Cointegration and Long-Run Asset Allocation

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Abstract

We show that economic restrictions of cointegration between asset cash flows and aggregate consumption have important implications for return dynamics and optimal portfolio rules, particularly for long investment horizons. When cash flows and consumption share a common stochastic trend (i.e., are cointegrated), temporary deviations between their levels forecast long-horizon dividend growth rates and returns and, consequently, alter the profile, by investment horizon, of variances and covariances of asset returns. We show that the optimal asset allocation based on the cointegration-based EC-VAR specification can be quite different relative to a traditional VAR that ignores the error-correction dynamics. Unlike the EC-VAR, the commonly used VAR approach to model expected returns focuses on the short-run forecasts and can considerably miss on long-horizon return dynamics and, hence, the optimal portfolio mix in the presence of cointegration. We develop and implement methods to account for parameter uncertainty in the EC-VAR setup and highlight the importance of the error-correction channel for optimal portfolio decisions at various investment horizons.

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1 Introduction

Risks facing a short-run and long-run investor can be quite different. While at very short horizons, the contribution of cash flow news to the variance of the return may be small, as the investment horizon increases, cash flow fluctuations become the dominant source of return variability. Hence, understanding and modeling the behavior of asset returns, especially at long horizons, critically depends on understanding and modeling the dynamics of their cash flows. In this paper we argue that deviations between cash flow levels and aggregate consumption (the error-correction term) contain important information about means and variances of future cash-flow growth rates and, consequently, returns. Incorporating this cointegration restriction in return dynamics yields interesting implications for the term-structure of expected returns and risks and, hence, asset allocations at various investment horizons. In particular, we show that the error-correction mechanism significantly alters the risk-return tradeoff and so does the shape of optimal portfolio rules implied by models where the long-run adjustment of cash flows is ignored.

Our motivation for including the error-correction mechanism is based on the ideas of long-run risks developed in Bansal and Yaron (2004), Hansen, Heaton, and Li (2005), Bansal, Dittmar, and Lundblad (2005), and Bansal, Dittmar, and Kiku (2007). These papers, both theoretically and empirically, highlight the importance of the long-run relation between cash flows and aggregate consumption for understanding the magnitude of the risk premium and its cross-sectional variation. Built on this evidence, our paper aims to explore the effect of long-run properties of asset cash flows on the optimal portfolio mix at various investment horizons. Intuitively, if the long-run dynamics of asset dividends is described by a cointegrating relation with aggregate consumption, then current deviations between their levels should forecast future dividend growth rates (see Granger and Engle (1987)). Further, as risks in long-horizon returns are dominated by cash flow news, predictability of asset dividends emanating from the error-correction mechanism may significantly alter the future dynamics of multi-horizon returns and their volatilities. This suggests that the error-correction channel may be quite important for determining the optimal asset allocation at intermediate and long investment horizons. Earlier portfolio choice literature, including Kandel and Stambaugh (1996), Barberis (2000), Chan, Campbell, and Viceira (2003), and Jurek and Viceira (2005), model asset returns via a standard vector-autoregression and, hence, ignore the consequences of the long-run dividend dynamics for the risk-return tradeoff and allocation decisions.
We measure the long-run relation between asset dividends and aggregate consumption via a stochastic cointegration. Based on the implications of the cointegrating relation, we model dividend growth rates, price-dividend ratios and returns using an error-correction specification of a vector autoregression (EC-VAR) model. Our time-series specification allows us to compute the term profile of conditional and unconditional means and the variance-covariance structure of asset returns, which we consequently use to derive the optimal conditional and unconditional portfolio rules. To highlight the importance of the error-correction dynamics in dividends, we compare the resulting allocations with those implied by a standard VAR model, which excludes the error-correction variable from investors’ information set.

We solve the portfolio choice problem for buy-and-hold mean-variance investors with different investment horizons, ranging from one to 15 years, and different levels of risk aversion. To emphasize the implications of long-run cash flow dynamics for the risk-return tradeoff and optimal portfolio mix, we focus on equities that are known to display large dispersion in average returns and opposite long-run (cointegrating) characteristics. In particular, we consider investors who allocate their wealth across value and growth (i.e., high and low book-to-market) stocks, and the one-year Treasury bond. We distinguish between conditional and unconditional portfolio choice problems and highlight the differences between the two. The importance of conditioning information in the context of asset allocation has also been analyzed in Ferson and Siegel (2001), and Brandt and Santa-Clara (2006) among others.\footnote{See, in addition, Barberis (2000), Lynch and Balduzzi (2000), Brandt and Ait-Sahalia (2001), Ang and Bekaert (2002), and Bansal, Dahlquist, and Harvey (2004).}

To keep the analysis simple and transparent, we abstract from any types of dynamic rebalancing and focus on the first-order effect of the error-correction mechanism captured by the solution to the mean-variance problem at various horizons. As shown in Jurek and Viceira (2005), regardless of investors’ risk aversion, the intertemporal hedging demand contributes a very small portion (less than 5%) to the variation of the overall portfolio weights across time. Thus, the volatility of the optimal portfolio is largely dominated by its myopic component which our approach captures. Considering reasonable alternative preference specifications, while straightforward, is unlikely to materially alter our evidence.

We establish several interesting results. Consistent with Bansal, Dittmar, and Lundblad (2001), Hansen, Heaton, and Li (2005), and Bansal, Dittmar, and Kiku (2007), we find that value and growth stocks significantly differ in their exposures to long-run consumption risks.
While cash flows of value firms respond positively to low-frequency consumption fluctuations, growth firms display a negative response in the long run. Importantly, we find that current deviations in the dividend-consumption pair (the cointegrating residual) contain distinct information about future dynamics of both cash flows and multi-horizon returns, which is missing in the VAR setup. In particular, if the error-correction dynamics is ignored and returns are modelled via the standard VAR, one is able to account for about 11% and 52% of the variation in growth and value returns at the 10-year horizon. With the cointegration-based specification, the long-run predictability of value and growth returns rises to striking 42% and 65% respectively.

The forecasting ability of the error-correction term significantly alters variances (and correlations) of asset returns relative to the growth-rates based VAR, especially at intermediate and long horizons. As expected, the EC-VAR model generates a declining pattern in the term structure of unconditional volatilities of both value and growth stocks. The standard deviation of value returns in the VAR specification, on the other hand, is slightly increasing with the horizon. Hence, the EC-VAR model potentially is able to capture much larger benefits of time-diversification relative to the traditional VAR approach. Indeed, if the error-correction channel is ignored, the unconditional allocation to value stocks steadily declines: the VAR investors reduce their holdings of value stocks from 66% to about 52% as the investment horizon changes from one to ten years. This pattern is consistent with the VAR-based evidence of Jurek and Viceira (2005). In contrast, relying on the cointegration-based specification, investors tend to allocate a larger fraction of their wealth to value stocks as the investment horizon lengthens. In particular, the holding of value firms increases from 76% at the one-year horizon to about 96% for the ten-year investment. Thus, optimal portfolio prescriptions based on the standard VAR and EC-VAR models can be very different – these differences are a reflection of the error-correction mechanism between asset cash flows and aggregate consumption, and the ensuing time-diversification effect. Given a strong economic appeal of cointegration in dividend-consumption relation, our evidence suggests that investors should rely on the optimal portfolio mix based on the EC-VAR model.

It is well recognized in the literature that asset allocation decisions may be quite sensitive to parameter uncertainty. To ensure that our results are robust to any estimation errors, we supplement our evidence by deriving optimal allocations of a bayesian-type investor who recognizes and accounts for uncertainty about model parameters. The impact of parameter uncertainty in a standard VAR framework has been earlier analyzed in Kandel and Stambaugh (1996) and Barberis (2000). We extend their approach and develop a method that allows us to handle parameter uncertainty in the cointegration setup. We find
the bayesian-based evidence to be qualitatively similar to the no-uncertainty case. To be specific, even after accounting for uncertainty in model parameters, the EC-VAR and VAR specifications deliver quite different portfolio rules, particularly in the intermediate and long runs. As the horizon increases, the allocation to value continues to rise within the EC-VAR specification (from 47% to 64% at the horizon extends from one to ten years) and keeps on falling in the growth-rates VAR framework (from 43% to 21% respectively). Further, similar to Barberis (2000), we find that investors that doubt reliability of the estimated model parameters tend to shift their wealth away from equities towards safer securities. Depending on the horizon, the allocation to the Treasury bond increases by 20-40% compared to the no-uncertainty case. Taken together, our evidence suggests that parameter uncertainty affects the scale but not the shape of optimal asset allocations.

