Rational Pessimism, Rational Exuberance, and Markets for Macro Risks

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Abstract

The paper examines two asset pricing models that attempt to explain features of financial markets such as the size of the equity premium and the volatility of the stock market. One model, based on Bansal and Yaron (2004), relies on low frequency movements in aggregate consumption growth, cash-flows, and economic uncertainty as the key channels to interpret asset market behavior. The other, as typified by Campbell and Cochrane (1999), relies on time-varying risk-aversion and consequently time-variation in the risk-premia as the key channel to interpret asset markets. The models are fitted to data using a simulation-based procedure similar to that proposed by Smith (1993). Both models are found to fit the data equally well at conventional significance levels, and they can track quite closely a new measure of realized annual volatility. Their economic implications, however, are different in some important respects. For instance, the models predict quite different characteristics for the price of a perpetual consumption claim in a macro market of the sort proposed by Shiller (1998).

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1 Introduction

Economic agents face uncertain income streams and thereby face uncertain future consumption and utility. A significant portion of this uncertainty is attributable to macroeconomic risks, i.e., long term trend and business cycle fluctuations. Shiller (1998) notes that at present there is no effective way for agents to hedge macroeconomic risks. That is, there are no readily available markets that agents can use to shift macro risks from those less willing to bear the risks to others more able to take on such risks. Of course, existing markets in capital assets, such as individual stocks or stock indexed mutual funds, provide very effective means for agents to shift financial risks. Derivatives on these assets, such as puts and calls, break financial risks into pieces and thereby allow agents to shift portions of the risks. Nonetheless, these financial risks are only mildly correlated with actual macroeconomic risks, and they therefore provide very weak hedging opportunities against macroeconomic risks.

Shiller (1998, pp. 38–46) calls for markets on macro aggregates and describes in detail the proposed markets. These markets would be for instruments on perpetual income flows, not on capital assets. For example, consider an asset that pays, at the end of each quarter, an amount proportional to U.S. GDP for that quarter. Contractually, the holder of one unit of this asset would receive a cash payment equal to the index number times quarterly GDP for each quarter that the contract is held. The cash payments would be made by holders of the other side of the contract, and the price of the instrument would adjust to whatever value equilibrates the positions on the two sides of the contract. In practice, the contract would be cash settled and marked to the market each day, which greatly facilitates contract compliance. Shiller describes a cash (spot) market in such an instrument and also a futures market. The two are essentially equivalent, because arbitrage would enforce a parity condition that the futures price is the spot price adjusted for accrued interest and intervening cash pay outs.

Such macro markets do not actually exist. Thus, initial consideration and quantitative evaluation of macro markets must be done within the context of an asset pricing model. We do this using two modern nonlinear rational expectations models.

The first, termed the Long Run Risks (LRR) Model, is patterned after Bansal and
Lundblad (2002) and Bansal and Yaron (2004). In the LRR model, growth rates of cash flows contain a statistically small but economically important slow moving component. As is well known from the familiar Gordon growth formula, small, near permanent movements in the forecasted growth rates of cash flows can be expected to generate large movements in asset valuations relative to current cash flows. The LRR model couples these cash flow dynamics with preferences of the Epstein-Zin-Weil type. One important extension considered in this paper is to incorporate and test for cointegration between aggregate stock market dividends and consumption.

The second model we consider is one with habit persistence (HAB) patterned rather closely after Campbell and Cochrane (1999). Their dynamics must be modified slightly to be compatible with the cointegration relationships in the data. The HAB model presumes quite simple cash flow dynamics, in contrast to the LRR model, but it assumes an involved preference structure with a time-varying habit stock that evolves with conditionally heteroskedastic innovations. Like the LRR model, the HAB model also generates long-term swings in stock market valuations.

We undertake econometric estimation of both the LRR and HAB models using a simulation-based procedure similar to that of Smith (1993). Intuitively, the estimation technique minimizes a statistical measure of distance between a vector autoregression model fitted to observed data and to simulated data at the annual frequency. The simulations are generated by operating the models in monthly time, numerically solving for the equilibrium, and then aggregating appropriately to the annual frequency. The estimation is a type of GMM estimation, and it therefore delivers a chi squared measure of fit to the data. We also undertake extensive additional diagnostic assessments using reprojection (Gallant and Tauchen, 1998, 2002). As part of this effort, we develop a new measure of realized annual volatility, which is patterned after the measure developed by Andersen, Bollerslev, and Diebold (2002), and we assess the models’ capabilities to track this new variable.

We then proceed to study the properties of a macro market in a perpetual claim to the monthly consumption endowment. This is the natural market to consider within the context of the endowment economies underlying nearly all asset pricing models. The claim represents per capita wealth in these models and is referred to as such below. Although Shiller calls for
markets in aggregate income or production measures, a market in a perpetual consumption
claim would be an especially useful hedging device for agents. Indeed, one might argue that
agents could make better hedging use of market on consumption claims than of a market on
aggregate GDP or national income.

A natural outgrowth of such a macro market would be derivatives markets, i.e., markets
in calls and puts on the value of the claim. Markets in calls and puts on the S&P Index
arose directly with the creation of the S&P futures market. Calls and puts have a nonlinear
payoff structure that facilitate interesting payoff patterns that cannot be readily produced
with positions in the underlying asset or futures. We start the analysis of derivatives by
computing calls and put prices on more traditional derivatives written on the stock market
index itself. We then go on to consider less familiar calls and puts written on consumption
and on the perpetual claims to consumption.

The analysis of a perpetual claim on consumption and derivatives on this claim is con-
ducted using the \textbf{LRR} and \textbf{HAB} models, both of which are cast within the representative
agent framework. Needless to say, this is not the framework Shiller (1998) has in mind.
He clearly believes the absence of such markets indicates an inefficiency and their creation
would expand agents’ opportunity sets. In contrast, our derivative pricing computations
are done by computing the price at which the agent is, at the margin, indifferent between
taking a position in the contract or not. The claim, and its derivatives, are redundant as-
sets. Nonetheless, the calculations provide an initial benchmark against which one could
compare prices computed in a more elaborate model with appropriate frictions. Further-
more, the analysis sheds interesting new insights into the contrasts between asset pricing
models. These insights appear to be especially useful for understanding the characteristics
and ascertaining the credibility of asset pricing models.

Section 2 below sets out the Long Run Risks model and Section 3 sets forth the Habit
Persistence model. Section 4 develops the observation equations along with the new mea-
sure of realized annual volatility. Section 5 describes the data and the cointegration analysis.
Section 6 describes the estimation methodology and Section 7 the empirical results. Sec-
tion 8 contains the analysis of derivatives written on the stock price, consumption, and the
perpetual consumption claim (wealth). Section 9 contains concluding remarks.
2 Long Run Risks (LRR)

We develop an asset pricing model that is extension of Bansal and Yaron (2004). Some key features of the model are that consumption and dividends are separate stochastic processes but are tied together by a long run cointegrating relationship. Other features include preferences of the Epstein-Zin (1989) and Weil (1989) type along with time varying stochastic volatility.

2.1 Dynamics of Driving Variables

Let \( d_t = \log(D_t) \), and \( c_t = \log(C_t) \) denote log real per capita values of the stock dividend and the consumption endowment. The log endowment \( c_t \) is assumed to be generated as

\[
c_t = c_{t-1} + \mu_c + x_{t-1} + \epsilon_{ct} \tag{1}
\]

where \( \mu_c \) is the average growth rate of \( c_t \), \( x_t \) is a mean zero process, and \( \epsilon_{ct} \) is a mean zero serially uncorrelated process. Put \( \Delta c_t = c_t - c_{t-1} \) and write the above as

\[
\Delta c_t = \mu_c + x_{t-1} + \epsilon_{ct}
\]

We assume

\[
x_t = \rho_x x_{t-1} + \epsilon_{xt}
\]

where \( 0 < \rho_x < 1 \) and \( \epsilon_{xt} \) is a serially uncorrelated process. The cointegrating assumption is that the \( d_t - c_t \) process is \( I(0) \). Hence let

\[
d_t - c_t = \mu_{dc} + s_t
\]

where \( \mu_{dc} \) is a constant and \( s_t \) is an \( I(0) \) process:

\[
s_t = \rho_s s_{t-1} + \lambda_{sx} x_{t-1} + \epsilon_{st} \tag{2}
\]

It is assumed that \( 0 < \rho_s < 1 \). It follows that

\[
\Delta d_t := d_t - d_{t-1} = \mu_c + (\rho_s - 1) s_{t-1} + (1 + \lambda_{sx}) x_{t-1} + \epsilon_{ct} + \epsilon_{st}
\]
Another way to view these dynamics is to note that the variables $x_t$, $s_t$, and $\Delta c_t$ form a stationary VAR process:

\[
\begin{align*}
\Delta c_t &= \mu_c + x_{t-1} + \epsilon_{ct} \\
\Delta s_t &= \rho_s s_{t-1} + \lambda_{sx} x_{t-1} + \epsilon_{st} \\
\Delta x_t &= \rho_x x_{t-1} + \epsilon_{xt}
\end{align*}
\]

and the $c_t$ and $d_t$ series are generated as

\[
\begin{align*}
ct &= c_{t-1} + \Delta c_t \\
d_t &= \mu_d + c_t + s_t
\end{align*}
\]

with $c_0, d_0$ given as initial conditions.

The system (3) and is a special cases of the general VAR(1) dynamics

\[
\begin{pmatrix}
\Delta c_t \\
\Delta s_t \\
\Delta x_t
\end{pmatrix} =
\begin{pmatrix}
a_c \\
a_s \\
a_x
\end{pmatrix}
+ 
\begin{pmatrix}
a_{cc} & a_{cs} & a_{cx} \\
a_{sc} & a_{ss} & a_{sx} \\
a_{xc} & a_{xs} & a_{xx}
\end{pmatrix}
\begin{pmatrix}
\Delta c_{t-1} \\
\Delta s_{t-1} \\
\Delta x_{t-1}
\end{pmatrix}
+ 
\begin{pmatrix}
\epsilon_{ct} \\
\epsilon_{st} \\
\epsilon_{xt}
\end{pmatrix}
\]

with $c_t$ and $d_t$ still obtained from (4).

