, incorporating economic uncertainty leads to an interesting channel for interpreting fluctuations in the real risk-free rate. In particular, Bansal and Yaron show that this has serious implications for the measurement of the IES in the data. In the presence of varying volatility, the estimates of the IES based on the projections considered in Hall (1988) and Campbell (1999) are seriously biased downwards.

2.4. Data and Model Implications

2.4.1. Data and the Growth Rate Dynamics

Bansal and Yaron (2004) calibrate the model described in (4) and (9) at the monthly frequency. From this monthly model they derive time-aggregated annual growth rates of consumption and dividends to match key aspects of annual aggregate consumption and dividends data. Further, as in Campbell and Cochrane (1999) and in Kandel and Stambaugh (1991), they assume that the decision interval of the agent is monthly, but the targeted data to match is annual.[9]

[8]Given the conditional normality of the logarithm of the IMRS, the maximal Sharpe ratio is simply the conditional standard deviation of the logarithm of the IMRS.

[9]The evidence regarding the model is based on numerical solutions using standard polynomial-based projection methods discussed in Judd (1998). The numerical results are quite close to those based on the approximate analytical solutions.

For consumption, BEA data on real per capita annual consumption growth of non-durables and services for the period 1929–1998 is utilized. This is the longest single data source of consumption data. Dividends and the value-weighted market return data are taken from the CRSP. All nominal quantities are deflated using the CPI. To facilitate comparisons between the model, which is calibrated to a monthly decision interval, and the annual data, the monthly model is time-aggregated to the annual frequency to derive annual statistics.

The annual real per capita consumption growth mean is 1.8 percent, and its standard deviation is about 2.9 percent. This volatility is somewhat lower for our sample than for the period considered in Mehra and Prescott (1985), Kandel and Stambaugh (1991), and Abel (1999). Table 1, adapted from Bansal and Yaron (2004), shows that in the data, consumption growth has a large first-order autocorrelation coefficient and a small second-order one. The standard errors in the data for these autocorrelations are sizeable. An alternative way to view the long horizon properties of the consumption and dividend growth rates is to use variance ratios, which themselves are determined by the autocorrelations (see Cochrane (1988)). In the data the variance ratios first rise significantly and at about 7 years start to decline. The standard errors on these variance ratios, not surprisingly, are quite substantial. There is considerable evidence of small sample biases in estimating autoregression coefficients and variance ratios (see Hurwicz (1950) and Ansley and Newbold (1980)). To account for any small sample biases, Bansal and Yaron also report statistics based on 1000 Monte Carlo experiments

TABLE 1
Time-Series Properties of Data

Variable	Estimate	S.E.
$\sigma(g)$	2.93	(0.69)
$AC(1)$	0.49	(0.14)
$AC(2)$	0.15	(0.22)
$AC(5)$	−0.08	(0.10)
$AC(10)$	0.05	(0.09)
$VR(2)$	1.61	(0.34)
$VR(5)$	2.01	(1.23)
$VR(10)$	1.57	(2.07)
$\sigma(g_d)$	11.49	(1.98)
$AC(1)$	0.21	(0.13)
$corr(g, g_d)$	0.55	(0.34)

Table 1 displays the time-series properties of aggregate consumption and dividend growth rates: g and g_d, respectively. The statistics are based on annual observations from 1929 to 1998. Consumption is real per capita consumption of non-durables and services; dividends are the sum of real dividends across all CRSP firms. $AC(j)$ is the jth autocorrelation, $VR(j)$ is the jth variance ratio, σ is the volatility, and $corr$ denotes the correlation. Standard errors are Newey and West (1987) corrected using 10 lags.

each with 840 monthly observations corresponding to the 70 annual observations available in the annual data set.

In terms of the specific parameters, Bansal and Yaron (2004) calibrate ρ at 0.979, which determines the persistence in the long-run component in growth rates. Their choice of φ_e and σ ensures that the model matches the unconditional variance and the autocorrelation function of annual consumption growth. The standard deviation of the one-step-ahead innovation in consumption, that is σ, equals 0.0078. This parameter configuration implies that the predictable variation in monthly consumption growth is very small, as the implied R^2 is only 4.4 percent. The exposure of the corporate sector to long-run risks is governed by ϕ, and its magnitude is similar to that in Abel (1999). The standard deviation of the monthly innovation in dividends, $\varphi_d\sigma$, is 0.0351.

Bansal and Yaron also consider the consumption and dividend dynamics that incorporate time-varying volatility (see Eq. (9)). The parameters of the volatility process are chosen to capture the persistence in consumption volatility. Based on the evidence of slow decay in volatility shocks, they calibrate ν_1, the parameter governing the persistence of conditional volatility, at 0.987. The shocks to the volatility process have very small volatility, and σ_w is calibrated at 0.23×10^{-5}. Bansal and Yaron show that with this configuration, the assumed consumption and dividend growth rates very closely match the key consumption and dividends data features reported in Table 1.

Table 2 presents the targeted asset market data for 1929 to 1998. The equity risk premium is 6.33 percent per annum, and the real risk-free rate is 0.9 percent. The annual market return volatility is 19.42 percent, and that of the real risk-free is quite small,

TABLE 2
Asset Market Data

Variable	Estimate	S.E.
	Returns	
$E(r_m - r_f)$	6.33	(2.15)
$E(r_f)$	0.86	(0.42)
$\sigma(r_m)$	19.42	(3.07)
$\sigma(r_f)$	0.97	(0.28)
	Price-dividend ratio	
$E(\exp(p - d))$	26.56	(2.53)
$\sigma(p - d)$	0.29	(0.04)
$AC1(p - d)$	0.81	(0.09)
$AC2(p - d)$	0.64	(0.15)

Table 2, adapted from Bansal and Yaron (2004), presents descriptive statistics of asset market data. $E(r_m - r_f)$ and $E(r_f)$ are, respectively, the annualized equity premium and mean risk free-rate. $\sigma(r_m)$, $\sigma(r_f)$, and $\sigma(p - d)$ are the annualized volatilities of the market return, the risk-free rate, and the log price-dividend, respectively. $AC1$ and $AC2$ denote the first and second autocorrelations. Standard errors are Newey and West (1987) corrected using 10 lags.

about 1 percent per annum. The volatility of the price-dividend ratio is quite high, and it is a very persistent series. In addition to these data dimensions, Bansal and Yaron also evaluate the ability of the model to capture the predictability of returns and the new evidence (see Bansal, Khatchatrian, and Yaron (2005)) that price-dividend ratios are negatively correlated with consumption volatility at long leads and lags.