The rest of the paper is structured as follows. Section 2 outlines the portfolio choice problem, highlights the implications of cointegration and describes the dynamic model for asset returns. Our empirical results and their discussion are presented in Section 3. Finally, Section 4 concludes.

2 Asset Allocation Framework

2.1 Portfolio Choice Problem

We consider investors with CRRA preferences who follow a buy-and-hold strategy over different holding horizons. At time $t$, an investor chooses an allocation that maximizes her expected end-of-period utility and is locked into the chosen portfolio till the end of her investment horizon. Specifically, the $s$-period investor solves,

$$\max_{\alpha_{s,t}} E_t[U_{t+s}] = \max_{\alpha_{s,t}} E_t \left[ \frac{W_{t+s}^{1-\gamma}}{1-\gamma} \right],$$

where $\alpha_{s,t}$ is the vector of portfolio weights, $W_{t+s}$ is the terminal wealth, and $\gamma$ is the coefficient of risk aversion. Letting $R_{t+1-t+s}^p$ denote the (gross) return on the portfolio held by the investor,

$$R_{t+1-t+s}^p = \alpha'_{s,t}R_{t+1-t+s},$$

where $R_{t+1-t+s}$ is the vector of compounded asset returns, the evolution of wealth is described
by,

\[ W_{t+s} = W_t \ast R^p_{t+1-t+s}. \]  

(3)

We distinguish between the conditional and unconditional stock allocation problems. The conditional problem is stated above and uses the conditional distribution of future returns. The unconditional asset allocation relies on the unconditional distribution of asset returns to maximize expected utility.

To make the problem tractable, we will assume throughout that gross asset returns are log normally distributed. As shown in Campbell and Viceira (2002), the investor’s objective function in this case can be written as,

\[
\max_{\alpha, t} \left\{ \left[ E_t[r^p_{t+1-t+s}] + \frac{1}{2} Var_t(r^p_{t+1-t+s}) \right] - \frac{\gamma}{2} Var_t(r^p_{t+1-t+s}) \right\},
\]

(4)

where \( r^p_{t+1-t+s} \) is the log return on a portfolio bought at time \( t \) and held up to \( t + s \). The unconditional problem can be restated analogously by dropping time subscripts in the expression above. We will refer to \( E_t[r^p_{t+1-t+s}] \) as the expected log return, and \( E_t[r^p_{t+1-t+s}] + \frac{1}{2} Var_t(r^p_{t+1-t+s}) \) as the arithmetic mean return. In the empirical section, the reported mean returns correspond to arithmetic means. To enhance the comparison across different holding periods, we measure and express all asset return moments per unit time, that is, we scale both means and variances by horizon.

There are three assets available to investors: in addition to the 1-year Treasury bond, they allocate their wealth between growth and value stocks. We focus on stocks with opposite book-to-market characteristics that, historically, are known to display large dispersion in average returns. In this respect, our asset menu is similar to that in Jurek and Viceira (2005). The data employed in our empirical work are sampled on an annual frequency, thus, a single investment period corresponds to one year. We consider investors with different degrees of risk aversion. We set the RA parameter at 5 in our benchmark case. To highlight the sensitivity of the resulting weights to investors’ preferences we, in addition, entertain a risk aversion level of 10.
2.2 Modelling Asset Returns

2.2.1 Cointegration Specification

We describe the long-run dynamics of dividends and consumption via a cointegrating relation,\(^2\)

\[
d_t = \tau_0 + \tau_1 t + \delta c_t + \epsilon_{d,t},
\]

where \(d_t\) is the level of an asset’s dividends, \(c_t\) is the level of aggregate consumption, and \(\epsilon_{d,t} \sim I(0)\) is the cointegrating residual or the error-correction term. It follows from (5) that dividend growth evolves as \(\Delta d_t \equiv \tau_1 + \delta \Delta c_t + \Delta \epsilon_{d,t}\). Hence a time-series model for \(\epsilon_{d,t}\) and \(\Delta c_t\) is sufficient to model the dynamics of cash flow growth rates.

Our specification implies that at low frequencies, dividends and consumption share a common stochastic trend. The two, however, may exhibit different exposures to the underlying long-run risks as we do not impose a unit restriction on the cointegration parameter, \(\delta\). In addition, by including the time-trend in (5), we allow for differences in deterministic trends in asset dividends and aggregate consumption. As we argue below, imposing restrictions on either \(\tau\) or \(\delta\) may not be appropriate for dividend series we rely on.

Following the existing asset pricing literature, we focus on dividends constructed on the per-share basis. Specifically, the level of dividends at time \(t + 1\) is computed as,

\[
D_{t+1} = Y_{t+1} P_t,
\]

where \(Y\) and \(P\) are the dividend yield and the price index respectively; the latter evolves according to \(P_{t+1} = H_{t+1} P_t\), \(P_0\) is normalized to 1, and \(H\) denotes the price gain. These dividends correspond to a trading strategy of holding one share of a firm’s stock at each point in time. Thus, an investor following the one-share strategy will consume all the dividends and reinvests only capital gains. Consider, alternatively, an investor who plow a portion of the received cash back into the firm to sustain its further growth. If the amount of reinvested income matches the net share issuance, such an investor will hold a claim to the total equity capital of the firm. Consequently, payout series associated with this alternative investment, which we refer to as aggregate dividends, are proportional to the firm’s market capitalization \((K)\),

\[
D_{t+1}^{agg} = Y_{t+1} K_t.
\]

\(^2\)Similar specifications are considered in Bansal, Dittmar, and Lundblad (2001), Hansen, Heaton, and Li (2005), and Bansal, Dittmar, and Kiku (2007).
Notice the difference between the two measures – while the per-share series account for the growth of the share price, aggregate dividends reflect the appreciation of the firm’s equity capital.\(^3\)

It may be theoretically appealing to omit the time trend and restrict the cointegration parameter of aggregate dividends on the market (or a particular sector of the economy) to one, as it would yield balanced growth paths of aggregate payouts and aggregate consumption. There is, however, no economic rationale for such restrictions for dividends per share. In order to illustrate and reinforce this important point, Figure 1 plots the log of dividends-to-consumption ratio for two alternative dividend measures over 1954-2003 period. As Panel (a) shows, the ratio of aggregate dividends to consumption is quite stationary as the average growth rate of aggregate dividends (of 2.9%) is comparable to that of aggregate consumption. Consequently, restricting aggregate series to be cointegrated with consumption with a parameter of one and omitting a time trend appears to be quite reasonable. In contrast, the ratio of market per-share dividends to consumption displays a dramatic decline over time, as shown in Figure 1(b). The average growth rate in dividends per share is only 0.9%, which is considerably lower than the mean growth of consumption. Thus, the cointegrating relation between per-share dividends and aggregate consumption cannot be established under \(\{\delta = 1, \tau = 0\}\) restrictions. The reason per-share dividends fail to catch up with the level of aggregate consumption is due to the fact that per-share series, by construction, do not account for capital inflow in equity markets. As mentioned above, the growth in dividends per share reflects the growth in the share price, whereas the growth in aggregate dividend series reflects the growth in market capital. The average growth of aggregate market capitalization over the post-war period is about 3.5%, while the growth in the share price is only 1.7%.

From the econometric perspective, the distinction between aggregate and per-share dividends has important implications for modelling the dynamics of asset returns. As Figure 1(b) shows, per-share dividends and aggregate consumption tend to drift apart over time. Thus, omitting the time trend and imposing unit cointegration restriction \((\delta = 1 \text{ and } \tau = 0)\), in the data, leads to explosive/non-stationary return dynamics.\(^4\) This issue is also discussed in Bansal, Dittmar, and Kiku (2007).

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\(^3\)For further discussion of the difference between the two trading strategies and implied dividend series see Bansal and Yaron (2006).