To introduce stochastic volatility, let $\nu_t$ denote a common log-volatility shock such that

\[
\nu_t = \mu_\sigma + \rho_\sigma \nu_{t-1} + \epsilon_{\sigma t}.
\]

We assume that

\[
\epsilon_t = \text{diag}(b_t) \Psi \eta_t
\]

where $\epsilon_t = (\epsilon_{ct}, \epsilon_{st}, \epsilon_{xt})'$,

\[
b_t = \begin{pmatrix}
\exp(b_c + b_{cc} \nu_t) \\
\exp(b_s + b_{ss} \nu_t) \\
\exp(b_x + b_{xx} \nu_t)
\end{pmatrix},
\]

$\Psi$ is the upper triangular matrix

\[
\Psi = \begin{pmatrix}
\Psi_{cc} & \Psi_{cs} & \Psi_{cx} \\
0 & \Psi_{cc} & \Psi_{cx} \\
0 & 0 & \Psi_{xx}
\end{pmatrix},
\]
and \( \text{Var}(\eta_t) = I \). The state vector is, then,

\[
\mathbf{u}_t = \begin{pmatrix}
    s_t \\
    x_t \\
    \nu_t
\end{pmatrix}.
\]  

(10)

### 2.2 Asset Pricing

Let \( P_{ct} \) denote the price of an asset that pays the consumption endowment and let

\[
v_{ct} = \frac{P_{ct}}{C_t}
\]

(11)

denote the corresponding price dividend ratio. The Epstein-Zin-Weil utility function is

\[
U_t = \left[ (1 - \delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_tU_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{1}{1-\gamma}}
\]

(12)

where \( \gamma \) is the coefficient of risk aversion,

\[
\theta = \frac{1 - \gamma}{1 - 1/\psi},
\]

(13)

and \( \psi \) is the elasticity of intertemporal substitution. As noted in Campbell (2002), Bansal and Yaron (2004), and elsewhere, the first order conditions derived in Epstein and Zin (1989, 1991) imply that the price dividend ratio, \( v_{ct} \), is the solution to the nonlinear expectational equation

\[
v_{ct} = E_t \left\{ \delta^\theta \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}] (1 + v_{c,t+1}) \exp(\Delta c_{t+1}) \right\}.
\]

(14)

where

\[
r_{c,t+1} = \log \left[ \frac{1 + v_{c,t+1}}{v_{ct}} \exp(\Delta c_{t+1}) \right]
\]

(15)

is the geometric return on the asset. Evidently, the one-period marginal rate of substitution is

\[
M_{t,t+1} = \delta^\theta \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}].
\]

(16)

The price dividend ratio \( v_{dt} = P_{dt}/D_t \) on the asset that pays \( D_t \) is the solution to

\[
v_{dt} = E_t \left\{ \delta^\theta \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}] (1 + v_{d,t+1}) \exp(\Delta d_{t+1}) \right\}.
\]
The one-step-ahead risk-free rate $r_{ft}$ is the solution to

$$e^{-r_{ft}} = \mathcal{E}_t \{ \delta^\theta \exp \left[ -\left( \frac{\theta}{\psi} \right) \Delta c_{t+1} + (\theta - 1) r_{c,t+1} \right] \}.$$

The outcome variables of interest are

$$
\begin{pmatrix}
  r_{ct} \\
  v_{ct} \\
  r_{dt} \\
  v_{dt} \\
  r_{ft}
\end{pmatrix}.
$$

(17)

These are obtained by evaluating the pricing functions

$$
\begin{align*}
  v_{ct} &= v_c(u_t) \\
  v_{dt} &= v_d(u_t) \\
  r_{ft} &= r_f(u_t),
\end{align*}
$$

(18)

which are determined using the solution method described in Section 6, and then setting

$$
\begin{align*}
  r_{ct} &= \log \left[ \frac{1 + v_{ct} \exp(\Delta c_t)}{v_{ct,-1}} \right] \\
  r_{dt} &= \log \left[ \frac{1 + v_{dt} \exp(\Delta d_t)}{v_{dt,-1}} \right].
\end{align*}
$$

(19)

3 Habit Persistence (HAB)

We now set forth a model of habit persistence that follows rather closely the specification of Campbell and Cochrane (1999). We employ a cointegrated variant of their dynamics for the driving variables, and, as in Section 2, we think of the theoretical model as operating in monthly time and then aggregate to the annual level as described in Section 4.

The agent is assumed to maximize the utility function

$$
\mathcal{E}_t \sum_{i=0}^{\infty} \delta^i \left( C_{t+i} - X_{t+i} \right)^{1-\gamma} - 1
$$

where $C_t$ denotes real consumption, $X_t$ denotes the habit stock, $\delta$ is a discount rate, and $\gamma$ is a risk aversion parameter, although relative risk aversion itself depends on $X_t$ as well and will be much higher than $\gamma$ for plausible values of $X_t$. Define the surplus ratio

$$H_t = \frac{C_t - X_t}{C_t}$$

8
and assume that

\[ h_t = \log(H_t) \]

evolves as

\[ h_{t+1} = (1 - \rho_h)\bar{h} + \rho_h h_t + \lambda(h_t)\epsilon_{c,t+1}, \]

where \( \rho_h \) and \( \bar{h} \) are parameters, \( \epsilon_{c,t+1} \) is the consumption innovation defined in (1) above, and the sensitivity function \( \lambda(h_t) \) is

\[ \lambda(h_t) = \begin{cases} \frac{1}{H} \sqrt{1 - 2(h_t - \bar{h})} - 1 & h_t \leq h_{\text{max}} \\ 0 & h_t > h_{\text{max}} \end{cases} \]

where

\[ H = \sigma_c \sqrt{\frac{\gamma}{1 - \phi_h}} \]

and \( \sigma_c \) is the standard deviation of \( \epsilon_{ct} \). The value \( h_{\text{max}} \) is the value at which \( \lambda(h_t) \) first touches zero; by inspection,

\[ h_{\text{max}} = \bar{h} + \frac{1}{2} \left[ 1 - (\bar{H})^2 \right] \]

and, in the continuous time limit, \( h_{\text{max}} \) is an upper bound on \( h_t \). These dynamics for \( h_t \) imply, among other things, that the risk free rate is constant to a first approximation.

Under the assumption of external habit, the intertemporal marginal rate of substitution is

\[ M_{t+1} = \delta \left( \frac{H_{t+1} C_{t+1}}{H_t C_t} \right)^{-\gamma}, \]

or, equivalently,

\[ M_{t+1} = \delta \exp \left[ -\gamma (\Delta h_{t+1} + \Delta c_{t+1}) \right] \] (20)

where \( \Delta h_{t+1} = h_{t+1} - h_t \) and \( \Delta c_{t+1} \) is as defined in Section 2 above. Thus, in notation consistent with the preceding sections, the expectational equation for the price dividend ratio \( v_{ct} = P_{ct}/C_t \) for the asset that pays the consumption endowment \( C_t \) is

\[ v_{ct} = \mathcal{E}_t \left\{ \delta \exp \left[ -\gamma (\Delta h_{t+1} + \Delta c_{t+1}) \right] (1 + v_{c,t+1}) \exp(\Delta c_{t+1}) \right\}, \] (21)

and the expectational equation for the price dividend ratio \( v_{dt} = P_{dt}/D_t \) for the asset that pays the dividend \( D_t \) is

\[ v_{dt} = \mathcal{E}_t \left\{ \delta \exp \left[ -\gamma (\Delta h_{t+1} + \Delta c_{t+1}) \right] (1 + v_{d,t+1}) \exp(\Delta d_{t+1}) \right\} \] (22)
The risk free rate $r_{ft}$ is the solution to

$$\exp(-r_{ft}) = \mathcal{E}_{t}\{\delta \exp[-\gamma(\Delta h_{t+1} + \Delta c_{t+1})]\}. \quad (23)$$

The above three equations correspond directly to their counterparts above for Epstein-Zin-Weil preferences. The geometric return is

$$r_{c,t+1} = \log\left[1 + \frac{v_{c,t+1}}{v_{ct}} \exp(\Delta c_{t+1})\right] \quad (24)$$
on the consumption asset and

$$r_{d,t+1} = \log\left[1 + \frac{v_{d,t+1}}{v_{dt}} \exp(\Delta d_{t+1})\right] \quad (25)$$
on the equity asset.

Campbell and Cochrane (1999) assume that log consumption $c_t$ and the log dividend $d_t$ processes are each I(1) with the same growth rates and correlated innovations. We can easily modify the dynamics (5) of Section 2 to retain the essential features of their model while incorporating cointegration between consumption and dividends. We retain the equation

$$d_t = \mu_{dc} + c_t + s_t.$$

We set $x_t = 0$ for all $t$; so that

$$\Delta c_t = \mu_c + \epsilon_{ct} \quad \Delta d_t = \mu_c + (\rho_s - 1)s_{t-1} + \epsilon_{ct} + \epsilon_{st} \quad (26)$$

and turn off stochastic volatility, i.e., $\nu_t = 0$ for all $t$, so that $\epsilon_{ct}$ and $\epsilon_{st}$ immediately above are iid Gaussian. This setup, with correlation between $\epsilon_{ct}$ and $\epsilon_{st}$, is a direct extension of Campbell and Cochrane (1999). Here $c_t$ and $d_t$ are each I(1), $\Delta c_t$ is iid, and the two series are cointegrated except if $\rho_s = 1$, in which the setup reduces to that of Campbell and Cochrane (1999).

The outcome variables of interest are the same as in (17) above. Using the same solution technique we solve for the pricing functions

$$v_{ct} = v_c(u_t)$$
$$v_{dt} = v_d(u_t)$$
$$r_{ft} = r_f(u_t). \quad (27)$$
Here we take the state vector $u_t$ as

$$u_t = \begin{pmatrix} s_t \\ h_t \\ \lambda(h_t) \end{pmatrix}.$$  \hspace{1cm} (28)

We include $\lambda(h_t)$ in the state vector because of the central role $\lambda(h_t)$ plays in the model, although it turns out that the solution is essentially a quadratic function of $h_t$.

Observe that the outcome variables for the HAB Model are the same as those of the LRR model defined in (17) above with analogous pricing functions (18) and one period holding returns definitions (19).

4 Time Aggregation and the Observation Equations

The LRR and HAB models defined in Sections 2 and 3 operate in monthly time while we observe annual consumption data and annual summary measures from the financial markets. The monthly levels of consumption and dividends are obtained by iterating the monthly growth sequences

$$C_t = e^{\Delta c_t} C_{t-1} \quad t = 1, 2, \ldots$$

$$D_t = e^{\Delta d_t} D_{t-1} \quad t = 1, 2, \ldots$$

starting from

$$C_0 = e^{c_0}$$

$$D_0 = e^{d_0}.$$  

The annual consumption series is, then,

$$C^a_t = \sum_{k=0}^{11} C_{t-k} \quad t = 12, 24, 36, \ldots$$

$$c^a_t = \log(C^a_t) \quad t = 12, 24, 36, \ldots$$

and the annual cumulative dividend series is

$$D^a_t = \sum_{k=0}^{11} D_{t-k} \quad t = 12, 24, 36, \ldots$$

$$d^a_t = \log(D^a_t) \quad t = 12, 24, 36, \ldots$$
The annual price dividend ratio is computed as

\[ p_{dt}^a - d_t^a = \log \left( \frac{v_{dt}}{v_{dt}^c} \right) \quad t = 12, 24, 36, \ldots \]

We also compute the annual geometric returns

\[ r_{dt}^a = \sum_{k=0}^{11} r_{d,t-k} \]

\[ r_{ft}^a = \sum_{k=0}^{11} r_{f,t-k} \]

and, following Andersen, Bollerslev, and Diebold (2002), the within-year quadratic variation

\[ Q_t^a = \sum_{k=0}^{11} r_{d,t-k}^2. \]

We use its logarithm

\[ q_t^a = \log(Q_t^a) \quad (29) \]

as the measure of within year variation in statistical analyses because it is nearly normally distributed (Andersen, Bollerslev, and Diebold, 2002). A more familiar measure is volatility computed as

\[ \text{std}_t^a = \sqrt{Q_t^a}, \quad (30) \]

which we shall often use when reporting results.