It is often argued that consumption and dividend growth, in the data, is close to being i.i.d. Bansal and Yaron show that their model of consumption and dividends is also consistent with the observed data on consumption and dividends growth rates. However, while the financial market data is hard to interpret from the perspective of the i.i.d. growth rate dynamics, Bansal and Yaron show that it is interpretable from the perspective of the growth rate dynamics that incorporate long-run risks.

This issue is further considered in Shephard and Harvey (1990), Barsky and Delong (1993), and Bansal and Lundblad (2002), who show that discrimination across the i.i.d. growth rate specification and the one that incorporates long-run components is extremely difficult in finite samples. Given these difficulties in discrimination across models, Anderson, Hansen, and Sargent (2003) utilize features of the long-run growth rate dynamics developed in Bansal and Yaron for motivating economic models that incorporate robust control.

2.4.2. Preference Parameters

The preference parameters take account of economic considerations. The time preference parameter $\delta < 1$ and the risk aversion parameter γ in Bansal and Yaron is either 7.5 or 10. Mehra and Prescott (1985) do not entertain risk aversion values larger than 10. Bansal and Yaron focus on an IES of 1.5—an IES value larger than one is important for their quantitative results.

There is considerable debate about the magnitude of the IES. Hansen and Singleton (1982) and Attanasio and Weber (1989) estimate the IES to be well in excess of 1. More recently, Guvenen (2006) and Attanasio and Vissing-Jorgensen (2003) also estimate the IES over one—they show that their estimates are close to that used in Bansal and Yaron. However, Hall (1988) and Campbell (1999) estimate the IES to be well below one. Bansal and Yaron (2004) argue that the low IES estimates of Hall and Campbell are based on a model without time-varying volatility. They show that ignoring the effects of time-varying consumption volatility leads to a serious downwards bias in the estimates of the IES. If the population value of the IES in the Bansal and Yaron model is 1.5, then the estimated value of the IES using Hall estimation methods will be less than 0.3. Bansal and Yaron show that with fluctuating consumption volatility, the projection of consumption growth on the level of the risk-free rate does not equal the IES, leading to the downwards bias. This suggests that Hall and Campbell's estimates are not a robust guide for calibrating the IES.

In addition to the above arguments, the empirical evidence in Bansal, Khatchatrian, and Yaron (2005) shows that a rise in consumption volatility sharply lowers asset prices at long leads and lags and that higher asset valuations today predict higher corporate earnings growth. Figures 1 to 4 use data from the U.S., the U.K., Germany, and Japan to highlight the volatility channel. The asset valuation measure is the price to earnings ratio, and the consumption volatility measure is constructed by averaging eight lags of the absolute value of consumption residuals; see Bansal, Khatchatrian, and Yaron (2005) for additional details. It is evident from the graphs that a rise in consumption volatility lowers asset valuations for all counties under consideration—this highlights the volatility channel and motivates the specification of the IES larger than one. In terms of growth rate predictability, Ang and Bekaert (2007) and Bansal, Khatchatrian, and Yaron (2005) report a positive relation between asset valuations and expected earnings growth. These data features, as discussed in the theory sections above, again require an IES larger than one.

2.4.3. Asset Pricing Implications

To underscore the importance of two key aspects of the model, preferences and long-run risks, first consider the genesis of the risk premium on $r_{a,t+1}$—the return on the asset that delivers aggregate consumption as its dividends. The market risk premium magnifies these risk compensations due to leverage and, consequently, larger exposure to the various risk components.

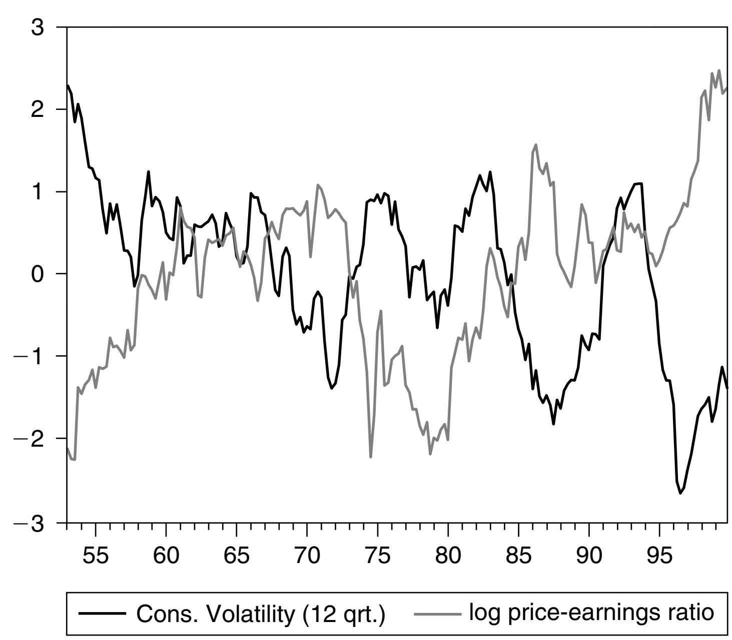

FIGURE 1 P/E ratio and consumption volatility for the U.S. Both series are standardized. (*Source*: Bansal, Khatchatrian, and Yaron (2005)).

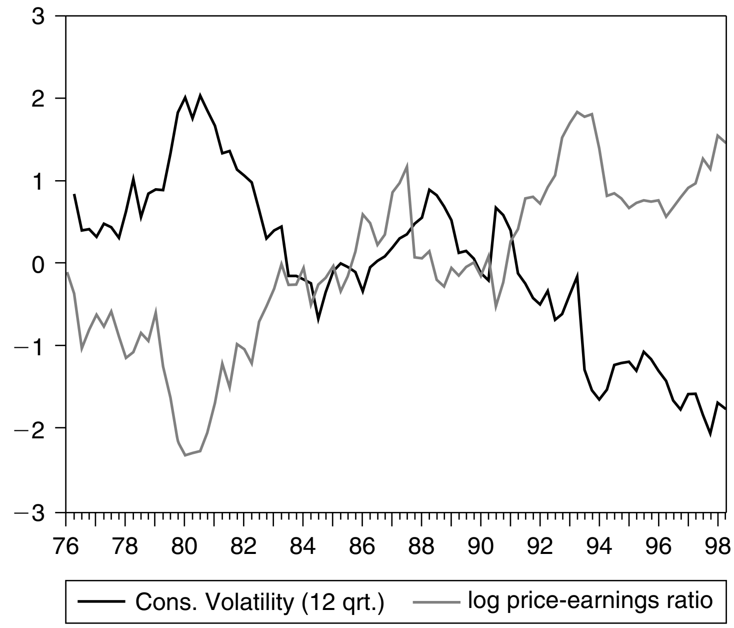

FIGURE 2 P/E ratio and consumption volatility for the U.K. Both series are standardized.