\(^4\)The augmented Dickey-Fuller test confirms the existence of a unit root in the ratio of per-share dividends to aggregate consumption for the market portfolio as well as for value and growth stocks that we consider in our empirical work.
The asset pricing literature typically focuses on the per-share dividends as their present value corresponds to the price of the asset, which is not true for aggregate dividend series. We follow this tradition and use series constructed on the per-share basis. Given the above discussion, we do not impose any restrictions on parameters that govern their long-run dynamics, letting the data decide on the underlying cointegrating relation between per-share dividends and aggregate consumption.

2.2.2 Return Dynamics

To describe the distribution of asset returns at various investment horizons, we model the dynamics of single-period returns and state variables jointly via the following error-correction VAR,

\[
\begin{pmatrix}
  b_{t+1} \\
  \Delta c_{t+1} \\
  \epsilon_{d,t+1} \\
  z_{t+1} \\
  r_{t+1}
\end{pmatrix} = \begin{pmatrix}
  a_b & 0 & 0 & 0 & 0 \\
  a_c & a_e & 0 & 0 & 0 \\
  a_e & 0 & \cdots & \cdots & 0 \\
  a_z & 0 & \cdots & \cdots & 0 \\
  a_r & 0 & \cdots & \cdots & 0
\end{pmatrix} \begin{pmatrix}
  b_t \\
  \Delta c_t \\
  \epsilon_{d,t} \\
  z_t \\
  r_t
\end{pmatrix} + \begin{pmatrix}
  u_{b,t+1} \\
  u_{c,t+1} \\
  u_{\epsilon,t+1} \\
  u_{z,t+1} \\
  u_{r,t+1}
\end{pmatrix}.
\] (8)

That is, we project log bond return, \(b_t\), and consumption growth, \(\Delta c_t\), on their own lags, and regress cointegrating residual, \(\epsilon_{d,t}\), log price-dividend ratio, \(z_t\), and log return, \(r_t\), on their lags (excluding lagged return) and past consumption growth. Denoting \(X_t = (b_t \ \Delta c_t \ \epsilon_{d,t} \ z_t \ r_t)\), we can rewrite the EC-VAR in a compact matrix form,

\[X_{t+1} = a + AX_t + u_{t+1},\] (9)

where \(a\) is the vector of intercepts, the matrix \(A\) is defined above, and \(u\) is a \((5 \times 1)\)-matrix of shocks that follow a normal distribution with zero mean and variance-covariance matrix \(\Sigma_u\).5

The error-correction specification is the key dimension that differentiates this paper from the existing portfolio choice literature. The latter typically models asset returns via a simple VAR incorporating information on the price-dividend ratio (see Kandel and Stambaugh (1996), Barberis (2000), Chan, Campbell, and Viceira (2003), and Jurek and Viceira (2005)).

5To keep the focus on the effect of long-run predictability of asset returns, we assume that their second moments are constant. This assumption is unlikely to significantly affect our empirical results as we consider investors with quite lengthy decision intervals. The shortest investment horizon in our empirical work equals one year.
among others). In contrast to the traditional VAR approach, we describe the dynamics of asset returns using the error-correction framework that exploits the implications of long-run relation between dividends and consumption.

The conceptual difference between a standard VAR and the EC-VAR specifications is summarized by the error-correction variable, $\epsilon_{d,t}$. From an econometric perspective, the Granger and Engle (1987) representation theorem states that the error-correction variable should forecast future dividend growth rates. To see its implications for returns consider a Taylor series approximation of log returns (as in Campbell and Shiller (1988)),

$$r_{t+1} = \kappa_0 + \Delta d_{t+1} + \kappa_1 z_{t+1} - z_t,$$

where $\kappa$’s are constants of linearization. As long-horizon returns can be computed via summing up both sides of equation (10), multi-period returns will depend on the dynamics of long-horizon dividend growth rates. Thus, long-run predictable variation in dividend growth via the cointegrating residual has the potential to alter predictability and, hence, distribution of multi-period returns. In fact, cointegration between dividends and consumption has potentially the same economic consequences for returns as the unit cointegration restriction between prices and dividends (that is, stationarity of the price-dividend ratio). While price-dividend ratios are commonly used to forecast long-horizon returns, we argue that the error-correction residual, $\epsilon_{d,t}$, may be equally (if not more) important for predicting future returns; as such, it may significantly affect volatilities and correlations of multi-period returns. Consequently, including the error-correction variable in the return dynamics may alter our views of the optimal asset allocation mix at both intermediate and long investment horizons.

To highlight the importance of the error-correction mechanism in cash flows for the risk-return tradeoff and optimal portfolio decisions, we will compare the implications of the cointegration-based EC-VAR to those implied by the traditional VAR specification. In the VAR setup, the error correction variable, $\epsilon_{d,t}$, is simply replaced by the dividend growth, $\Delta d_t$. Price-dividend ratio is included as one of the predictive variables in both cases.

### 2.3 Term Structure of Expected Returns and Risks

The solution to the portfolio choice problem (4) hinges on the distribution of multi-period returns, in particular, its first two moments. The required term-profile of expected returns and risks can be easily computed by exploiting the recursive structure of the EC-VAR.
We first outline the construction of conditional moments, turning to the measurement of unconditional means and variance-covariances afterwards.

### 2.3.1 Conditional Analysis

In the conditional problem, we rely on the above EC-VAR to measure both expected values and variance-covariance structure of asset returns. Specifically, the mean of the continuously compounded return is computed as,

$$
E_t[r_{t+1\rightarrow t+s}] = \frac{1}{s} \sum_{j=1}^{s} (G_j a + A_j X_t),
$$

where $G_j = G_{j-1} + A_{j-1}$ and $G_0 = 0$, for $j = 1, ..., s$. Further, for a given horizon $s \geq 1$, the innovation in the sum of $s$ consecutive $X$’s can be extracted as follows,

$$
\sum_{j=1}^{s} X_{t+j} - E_t[\sum_{j=1}^{s} X_{t+j}] \equiv \zeta_{t,t+s},
$$

where $\zeta_{t,t+s}$ is

$$
\zeta_{t,t+s} = \sum_{j=1}^{s} G_j u_{t+1+s-j}.
$$

Exploiting the fact that errors are identically distributed and serially uncorrelated, the covariance matrix of $\zeta_{t,t+s}$ for any given horizon $s$ is

$$
\Sigma_s^* = G_s \Sigma_u G_s' + \Sigma_{s-1}^*,
$$

where $\Sigma_0 = G_0 \Sigma_u G_0' = 0$. As $s$ increases, $\Sigma_s^*$ grows without bound; hence we consider $\Sigma_s \equiv \frac{\Sigma_s^*}{s}$, that is the covariance matrix of $\zeta_{t,t+s}$ scaled by the horizon. Given $\Sigma_u$ and $G_s$, the evolution of $\Sigma_s$ is given by,

$$
\Sigma_s = \frac{1}{s} G_s \Sigma_u G_s' + \left(1 - \frac{1}{s}\right) \Sigma_{s-1}.
$$

The term structure of risks in returns can now be extracted by taking the corresponding element of the $s$-horizon matrix,

$$
Var_t(r_{t+1\rightarrow t+s}) = \iota_r' \Sigma_s \iota_r,
$$
where $r$ is a $(5 \times 1)$-indicator vector with the last element (corresponding to return) set equal to 1. The arithmetic mean return can be constructed by adding half the variance to the expected log return given by expression (11). The covariance between assets returns is calculated by stacking individual EC-VAR models and applying the same recursive procedure to the augmented system.

Notice that the solution to the conditional problem incorporates both horizon and time dimensions, allowing us to trace the impact of time-diversification as well as time-varying economic conditions on optimal allocations. Investors, in this case, are said to time the market by choosing their portfolio holdings according to the current level of state variables $X_t$. This is different from the strategy of the unconditional investor who discards the information in the forecasting variables and holds the same mix of firms at each point in time.