In results reported in Section 7, we estimate models using an observation equation comprised of the four variables

\[ y_t = \begin{pmatrix} d_t^a - c_t^a \\ c_t^a - c_{t-12}^a \\ p_{dt}^a - d_t^a \\ r_{dt}^a \end{pmatrix} \quad (31) \]

for \( t = 12, 24, \ldots \). Were the observations monthly, including \( r_{dt} \) in the observation equation (31) would induce a nonlinear intertemporal redundancy. But at an annual frequency, the aggregation protocol just described implies that including \( r_{dt}^a \) adds additional information.
We evaluate models, by studying their implications for the five-variable observation equation

\[
y_t = \begin{pmatrix}
d_t^a - c_t^a \\
c_t^a - c_{t-12}^a \\
p_{dt}^a - d_t^a \\
r_{dt}^a \\
q_t^a
\end{pmatrix}.
\]  
(32)

5 Data

5.1 Raw Data

Our data set consists of annual observations 1929–2001. All variables, except population and consumption, are computed using monthly data from CRSP and then converted to the annual frequency. Annual real ($1996) per capita consumption, nondurables and services, along with the mid-year population data are taken from the Bureau of Economic Analysis (BEA) web site.

To construct the annual per capita stock market valuation series, we start with the month-end combined nominal capitalizations of the NYSE and the AMEX, convert to $1996 using the monthly CPI, take the year-end value, and divide by the BEA population figure. To compute the annual dividend series, we use the difference between the nominal value weighted return and the capital return (i.e. the return excluding dividend) to compute an implied monthly nominal dividend yield on the NYSE+AMEX. Applying this dividend yield to the preceding month’s market capitalization gives an implied monthly nominal dividend series. This series is converted to a real ($1996) monthly dividend series using the monthly CPI, aggregated over the year, and then divided by the BEA population figure. To compute the annual real return series, we use the monthly nominal value weighted return on the NYSE+AMEX and the CPI to compute a monthly real geometric return, which is then cumulated over the year to form a real annual geometric return. The annual quadratic variation is the sum of the monthly squared real geometric returns.

For consistency with the presentation of the model, we let \( t \) denote the time index in months, so that \( P_{dt}^a, t = 12, 24, \ldots \) denotes the end-of-year per capita stock market value
observations; $D^n_t$, $t = 12, 24, \ldots$ denotes the annual aggregate per capita dividend observations; $C^n_t$, $t = 12, 24, \ldots$ denotes the annual per capita observations; $r^n_{dt}$, $t = 12, 24, \ldots$ denotes the annual real geometric return observations; $Q^n_t$, $t = 12, 24, \ldots$ denotes the annual quadratic variation observations.

Figure 1 shows time series plots of the annual observations on the logged series $p^n_{dt} = \log(P^n_{dt})$, $d^n_t = \log(D^n_t)$, $c^n_t = \log(C^n_t)$, $r^n_t$, and $q^n_t = \log(Q^n_t)$. The three series $p^n_{dt}$, $d^n_t$, and $c^n_t$, are upward trending series, while $r^n_t$ and $q^n_t$ appear stationary.

### 5.2 Cointegrating Relationships

One would expect there to be cointegrating relationships among the three trending variables. A simple regression of $(p^n_{dt}, d^n_t, c^n_t)$ on $(p^n_{dt-12}, d^n_{t-12}, c^n_{t-12})$, annual data 1930–2001, yields an autoregressive matrix with one eigenvalue nearly exactly equal to unity and two others about 0.90 in magnitude. Two eigenvalues separated from unity suggests that there are two cointegrating relationships among the three variables. One relationship, which has been explored extensively in the literature and which we imposed $a priori$ in the development of the model in Section 2, is that the log price dividend ratio $v^n_{dt} = p^n_{dt} - d^n_t$ is stationary. It is natural to presume that there is also a cointegrating relationship between the log dividend and the log consumption variable. Thus we conjecture the relationship

$$d^n_t - \lambda_d c^n_t = I(0)$$
where $\lambda_{dc}$ is a parameter that we conjecture equals unity. To estimate $\lambda_{dc}$, we run the reduced rank regression (Anderson, 2001)

\[
\begin{pmatrix}
    p_t^{o} - p_{d,t-12}^{o} \\
    d_t^{a} - d_{t-12}^{a} \\
    c_t^{a} - c_{t-12}^{a}
\end{pmatrix} = 
\begin{pmatrix}
    a_{p0} \\
    a_{d0} \\
    a_{c0}
\end{pmatrix} + 
\begin{pmatrix}
    a_{p1} & a_{p2} \\
    a_{d1} & a_{d2} \\
    a_{c1} & a_{c2}
\end{pmatrix} 
\begin{pmatrix}
    1 & -1 & 0 \\
    0 & 1 & -\lambda_{dc}
\end{pmatrix} 
\begin{pmatrix}
    p_{d,t-12}^{o} \\
    d_{t-12}^{a} \\
    c_{t-12}^{a}
\end{pmatrix} + 
\begin{pmatrix}
    \epsilon_{pt} \\
    \epsilon_{dt} \\
    \epsilon_{ct}
\end{pmatrix}
\]

(33)

The above is a nonlinear SUR system (Gallant, 1987) that is nonlinear in parameters but linear in variables.

Table 1 shows the results of the nonlinear SUR estimation with annual data; the range of the dependent variable is 1931–2001. With the exception of the estimate of $\lambda_{dc}$, all coefficients multiply I(0) variables and conventional asymptotics apply to the corresponding point estimates. The estimate of $\lambda_{dc}$ is, on the other hand, influenced by unit-root effects. The estimate of $\lambda_{dc}$, however, is very close to unity, especially relative to its (conventionally computed) standard error so that by any reasonable theory of inference the presumption that $\lambda_{dc} = 1$ is consistent with the data.

In what follows we shall take the series

\[ p_t^{o} - d_t^{n} \]
\[ d_t^{a} - c_t^{a} \]
\[ c_t^{a} - c_{t-12}^{a} \]

as three basic jointly I(0) variables embodied in the three trending series. Of course the above analysis only identifies the two dimensional subspace of $\mathbb{R}^{3}$ that determines the two
cointegrating relationships among \((p^a_{dt} \; d^a_t \; c^a_t)\). Put another way, given any nonsingular \(2 \times 2\) matrix times the vector
\[
\begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1
\end{bmatrix}
\begin{pmatrix}
p^a_{dt} \\
d^a_t \\
c^a_t
\end{pmatrix}
\]
could, on statistical grounds, also be taken as the I(0) linear combinations of the three trending series. However, the normalization above leads to very simple and easy to interpret variables: the log price dividend ratio, the log dividend consumption ratio, together with consumption growth are jointly I(0).

6 Estimation Methodology

We use an estimation/calibration methodology due to Smith (1993), described below, which is implemented using a score-based computational strategy proposed by Gallant and Tauchen (1996). Central to the methodology is the use of long simulations to compute unconditional expectations of the functions of the state vector involving one or more leads and lags. In particular, an expectation of a random variable of the form \(g(u_{t+1}, u_t)\), which depends on a contemporaneous and a one-step-ahead value of the state vector, can be computed from a simulation \(
\{\hat{u}_t\}_{t=1}^{N+1}
\) as
\[
\mathcal{E}(g) \doteq \frac{1}{N} \sum_{t=1}^{N} g(\hat{u}_{t+1}, \hat{u}_t)
\]
to any desired degree of accuracy by taking \(N\) sufficiently large. Because one must bear the cost of generating the simulation \(
\{\hat{u}_t\}_{t=1}^{N+1}
\) for the purpose of estimation/calibration anyway, it becomes convenient to use this same simulation to determine the pricing functions \(v_c(u)\), \(v_d(u)\), and \(r_f(u)\) as well.

Consider, for example, the determination of \(v_c(u)\) for the LRR model, which, for this purpose, we presume can be adequately approximated by a quadratic
\[
v_c(u) \doteq a_0 + a_1' u + u' A_2 u.
\]
Using (14) and (15), \(v_c(u)\) must satisfy the conditional expression
\[
\mathcal{E}\{v_c(u_t) - M(u_{t+1}, u_t)[1 + v_c(u_{t+1})] \exp(\Delta c_{t+1}) \mid u_t = u\} = 0 \quad (34)
\]
for all $u \in \mathbb{R}^3$, where

$$M(u_{t+1}, u_t) = \delta^0 \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_c(u_{t+1}, u_t)]$$

$$r_c(u_{t+1}, u_t) = \log \left[ \frac{1 + v_c(u_{t+1})}{v_c(u_t)} \exp(\Delta c_{t+1}) \right].$$

Let

$$g(u_{t+1}, u_t) = \{v_c(u_t) - M(u_{t+1}, u_t)[1 + v_c(u_{t+1})] \exp(\Delta c_{t+1})\} Z_t$$

where $Z_t = [1, u'_t, \text{vech}'(u_t, u_t)]'$ and $\text{vech}(A)$ denotes the vector comprised of the elements of the upper triangle of a symmetric matrix $A$. Then, by the law of iterated expectations, the conditional expression (34) implies that the unconditional expression $\mathcal{E}g = 0$ must also be satisfied by $v_c(u)$.

Let $g(u_{t+1}, u_t, \alpha_c)$ denote $g(u_{t+1}, u_t)$ with $v_c(u)$ replaced by its quadratic approximation and let $\alpha_c = [a_0, a'_0, \text{vech}'(A_2)]'$ denote the ten distinct coefficients of the quadratic approximation. Because both $\alpha_c$ and $Z_t$ are vectors of length ten, the unconditional expression $\mathcal{E}g(\cdot, \alpha_c) = 0$ becomes a system of ten nonlinear equations that can be solved for $\alpha_c$.

As integrals are computed by averaging over a long simulation $\{\hat{u}_t\}_{t=1}^{N+1}$, it is actually the system

$$\frac{1}{N} \sum_{t=1}^N g(\hat{u}_{t+1}, \hat{u}_t, \alpha_c) = 0 \quad (35)$$

that is to be solved for $\alpha_c$. Bear in mind that at the time of the simulation the values of all parameters of the model are known so that, in particular, the values of $(\Delta c_{t+1}, \Delta c_t)$ that appear in the definition of $g(u_{t+1}, u_t, \alpha_c)$ can be computed from $(u_{t+1}, u_t)$ by knowing the values of the parameters of the VAR given by equation (5).

Now (35) is immediately recognized as a system of equations of the sort that defines a GMM or NL2SLS estimator, albeit for a very large data set. Thus it can be solved either by using a nonlinear equation solver or by using standard statistical software for GMM or NL2SLS estimation. Issues of statistical efficiency involving the appropriate weighting matrix do not arise because we are just using the software to solve a system of equations; we are not really estimating anything. Therefore, any convenient, well-tested, stable software will do. For specificity hereafter, we shall presume that NL2SLS software is used.