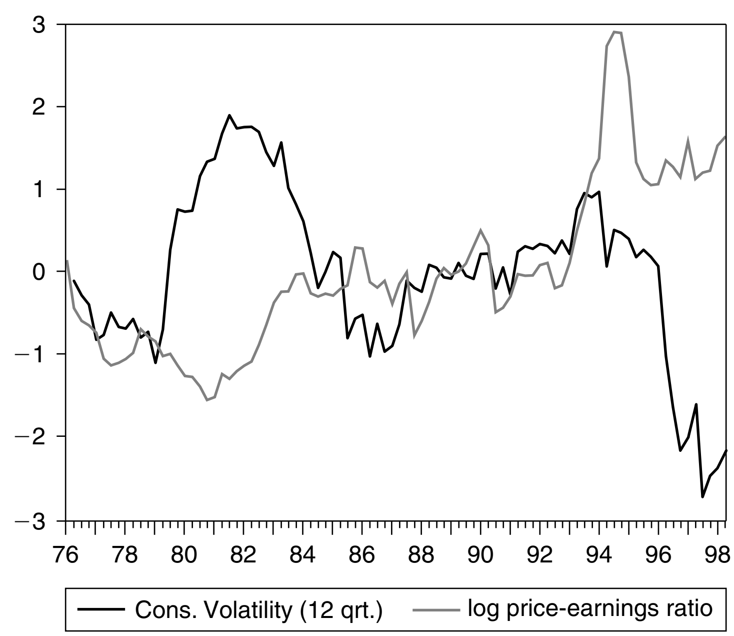

FIGURE 3 P/E ratio and consumption volatility for Germany. Both series are standardized.

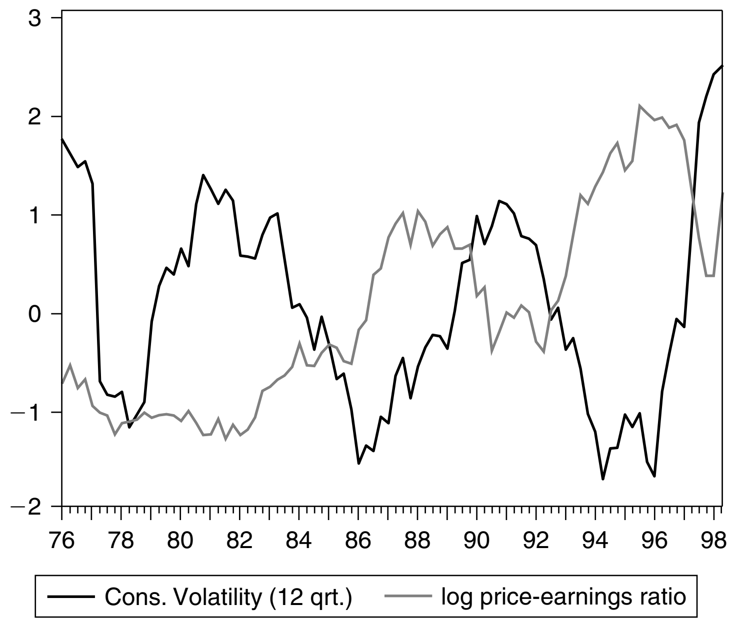

FIGURE 4 P/E ratio and consumption volatility for Japan. Both series are standardized.

Table 3 shows the market price of risk and the breakdown of the risk premium from various risk sources. Column 1 considers the case of power utility as the IES equals the reciprocal of the risk aversion parameter. The prices of long-run risks and economic uncertainty are zero—with power utility, long-run risks and volatility risks are not priced separately. In this case, the risk premium on the consumption asset equals $\gamma\sigma^2$ and is 0.7 percent per annum—that is, only short-run risks are priced.

Column 2 of Table 3 considers the case of Epstein and Zin preferences with an IES less than one (set at 0.5). The price of long-run growth rate risks is positive and negative for volatility risks. However, the consumption asset's beta for the long-run risks is negative. This, as discussed earlier, is because A_1 is negative (see Eq. (5)), implying that a rise in expected growth lowers the wealth to consumption ratio. Consequently, long-run risks in this case contribute a negative risk premium of −1.96 percent per annum. The market price of volatility risk is negative and small; however, the asset's beta for this risk source is large and positive, reflecting the fact that asset prices rise when economic uncertainty rises (see Eq. (10)). In all, when the IES is less than one, the risk premium on the consumption asset is negative, which is highly counterintuitive.

Column 3 of Table 3 shows that when the IES is larger than one (set at 1.5), the price of long-run growth risk rises. More importantly, the asset's beta with respect to the long-run growth risk is positive and that for volatility risk is negative—hence, both risk sources contribute toward a positive risk premium. The risk premium from long-run growth is 0.76 percent and that for the short-run consumption shock is 0.73 percent. The overall risk premium for this consumption asset is 1.52 percent. This evidence shows

TABLE 3
Risk Components and Risk Compensation

	$\psi = 0.1$	$\psi = 0.5$	$\psi = 1.5$
mpr_η	93.60	93.60	93.60
mpr_e	0.00	137.23	160.05
mpr_w	0.00	−27.05	−31.56
β_η	1.00	1.00	1.00
β_e	−16.49	−1.83	0.61
β_w	11026.45	1225.16	−408.39
prm_η	0.73	0.73	0.73
prm_e	0.00	−1.96	0.76
prm_w	0.00	−0.08	0.03

Table 3 presents model-implied components of the risk premium on the consumption asset for different values of the intertemporal elasticity of the substitution parameter, ψ. All entries are based on $\gamma = 10$. The parameters that govern the dynamics of the consumption process in Eq. (9) are identical to Bansal and Yaron (2004): $\rho = 0.979$, $\sigma = 0.0078$, $\varphi_e = 0.044$, $\nu_1 = 0.987$, $\sigma_w = 0.23 \times 10^{-5}$, and $\kappa_1 = 0.997$. The first three rows report the annualized percentage prices of risk for innovations in consumption, the expected growth risk, and the consumption volatility risk—mpr_η, mpr_e, and mpr_w, respectively. These prices of risks correspond to annualized percentage values for $\lambda_{m,\eta}\sigma$, $\lambda_{m,e}\sigma$, $\lambda_{m,w}\sigma_w$ in Eq. (11). The exposures of the consumption asset to the three systematic risks, β_η, β_e, and β_w, are presented in the middle part of the table. Total risk compensation in annual percentage terms for each risk is reported as prm_* and equals the product of the price of risk, the standard deviation of the shock, and the beta for the specific risk.

that an IES larger than one is required for the long-run and volatility risks to carry to a positive risk premium.