### 2.3.2 Unconditional Analysis

The solution to the unconditional problem is derived by fixing expected log returns on individual assets at their sample means, i.e.,

$$E[r_{t+1|t,s}] = \frac{1}{s} \sum_{j=1}^{s} \bar{r} = \bar{r}. \quad (15)$$

To compute the unconditional variance of asset returns at various investment horizons, we exploit the stationarity property of EC-VAR variables and present the original specification as an infinite-order moving average,

$$X_{t+1} = (I - AL)^{-1} u_{t+1} = \sum_{j=0}^{\infty} A^j u_{t+1-j}. \quad (16)$$

It follows, then, that the unconditional variance of $X_t$ is,

$$\Omega_0^s = \sum_{j=0}^{\infty} A^j \Sigma u A^j, \quad (17)$$

and the variance of the sum of $s$ consecutive $X$’s is given by,

$$\Omega_s^s = s \Omega_0 + \sum_{j=1}^{s-1} (s - j) [V_j + V'_j], \quad (18)$$
where $V_j$ is the $j$-order autocovariance of $X_t$ defined as $V_j = A^j \Omega_0$. Scaling $\Omega^*$ by horizon, $\Omega_s \equiv \frac{\Omega^*}{s}$, the unconditional variance of multi-period returns (expressed per-unit time) can be extracted via,

$$Var(r_{t+1 \rightarrow t+s}) = \xi_s' \Omega_s \xi_r. \quad (19)$$

As pointed above, the expected log returns $E[r_{t+1 \rightarrow t+s}]$ are constant across horizon. However, the term $E[r_{t+1 \rightarrow t+s}] + 0.5Var(r_{t+1 \rightarrow t+s})$, which corresponds to the arithmetic mean, may vary as the unconditional variance may change with horizon. Thus, although the unconditional problem does not accommodate market timing, it does nevertheless exploit return predictability via horizon-dependent variances and correlations in designing the optimal portfolio mix.

### 2.4 Incorporating Parameter Uncertainty

Despite the growing evidence of time-variation in expected returns, it is well recognized that the true predictability of asset returns is highly uncertain. Indeed, adjusted-$R^2$'s in short-horizon return projections, reported in the literature, are typically less than 5%. Furthermore, the predictive power of popular forecasting instruments, such as dividend yields, price-earning ratios or interest rates, is highly unstable across sample periods and sampling frequencies.\(^6\) This raises some concerns as to what extent investors would incorporate the data evidence on return predictability in their investment decisions. We address this issue in a Bayesian framework similar to Kandel and Stambaugh (1996) and Barberis (2000).

The difference between a frequentist and a Bayesian approaches lies in the probability distribution of asset returns which they rely on. In the former case, the term-structure of the risk-return relation is measured using the distribution conditioned on both the data and the point estimates, $\varphi(r_{t+1 \rightarrow t+s}|\text{Data}, \hat{\theta})$, where $\theta$ is the vector of model parameters. The Bayesian analysis, on the other hand, relies on the so-called predictive distribution of future returns conditioned only on the observed sample, i.e., $\varphi(r_{t+1 \rightarrow t+s}|\text{Data})$.

To integrate out parameter uncertainty, we employ a standard Bayesian technique that can be summarized as follows. First, we combine investors’ prior views on model parameters, $f_0(\theta)$, with the conditional likelihood of the observed sample to derive the posterior distribution of regression coefficients, $f(\theta|\text{Data})$. The predictive distribution of

\(^6\)See, for example, Stambaugh (1999) and Goyal and Welch (2003).
multi-horizon asset returns is then given by,

\[ \varphi(r_{t+1-t+s} | \text{Data}) = \int \varphi(r_{t+1-t+s} | \text{Data}, \theta) f(\theta | \text{Data}) d\theta. \quad (20) \]

To construct the first two moments of the predictive distribution (20), we generate a long sequence of parameter draws from the posterior \( f(\theta | \text{Data}) \). Using expressions (13), (11), (15) and (18), we iterate on each realization and compute the profile of expected returns and risks across various horizons. Averaging them over all draws yields the moments of future asset returns that we subsequently rely on to solve the portfolio choice problem. Technical details of this procedure are provided in Appendix.

Our analysis differs from Kandel and Stambaugh (1996) and Barberis (2000) as they design optimal allocations using the standard VAR approach and, thus, do not entertain parameter uncertainty emanating from estimating cointegration parameters. Incorporating uncertainty in our EC-VAR setup leads to two layers of estimation risk. The first is induced by the uncertainty in the cointegrating relation, the second arises from the uncertainty about the EC-VAR parameters. Following the literature, we impose a flat prior on the EC-VAR model parameters, but consider an informative prior on the cointegrating relation between asset dividends and aggregate consumption. In particular, we assume that the prior distribution of the cointegration parameter is normal and centered at 1.\(^7\) To highlight the sensitivity of optimal asset allocations to prior uncertainty about the cointegration parameter, we allow for various degrees of investors’ confidence. In the first case, that we refer to as “tight” prior, we assume that 95% of the probability mass of the distribution of the cointegration parameter lies in 0.5 to 1.5 range. In the second, “loose”-prior case, we expand the confidence interval from -1 to 3. In the case of the standard VAR we assume a non-informative prior.\(^7\)

\(^7\)Our assumption that investors’ beliefs are biased towards unit cointegration is not as arbitrary as it may seem at first. Similar to the existing asset pricing literature, our empirical work relies on dividends constructed on the per-share basis. As discussed above, there is no theoretical reason to expect these dividend series have unit cointegration with consumption. In fact, as documented in Bansal, Dittmar, and Lundblad (2001), Hansen, Heaton, and Li (2005), and Bansal, Dittmar, and Kiku (2007), there is considerable cross-sectional variation in long-run risk exposures measured by the cointegration parameter. Yet, the prevailing view in the profession is heavily skewed towards unit cointegration. Such a misconception arises due to overlooking the important differences between the per-share dividends and aggregate payouts (see the discussion in Section 2). Our prior, centered at 1, tries to capture this stale but still common view on the cointegrating relation.
3 Empirical Results

3.1 Data

As mentioned above, we consider investors who allocate their wealth across value and growth stocks, and the one-year Treasury bond. Value and growth stocks are defined as top and bottom quintile portfolios of assets sorted by their book equity to market equity ratios. Portfolios are constructed as in Fama and French (1993) using the data from the Compustat and CRSP databases. Specifically, we rank all firms on their book-to-market ratios as of end of June of each year using NYSE breakpoints. The book-to-market ratio in year \( t \) is computed as the ratio of book value at fiscal year end \( t - 1 \) to market value of equity at calendar year end \( t - 1 \). For each portfolio, we construct value-weighted returns and per-share dividend series following the standard procedure as in Campbell and Shiller (1988), Bansal, Dittmar, and Lundblad (2005), and Hansen, Heaton, and Li (2005). Time-series for 1-year Tbond are taken from the CRSP Fama-Bliss Discount Bonds files. All data, are converted to real using the personal consumption deflator. Finally, we collect seasonally adjusted data on real per capita consumption of nondurables and services from the NIPA tables available from the Bureau of Economic Analysis. Consumption and stock market data cover the period from 1954 to 2003.

Summary statistics for returns, dividends and price-dividend ratios are presented in Table I. The data evidence a well-known value spread – high book-to-market firms earn average real annual returns of 14.1%, whereas growth firms average only about 8.5%. Value stocks also have a higher mean growth of their dividends and are relatively more volatile than growth firms. Both equity returns have a higher average real return relative to the real return on the t-bond. The volatility of value returns is slightly higher than that of growth. The t-bond has the smallest volatility.

3.2 Sources of Risks at Different Horizons

One important motivation for incorporating the EC-VAR restrictions is the changing nature of risks by investment horizon. The return expression in equation (10) reveals that return variance arises from variation in both dividend growth rates and price-dividend ratios. However, the relative contribution of dividend and price news to return variation changes considerably with the horizon. While at short horizons, price risks may be very
important, their impact gradually diminishes due to stationarity of the price-dividend ratio. Consequently, at long horizons, the per unit time variance of accumulated returns (i.e., variance of the sum of log returns divided by horizon) is dominated by dividend growth risks.