In NL2SLS terminology, $g(u_{t+1}, u_t, \alpha_c)$ is an exactly identified system. This is important
because it means that one can determine whether or not the nonlinear optimizer implementing the NL2SLS software has in fact found coefficients $\alpha_c$ that solve (35). That is, the computed value for the left hand side of (35) reported by the program must actually be zero. Whether or not this is accomplished is, of course, a matter of correctly choosing tuning parameters and starting values for $\alpha_c$. In the work reported here, we set tuning parameters such that the Euclidean norm of the left hand side of (35) is driven to less than machine epsilon. Our starting value strategy is described below in connection with the estimation/calibration methodology.

One solves for $v_d(u)$ and $r_f(u)$ similarly. As the marginal rate of substitution $M(u_{t+1}, u_t)$ depends on $v_c(u_{t+1})$ and $v_c(u_t)$, one must solve for $v_c(u)$ first. The solution strategy for $\text{HAB}$ is exactly the same using (20) as the marginal rate of substitution and (28) as the state vector. Because $v_c(u)$ is needed to price macro risks as described later, we compute it for the habit model.

At this stage of the computations, both the simulation $\{\hat{u}_t\}_{t=1}^{N+1}$ and the values $v_c(\hat{u}_t)$, $v_d(\hat{u}_t)$, and $r_f(\hat{u}_t)$ at the monthly frequency become available. The corresponding values for $\{\hat{y}_t\}_{t=12,24,\ldots}$ at the annual frequency can now be computed by applying the expressions for aggregation set forth in Section 4.

This method for determining $v_c(u)$, $v_d(u)$, and $r_f(u)$ works quite well. It is also efficient provided the starting values are chosen adaptively as described below.

We estimate the parameters of the model by matching the population values of statistics $\theta(\rho)$ that are implied by the model of Section 2 to statistics $\tilde{\theta}$ that are computed from the observed data $\{\tilde{y}_t\}_{t=12,24,\ldots}$. Usually $\theta(\rho)$ is computed from a long simulation $\{\hat{y}_t\}_{t=12,24,\ldots}$ of the model rather than analytically. We distinguish between data and simulations in this section by writing $\tilde{y}_t$ for data and $\hat{y}_t$ for simulations.

Our calibrations strategy aims to match the transition dynamics implied by a VAR. We match the parameters $\theta(\rho)$ of a VAR for $y_t$ that are implied by the model to the parameters $\tilde{\theta}$ of a VAR that are estimated from the data. Because the match cannot be exact, it is achieved approximately by minimizing a GMM metric. This is a strategy proposed by Smith (1993).

Let the transition density of the VAR be written as $f(y_t|y_{t-12}, \theta), t = 12, 24, \ldots$, let $\hat{\theta}$ denote the maximum likelihood estimate of $\theta$ computed from $\{\hat{y}_t\}_{t=12,24,\ldots}$, and let the
stationary density implied by the model described in Section 2 be written as \( p(y_t, y_{t-12}|\rho) \). Then, as shown by Gouriéroux, Monfort, and Renault (1993), the estimator computed by solving the score-based orthogonality condition for \( \rho \)

\[
0 = m(\rho, \tilde{\theta}) = \int \int \frac{\partial}{\partial \theta} \log f(y_t|y_{t-12}, \theta) \, p(y_t, y_{t-12}|\rho) \, dy_t \, dy_{t-12},
\]

(36)

is asymptotically equivalent to Smith’s (1993) estimator. As above, (36) is not computed analytically but rather by averaging over a simulation \( \{\tilde{y}_t\}_{t=12,24,...} \) from the model. Because (36) cannot be solved exactly, \( \rho \) is estimated by minimizing the GMM criterion

\[
s(\rho) = m'(\rho, \tilde{\theta})(\tilde{I})^{-1}m(\rho, \tilde{\theta})
\]

(37)

for a suitable weighting matrix \( \tilde{I} \).

The advantage to the score-based approach is computational. For more complicated representations of the dynamics \( f(y_t|y_{t-12}, \theta) \), such as a multivariate GARCH specification, the statistics to be matched to \( \tilde{\theta} \) become implicitly defined nonlinear functions of the simulation which puts an inner nonlinear optimization to compute \( \theta(\rho) \) within an outer GMM optimization to compute \( \rho \) making computations following Smith’s (1993) approach to be so computationally intensive and unstable as to be nearly infeasible. In contrast, the computational burden of the score-based approach does not increase at all. The upshot is that the score-based approach is more widely used, especially in financial econometrics, and therefore stable, well-tested software implementing it is available.

It is easy to give the score-based approach an independent justification without relying on its equivalence with Smith’s (1993) estimator as follows. The orthogonality conditions defining the (quasi) maximum likelihood estimator are

\[
0 = \frac{1}{n/12} \sum_{t=12,24,...}^{n/12} \frac{\partial}{\partial \theta} \log f(y_t|y_{t-12}, \theta).
\]

If the data is actually generated according to \( p(y_t, y_{t-12}|\rho^o) \), then, in the limit as \( n \to \infty \), \( \tilde{\theta} \) tends to a value \( \theta^o \) that satisfies the population orthogonality conditions

\[
0 = m(\rho, \theta^o) = \int \int \frac{\partial}{\partial \theta} \log f(y_t|y_{t-12}, \theta^o) \, p(y_t, y_{t-12}|\rho^o) \, dy_t \, dy_{t-12}.
\]

It is then natural to define an estimator as \( \hat{\rho} \) that satisfies the population orthogonality conditions as nearly as possible in a suitable GMM metric with sample estimates of \( \theta^o \) substituted for population values. Being a GMM estimator, this estimator comes equipped with
a test of over-identifying restrictions and methods for computing standard errors. Regularity
conditions and details are in Gallant and Tauchen (1996), where this score-based approach
is termed the EMM estimator. We remark here that although it is desirable that the aux-
iliary model \( f(y_t|y_{t-12}, \theta) \) be a good approximation to the dynamics in the data, this is not
required by the regularity conditions of Gallant and Tauchen (1996).

If the auxiliary model \( f(y_t|y_{t-12}, \theta) \) fits the data well, then \( \left\{ \frac{\partial}{\partial \theta} \log f(y_t|y_{t-12}, \theta) \right\} \) is effect-
vively a martingale difference sequence and the GMM weighting matrix appearing in (37)
above can be taken as

\[
\tilde{I} = \sum_{t=12,24,...}^{n/12} \left\{ \frac{\partial}{\partial \theta} \log f(\tilde{y}_t|\tilde{y}_{t-12}, \tilde{\theta}) \right\} \left\{ \frac{\partial}{\partial \theta} \log f(\tilde{y}_t|\tilde{y}_{t-12}, \tilde{\theta}) \right\}^t,
\]

If \( f(y_t|y_{t-12}, \theta) \) does not fit the data well, then a more complicated weighting matrix described
in Gallant and Tauchen (1996) is used.

Note that \( \tilde{\theta} \) and \( \tilde{I} \) and are computed once and for all from the data \( \{\tilde{y}_t\}_{t=12,24,...} \) at the
commencement. The computational burden in computing \( \hat{\rho} \) is due to the need to determine
\( v_c(u), v_d(u), \) and \( r_f(u) \). The computational cost of generating \( \{\hat{u}_t\} \) and evaluating \( v_c(\hat{u}_t),
\( v_d(\hat{u}_t), \) and \( r_f(\hat{u}_t) \) to get \( \{\hat{y}_t\} \) and thence

\[
m(\rho, \hat{\theta}) = \frac{1}{N/12} \sum_{t=12,24,...}^{N/12} \frac{\partial}{\partial \theta} \log f(\hat{y}_t|\hat{y}_{t-12}, \hat{\theta}) p(\hat{y}_t, \hat{y}_{t-12}|\rho)
\]

and

\[
s(\rho) = m'(\rho, \hat{\theta})(\tilde{I})^{-1} m'(\rho, \hat{\theta})
\] (38)
is trivial.

We remark in passing that in all optimizations of (38) reported later the constraint on
the risk free rate

\[
\frac{1}{N} \sum_{t=1}^{N} r_{ft} = 0.000743618
\]
is imposed. A monthly risk free rate of 0.000743618 corresponds to an annual rate of 0.00896,
which is the rate reported by Campbell (2002). This is a constraint on the average over the
entire simulation so that the risk free rate is free to vary within the simulation. Full details
are in Section 7.

We return now to considerations involving the computation of \( v_c(u), v_d(u), \) and \( r_f(u) \).
One uses a nonlinear optimizer to minimize \( s(\rho) \). Thus, the starting value \( \hat{\rho}_0 \) for which \( v_c(u),\)
\( v_d(u) \), and \( r_f(u) \) are to be determined is under the user’s control, but the remaining values \( \hat{\rho}_i, \ i = 1, 2, \ldots \) are determined by the optimizer and are not under the user’s control. For \( \hat{\rho}_0 \), (35) is solved using the NL2SLS software to obtain the corresponding \( \hat{\alpha}_{c,0} \), similarly for \( v_d(u) \), and \( r_f(u) \). The iterations to compute \( \hat{\rho}_i, \ i = 1, 2, \ldots \), use these \( \hat{\alpha}_{j,0}, \ j = c, d, f \), as starting values for the NL2SLS code throughout because it is important for the optimization algorithm’s stability that these starting values not be changed during the optimization. The optimizer’s iterations, or at least the last of them, should be inspected to be certain that (35) is indeed solved. The number of iterations to solve (35) taken by the NL2SLS algorithm will typically increase as the sequence \( \hat{\rho}_i, \ i = 1, 2, \ldots \) progresses. It is advantageous to terminate the sequence \( \hat{\rho}_i, \ i = 1, 2, \ldots \) relatively quickly by using a loose convergence criterion or a small iteration limit so as to gain the opportunity to recompute and reset \( \hat{\alpha}_{j,0}, \ j = c, d, f \). Iterations recommence with \( \rho_0 \) put to the last value returned by the optimizer and \( \hat{\alpha}_{j,0}, \ j = c, d, f \) recomputed for that value of \( \rho_0 \). Initially, the optimizer makes large moves and the values of \( \hat{\rho}_i, \ i = 1, 2, \ldots \) are widely spaced. We find it advantageous to use a linear specification of \( v_c(u), v_d(u) \), and \( r_f(u) \) during this initial phase (i.e. put the quadratic terms of \( \hat{\alpha}_{j,0}, \ j = c, d, f \) to zero) so as to enlarge the radius of convergence of the NL2SLS algorithm. As convergence nears, the spacing in the sequence \( \hat{\rho}_i, \ i = 1, 2, \ldots \) decreases and the quadratic terms can be introduced. This hands-on intervention is, perhaps, overly cautious, but it does dramatically reduce the number of iterations performed by the NL2SLS algorithm thereby reducing running times equally dramatically.

## 7 Empirical Results

The most immediate issue for estimating the models described in Section 2 and 3 is that of handling the ex ante real rate risk free rate of interest, which is not directly observable. Campbell (2002) notes that any reasonable asset pricing model must accommodate the indirect evidence that the mean risk-free rate \( r_{ft} \) is very low with low volatility. Campbell’s evidence suggests that the mean risk-free rate for the U.S. is 0.896 percent per annum. Thus, as noted in Section 6, we impose that restriction on \( \mathcal{E}(r_{ft}) \) in the simulations. We actually constrain \( \mathcal{E}(r_{ft}) \) to lie within a small band of approximately 50 annual basis points around 0.896 percent per annum. Among other things, treating the constraint on the risk-free rate
as a small inequality constraint lightens the burden of finding start values for the numerical optimization, and it also takes account of the uncertainty regarding Campbell’s number.