It is clear from Table 3 that the price of risk is highest for the long-run risks (see columns 2 and 3) and smallest for the volatility risks. A comparison of columns 2 and 3 also shows that raising the IES increases the prices of long-run and volatility risks in absolute value. The magnitudes reported in Table 3 are with $\rho = 0.979$—lowering this persistence parameter also lowers the prices of long-run and volatility risks (in absolute value). Increasing the risk aversion parameter increases the prices of all consumption risks, as shown in Eq. (12). Hansen and Jagannathan (1991) document the importance of the maximal Sharpe ratio, determined by the volatility of the IMRS, in assessing asset pricing models. Bansal and Yaron show that incorporating long-run risks increases the maximal Sharpe ratio for their model, and it satisfies the non-parametric bounds of Hansen and Jagannathan (1991).

The risk premium on the market portfolio (i.e., the dividend asset) is also affected by the presence of long-run risks. To underscore their importance, assume that consumption and dividend growth rates are i.i.d. This shuts off the long-run risk channel. The market risk premium in this case is

$$E_t(r_{m,t+1} - r_{f,t}) = \gamma \mathrm{Cov}(g_{t+1}, g_{d,t+1}) - 0.5\mathrm{Var}(g_{d,t+1}), \tag{14}$$

and market return volatility equals the dividend growth rate volatility. If shocks to consumption and dividends are uncorrelated, then the geometric risk premium is negative and equals $-0.5\text{Var}(g_{d,t+1})$. If the correlation between monthly consumption and dividend growth is 0.25, then the equity premium is 0.08 percent per annum—similar to the evidence documented in Mehra and Prescott (1985) and Weil (1989). Bansal and Yaron show that incorporating long-run growth rate risks (see Eq. (4)) produces an annual equity risk premium of 4.2 percent and a risk-free rate of 1.34 percent along with market return and price-dividend volatility of 16.21 percent and 0.16, respectively. These are fairly comparable to what we see in the data (see Table 2) and highlight the importance of long-run growth rate risks.

Bansal and Yaron show that the full model that incorporates long-run growth rate risks and fluctuating economic uncertainty provides a very close match to the asset market data reported in Table 2. That is, this model can account for the *low risk-free rate*, *high equity premium*, *high asset price volatility*, and *low risk-free rate volatility*. This model also quantitatively matches additional data features, such as (i) predictability of returns at short and long horizons using the dividend yield as a predictive variable, (ii) time-varying and persistent market return volatility, (iii) negative correlation between market return and volatility shocks, i.e., the volatility feedback effect, (iv) negative relation between consumption volatility and asset prices at long leads and lags, documented in Bansal, Khatchatrian, and Yaron (2005).

In all, this evidence shows that incorporating long-run risks in growth rates and fluctuating economic uncertainty can help interpret a wide array of the asset market puzzles.

2.4.4. Value of Contingent Claims and Macro Markets

Using the Efficient Method of Moments (EMM), Bansal, Gallant, and Tauchen (2007) consider the implications of alternative asset pricing models presented in Bansal and Yaron and in Campbell and Cochrane (1999). A unique dimension of this paper is that they model the consumption and dividends as being cointegrated, a feature that is missing in earlier work of Campbell and Cochrane (1999) and Bansal and Yaron (2004). Bansal, Gallant, and Tauchen evaluate the value of contingent claims on the aggregate wealth of the economy. This is a first step in assessing the plausibility of introducing contingent claims on macro variables for better risk sharing, as espoused by Shiller (1998). This valuation exercise also helps understand the different channels operating in these models.

Bansal, Gallant, and Tauchen document large differences in the prices of put options on the consumption claim across the two asset pricing models. The volatility of the consumption claim in the Bansal and Yaron model is about one-fourth that of the market return, while in the Campbell and Cochrane model it is about as volatile as the market portfolio.

3. CROSS-SECTIONAL IMPLICATIONS

3.1. Value, Momentum, Size, and the Cross-Sectional Puzzle

Table 4, taken from Bansal, Dittmar, and Lundblad (2005), shows that there are sizeable differences in mean real returns across assets. Bansal, Dittmar, and Lundblad (2005) use quarterly data from 1967 to 2001. They rely on standard book-to-market, size, and momentum sorted portfolios. For the first two sorts, firms are sorted into different deciles once a year, and the subsequent return on these portfolios is used for empirical work. For momentum assets, CRSP-covered NYSE and AMEX firms are sorted on the basis of their cumulative return over months $t-12$ through $t-1$. The loser portfolio (M1) includes firms with the worst performance over the last year, and the winner portfolio (M10) includes firms with the best performance. The data shows that subsequent returns on these portfolios have a large spread (i.e., M10 return − M1 return), of about 4.62 percent per quarter—this is the *momentum spread puzzle*. Similarly, the highest book-to-market firms (B10) earn average real quarterly returns of 3.27 percent, whereas the lowest book-to-market (B1) firms average 1.54 percent per quarter. The value spread (return on B10 − return on B1) is about 2 percent per quarter—this is the *value spread puzzle*. What explains these big differences in mean returns across portfolios?

TABLE 4
Portfolio Returns

	Mean	Std. Dev.		Mean	Std. Dev.		Mean	Std. Dev.
S1	0.0230	0.1370	**B1**	0.0154	0.1058	**M1**	−0.0104	0.1541
S2	0.0231	0.1265	**B2**	0.0199	0.0956	**M2**	0.0070	0.1192
S3	0.0233	0.1200	**B3**	0.0211	0.0921	**M3**	0.0122	0.1089
S4	0.0233	0.1174	**B4**	0.0218	0.0915	**M4**	0.0197	0.0943
S5	0.0242	0.1112	**B5**	0.0200	0.0798	**M5**	0.0135	0.0869
S6	0.0207	0.1050	**B6**	0.0234	0.0813	**M6**	0.0160	0.0876
S7	0.0224	0.1041	**B7**	0.0237	0.0839	**M7**	0.0200	0.0886
S8	0.0219	0.1001	**B8**	0.0259	0.0837	**M8**	0.0237	0.0825
S9	0.0207	0.0913	**B9**	0.0273	0.0892	**M9**	0.0283	0.0931
S10	0.0181	0.0827	**B10**	0.0327	0.1034	**M10**	0.0358	0.1139

Table 4, reported in Bansal, Dittmar, and Lundblad (2005), presents descriptive statistics for the returns on the 30 characteristic-sorted decile portfolios. Value-weighted returns are presented for portfolios formed on momentum (M), market capitalization (S), and book-to-market ratio (B). M1 represents the lowest momentum (loser) decile, S1 the lowest size (small firms) decile, and B1 the lowest book-to-market decile. Data are converted to real using the PCE deflator. The data are sampled at the quarterly frequency, and cover the first quarter of 1967 through the fourth quarter of 2001.