To formalize this intuition, we perform a variance decomposition for returns using a traditional VAR(1) model for dividend growth rates and log price-dividend ratios. Specifically, the dividend growth is projected on its own lag, whereas the price-dividend ratio depends on one lag of the dividend growth, as well as its own lag. That is,

\[
\begin{pmatrix}
\Delta d_{t+1} \\
z_{t+1}
\end{pmatrix} =
\begin{bmatrix}
a_{11} & 0 \\
a_{12} & a_{22}
\end{bmatrix}
\begin{pmatrix}
\Delta d_t \\
z_t
\end{pmatrix} +
\begin{pmatrix}
u_{1,t+1} \\
u_{2,t+1}
\end{pmatrix}. 
\tag{21}
\]

In general, the innovation in the price-dividend ratio, \(u_{2,t+1}\), could be correlated with that in dividend growth rates, \(u_{1,t+1}\). To provide a clean interpretation of the role of price shocks versus dividend shocks we regress \(u_{2,t+1}\) onto \(u_{1,t+1}\) and treat the residual from this projection as price innovation. In essence, we are assuming that dividend news leads to price movements, but price innovations do not lead to (contemporaneous) responses in dividends.

Table II presents the percentages of the return variance that are attributed to dividend and price risks for growth and value stocks. As the table shows, the importance of dividends and price-dividend ratios to return variation changes significantly with the horizon. In the short run, price risks dominate – about 98% of the return variation over the one-year horizon is attributable to price news. This pattern, however, reverses as the holding interval lengthens. By the five-year horizon, about 24-30% of return variation is due to dividend shocks, and at the 10-year horizon, more than a half of the variation is due to dividend shocks. Notice also the difference in risk properties of value and growth stocks. As the horizon increases, risks in value returns shift rapidly towards risks in dividends. In contrast, even at long horizons, price fluctuations still noticeably contribute to the variation in low book-to-market stock returns.

The results in this section imply that asset allocation at longer horizons is mostly about managing dividend risks – an idea that is developed in the long-run risks model. Cointegration between dividends and consumption and the ensuing error-correction model implies that long-horizon movements in dividend growth rates (thus, returns) are predicted by the cointegrating residual. Hence, the error-correction channel may have significant

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8The evidence based on the EC-VAR is even stronger. However, as we have not yet discussed the cointegration evidence, we illustrate the argument using a more conservative VAR-based approach.
consequences for designing asset allocation at both intermediate and long horizons.

### 3.3 Return Dynamics

In this section we discuss the dynamics of the state variables and returns across various investment horizons implied by our first-order EC-VAR specification. We start by presenting empirical evidence on cointegration and analyzing the ability of the error-correction variable to predict future dividend growth rates and returns.

#### 3.3.1 Cointegration Evidence

In Table III, we report point estimates of the cointegration parameters between dividends and consumption and present the sample autocorrelation functions (ACF) of cointegrating residuals. We estimate cointegration parameters via OLS by regressing log dividends on log consumption and a time trend. First notice that the sample autocorrelations of the resulting cointegrating residuals exhibit a rapid decline turning negative at the fifth lag. The Dickey-Fuller unit root test on the error-correction term is significantly rejected for both, value and growth, assets. This supports our assumption that the dynamics of portfolios’ dividends and aggregate consumption are tied together in the long run.

Long-run risk properties of value and growth firms, however, are very different as suggested by the estimated cointegration parameters reported in the first column of Panel A. While cash flows of value firms respond positively to persistent shocks in aggregate consumption, growth firms’ dividends exhibit an opposite, negative exposure to low-frequency consumption fluctuations. In particular, $\delta$ is estimated at 1.94 (SE=2.30) for high book-to-market firms and -4.84 (SE=0.97) for low book-to-market firms. This is consistent with the cross-sectional pattern in long-run dividend betas documented in Bansal, Dittmar, and Lundblad (2001), Hansen, Heaton, and Li (2005) and Bansal, Dittmar, and Kiku (2007), who consequently elaborate on this evidence to highlight the importance of low-frequency risks for understanding the cross-section risk-return tradeoff.

From the cointegrating relation it follows that dividend growth rates satisfy $\Delta d_t = \tau_1 + \delta \Delta c_t + \Delta \epsilon_{d,t}$. Hence, joint time-series dynamics for $\Delta c_t$ and $\epsilon_{d,t}$ is enough to describe the time-series behavior of the dividend growth. The implications of cointegration for future growth rates can be intuitively explained via the error-correction mechanism. Assume that dividends are unusually high today (i.e., $\epsilon_{d,t}$ is high). As the cointegrating residual is
stationary, dividend growth rates are expected to fall since dividends have to adjust back to their long-run equilibrium with consumption. Thus, the variation in future dividend growth rates should be accounted for by the variation in the error-correction variable. Further, the slower the adjustment of dividends to the consumption level, the longer the effect of the cointegrating residual on future growth rates. Given that dividend growth is a key input in thinking about multi-horizon returns, predictability of dividends emanating from the cointegrating relation may have important consequences for predictability and volatility of future returns.

### 3.3.2 Predictability Evidence

In this section, we examine the ability of the cointegrating residual to forecast asset returns at various horizons. To highlight the importance of the cointegrating relation, we compare $R^2$'s for return projections implied by the EC-VAR model with the corresponding $R^2$'s from the growth-rate based VAR. The predictive state variables in the VAR are consumption growth, the price-dividend ratio and dividend growth of the asset. In the EC-VAR specification, we replace the asset’s dividend growth with the asset’s cointegrating residual, $\epsilon_{d,t}$. Hence, in both cases we have three variables that forecast future equity returns.

As in Hodrick (1992), long-horizons $R^2$'s are calculated as one minus the ratio of the innovation variance in the return compounded over a given horizon $s$ to the total variance of $s$-period returns, i.e.,

$$R^2_s = 1 - \frac{\nu_r^s \Sigma_{s} \nu_r^s}{\nu_r^s \Omega_s \nu_r^s}, \quad (22)$$

where $\Sigma_s$ and $\Omega_s$ are defined above.

Table IV presents $R^2$'s implied by the EC-VAR and the alternative, growth-rates model. Numbers reported in parentheses are the 2.5% and 97.5% percentiles of the corresponding bootstrap distributions. Notice first that return predictability, implied by our EC-VAR specification, improves considerably with the horizon. As Panel A shows, the EC-VAR model accounts for only about 12% – 17% of the one-period return variation. However, by the 10-year horizon its predictive ability increases to striking 42% and 65% for growth and value stocks, respectively. Second, while none of the models seems to outperform the other in the short run, the growth-rates VAR is noticeably dominated by the error-correction specification at longer horizons. This evidence, illustrated further in Figure 2, suggests that the cointegrating residual incorporated in the EC-VAR specification contains distinct and important information about return dynamics, especially in the long run. Consequently, it
has a potentially important bearing on the evolution of conditional (as well as unconditional) arithmetic mean returns and return variances at various horizons. Note that the reported multi-horizon $R^2$'s are read from the first-order EC-VAR or VAR; to ensure that the evidence is sound, we also considered direct projections of multi-period returns (sum of log returns) on the EC-VAR and VAR predictive variables. The $R^2$'s from these regressions are very similar to those computed from the time-series EC-VAR and VAR specifications. We find that the cointegrating residual is statistically significant for predicting value and growth returns, especially at long horizons.

To identify the sources of return predictability we also compute the $R^2$'s in dividend growth projections for the two alternative specifications. For the VAR specification, the adjusted $R^2$ for predicting dividend growth rates at horizons of 1, 5, and 10 are 24%, 22% and 22% for growth, and 8%, 4%, and 4% for value firms. For the EC-VAR, the corresponding numbers are 13%, 39%, and 44% for growth and 9%, 18%, and 23% for value. Hence, there is a sizable improvement in predicting dividend growth rates by relying on the EC-VAR model. As discussed earlier, dividend risks are an important component of return variation, especially at long horizons. Our findings suggests that the error-correction specification has an ability to forecast these risks much better than the standard VAR. In the next section, we examine the effect of the error-correction dynamics on the evolution of the expected return-risk relation across various investment horizons.

### 3.3.3 Term Profile of Means and Variances

The profile of arithmetic means and unconditional volatilities of asset returns is presented in Table V. To emphasize the differences between the EC-VAR and the alternative VAR setup we display return moments for both models. As Panel A shows, the term structure of arithmetic mean returns on low and high book-to-market firms is declining with the horizon for the EC-VAR specification. In contrast, there is almost no decline in mean returns of value stocks in the VAR specification (see Panel B). Recall that in the unconditional case, the arithmetic mean return for a given horizon is defined as the mean log return plus one-half of the scaled variance of the multi-horizon return. Clearly, the first component is the same in both the EC-VAR and VAR specifications, as it is simply determined by the historical average of log asset returns. The variance component, however, depends on the time-series dynamics and predictability of long-horizon returns and, consequently, may significantly differ under the competing models.
Indeed, we find that volatilities of asset returns at various horizons are quite different across the two specifications. As expected, the standard deviation of long-run returns is reduced within the error-correction framework for both value and growth stocks. The volatility of returns implied by the alternative VAR specification similarly decreases for growth firms, but displays a generally flat (slightly increasing) pattern for value firms. In particular, the standard deviation of value returns declines from 20% at the one-year horizon to about 15% at the long horizon for the EC-VAR specification, but stays at the initial 20% for the VAR model. Thus, the EC-VAR specification captures considerable time-diversification benefits in value returns that are missing in the VAR model.