In results reported below, we use as the auxiliary model a four-variable VAR(1) at the annual frequency:

\[ y_t = b_0 + By_{t-12} + e_t \]  

where

\[ y_t = \begin{pmatrix} d^a_t - e^a_t \\ c^a_t - c^a_{t-12} \\ p^a_{it} - d^a_t \\ r^a_t \end{pmatrix} \]  

The definition of \( y_t \) above corresponds exactly with that of the observation equation (31) for the theoretical model. The period of the dependent variable in (39) is 1931–2001. We do not include the log quadratic variation variable \( q^a_t \), but we do check how well the asset pricing model can match the dynamics of that variable.

In some initial work, we used an unrestricted 4 × 4 VAR(1). However, many of the estimates of the off-diagonal elements of \( B \) are statistically insignificant and the diagonal elements clearly dominate; rather than adopt some arbitrary scheme for dropping variables, we use a VAR that is diagonal in both location and scale for estimation. The intercept and AR(1) coefficients are shown in the columns labeled “Observed” in Table 5, which discussed more fully below. The simulation estimator described in Section 6 above remains consistent and asymptotically normal with this choice of the auxiliary model.

7.1 Parameter Estimates: Short and Long Run Risk Models

We start with simplified dynamics for the driving processes; this version corresponds closely to the model of Hall (1978). In this version, labeled the Short-Run Risk (SRR) model, the \( x_t \) process is set to zero, so consumption growth is a random walk with drift, and the driving processes in (3) are homoskedastic, so that the risk premia are constant. The restrictions are implemented by setting \( \rho_x = 0 \), by setting the elements of \( \Psi \) corresponding to \( x_t \) to zero, and by setting to zero the parameters governing the stochastic volatility in (8). The assumption that consumption growth is iid is quite common, e.g., Campbell and Cochrane (1999), as is
the assumption of homoskedastic driving variables. We use the term Short Run Risk (SRR) because the only exposure of the dividend to consumption risk arises through innovation correlations. We use linear functional forms for the solution functions of the SRR model in the numerical solution of the model as described in Section 6; we could find little indication of a need for quadratic terms. The use of a linear form is typical in the literature. A major difference is that we maintain cointegration between the consumption and dividend process.

The second model, labeled the Long Run Risk (LRR) model, is more general with underlying dynamics given by (3) above. This model incorporates a stochastic mean, $\mu_c + x_t$, for consumption growth and it includes stochastic volatility as given by (8). A normalization for identification is that $b_c = b_s = b_x = 0$. Initial estimation was done with both linear and quadratic terms in the solution functions. Inspection of the results indicated that the only important quadratic term was the interaction term of $x_t$ with the log-volatility variable, $\nu_t$, and that other quadratic terms could be set to zero without altering the results.

The initial work suggested that freely estimating the LRR model using an unconstrained $4 \times 4$ VAR(1) as the auxiliary model gives results almost exactly the same as those reported below, except that consumption volatility tends to be underestimated somewhat and therefore returns volatility is underestimated. We elected to fix the scale parameters of the consumption process in (8) and (9) to values that, after some experimentation, would give rise to consumption volatility matching that of the observed data. These values are

$$
\begin{align*}
   b_{cc} &= 0.14320 \\
   b_{xx} &= 0.11000 \\
   \Psi_{cc} &= 0.00340 \\
   \Psi_{xx} &= 0.00012
\end{align*}
$$

and they remain fixed throughout the estimation, so they are calibrated parameters; $\Psi_{ss}$ is a free parameter while the off-diagonal elements of (9) are set to zero. Also, estimates of the elasticity of substitution parameter, $\psi$, typically land in the region 1.50–2.50, but the objective function is very flat in $\psi$, so we constrain $\psi = 2.00$. The remaining parameters, including the dynamic parameters, $\rho_x$ and $\rho_\sigma$, and the leverage parameter, $\lambda_{sx}$, are freely estimated.

Table 2 displays parameter estimates for the SRR and LRR models along with the chi-
squared measure of fit. The chi-squared statistics suggest that the \textbf{SRR} model is rejected, as might be expected, though the \textbf{LRR} model appears to give an adequate fit. Both point estimates of $\rho_s$ are close to unity, indicating that shocks to the log dividend consumption ratio are highly persistent in both models. The estimate of $\rho_x$ is close to unity in the \textbf{LRR} model, which is evidence for a very persistent component $x_t$ to consumption growth, though the small conditional standard deviation suggests that this component is small. Likewise, the estimate of $\rho_x$ is very close to unity, a finding consistent with all other empirical evidence on volatility persistence. Interestingly, the estimate of $\rho_x$ close to unity is obtained only using mean dynamics without regard to the log quadratic variance process, $q_t^a$, as that process was not included in the VAR on which \textbf{LRR} was estimated.

Perhaps the sharpest difference between \textbf{SRR} and \textbf{LRR} is the estimates of the risk aversion parameter $\gamma$, which is huge in the \textbf{SRR} model but much more reasonable in the \textbf{LRR} model. The difference can be traced directly to very different dynamics of the driving variables across the models. In the \textbf{SRR} model, consumption shocks are completely transient, while in the \textbf{LRR} model the transient consumption shocks are superimposed with the very persistent process $x_t$. Thus, in the latter model, the asset that pays the consumption endowment $C_t$ is much riskier and thereby commands a higher risk premium, $\mathcal{E}_t(r_{c,t+1} - r_{ft})$, other things equal. In addition, the near permanent shocks to $x_t$ affect the mean growth rate of the dividend in (2) with leverage parameter $\lambda_{sx}$, estimated to be about 2.5. Thus, the asset paying the dividend stream $D_t$ is likewise much riskier in the \textbf{LRR} model and commands a higher risk premium, $\mathcal{E}_t(r_{d,t+1} - r_{ft})$. Taken together, these characteristics of the dynamics of the driving processes imply that the \textbf{LRR} model can generate higher equity risk premia without appeal to an (implausible) high coefficient of risk aversion $\gamma$.

Another model we considered was one with long run dynamics but with the utility function constrained to be the constant relative risk aversion (\textbf{CRR}) instead of the more general Epstein-Zin-Weil form considered in the \textbf{LRR} model. This estimation is achieved by setting $\theta = 1$ in (12) and estimating risk aversion, $\gamma$, which corresponds to $1/\psi$, along with the other free parameters. The empirical failures of \textbf{CRR} utility have been extensively documented in the literature, so we not report the results in detail. The model is sharply rejected (p-value = 4.98e-13). Interesting, the fitted model estimates a modest value for the risk aversion
parameter, $\hat{y} = 7.285$ (2.223), but it also estimates a very small value for $\mu_c$, and thereby predicts an absurdly low value for mean annual average consumption growth. The low estimate of $\mu_c$ is the only way this model can satisfy the real interest constraint; of course it cannot then generate a sufficient equity premium, as has been long understood.

### 7.2 Parameter Estimates: Habit Persistence Model

Table 3 shows parameter estimates and the chi-squared statistic for the model of habit persistence ($HAB$) described in Section 3 above. Initial work suggested that $\Psi_{cs}$ is difficult to estimate as a free parameter, so we constrain $\Psi_{cs} = 0$. Keep in mind that the consumption and dividend growth innovations are still correlated in (26); the parameter estimates imply correlations very close to 0.20, which is the value imposed by Campbell and Cochrane (1999). As seen in the table, the HAB model does well on the $\chi^2$ criterion and the point estimates appear reasonable. As expected, the estimates of $\rho_h$ and $\rho_s$ are close to unity as is $\delta$. The risk aversion parameter $\gamma$ is estimated to be close to unity and is reasonably precisely estimated. A conventional Wald confidence interval would include a range of values above unity but would exclude the value 2.00 imposed by Campbell and Cochrane (1999).

### 7.3 Model Comparisons

Table 4 shows unconditional means and standard deviations computed from the data and the predicted values under the models SRR, LRR, and HAB. In this, and in subsequent tables, the predicted values should be regarded as population values implied by the model at the estimated parameter values corrupted by very small Monte Carlo noise. All three models agree rather closely with the data on the means and standard deviations of the log dividend/consumption ratio and also for consumption growth, except that the HAB model underpredicts consumption growth volatility somewhat. Also, the three models agree quite closely with the data on the mean of the price dividend ratio, though the SRR model seriously underestimates its volatility, while the LRR and HAB models are much closer, though still below that of the data.

All three models predict an annual return on the stock market, i.e., the dividend asset, at just over 6 percent per year, which is consistent with the data. Likewise, since the mean of
the risk-free is tightly constrained, the three models also predict about the same equity premium. The mechanisms, however, are quite different. The unrealistically high degree of risk aversion of the SRR model generates an expected return on the consumption asset, \( E(r_{ct}) \), of about 5.60 percent per year, which, via the correlation between consumption innovations and dividend innovations, is stepped up to a predicted value of 6.15 percent per year for the dividend asset. On the other hand, for the LRR model, the risk aversion parameter is much smaller, but the inherent higher riskiness of the consumption asset implies an expected return \( E(r_{ct}) \) on the consumption asset of about 2.33 percent per year. Cointegration, innovation correlation, and the leveraging of \( x_t \) innovations implied by a positive estimate of \( \lambda_{sx} \) (see (2)) increase the expected return on the equity \( E(r_{dt}) \) to the predicted value of 6.29 percent per year. Finally, as explained at length in Campbell and Cochrane (1999), the dynamics of the surplus consumption ratio make stocks rather unappealing to investors who therefore require a relatively large premium over cash. As seen from the table, the unconditional first two moments of the dividend asset and the consumption asset are about the same under the HAB model, and for the dividend asset, are in close agreement with the data, which is largely consistent with Campbell and Cochrane (1999, Table 2, p. 225).

One quite remarkable finding in Table 4 is how well both the LRR and HAB models do in terms of matching the unconditional moments of the volatility measure defined by the square root of the quadratic variance variable (30). Both models are nearly right on the observed values despite the fact that this variable is not used in the estimation.

Table 5 shows observed and predicted univariate AR(1) models. It includes an AR(1) model for the log quadratic variance variable, \( q_t^q \) defined in (29), which was not used in estimation but is still available for model assessment of return volatility dynamics. The shortcomings of the SRR model are readily apparent. This model under predicts the persistence in annual consumption growth, \( c_t - c_{t-12} \), and it misses very badly on volatility dynamics.

The LRR model, on the other hand, matches quite well nearly all of the univariate features of the data. Interestingly, this model captures the serial correlation properties of return volatility quite while only slightly underestimating the level of volatility. Overall, the HAB does just about as well, though there is one important exception worth noting.
The HAB model, just like the SRR model, predicts a value of 0.25 for the first order autocorrelation in consumption growth, while the observed value is about 0.45. This underprediction can be traced directly to the presumption that consumption growth process lacks the long-term component \(x_t\) of the LRR model.