The central idea, in theory, is that differences in exposure to systematic risk should justify this puzzling difference in mean returns. The static CAPM (see Sharpe (1964)) implies that these differences mirror differences in market betas of assets, while Lucas (1978) and Breeden (1979) argue that they reflect differences in exposure to aggregate consumption movements. Evidence presented in Hansen and Singleton (1982) for the consumption-based models, and in Fama and French (1992) for the CAPM, shows that these models have considerable difficulty in accounting for the differences in observable rates of return across assets. Consequently, identifying economic sources of risks that justify differences in the measured risk premia continues to be an important economic issue.

Bansal, Dittmar, and Lundblad (2002, 2005) connect systematic risks to cash-flow risks. They build on the intuition developed in the Bansal and Yaron (2004) model. This intuition is provided in Eq. (8). Assume that the dividend growth rate dynamics for equity i follow Eq. (4). In equilibrium, the differences in the risk premium across assets mirror the differences in their long-run risks beta, β_i,

$$\beta_i = \left[\left(\frac{\phi_i \varphi_e}{1-\kappa_1\rho}\right) - \left(\frac{1}{\psi}\frac{\varphi_e}{1-\kappa_1\rho}\right)\right]\kappa_1. \tag{15}$$

An important insight is that β_i risk measure differs across assets only because of the differences in ϕ_i—the exposure of dividends to long-run risks. In the cross section of assets, the systematic risks in cash flows determine the differences in the systematic risks in the asset. Assets with higher ϕ_i are more exposed to systematic risks, and agents, consequently, should demand a higher risk premium of such assets.

Empirical work for cross-sectional differences in mean returns, typically, explores the link between cross-sectional mean return and the systematic risk measure, β_i. That is, it considers the projection

$$E(r_{i,t+1}) = \lambda_0 + \lambda_c \beta_i. \tag{16}$$

As the β_i and the cash-flow risks measures, ϕ_i, are correlated in the cross section, it follows that cross-sectional differences in mean returns must mirror differences in ϕ_i. Given this connection, Bansal, Dittmar, and Lundblad ask if cash-flow betas can explain differences in mean return across assets.

3.1.1. Measuring Risks in Cash Flows

To measure consumption risks in cash flows (i.e., cash-flow betas), Bansal, Dittmar, and Lundblad model the joint dynamics of observed cash flow and aggregate consumption growth rates as an autoregression (VAR). I briefly discuss this below.

For any asset i, using the return approximation in Eq. (3), the log price minus log cash flow, $p_{i,t} - d_{i,t}$, satisfies

$$p_{i,t} - d_{i,t} = \frac{\kappa_{i,0}}{1 - \kappa_{i,1}} + E_t\left[\sum_{j=0}^{\infty} \kappa_{i,1}^{j} g_{i,t+1+j} - \sum_{j=0}^{\infty} \kappa_{i,1}^{j} r_{i,t+1+j}\right]. \tag{17}$$

The log price – cash flow ratio is determined by the discounted expected cash-flow growth rates and discounted expected returns. The discount rate, $\kappa_{i,1}$, is less than one by construction. Exploiting (3) and (17), it follows that return innovations are related to innovations in expectations of future cash flows and returns:

$$\begin{aligned} r_{i,t} - E_{t-1}[r_{i,t}] &= \{E_t - E_{t-1}\}\left[\sum_{j=0}^{\infty} \kappa_{i,1}^{j} g_{i,t+j}\right] - \{E_t - E_{t-1}\}\left[\sum_{j=1}^{\infty} \kappa_{i,1}^{j} r_{i,t+j}\right] \\ &= \eta_{g_i,t} - \eta_{e_i,t}. \end{aligned} \tag{18}$$

The piece $\eta_{g_i,t} = \{E_t - E_{t-1}\}[\sum_{j=0}^{\infty} \kappa_{i,1}^{j} g_{i,t+j}]$ is cash-flow news and represents the revision in expectations of the sum of future dividend growth rates. Analogously, $\eta_{e_i,t}$ represents discount rate news.

Given the return decomposition, the consumption beta can be described as

$$\beta_i = \frac{\mathrm{Cov}(r_{i,t} - E_{t-1}(r_{i,t}), \eta_t)}{\mathrm{Var}(\eta_t)} = \frac{\mathrm{Cov}(\eta_{g_i,t} - \eta_{e_i,t}, \eta_t)}{\mathrm{Var}(\eta_t)} = \beta_{i,g} - \beta_{i,e}, \tag{19}$$

where η_t is the time t innovation in consumption growth. The consumption beta is governed by two components—the cash-flow beta and the beta of discount rate news. Bansal, Dittmar, and Lundblad ask if the cash-flow beta, $\beta_{i,g}$, can explain differences in risk premia across assets.

To estimate the cash-flow betas, they model the de-meaned log consumption growth, $g_{c,t}$, as a simple AR(1) process:

$$g_{c,t} = \rho_c g_{c,t-1} + \eta_t, \tag{20}$$

with ρ_c being the AR(1) coefficient and η_t the consumption news at date t. Further, they assume that the relationship between de-meaned dividend and consumption growth rates is

$$g_{i,t} = \phi_i\left(\frac{1}{K}\sum_{k=1}^{K} g_{c,t-k}\right) + u_{i,t}, \tag{21}$$

$$u_{i,t} = \sum_{j=1}^{L} \rho_{j,i} u_{i,t-j} + \zeta_{i,t}. \tag{22}$$

The expression $1/K\sum_{k=1}^{K} g_{c,t-k}$ represents a trailing K-period moving average of past consumption growth—this a measure of x_t discussed above in the Bansal and Yaron (2004) model. The parameter ϕ_i measures the leverage of the dividends, as discussed in Eq. (4). This specification allows for cash-flow growth rates to depend on the current consumption innovation through the process for $\zeta_{i,t}$. This contemporaneous covariance is reflected in the measured cash-flow betas.