Differences between the error-correction and growth-rates specifications are pronounced not only for volatilities but also correlations of asset returns as shown in Figure 3. In particular, for the EC-VAR, the correlations of returns are much higher than for the VAR model. In the latter setup, the correlations decline from 0.63 to about 0.30 from the one-to 15-year horizon. In contrast, for the EC-VAR, the correlation starts at 75% gradually decreasing to about 52% only. We should emphasize that these differences are not solely driven by the differences in returns’ variances across the two models – the EC-VAR and VAR-implied covariances likewise significantly deviate from each other. These evidence implies that from the VAR perspective, the diversification across assets can be quite important – the growth asset can be valuable at long horizons, despite its lower mean for purposes of reducing the overall volatility of the optimal portfolio. The cross-sectional diversification seems to be less valuable from the EC-VAR perspective.

To investigate the impact of the estimation error on return moments, in Table VII we report return volatilities after accounting for parameter uncertainty. Mean returns are not reported for brevity – across all horizons, they are about 1% lower their counterparts in the case when parameter uncertainty is ignored. Not surprising, the volatility of asset returns is somewhat higher when estimation errors are taken into account. However, the general pattern is similar to the case when parameter uncertainty is ignored.

To summarize, empirical evidence presented in this section underscores the importance of the cointegration specification for risks and expected returns. Temporary deviations of cash flows from the permanent component in consumption contain important information about future dynamics of asset returns and, thus, represent a pivotal component in measuring the term structure of the risk-return tradeoff. Furthermore, cointegration alters the risk diversification properties of value and growth assets relative to the standard VAR model that, we expect, my significantly affect wealth allocation across the two stocks.
3.4 Asset Allocation Decisions

Using the profile of constructed return moments, we solve for the optimal allocations of investors with different holding intervals. For brevity, in the benchmark case of no-parameter uncertainty, we report all the results for the risk aversion level of 5. The impact of investors’ preferences is illustrated later on, for the case that incorporates estimation uncertainty in model parameters. In particular, we consider two values of risk aversion, 5 and 10, and two levels of prior confidence, “loose” and “tight”, defined above. To keep the focus on highlighting the differences in optimal portfolios between EC-VAR and VAR models, we ignore short selling constraints; the essential message is similar if one were to impose these restrictions.

3.4.1 Unconditional Analysis

Panel A of Table VI reports the asset allocation of the EC-VAR based investors. We find that their investment strategy is considerably tilted towards value stocks at both short and long horizons. In particular, the allocation to value firms starts at about 76% at the one-year horizon, increasing to 95% at the 15-year horizon. The allocation to growth increases as well but it starts with a negative position. In addition, the level of growth investment is significantly lower than the allocation to value stocks at any horizon.

The horizon effect has an opposite pattern for the alternative VAR specification, as suggested by the entries in Panel B. At the very short horizon, the VAR-based investors still allocate more to value than to growth stocks. Their preferences towards the two assets, however, reverse as the holding period lengthens – as the horizon increases, the allocation to value declines and that to growth increases. This is consistent with the evidence in Jurek and Viceira (2005). In a similar VAR setup, they also find that long-run investors gradually shift their wealth away from value stocks.

The documented differences in the optimal portfolio mix across the two models arise due to different patterns in return volatilities and their correlations. The VAR-based investors perceive value stocks as quite risky, especially in the long run, thus, steadily reducing their allocations to high book-to-market firms. In contrast, the EC-VAR investors recognize that, via the error-correction mechanism, the relative riskiness of value investment shrinks over time. At short horizons, transitory risks in dividends and prices make value stocks look quite risky.\textsuperscript{9} In the long run, transitory fluctuations are washed away and all risks in

\textsuperscript{9}Notice that even at short horizons, value stocks are much better deal as they offer significantly higher
value returns come from permanent risks in their dividends. Importantly, the adjustment of value dividends and, thus, value returns to their long-run equilibrium relation with aggregate consumption is strongly predicted by the error-correction variable. This long-run predictability reduces the volatility of multi-horizon returns making value firms more attractive for long-horizon investments. Further, while in the VAR, the cross-sectional diversification via growth returns increases (as the correlation between value and growth returns rapidly declines with the horizon), its benefits are significantly reduced in the cointegration-based framework. Thus, the EC-VAR-based investors do not view growth stocks as good substitutes for value.

We now turn the discussion to the portfolio choice in the presence of parameter uncertainty. In Table VIII, we show asset allocation weights in the case of the EC-VAR and the alternative VAR specifications with risk aversion of 5. In the cointegration specification we use “tight” prior centered at one. First notice that incorporating parameter uncertainty significantly lowers positions in risky assets, which is similar to the findings of Barberis (2000). In the VAR specification, investors continue to cut down their allocations to value stocks as the horizon increases – starting at 43%, investment in value firms declines to 21% at the 15-year horizon. Similar to the no-parameter uncertainty case, the allocation to growth increases with the horizon due to the cross-sectional diversification effect. With the EC-VAR specification, the allocation to value increases with the investment horizon: from 47% at the one-year horizon to 69% under the 15-year buy-and-hold strategy. Consequently, for both specifications, parameter uncertainty affects the scale of the position but does not affect the horizon pattern of allocations.

In Table IX we analyze the effect of parameter uncertainty under various assumptions of the prior confidence. In addition, we consider investors with different degrees of risk aversion. The optimal portfolio weights are reported for four configurations based on high and low risk aversion, and “tight” and “loose” priors about the cointegration parameter. As expected, for a given prior, increasing the risk aversion shifts the allocation away from risky assets towards the T-bond. At the same time, both high and low risk aversion investors still continue to hold on to value stocks. Although the allocation to growth increases with the horizon, it fails to keep up with the value investment. Further, for a given risk aversion, different prior beliefs do not dramatically affect portfolio weights at short horizons. The prior uncertainty, however, seems to matter for longer holding periods. With lax beliefs about the cointegration parameter, the long-horizon allocation to value stocks scales down Sharpe ratio than growth stocks.
relative a tighter prior.

The key message of the evidence presented above is that the EC-VAR view of return dynamics significantly alters asset allocations, particularly at long horizons. Specifically, in the long run, value stocks seem to be far more attractive relative to the traditional VAR model for returns. Moreover, this result continues to hold even after accounting for uncertainty about model parameters. In the next section, we discuss the effect of the error-correction channel within the conditional framework.

### 3.4.2 Conditional Analysis

The optimal allocation of a conditional-type investor with the ten-year horizon is presented in Figure 4. We report the evidence for the case where parameter uncertainty about cointegration is incorporated using “loose” prior around unity and risk aversion of 10. The portfolio choice problem is solved for each date using the predictive density and the current value of the predictive state variables that are observable to the investor. The conditional problem, as is usually the case, exploits the market timing feature of asset allocation.\(^\text{10}\)

Figure 4 suggests a number of interesting observations. The first-order difference between the EC-VAR and the VAR specifications is the difference in level allocations to value and growth firms. Similar to the unconditional analysis, the allocation to value is much higher under the EC-VAR specification. Second, a visual inspection of EC-VAR weights reveals an interesting business-cycle pattern in investment decisions of long-horizon investors. The holding of growth stocks tends to significantly decline at the outset of economic troughs. For example, the 10-year allocation to growth falls right before the 1970, 1973, 1990-91 and 2001’s recessions. A lag between a decrease in growth weight and an up-coming downturn measured by the NBER business cycle indicator is about 2-3 years. Value holdings, on the other hand, strengthen during recessions. Fluctuations in the weight put on the T-bond closely resemble time-variation pattern in the growth allocation, increasing during expansions and declining when the economy moves down. The market timing dimension is much less pronounced in the VAR setup. In particular, the correlation between the 10-year allocation to value and consumption growth is -17% in the EC-VAR specification but only -6% in the case of the VAR specification. The corresponding numbers for the 5-year horizon are -30% and -15%.