Figures 3–5 show time series plots of the observed data along with the predicted values from an unrestricted VAR estimate and the restricted VAR implied by the SRR, LRR, HAB models. The unrestricted VAR is a four-variable VAR estimated on \(y_t\) in (40) augmented by the log quadratic variance variable \(q_t^a\), but set up to be block diagonal in \((y_t, q_t^a)\). The restricted VAR is the same thing except it is estimated on a long simulation (based on 50,000 months) from the model evaluated at the fitted parameters and is therefore the reprojected VAR; the one-step ahead predictions are the reprojected data series. To ease interpretation, the values for \(q_t^a\) are converted from logs to levels and annualized.

The figures are quite conclusive on the abilities of the models to track the data. The reprojected series from the SRR model are a disaster, as seen in Figure 3. The reprojected series from the LRR and HAB model, however, are seen in Figures 4 and 5 to track very closely the predictable components of the log dividend consumption ratio, consumption growth, the log price dividend ratio, and the equity return. Indeed, the LRR and HAB predict well ahead of time the drop in equity returns that occurs between 1999 and 2001. Also, the LRR and HAB models track rather well the conditional volatility of the return as is evident from the bottom panels of these figures.

It seems quite striking that two models with such quite different internal structures as the LRR and HAB can come to such a close agreement on the data, at least on a one-step-ahead basis; we thus investigate further the multi-step characteristics of these two models using selected pairwise projections of financial and macro variables analogously to Wachter (2002).

Table 6 shows projections of the end-of-year log price dividend ratio on contemporaneous and five (annual) lags of the consumption growth variable. The table suggests a rather weak link between the log price dividend ratio and the history of consumption in both the observed data and under the LRR model: the \(R^2\)’s are nearly negligible. On the other, the HAB model suggests a much tighter link than is consistent with the data. Table 7 shows linear
projections of cumulative $n$-year-ahead geometric stock returns on the log price dividend ratio, for $n = 1, 2, ..., 5$ years out. The $R^2$'s increase with the horizon for all three sets of projections. For those done on the observed data the increase is rather mild, while for those done for the LRR model there is steeper increase with horizon, while for the HAB model, there is an even steeper increase with horizon. Taken together, Tables 6 and 7 appear to contain evidence generally supportive of the LRR model over the HAB model.

Table 2 about here

Table 3 about here

Table 4 about here

Table 5 about here

Table 6 about here

Table 7 about here
8 Derivatives Pricing

We now examine the implication of the LRR and HAB models for the prices and returns on some interesting contingent claims. In general, the price $P_{gt}$ at time $t$ of a financial instrument with payoff $G_{t+k}$ at $t+k$ is given by

$$P_{gt} = \mathcal{E}_t \left[ \exp \left( \sum_{j=1}^{k} m_{t+j-1,t+j} \right) G_{t+k} \right]$$

(42)

where $m_{t,t+1}$ is the log of the one-period marginal rate of substitution given above by (16) for the the LRR model or (20) for the the HAB model. Define the pricing operator

$$V_{tk}(\cdot) = \mathcal{E}_t \left[ \exp \left( \sum_{j=1}^{k} m_{t+j-1,t+j} \right) (\cdot) \right]$$

(43)

so the valuation can be expressed simply as $P_{gt} = V_{tk}(G_{t+k})$.

8.1 Stock Price Derivatives

We start with derivatives on the stock price process $P_{dt}$. Consider a European call option with strike price $X$ and expiration $k$ steps ahead. The payoff is

$$G_{t+k} = \max \left( P_{d,t+k} - X, 0 \right) .$$
For the purpose of computation, we need to express everything in relative terms. Let
\[ v_{gt} = \frac{P_{gt}}{P_{dt}} \]
denote the current price of the call relative to the current stock price, and let \( x_t^* = X/P_{dt} \) denote the strike-to-underlying of the call. The relative payoff on the call is
\[ \frac{G_{t+k}}{P_{dt}} = \max \left( \frac{P_{d,t+k}}{P_{dt}} - x_t^*, 0 \right). \]
Its current relative price is
\[ v_{gt} = V_{tk} \left( \frac{G_{t+k}}{P_{dt}} \right). \]
The annualized \( k \)-period holding return on the call is
\[
\frac{12}{k} \left[ \frac{G_{t+k}/P_{dt}}{v_{gt}} - 1 \right].
\]
(44)

Having alternative expressions for the relative payoff proves convenient. In terms of the log stock price \( P_{dt} \) the relative payoff is
\[ \frac{G_{t+k}}{P_{dt}} = \max (\exp[p_{d,t+k} - p_{d,t}] - x_t^*, 0). \]
From the definition of the price dividend ratio, \( v_{dt} = P_{dt}/D_t \), one has that \( \log(v_{dt}) = p_{dt} - d_t \), or \( p_{dt} = \log(v_{dt}) + d_t \), so that
\[ p_{d,t} - p_{d,t-1} = \log(v_{dt}) - \log(v_{d,t-1}) + \Delta d_t. \]
Hence
\[ \frac{G_{t+k}}{P_{dt}} = \max \left( \exp \left[ \sum_{j=1}^{k} \log(v_{d,t+j}) - \log(v_{d,t-j-1}) + \Delta d_{t+j} \right] - x_t^* \right), \]
(45)
which is the expression most useful in actual computations. In what follows, we compute \( v_{gt} \) by the same solution technique used to compute \( v_{dt} \) described in Section 6 and then compute the holding period return using (44).

Evidently, the computations proceed in analogous manner for a European put option starting from
\[ G_{t+k} = \max (X - P_{d,t+k}, 0). \]
Because of the stochastic dividend, the put-call parity formula becomes rather awkward. If \( v_{gl}^{\text{call}} \) denotes the relative price of the call and \( v_{gl}^{\text{put}} \) that of the put, then the parity condition is

\[
1 + v_{gl}^{\text{put}} = v_{gl}^{\text{call}} + \sum_{j=1}^{k} V_{tk}(x^*_t) + \sum_{j=1}^{k} \mathcal{V}_{ij} \left[ v_{dt} \exp \left( \sum_{l=1}^{j} \Delta d_{t+l} \right) \right].
\] (46)

8.2 Consumption and Wealth Derivatives

We now consider derivatives written directly on consumption \( C_t \) or on the consumption asset \( P_{ct} \). Although data on prices of such securities are not readily available, we can still learn about the properties of the models and make policy proscriptions by studying the implied prices of such derivatives.

Consider first a call option written on \( C_{t+k} \), so the payoff is

\[
G_{t+k} = \max (C_{t+k} - X, 0).
\]

The most convenient normalization is to use \( C_t \), so that

\[
\frac{G_{t+k}}{C_t} = \max \left[ \exp \left( \sum_{k=1}^{k} \Delta c_{t+k} \right) - x^*_t, 0 \right]
\]

where \( x^*_t = X/C_t \) here represents the consumption strike-to-underlying of the call. The relative price of this call is

\[
v_{gt} = \mathcal{V}_{tk} \left( \frac{G_{t+k}}{C_t} \right)
\]

The relative price \( v_{gt} \) represents price of the call as a fraction of current consumption. Evidently, the same can be done for a consumption put with payoff \( \max (X - C_{t+k}, 0) \).

For derivatives written on the consumption asset \( P_{ct} \), note that \( P_{ct} \) is just wealth in the economies considered above, so derivatives written on \( P_{ct} \) represent derivatives on wealth. Consider a European call option written on \( P_{c,t+k} \) with payoff

\[
G_{t+k} = \max (P_{c,t+k} - X, 0).
\]

Normalizing gives

\[
\frac{G_{t+k}}{P_{ct}} = \max \left( \frac{P_{c,t+k}}{P_{ct}} - x^*_t, 0 \right),
\]
where now \( x_t^* \) represents the wealth strike-to-underlying of the call. Following steps analogous to those for a call written on the dividend asset we have that

\[
\frac{G_{t+k}}{P_{ct}} = \max \left( \exp \left[ \sum_{j=1}^{k} \log (v_{c,t+j}) - \log (v_{c,t+j-1} + \Delta c_{t+j}) \right] - x_t^* 0 \right).
\]

The relative price of this call is

\[
v_{gt} = v_t \left( \frac{G_{t+k}}{P_{dt}} \right)
\]

and would interpreted as the fraction of wealth that must be paid for a call with payoff \( \max \left( \frac{P_{ct+k}}{P_{ct}} - x_t^*, 0 \right) \). The analysis is completely analogous for wealth puts starting from the scaled payoff \( \max \left( x_t^* - \frac{P_{ct+k}}{P_{ct}}, 0 \right) \).

### 8.3 Consumption Insurance

Since agent already owns the stream \( \{C_{t+k}\}_{k \geq 0} \), the purchase of a put option creates a protective put (underlying plus put option) which has scaled payoff

\[
\max \left( x_t^*, \frac{C_{t+k}}{C_t} \right).
\]

This strategy provides a form of consumption growth insurance, but one must keep in mind the associated reduction in the consumption due to the cost of the put itself. This reduction could occur in one lump sum in period \( t \), or it could be spread across several periods should the agent elect to use the fixed income market to finance the put.

An interesting strategy is protecting the path of consumption growth over a sequence of periods. A portfolio of puts with expirations \( k = 1, 2, \ldots, K \) has scaled payoffs

\[
\max \left( \frac{C_{t+k}}{C_t}, x_t^* \right), \ k = 1, 2, \ldots, K
\]

and thereby protects the path of consumption growth between \( t + 1 \) and \( t + K \). Of course the protection would be achieved at cost

\[
\sum_{k=1}^{K} \nu_{tk} \left[ \max \left( x_t^* - \frac{C_{t+k}}{C_t}, 0 \right) \right]
\]

in period \( t \) consumption, which could be financed via the fixed income market.

We note briefly that the agent can also achieve consumption hedging by trading forward (futures) contracts as well. The forward price of consumption at \( t + k \) relative to current
consumption is
\[ f_{tk} = V_{tk} \left( \frac{C_{t+k}}{C_t} \right) \left[ V_{tk}(1) \right]^{-1}. \]
If the agent sells a fraction \( \alpha \) of consumption forward then realized consumption is \( C_t + \alpha f_{tk} C_t \) at time \( t \) and \( (1-\alpha)C_{t+k} \) at time \( t+k \). While such trading strategies are somewhat interesting, the nonlinear payoff functions of put and call options provide more revealing patterns of risk shifting and risk analysis.

8.4 Wealth Insurance

Since lifetime utility is very closely tied to wealth in the models above, it is natural to think about wealth insurance as well. The agent owns the uncertain wealth stream \( \{P_{c,t+k}\}_{k=1}^{\infty} \).

If the agent does nothing, wealth in period \( t+k \) relative to period \( t \) will be \( \frac{P_{c,t+k}}{P_{ct}} \). The purchase of a put option alters realized relative wealth to
\[ \max \left( x_t^*, \frac{P_{c,t+k}}{P_{ct}} \right). \]
The cost is
\[ V_{tk} \left[ \max \left( x_t^* - \frac{P_{c,t+k}}{P_{ct}}, 0 \right) \right]. \]
A portfolio of put options that ensures that \( \frac{P_{c,t+k}}{P_{ct}} \geq x_t^*, k = 1, 2, \ldots, K \) costs
\[ \sum_{k=1}^{K} V_{tk} \left[ \max \left( x_t^* - \frac{P_{c,t+k}}{P_{ct}}, 0 \right) \right]. \]
Such a portfolio might be prohibitively expensive and represent a very inefficient way to hedge wealth, however. For example, a single put option with expiration, say \( t+12 \), provides some hedge for wealth at \( t+1 \); if the delta of the put is -0.50 then for each $1 fall in wealth the put will appreciate by $0.50.