Equations (20), (21), and (22) characterize a simple VAR. The q-vector, Y_t, is

$$Y_t = [g_{i,t}\ u_{i,t} \cdots u_{i,t-(L-1)}\ g_{c,t} \cdots g_{c,t-(K-1)}]', \tag{23}$$

and the dynamics of the state variables and portfolio cash flow growth can then be expressed as

$$Y_t = AY_{t-1} + v_t, \tag{24}$$

where A is the $q \times q$ matrix of coefficients. Let the first element of Y_t be $g_{i,t}$ such that $e_1' z_t = g_{i,t}$, where e_1 is a $q \times 1$ vector with first element 1 and remaining elements 0. From Eq. (18), it follows that $\eta_{g_i,t}$ is equal to

$$\begin{aligned} \eta_{g_i,t} &= \{E_t - E_{t-1}\}\left[\sum_{j=0}^{\infty} \kappa_{i,1}^j g_{i,t+j}\right] \\ &= e_1'\left[\sum_{j=0}^{\infty} \kappa_{i,1}^j A^j v_t\right] \\ &= e_1'\left[I - \kappa_{i,1} A\right]^{-1} v_t. \end{aligned} \tag{25}$$

This residual represents the innovation to current and expected future cash-flow growth rates. The exposure of this innovation to consumption growth is measured by projecting it on the innovation in consumption growth, specifically,

$$\eta_{g_i,t} = \beta_{i,g}\eta_t + \xi_{g_i,t}. \tag{26}$$

The resulting projection coefficient, $\beta_{i,g}$, is the asset's cash-flow beta developed in Bansal, Dittmar, and Lundblad (2002, 2005). Note that if $u_{i,t}$ is uncorrelated with consumption innovation, then the cross-sectional differences in the cash-flow beta based on Eq. (26) solely reflect differences in ϕ_i. Hence, if one imposes the restriction that $u_{i,t}$ is uncorrelated with consumption innovations, then it is sufficient to focus on ϕ_i. Given the cash flow's consumption beta, Bansal, Dittmar, and Lundblad run the cross-sectional regression,

$$E[R_{i,t}] = \lambda_0 + \beta_{i,g}\lambda_c \tag{27}$$

to evaluate the empirical plausibility of the cash-flow beta model.

3.1.2. Dividends and Cash Flows

Bansal, Dittmar, and Lundblad measure the cash flows as dividends for each portfolio in a standard manner, specifically,

$$R_{t+1} = h_{t+1} + y_{t+1}, \tag{28}$$

where h_{t+1} is the price appreciation and y_{t+1} the dividend yield (i.e., dividends at date $t+1$ per dollar invested at date t). We observe R_{t+1} (RET in CRSP terminology) and the price gain series h_{t+1} ($RETX$) for each portfolio; hence, $y_{t+1} = R_{t+1} - h_{t+1}$. The level of the dividends we use in the paper is computed as

$$D_{t+1} = y_{t+1}V_t, \tag{29}$$

where

$$V_{t+1} = h_{t+1}V_t, \tag{30}$$

with $V_0 = 1$. Hence, the dividend series that we use, D_t, corresponds to the total *cash* dividends given out by a mutual fund at t that extracts the dividends and reinvests the capital gains. The ex-dividend value of the mutual fund is V_t, and the per dollar return for the investors in the mutual fund is

$$R_{t+1} = \frac{V_{t+1} + D_{t+1}}{V_t} = h_{t+1} + y_{t+1}. \tag{31}$$

From this equation, it is evident that V_t is the discounted value of the dividends that we use.

Bansal, Dittmar, and Lundblad (2005) also use repurchase adjusted dividends and earnings as a measure of cash flows. The empirical evidence is similar to that found using cash dividends. More recently, Bansal and Yaron (2006) use market clearing restrictions as a way to identify the appropriate trading strategy to use for measuring aggregate dividends. This measure, relative to the per share-based traditionally used measure (as in Eq. (29)), incorporates the relative shift in scale of the different sectors and yields different insights into the sources of the asset price variation.

3.1.3. Performance of Cash-Flow Beta Model

As predicted by the Bansal and Yaron (2004) model, the consumption leverage of divided growth rates has great explanatory power. Bansal, Dittmar, and Lundblad show that using ϕ_i or $\beta_{i,g}$ as the risk measure yields a highly positive and significant price of risk estimates of λ_c, and the cross-sectional R^2 are well in excess of 60 percent. Using an alternative asset menu of 25 portfolios based on the Fama–French 5 × 5 two-way sort on market capitalization and book-to-market value also yields comparable results. Bansal, Dittmar, and Lundblad (2002, 2005) also ask if the cash-flow risks are related to the long-run risks of Bansal and Yaron (2004). They show that when, in Eq. (21), $K = 1$, the consumption risks are largely short-run risks and the cash-flow betas are not able to capture the cross-sectional differences in risk premia. Increasing K, and hence exaggerating the long-run risks, is important for the success of the cash-flow betas in capturing differences in mean returns across assets.

In contrast, alternative models find it quite hard to explain the differences in mean returns for the 30 asset menus used in Bansal, Dittmar, and Lundblad. The standard consumption betas (i.e., C-CAPM) and the market-based CAPM asset betas have close to zero explanatory power. The R^2 for the C-CAPM is 2.7 percent, and that for the market CAPM is 6.5 percent, with an implausible negative slope coefficient. The Fama and French three-factor empirical specification also generates point estimates with negative, and difficult to interpret, price of risk for the market and size factors—the cross-sectional R^2 is about 36 percent. Compared to all these models, the cash-flow risks model of Bansal, Dittmar, and Lundblad is able to capture a significant portion of the differences in risk premia across assets.

In addition, Bansal, Dittmar, and Lundblad (2001) and Bansal, Dittmar, and Kiku (2007) also consider a risk measure based on stochastic cointegration between the log level of dividends and consumption. This risk measure is estimated via the projection of the deterministically de-trended level of log dividends on the de-trended level of log consumption. Bansal, Dittmar, and Kiku (2007) derive new results that link this cointegration parameter to consumption betas by investment horizon and evaluate the ability of their model to explain differences in mean returns for different horizons. Hansen, Heaton, and Li (2006) inquire about the robustness of the stochastic cointegration-based risk measures considered in Bansal, Dittmar, and Lundblad (2001). Bansal, Dittmar, and Kiku provide new evidence regarding the robustness of the stochastic cointegration-based measures of permanent risks in equity markets. Parker and Julliard (2005) evaluate if long-run risks in aggregate consumption can account for the cross section of expected returns. Malloy, Moskowitz, and Vissing-Jørgensen (2006) evaluate if long-run risks in the consumption of stockholders has greater ability to explain the cross section of equity returns, relative to aggregate consumption measures.