\(^{\text{10}}\)Portfolio allocation problems that exploit market timing in somewhat different settings are also considered in Lynch and Balduzzi (2000), Ferson and Siegel (2001), Brandt and Ait-Sahalia (2001), Ang and Bekaert (2002), Bansal, Dahlquist, and Harvey (2004), and Brandt and Santa-Clara (2006) among others. The importance of conditioning information and predictability is highlighted in the earlier work of Hansen and Richard (1987).
for the EC-VAR and the standard VAR respectively.

4 Conclusion

In this paper, we argue that the long-run equilibrium relation, measured via a stochastic cointegration between aggregate consumption and dividends, has significant implications for dividend growth rates and return dynamics. The error-correction mechanism between dividends and consumption and the ensuing EC-VAR provide an alternative specification for describing the dynamics of equity returns relative to the traditional VAR that ignores the implications of the long-run equilibrium. The recent long-run risks literature (including Bansal and Yaron (2004), Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2005), Bansal, Dittmar, and Kiku (2007)) argues that the long-run relation between cash flows and consumption contains important information about asset risk premia. Our approach explores the implications of these ideas for optimal portfolio decisions across different investment horizons.

We show that the error-correction channel, incorporated by the EC-VAR representation, significantly alters return forecasts and the variance-covariance matrix of asset returns, and shifts the optimal portfolio mix relative to the traditional VAR. The standard VAR specification captures the short-run return dynamics, but in the presence of cointegration, considerably misses the long-horizon dynamics of asset returns. In contrast, the EC-VAR specification successfully accounts for both the short- and long-horizons dynamics. Consequently, as we show, the EC-VAR model is able to capture much higher benefits of time-diversification than the standard VAR framework.

We develop methods that allow us to incorporate parameter uncertainty in the co-integration based specification and highlight the effect of estimation errors on long-horizon asset allocations. We show that significant differences in the optimal portfolio decisions between the EC-VAR and VAR specifications persist even after accounting for parameter uncertainty. In sum, our evidence suggests that optimal allocations derived from the standard VAR may be quite suboptimal for investors with intermediate and long investment horizons.
A. Appendix

A.1 Incorporating Uncertainty in the VAR

To obtain the posterior distribution of the VAR parameters, we follow Bauwens, Lubrano, and Richard (1999) and re-write the model in the form of seemingly unrelated regressions. The SUR representation allows us to explicitly incorporate zero restrictions imposed on the matrix $A$. Specifically, we present the dynamics of each variable $i$ ($i=1,...,n$) as,

$$X_i = Z_i\beta_i + U_i,$$  \hspace{1cm} (A-1)

where $X_i$ is $((T-1) \times 1)$-vector of observations on $i$-th variable, $Z_i$ is the $((T-1) \times k_i)$-matrix of relevant predictors, $\beta_i$ is the $(k_i \times 1)$-vector of regression coefficients (including the intercept), and $U_i$ is the $((T-1) \times 1)$-vector of shocks. Stacking all equations together, we can express (A-1) compactly as,

$$x = z\beta + u,$$  \hspace{1cm} (A-2)

where $x = (X_1',... ,X_n')$, $\beta = (\beta_1',... ,\beta_n')$, $u = (U_1',... ,U_n')$, and $z = \text{diag}(Z_1,... ,Z_n)$. Alternatively, the EC-VAR model can be cast in the following matrix form,

$$X = ZB + U,$$  \hspace{1cm} (A-3)

where $X = (X_1,... ,X_n)$, $Z = (Z_1,... ,Z_n)$, $U = (U_1,... ,U_n)$, and $B = \text{diag}(\beta_1,... ,\beta_n)$. To derive the posterior distribution, we assume that investors have no well-defined beliefs about the model parameters and, therefore, use a non-informative prior,

$$\varphi(\beta, \Sigma_u) \propto |\Sigma_u|^{-(n+1)/2}.$$  \hspace{1cm} (A-4)

As shown in Bauwens, Lubrano, and Richard (1999), the posterior density of the parameters can be factorizes as,

$$\beta \mid \Sigma_u \sim N(\hat{\beta}, [z'(\Sigma_u^{-1} \otimes I_{T-1})z]^{-1})$$

$$\Sigma_u \mid \beta \sim IW(Q, T-1)$$  \hspace{1cm} (A-5)

where

$$\hat{\beta} = [z'(\Sigma_u^{-1} \otimes I_{T-1})z]^{-1}z'(\Sigma_u^{-1} \otimes I_{T-1})x$$

$$Q = (X - ZB)'(X - ZB)$$
Although marginal posterior densities of $\beta$ and $\Sigma_u$ are not available, the posterior analysis can be easily implemented by applying a two-block Gibbs sampling algorithm to the above conditional densities. Specifically, at the $j$-th simulation, we draw $\beta^j$ conditional on the previous $\Sigma_{u,j-1}$, and close the loop by sampling $\Sigma_{u,j} \mid \beta^j$ from the inverse Wishart distribution. The chain is initialized using point estimates of the model parameters. We make two adjustments to the described sampling procedure. We discard first 500 draws in order to eliminate the influence of the starting values. In addition, to ensure stationarity, we remove draws if the matrix $B$ has eigenvalues larger than 0.98.

Our final sample consists of 20,000 draws of parameter values from the posterior, $\{\beta^j, \Sigma_{u,j}\}$. We cast them back into the original VAR representation and calculate means and variance-covariance matrices of multi-horizon returns. The corresponding moments of the predictive distribution of asset returns are obtained by taking the average of the constructed quantities. This procedure delivers the term-structure of expected returns and risks conditioned only on the observed data, but no the VAR estimates. Notice that this approach can be similarly applied to the EC-VAR if the uncertainty in the cointegration vector is ignored. In this case, the predictive distribution will depend on both the actual data and the estimated error-correction term. As such, it is relevant for investors who rely on the OLS estimate of the cointegration parameter (perhaps, given its superconsistent properties) but doubt the credibility of sample regression coefficients of the rest of the EC-VAR model.

### A.2 Incorporating Uncertainty in the EC-VAR

The approach of incorporating uncertainty in the cointegration vector along with the EC-VAR parameters is similar to the one discussed in the previous section. After observing the sample, investors update their prior to form the posterior distribution of the model parameters. Letting $f_0(\alpha)$ denote the prior density of the elements of the cointegration vector $\alpha \equiv (\tau_0, \tau_1, \delta)'$ and maintaining the non-informative prior for the EC-VAR parameters, we can summarize investors’ ax-ante beliefs in a composite form,

$$\varphi(\alpha, \beta, \Sigma_u) \propto f_0(\alpha) |\Sigma_u|^{-(n+1)/2}. \quad (A-6)$$

The conditional posterior densities are then given by,

$$\beta \mid \Sigma_u, \alpha \sim N\left(\hat{\beta}, \left[z'\left(\Sigma_u^{-1} \otimes I_{T-1}\right)z\right]^{-1}\right)$$

$$\Sigma_u \mid \beta, \alpha \sim IW(Q, T-1) \quad (A-7)$$

$$f(\alpha) \propto f_0(\alpha) \frac{|\alpha'W_0\alpha|^{l_0}}{|\alpha'W_1\alpha|^{l_1}}$$
where $\hat{\beta}$ and $Q$ are defined as above, and matrices $W_0$, $W_1$ and constants $l_0$, $l_1$ depend on the observed sample (for detailed formulas see Bauwens and Lubrano (1996)). Notice that the analytical expression for the conditional density of the cointegration parameter is not available. As suggested in Bauwens, Lubrano, and Richard (1999), $f(\alpha)$, in this case, can be simulated by applying the Griddy-Gibbs sampling technique. The algorithm is implemented by evaluating the kernel over a grid of points and approximating the cumulative distribution using numerical integration. Making a normalization to obtain a proper distribution function, we sample the cointegration vector by inverting the constructed cdf. Conditionally on a given draw $\alpha^j$, we generate the EC-VAR parameters applying the two-block Gibbs sampler to the conditional densities of $\beta$ and $\Sigma_u$ as discussed above. Discarding initial draws and rejecting draws that fall on the edge or outside the stationary region, we end up with 20,000 values from the posterior distribution and use them to construct the moments of the predictive distribution of asset returns. The resulting distribution and the optimal allocation it implies are conditioned only on the actual data and do not depend on any fixed estimates of the predictive model for returns.
References