8.5 Computed Derivatives Prices

Table 8 shows for both the LRR and HAB models the unconditional means of the implied relative call and put prices written on the dividend asset \( P_{dt} \). The expiration dates range from one through twelve months ahead and the strike-to-underlying ratios are 0.99, 1.00, and 1.01. The derivative prices are in percent relative to the level of the stock price at the
time the option is written. For example, under the LRR model, a one month call option with strike-to-underlying ratio of 1.00 costs 1.65 percent of the price of a share of the stock, with a similar interpretation for the other reported average prices. The average put prices might seem high relative to the average call prices but one must keep in mind the dividend, which tends to increase the value of puts relative to calls. (The price of a put option on an asset that pays dividends is what it would have been if the asset did not pay dividends plus the discounted value of the dividend stream.) Overall, the average prices shown in Table 8 exhibit the usual properties of call and put options. There does not seem to be much difference between the prices computed under the LRR and HAB models, indicating that the models are generally in agreement on this dimension.

Table 8 shows for the models the average call and put prices written directly on the flow of consumption itself. The prices are expressed as a percent of current consumption. Thus, for example, under the LRR model, the agent would pay 0.36 percent of current consumption for a six month put option on consumption with strike-to-underlying ratio of 0.99. A purchase of \( C_t \) units of this put option would generate an agent a payoff six months later of

\[
\max \left( 0.99 - \frac{C_{t+6}}{C_t}, 0 \right) C_t,
\]

where \( \frac{C_{t+6}}{C_t} \) must be interpreted as the economy-wide growth in consumption between \( t \) and \( t+6 \). The payoff above represents the gross payoff before accounting for the cost of the option itself. The actual consumption of the agent at time \( t+6 \) is \( \max (0.99 \times C_t, C_{t+6}) \) because, as remarked earlier, the agent already owns the consumption stream. The table suggests that the LRR and HAB models generally predict the same relative prices for the shorter term puts and calls. The LRR model predicts somewhat higher prices than HAB for the more distant calls. The HAB model predicts essentially the same put prices irrespective of time to maturity.

The most interesting set of results is in Table 10, which shows the average prices for puts and calls written on the consumption asset \( P_{C_t} \). Recall from Subsection 2.2 that this asset pays the consumption stream \( \{C_{t+j}\}_{j=1}^{\infty} \). The asset represents aggregate wealth in endowment economies and its value is very tightly connected to expected lifetime utility in the model economies considered here. Also, it is exactly the kind of “perpetual claim” on a
Table 10 shows the relative prices of the puts and calls as a percentage of total wealth $P_c$. Thus, the \textbf{LRR} model implies that, on average, the cost of a twelve month at-the-money put option on wealth is 1.26 percent of current wealth. By way of contrast, the \textbf{HAB} models implies the same put option would cost 7.93 percent of current wealth. As is clear from the table, the \textbf{HAB} model implies a substantially higher costs across the board for the these derivatives. A major factor accounting for the differences appears to be the different volatilities of the return on the consumption asset under the two models. From Table 4 above, the \textbf{HAB} model implies that the annual volatility on the return on the consumption asset is 18.12 percent per year, which is about the same as that on the dividend asset, 17.72 percent per year. In contrast, the \textbf{LRR} model implies the much lower value of 3.95 percent per year for the volatility of the consumption asset. With the volatility being a factor of four smaller under the \textbf{LRR} model, it becomes clear why the options are so cheaper as seen on Table 10.

\section{Conclusion}

A simulated method of moments method proposed by Smith (1993) was used to estimate consumption based rational expectations asset pricing models using aggregate data on con-
sumption, dividends, and stock prices from 1929 to 2001, taking cognizance of the cointegrating relationships among these variables both with respect to the data and with respect to the dynamics of the models themselves.

The first model, patterned after Bansal and Yaron (2004), termed long run risks (LRR), has Epstein-Zin-Weil preferences and barely discernable, slowly time-varying growth dynamics for the consumption and dividend driving processes. A special case of this model, termed short run risks (SRR), is patterned after Hall (1978), and has the same preferences but constant growth dynamics for the consumption and dividend driving processes. The second, patterned after Campbell and Cochrane (1999), termed habit (HAB), has power utility preferences with external habit formation and constant growth dynamics for the consumption and dividend driving processes. For these models, a practicable numerical method for pricing consumption flows, dividend flows, and derivatives on consumption, on the stock market, and on wealth was developed in the paper.

SRR was overwhelmingly rejected by the test of overidentifying restrictions that corresponds to Smith’s method and did a poor job of matching conditional and unconditional moments of the data. It was dismissed from further consideration.

On the other hand, both LRR and HAB did well on the tests of overidentification and did an excellent job of matching conditional moments, unconditional moments, and first order dynamics in general. The models account for the equity premium, the long swings in stock market valuations relative to underlying cash flows, the high level of stock market volatility, and the persistence of long term swings in stock market volatility. The models were further validated when a close match of predicted to realized conditional and unconditional moments was discovered for a quadratic variation series, newly developed in this paper, that had been held out from the data used for estimation. Also of interest is the fact that both models predicted the stock market downturn around 2000 whereas VAR forecasts did not.

According to conventional determinants of the adequacy of macro models, both LRR and HAB are great successes. One concludes that is no need to abandon rationality. Behavioral macro models are unnecessary.

A choice between LRR and HAB must be made on the basis of a more extensive scrutiny of their structural characteristics. Regressions designed to reveal differences in multi-step-
ahead dynamics were undertaken. These are regressions of the log price-dividend ratio on current and lagged consumption growth and regressions of future stock returns on the price-dividend ratio. Regressions for **LRR** simulations agreed more closely with regressions for the data than did regressions for **HAB** simulations. In addition, the prices of put and call options on the stock market, consumption, and wealth were computed under both models for a variety of strike prices and expiration dates. These are derivatives of the sort suggested by Shiller (1998), for hedging macro risks, which can only be studied by means of general equilibrium models in the absence of actual markets. The most notable differences occurred in the options on wealth where the **LRR** prices were far more plausible than **HAB** prices.

On the basis of the more extensive scrutiny, one concludes that the long run risks model is preferred to the habit model.
10 References


Tables and Figures
Table 1. Reduced Rank Nonlinear Regression

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
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<td>$a_{p1}$</td>
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$\chi^2(5) = 41.051 \ (9e-7)$ \hspace{1cm} $\chi^2(3) = 10.501 \ (0.0148)$

* Notes: (1) $\gamma$ is a derived parameter computed from $\theta$ and $\psi$. 
(2) "c" indicates a calibrated parameter.
Table 3. Parameter Estimates for the Habit Persistence (HAB) Model

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$\chi^2(5) = 7.109$ (0.213)
Table 4. Comparison of Model Predictions with Observed Unconditional Moments

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<td></td>
<td>Mean</td>
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<td>Mean</td>
<td>Std Dev</td>
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<td>-3.414 0.163</td>
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<td>Return (% Per Year), dividend asset</td>
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<td>1.07 3.23</td>
</tr>
<tr>
<td>Return (% Per Year), consumption asset</td>
<td>$100r_{ct}^a$</td>
<td>5.60 2.33</td>
<td>2.34 3.95</td>
<td>6.60 16.99</td>
</tr>
<tr>
<td>Equity premium (% Per Year)</td>
<td>$100r_{dt}^a - r_{ft}^a$</td>
<td>5.76 2.82</td>
<td>5.51 16.09</td>
<td>5.46 17.14</td>
</tr>
</tbody>
</table>

Notes. Observed values are sample statistics computed from annual data, 1930–2001; predicted values are computed from a long simulation from the indicated model.
Table 5: AR(1) Models for Each Series

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observed</th>
<th>Predicted (SRR)</th>
<th>Predicted (LRR)</th>
<th>Predicted (HAB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_t^a - c_t^a$</td>
<td>$-0.6879$</td>
<td>$0.3542$</td>
<td>$-0.30281$</td>
<td>$-0.6784$</td>
</tr>
<tr>
<td></td>
<td>$0.7992$</td>
<td>$0.1034$</td>
<td>$0.91128$</td>
<td>$0.7985$</td>
</tr>
<tr>
<td>$c_t^a - c_{t-12}^a$</td>
<td>$0.0116$</td>
<td>$0.0031$</td>
<td>$0.01894$</td>
<td>$0.0185$</td>
</tr>
<tr>
<td></td>
<td>$0.4495$</td>
<td>$0.0909$</td>
<td>$0.2666$</td>
<td>$0.2645$</td>
</tr>
<tr>
<td>$p_{dt}^a - d_t^a$</td>
<td>$0.4926$</td>
<td>$0.3678$</td>
<td>$0.3467$</td>
<td>$0.54924$</td>
</tr>
<tr>
<td></td>
<td>$0.8542$</td>
<td>$0.1077$</td>
<td>$0.8966$</td>
<td>$0.8326$</td>
</tr>
<tr>
<td>$r_{dt}^a$</td>
<td>$0.0675$</td>
<td>$0.0639$</td>
<td>$0.0582$</td>
<td>$0.0689$</td>
</tr>
<tr>
<td></td>
<td>$0.0088$</td>
<td>$0.1454$</td>
<td>$0.0546$</td>
<td>$-0.0536$</td>
</tr>
<tr>
<td>$q_t^a$</td>
<td>$-0.8972$</td>
<td>$0.2545$</td>
<td>$-0.6796$</td>
<td>$-1.2400$</td>
</tr>
<tr>
<td></td>
<td>$0.5352$</td>
<td>$0.1275$</td>
<td>$0.0177$</td>
<td>$0.7114$</td>
</tr>
<tr>
<td></td>
<td>$0.2545$</td>
<td>$0.1275$</td>
<td>$-1.73212$</td>
<td>$-1.400$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. Each line of the table shows the coefficients for a regression of the form $z_t = \alpha_0 + \alpha_1 z_{t-1} + \epsilon_t$ for the variable named in the first column. The regressions in the columns labeled observed are for the annual data where the span of the dependent variable is 1931–2001. The regressions in the columns labeled predicted are for a simulation of the model.
Table 6. Predictability Projections: Price Dividend Ratios

<table>
<thead>
<tr>
<th>Observed Coef</th>
<th>Std Err</th>
<th>Predicted LRR</th>
<th>Predicted HAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>3.4168</td>
<td>0.0055</td>
<td>3.2520</td>
</tr>
<tr>
<td>$\Delta c_t$</td>
<td>-0.3021</td>
<td>0.1744</td>
<td>1.0716</td>
</tr>
<tr>
<td>$\Delta c_{t-24}$</td>
<td>-1.0560</td>
<td>0.1926</td>
<td>0.0383</td>
</tr>
<tr>
<td>$\Delta c_{t-48}$</td>
<td>-0.6209</td>
<td>0.1925</td>
<td>0.2521</td>
</tr>
<tr>
<td>$\Delta c_{t-60}$</td>
<td>-0.3480</td>
<td>0.1926</td>
<td>-0.0176</td>
</tr>
<tr>
<td>$\Delta c_{t-72}$</td>
<td>-0.1263</td>
<td>0.1926</td>
<td>-0.0442</td>
</tr>
<tr>
<td>$\Delta c_{t-84}$</td>
<td>-0.2006</td>
<td>0.1744</td>
<td>-0.0857</td>
</tr>
</tbody>
</table>

$R^2$ 0.036 0.011 0.422

Notes: Shown above are the linear projections of the log price dividend ratio, $v_d$ on contemporaneous and five annual lags of log consumption growth $\Delta c$. The period for the observed projection is 1935-2001. The predicted values are from long simulations from the Long Run Risks LRR Model and the Habit Persistence HAB Model.