Bansal, Dittmar, and Lundblad (2005) also discuss why cash-flow risks may capture the differences in risk premia when standard consumption betas fail to do so. The Bansal and Yaron model helps explain why this may be the case—in this model of multiple risks, the consumption beta is not sufficient to capture differences in risk premia across assets. Imagine that risk premia are determined by Eq. (13), that is, there are multiple sources of risks. Let the shocks to the various risk sources be correlated, and then the standard consumption beta will measure a weighted linear combination of the three different betas. While each individual beta may be important in capturing the risk premia across assets—a weighted linear combination may fail to do so. Bansal, Dittmar, and Lundblad (2005) provide simulation evidence wherein the C-CAPM betas fail to explain the difference in mean returns across assets, while the cash-flow betas capture a sizable portion of the cross-sectional differences in mean returns.

The approach pursued in Bansal, Dittmar, and Lundblad (2005) derives constant risk measures. Jagannathan and Wang (1996) and Lettau and Ludvigson (2001) focus on time-varying return betas. Deriving and measuring time-varying cash-flow betas would be a valuable extension for future research.

4. CONCLUSION

The work of Bansal and Lundblad (2002), Bansal and Yaron (2004), and Bansal, Gallant, and Tauchen (2004) show that long-run growth rate risks and varying economic uncertainty are important for quantitatively interpreting financial markets. These papers argue that investors care about the long-run growth prospects and the uncertainty surrounding the growth rate. Risks associated with changing long-run growth prospects and varying economic uncertainty drive the level of returns and asset price volatility in financial markets.

A key issue in terms of the preferences of investors is their attitude toward risk as measured by risk aversion and the magnitude of the parameter that determines intertemporal substitution. Based on recent empirical evidence on the magnitude of the IES, and on its economic implications for asset markets, Bansal and Yaron argue that the IES should be larger than one. They show that only when the IES is larger than one does increased economic uncertainty translate into a drop in asset prices. Bansal, Khatchatrian, and Yaron (2005) find very robust empirical evidence that a rise in economic uncertainty lowers asset prices across different samples and countries. This evidence suggests that the IES may indeed be larger than one.

Bansal, Dittmar, and Lundblad (2002, 2005) show that long-run risks in cash flows of portfolios should explain differences in mean returns across assets. They measure cash-flow risks via estimating cash-flow betas and find that the consumption risks in cash flows of portfolios explain very well the differences in mean returns across equity portfolios. Using cash flows and consumption, they devise ways to measure cash-flow betas that capture systematic risks in cash flows. These risk measures, they find, provide very sharp information about differences in risk premia across assets. They show that the cash-flow betas can explain the differences in mean returns for value, size, and momentum sorted portfolios.

All of this evidence and the economics underlying it support the view that the long-run risks and uncertainty channel contain very valuable insights about the workings of financial markets.

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Discussion of "Long-Run Risks and Risk Compensation in Equity Markets"

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1. SUMMARY

Bansal and his co-authors have produced a series of important and provocative papers that demonstrate how low-frequency risk can provide a justification for observed risk premia. Bansal summarizes this work and helps us understand the linkages between long-run risk in consumption and long-run risk in financial securities. Since he does such a nice job of explaining the economics of the approach, I focus on a few issues raised by Bansal's work and by some of my own.

2. A LOW-FREQUENCY COMPONENT IN CONSUMPTION?

An important aspect of the model of Bansal and Yaron (2004) is the presence of a low-frequency component in aggregate consumption. Shocks to the level of consumption are persistent, as are shocks to volatility. A substantial and time-varying equity premium results when dividends display exposure to the shocks. It is natural to ask whether there is any empirical support for the assumed model of consumption.

As Bansal argues, one piece of evidence may come from financial markets. Agents could use the model's structure along with signals from asset prices to detect the low-frequency component. This may take the rational expectations assumption to an untenable position, however. Unless agents directly observe the low-frequency shocks driving consumption, it is not clear how the shocks could be reflected in security prices. Appealing to the standard idea that the econometrician observes a smaller information set than agents is also a little delicate in this context. In the model of preferences considered by Bansal and Yaron (2004), the exact conditioning information of the agents is needed in order to derive the implications of the model.

An interesting way of introducing a concern for low-frequency components, even when agents have a hard time detecting those components, is provided by the recent work of Hansen and Sargent (2007). In their work the representative agent is uncertain about the probability model generating consumption. There are two alternative models: one where consumption is not predictable and one where consumption is predictable. Because the agent is worried about model uncertainty, she acts as if there is a very high probability attached to the model with predictable consumption. This model generates both the high-risk premia consistent with a low-frequency component in consumption, and consumption dynamics that are difficult to distinguish from an i.i.d. model.

There is evidence in the literature for an important predictable low-frequency component to consumption. This evidence is typically obtained using additional variables to predict consumption along with plausible economic linkages across variables. Examples include the work of Fisher (2006) and Mulligan (2002). In Hansen et al. (2006a) we consider a bivariate model for aggregate consumption and aggregate corporate earnings. We present evidence that corporate earnings are co-integrated with aggregate consumption as predicted by most models of business cycles and economic growth. Under the co-integration assumption, corporate earnings reveal important long-run shocks to consumption.

To illustrate this, Figure 1 reports the impulse response of aggregate consumption to the two shocks in the model.[1] We call the first shock a "consumption shock"; it impacts both consumption and earnings contemporaneously. In contrast, the second shock impacts earnings contemporaneously but has no immediate impact on consumption. We call this shock an "earnings shock." Shocks to corporate earnings predict consumption over many quarters and reveal an important low-frequency component of consumption similar to the setup considered by Bansal and Yaron (2004).

3. PREFERENCES

In his work, Bansal employs the recursive specification of preferences developed by Kreps and Porteus (1978), Epstein and Zin (1989b), Weil (1990), and others. In the context of models with long-run consumption risk, this model of preferences is useful because it induces a concern for the resolution of uncertainty. In a standard model with time-additive CRRA utility, risk prices are determined by risk aversion and the one-period or instantaneous impact of shocks on consumption. With recursive preferences, the long-run impact of shocks on consumption also influences risk prices. Shocks that strongly predict future consumption have larger risk premia.

Calibration of the models considered by Bansal does run into the problem that consumption is not that volatile even in the long run. Large levels of risk aversion are typically needed to fit aggregate and cross-sectional risk premia. An advantage of the assumption of recursive preference is that the effects of high risk aversion on the level of

[1]This figure is taken from Hansen et al. (2006b).

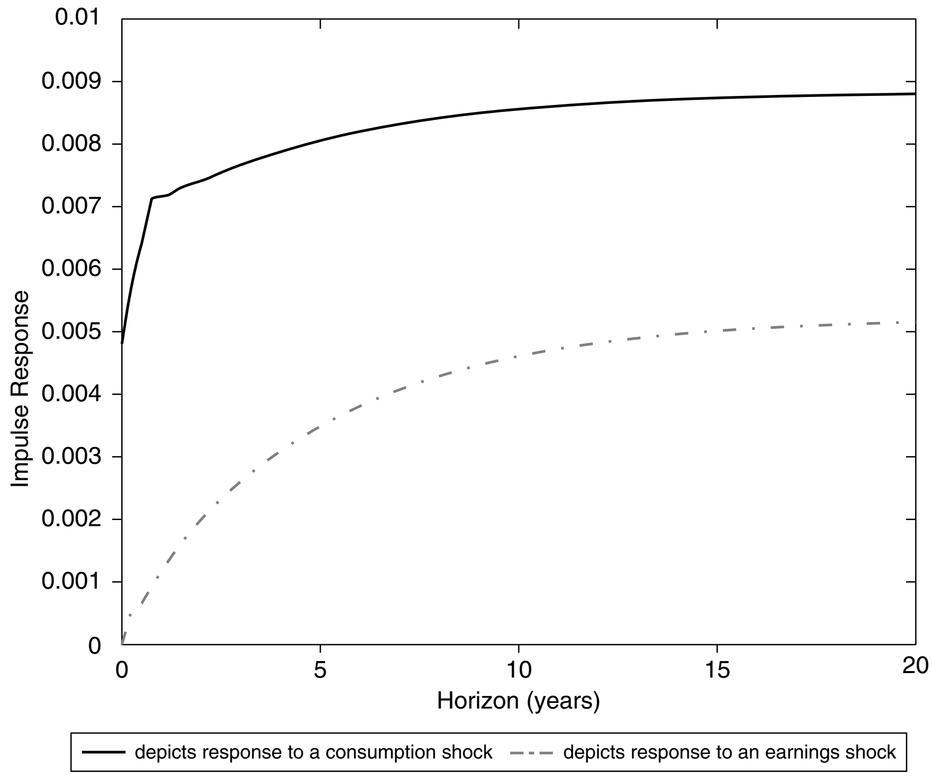

FIGURE 1 Impulse responses implied by the VAR of the Hansen–Heaton–Li model.

interest rates can be controlled by assuming the intertemporal elasticity of substitution (IES) is close to 1.

In the calibration exercise considered by Bansal and Yaron (2004), the IES parameter is assumed to be greater than 1 so that shocks that forecast increased future consumption also increase the price-to-dividend ratio or wealth-to-consumption ratio. This produces the desired effect that shocks that forecast future consumption have a magnified risk premium. In the model, this is captured by variation in the market return, which becomes part of the stochastic discount factor with recursive utility.

There is some controversy over the value of the IES parameter used by Bansal and Yaron (2004) because of other evidence in the literature. In particular, the typical assumption is that the IES parameter is less than 1. As Bansal points out, however, many of the existing estimates in the literature are sensitive to the assumption that consumption is homoskedastic. This identification problem is potentially magnified if the true underlying preferences are of the recursive form as shown by Hansen et al. (2006b).

An issue related to the assumption about the IES parameter is the solution method used by Bansal and many others in the literature. The general stochastic discount factor induced by recursive preferences has a term in consumption growth and a term in the "continuation value" due to future consumption. Following earlier work by Epstein and Zin (1989a), shocks to the "continuation value" are replaced by shocks to the return to holding a claim on aggregate consumption (the "market" return). This generates a proper solution to the model except for the case of logarithmic intertemporal risk preferences (IES = 1). At this point the wealth-to-consumption ratio is constant so that the market return covaries exactly with consumption. Shocks to future consumption still matter to the consumer, but the continuation value from future consumption plans must be calculated directly. Shocks to the derived continuation value then enter the stochastic discount factor.

In Hansen et al. (2006a), we show how the continuation value can be calculated in a log-linear stochastic environment with logarithmic intertemporal utility. Further, we develop an approximation to the case of a more general value of the IES parameter that works well for values of the IES near 1. This approach is extended to an environment with stochastic volatility in Hansen et al. (2006b). An advantage of this method of solution is that it allows for the consideration of values for the IES parameter close to 1, where the more typical use of the return to the market portfolio breaks down.

4. RETURNS AND LONG-RUN CASH FLOWS

In his work with Lundlad and Dittmar, Bansal develops the idea of a "cash flow" beta where the long-run covariance between consumption and dividends is assumed to drive risk premia. Measured differences in this long-run covariance across portfolio cash flows do coincide with observed differences in average returns to the portfolios. This is an important finding because there is very little heterogeneity in the contemporaneous covariance between shocks to consumption and the returns to the portfolios considered by Bansal. This is reflected in the well-established fact that the consumption CAPM cannot explain the value premium, the size effect, and other observed risk premia.

The contrast between the cash flow beta measure of risk and the contemporaneous covariance between returns and shocks to consumption creates a tension. One potential resolution of this tension could be that the short-run dynamics of risk exposure are difficult to model or measure. For example, there could be important market frictions, behavioral biases, or general model misspecification. In these situations, the long-run covariances between returns and consumption (as in Daniel and Marshall (1997)), or between long-run shocks to consumption and portfolio cash flows may be more appropriate measures of risk.

If short-run implications are to be ignored, what do we mean by the "short run?" One way to answer this question is to specify a complete model of pricing and then examine the model's predictions for the pricing of a portfolio's cash flows at different frequencies. For example, the holding period returns for each future cash flow can be calculated. The one-period portfolio return is just a weighted average of these individual returns. By ignoring the contribution from short-term cash flows, a long-run return is calculated. Alternatively, the rate at which the price of future cash flows declines relative to the predicted growth in cash flows can be used to infer a long-run return. These alternative specifications of long-run returns and their long-run limits are developed in Hansen et al. (2006a).[2] We apply our analysis to portfolios of growth and value stocks. Consistent with the results in Bansal et al. (2005), we find that portfolios of value stocks are predicted to have high long-run returns.

5. CONCLUSION

Bansal's paper does a nice job of summarizing his work and showing why the consideration of long-run risk can potentially help us to understand observed security prices. The paper also points the way toward the work to be done both in modeling the dynamics of consumption and in understanding how long-run risk is priced.

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[2]The analysis in Hansen et al. (2006a) is conducted in a log-linear environment. Hansen and Scheinkman (2006) extend this to more general environments.