Table 7. Long Horizon Predictability Projections: Cumulative Future Return on the Price Dividend Ratio

<table>
<thead>
<tr>
<th>Horizon(Years)</th>
<th>$R^2$</th>
<th>Observed</th>
<th>LRR</th>
<th>HAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.038</td>
<td>0.088</td>
<td>0.084</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.060</td>
<td>0.129</td>
<td>0.154</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.071</td>
<td>0.172</td>
<td>0.213</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.071</td>
<td>0.202</td>
<td>0.262</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.070</td>
<td>0.229</td>
<td>0.299</td>
</tr>
</tbody>
</table>

Notes: The table shows $R^2$’s from projections of cumulative annual geometric returns for 1,2,...,5 years ahead onto the log price dividend ratio for the observed data, 1935-2001, and for long simulations from the Long Run Risks LRR Model and the Habit Persistence HAB Model.
Table 8. Average Put and Call Prices on the Stock

<table>
<thead>
<tr>
<th>Strike-to-Underlying:</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expiration (months)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.19</td>
<td>1.65</td>
<td>1.22</td>
</tr>
<tr>
<td>2</td>
<td>2.79</td>
<td>2.28</td>
<td>1.84</td>
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<tr>
<td>3</td>
<td>3.25</td>
<td>2.75</td>
<td>2.31</td>
</tr>
<tr>
<td>4</td>
<td>3.61</td>
<td>3.13</td>
<td>2.69</td>
</tr>
<tr>
<td>5</td>
<td>3.93</td>
<td>3.45</td>
<td>3.01</td>
</tr>
<tr>
<td>6</td>
<td>4.21</td>
<td>3.73</td>
<td>3.30</td>
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<td>7</td>
<td>4.46</td>
<td>3.99</td>
<td>3.56</td>
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<td>4.69</td>
<td>4.23</td>
<td>3.80</td>
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<tr>
<td>9</td>
<td>4.90</td>
<td>4.44</td>
<td>4.01</td>
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<tr>
<td>10</td>
<td>5.08</td>
<td>4.63</td>
<td>4.21</td>
</tr>
<tr>
<td>11</td>
<td>5.25</td>
<td>4.80</td>
<td>4.38</td>
</tr>
<tr>
<td>12</td>
<td>5.40</td>
<td>4.96</td>
<td>4.54</td>
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</table>

Habit (HAB)

<table>
<thead>
<tr>
<th>Strike-to-Underlying:</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expiration (months)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.06</td>
<td>1.51</td>
<td>1.12</td>
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<tr>
<td>2</td>
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<tr>
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<td>3.68</td>
<td>3.16</td>
<td>2.70</td>
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<td>3.94</td>
<td>3.42</td>
<td>2.96</td>
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<td>7</td>
<td>4.18</td>
<td>3.66</td>
<td>3.20</td>
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<td>3.88</td>
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<td>4.08</td>
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<td>4.27</td>
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<tr>
<td>11</td>
<td>4.95</td>
<td>4.44</td>
<td>3.97</td>
</tr>
<tr>
<td>12</td>
<td>5.10</td>
<td>4.60</td>
<td>4.13</td>
</tr>
</tbody>
</table>

Note. Prices are percent of the stock price at the time the option is written.
<table>
<thead>
<tr>
<th>Strike-to-Underlying:</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>0.38</td>
<td>0.31</td>
<td>0.24</td>
</tr>
<tr>
<td>Put</td>
<td>0.12</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td>Expiration (months)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.59</td>
<td>0.51</td>
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</tr>
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<td>0.79</td>
<td>0.71</td>
<td>0.63</td>
</tr>
<tr>
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<td>0.98</td>
<td>0.90</td>
<td>0.82</td>
</tr>
<tr>
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<td>1.16</td>
<td>1.08</td>
<td>1.00</td>
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<td>1.34</td>
<td>1.26</td>
<td>1.18</td>
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</tr>
<tr>
<td>8</td>
<td>1.69</td>
<td>1.60</td>
<td>1.52</td>
</tr>
<tr>
<td>9</td>
<td>1.86</td>
<td>1.77</td>
<td>1.69</td>
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<tr>
<td>10</td>
<td>2.03</td>
<td>1.94</td>
<td>1.86</td>
</tr>
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<td>2.19</td>
<td>2.11</td>
<td>2.03</td>
</tr>
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<td>2.27</td>
<td>2.19</td>
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</table>

<table>
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<th>1.00</th>
<th>1.01</th>
</tr>
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<tbody>
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<td>0.02</td>
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<td>Put</td>
<td>0.01</td>
<td>0.18</td>
<td>0.87</td>
</tr>
<tr>
<td>Expiration (months)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.33</td>
<td>0.53</td>
<td>0.11</td>
</tr>
<tr>
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<td>1.50</td>
<td>0.71</td>
<td>0.22</td>
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<td>0.87</td>
<td>0.34</td>
</tr>
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<td>1.83</td>
<td>1.03</td>
<td>0.47</td>
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</tr>
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<td>0.73</td>
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<td>0.86</td>
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<td>1.00</td>
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</tr>
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<td>2.77</td>
<td>1.96</td>
<td>1.27</td>
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<tr>
<td>12</td>
<td>2.92</td>
<td>2.10</td>
<td>1.41</td>
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</tbody>
</table>

Note. Prices are percent of consumption at the time the option is written.
Table 10. Average Put and Call Prices on Wealth

<table>
<thead>
<tr>
<th>Strike-to-Underlying:</th>
<th>Call</th>
<th>Put</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>1.00</td>
<td>1.01</td>
<td>0.99</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expiration (months)</th>
<th>Strike-to-Underlying:</th>
<th>Call</th>
<th>Put</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.18 0.43 0.10</td>
<td>0.11</td>
<td>0.36</td>
<td>1.03</td>
</tr>
<tr>
<td>2</td>
<td>1.35 0.63 0.22</td>
<td>0.22</td>
<td>0.50</td>
<td>1.09</td>
</tr>
<tr>
<td>3</td>
<td>1.51 0.80 0.35</td>
<td>0.32</td>
<td>0.61</td>
<td>1.15</td>
</tr>
<tr>
<td>4</td>
<td>1.65 0.95 0.47</td>
<td>0.40</td>
<td>0.70</td>
<td>1.21</td>
</tr>
<tr>
<td>5</td>
<td>1.79 1.09 0.59</td>
<td>0.48</td>
<td>0.78</td>
<td>1.27</td>
</tr>
<tr>
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<td>1.92 1.23 0.71</td>
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<td>0.85</td>
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<td>0.92</td>
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<td>8</td>
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<td>0.98</td>
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<tr>
<td>9</td>
<td>2.30 1.61 1.04</td>
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<td>1.48</td>
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<td>0.81</td>
<td>1.12</td>
<td>1.54</td>
</tr>
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<td>0.88</td>
<td>1.18</td>
<td>1.60</td>
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<tr>
<td>12</td>
<td>2.65 1.95 1.37</td>
<td>0.96</td>
<td>1.26</td>
<td>1.66</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Strike-to-Underlying:</th>
<th>Call</th>
<th>Put</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>1.00</td>
<td>1.01</td>
<td>0.99</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expiration (months)</th>
<th>Strike-to-Underlying:</th>
<th>Call</th>
<th>Put</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.07 1.52 1.13</td>
<td>1.33</td>
<td>1.77</td>
<td>2.38</td>
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<tr>
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<td>2.64 2.10 1.68</td>
<td>2.15</td>
<td>2.61</td>
<td>3.19</td>
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<tr>
<td>3</td>
<td>3.06 2.53 2.09</td>
<td>2.84</td>
<td>3.31</td>
<td>3.86</td>
</tr>
<tr>
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<td>3.40 2.88 2.43</td>
<td>3.43</td>
<td>3.91</td>
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<td>3.69 3.17 2.72</td>
<td>3.99</td>
<td>4.47</td>
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<td>5.00</td>
<td>5.54</td>
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<td>4.79 4.28 3.81</td>
<td>6.54</td>
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<td>7.55</td>
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<tr>
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<td>7.01</td>
<td>7.49</td>
<td>8.01</td>
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<tr>
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<td>5.12 4.61 4.14</td>
<td>7.48</td>
<td>7.97</td>
<td>8.49</td>
</tr>
</tbody>
</table>

Note. Prices are percent of wealth at the time the option is written.
Figure 1. Raw Data. In the notation of the text, the variables shown are $p_{dt} = \log(P_{dt})$, $d_{it} = \log(D_{it})$, $c_{it} = \log(C_{it})$, $r_{dt}$, and $q_{it} = \log(Q_{it})$, in order.
Figure 2. Nominal Arithmetic and Real Geometric Returns. The figure compares the more familiar nominal arithmetic returns series with the real per-capita geometric returns series used to fit the model.
Figure 3. One Step Ahead Forecasts of the Data Confronted by the SRR Model

The variables shown are $d_t^a - c_t^a$, $c_t^a - c_{t-12}^a$, $p_{dt} - d_t^a$, $r_{dt}$, and $q_t^a$ of the text, in order. The dotted line is the data. The dashed line is a one-step-ahead forecast of a VAR fitted to the data. The solid line is the one-step-ahead forecast of a VAR computed from a simulation from the SRR Model.
Figure 4. One Step Ahead Forecasts of the Data Confronted by the LRR Model

The variables shown are $d_t^a - c_t^a$, $c_t^a - c_t^{a-12}$, $p_{dt} - d_t^a$, $c_{dt}^a$, and $q_t^a$ of the text, in order. The dotted line is the data. The dashed line is a one-step-ahead forecast of a VAR fitted to the data. The solid line is the one-step-ahead forecast of a VAR computed from a simulation from the LRR Model.
Figure 5. One Step Ahead Forecasts of the Data Confronted by the Habit Persistence (HAB) Model. The variables shown are $d_t^a - c_t^a$, $c_t^a - c_{t-12}^a$, $p_t^a - d_t^a$, $r_t^a$, and $q_t^a$ of the text, in order. The dotted line is the data. The dashed line is a one-step-ahead forecast of a VAR fitted to the data. The solid line is the one-step-ahead forecast of a VAR computed from a simulation from the HAB model.