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<th>Returns</th>
<th>Div Growth</th>
<th>Log(P/D)</th>
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<td>StdDev</td>
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<td>Growth</td>
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Table I presents descriptive statistics for returns, dividend growth rates and logarithms of price/dividend ratios on value and growth firms, as well as the return on the one-year Treasury bond. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Returns are value-weighted, price/dividend ratios are constructed by dividing the end-of-year price by the cumulative annual dividend, growth rates are constructed by taking the first difference of the logarithm of per-share dividend series. All data are annual, expressed in real terms, and cover the period from 1954 to 2003.
Table II
Variance Decomposition of Returns

| Horizon (yr) | Growth | | | | Value | | |
| | | | | | | | |
| 1 | 0.01   | 0.99   | | | 0.02   | 0.98   | |
| 2 | 0.09   | 0.91   | | | 0.02   | 0.98   | |
| 5 | 0.30   | 0.70   | | | 0.24   | 0.76   | |
| 10 | 0.53   | 0.47   | | | 0.76   | 0.24   | |
| 15 | 0.67   | 0.33   | | | 0.89   | 0.11   | |
| 20 | 0.75   | 0.25   | | | 0.92   | 0.08   | |

Table II reports fractions of the return variance that are accounted for by price and dividend growth shocks. The percentages are reported for value and growth stocks at various investment horizons. Variance decomposition is performed by fitting a VAR(1) model to portfolio’s cash-flow growth rates and price-dividend ratio. The coefficient matrix in the VAR is assumed to be lower triangular; dividend and price innovations are orthogonalized.
Table III
Cointegration Evidence

<table>
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<th>SE</th>
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<th>ACF(2)</th>
<th>ACF(5)</th>
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<tr>
<td>Growth</td>
<td>-4.84</td>
<td>0.97</td>
<td>0.82</td>
<td>0.49</td>
<td>-0.22</td>
</tr>
<tr>
<td>Value</td>
<td>1.95</td>
<td>2.30</td>
<td>0.76</td>
<td>0.42</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

Table III presents estimates of the cointegration parameter (\( \hat{\delta} \)) for value and growth portfolios. The estimates are obtained by regressing the deterministically de-trended log level of portfolio’s dividends on the de-trended log level of aggregate consumption. The last three columns, labelled “ACF”, report the sample autocorrelation function of cointegrating residuals for lags 1, 2 and 5.
Table IV
Predictability Evidence

Panel A: Error-Correction VAR

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.12 (0.07, 0.33)</td>
<td>0.17 (0.10, 0.39)</td>
</tr>
<tr>
<td>2</td>
<td>0.16 (0.11, 0.39)</td>
<td>0.26 (0.17, 0.45)</td>
</tr>
<tr>
<td>5</td>
<td>0.28 (0.17, 0.52)</td>
<td>0.46 (0.31, 0.62)</td>
</tr>
<tr>
<td>10</td>
<td>0.42 (0.22, 0.58)</td>
<td>0.65 (0.43, 0.73)</td>
</tr>
<tr>
<td>15</td>
<td>0.48 (0.23, 0.61)</td>
<td>0.73 (0.43, 0.79)</td>
</tr>
</tbody>
</table>

Panel B: Growth-rates VAR

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11 (0.07, 0.43)</td>
<td>0.18 (0.11, 0.44)</td>
</tr>
<tr>
<td>2</td>
<td>0.12 (0.07, 0.46)</td>
<td>0.27 (0.15, 0.57)</td>
</tr>
<tr>
<td>5</td>
<td>0.12 (0.05, 0.51)</td>
<td>0.45 (0.19, 0.75)</td>
</tr>
<tr>
<td>10</td>
<td>0.11 (0.03, 0.51)</td>
<td>0.52 (0.13, 0.70)</td>
</tr>
<tr>
<td>15</td>
<td>0.10 (0.02, 0.50)</td>
<td>0.47 (0.10, 0.60)</td>
</tr>
</tbody>
</table>

Table IV presents return $R^2$’s implied by the EC-VAR specification (Panel A) and the alternative growth-rates VAR model (Panel B). The latter ignores the implications of cointegration between asset cash flows and consumption. The entries are reported for value and growth portfolios across various holding horizons. Numbers in parentheses are the lower and upper bounds of the corresponding 95% bootstrap confidence intervals.
Table V
Term Structure of Expected Returns and Risks

**Panel A: Error-Correction VAR**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Expected Return</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Growth</td>
<td>Value</td>
</tr>
<tr>
<td>1</td>
<td>0.082 (0.014)</td>
<td>0.133 (0.019)</td>
</tr>
<tr>
<td>2</td>
<td>0.081 (0.014)</td>
<td>0.131 (0.019)</td>
</tr>
<tr>
<td>5</td>
<td>0.078 (0.014)</td>
<td>0.128 (0.019)</td>
</tr>
<tr>
<td>10</td>
<td>0.076 (0.014)</td>
<td>0.126 (0.020)</td>
</tr>
<tr>
<td>15</td>
<td>0.074 (0.014)</td>
<td>0.125 (0.021)</td>
</tr>
</tbody>
</table>

**Panel B: Growth-rates VAR**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Expected Return</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Growth</td>
<td>Value</td>
</tr>
<tr>
<td>1</td>
<td>0.080 (0.023)</td>
<td>0.132 (0.029)</td>
</tr>
<tr>
<td>2</td>
<td>0.080 (0.023)</td>
<td>0.132 (0.029)</td>
</tr>
<tr>
<td>5</td>
<td>0.078 (0.024)</td>
<td>0.132 (0.031)</td>
</tr>
<tr>
<td>10</td>
<td>0.077 (0.025)</td>
<td>0.133 (0.033)</td>
</tr>
<tr>
<td>15</td>
<td>0.076 (0.026)</td>
<td>0.133 (0.034)</td>
</tr>
</tbody>
</table>

Table V reports the profile of mean returns and volatilities by horizon. Expected returns and risks are presented for the EC-VAR specification (Panel A) and the alternative growth-rates VAR model (Panel B). The latter ignores the implications of cointegration between asset cash flows and consumption. Bootstrap standard errors are reported in parentheses.
Table VI
Optimal Allocation Strategy

**Panel A: Error-Correction VAR**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.30 (-0.94, 0.10)</td>
<td>0.76 (0.15, 1.27)</td>
<td>0.54 (0.42, 0.93)</td>
</tr>
<tr>
<td>2</td>
<td>-0.30 (-0.93, 0.15)</td>
<td>0.82 (0.19, 1.29)</td>
<td>0.48 (0.38, 0.91)</td>
</tr>
<tr>
<td>5</td>
<td>-0.26 (-0.93, 0.21)</td>
<td>0.91 (0.17, 1.51)</td>
<td>0.35 (0.15, 0.87)</td>
</tr>
<tr>
<td>10</td>
<td>-0.14 (-0.92, 0.36)</td>
<td>0.96 (0.16, 1.93)</td>
<td>0.18 (-0.35, 0.82)</td>
</tr>
<tr>
<td>15</td>
<td>-0.00 (-0.88, 0.49)</td>
<td>0.95 (0.18, 2.32)</td>
<td>0.05 (-0.94, 0.79)</td>
</tr>
</tbody>
</table>

**Panel B: Growth-rates VAR**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.13 (-0.66, 0.55)</td>
<td>0.66 (-0.21, 1.11)</td>
<td>0.47 (0.14, 0.98)</td>
</tr>
<tr>
<td>2</td>
<td>-0.07 (-0.64, 0.58)</td>
<td>0.64 (-0.18, 1.06)</td>
<td>0.43 (0.17, 0.95)</td>
</tr>
<tr>
<td>5</td>
<td>0.08 (-0.50, 0.64)</td>
<td>0.57 (-0.12, 1.10)</td>
<td>0.35 (0.00, 0.92)</td>
</tr>
<tr>
<td>10</td>
<td>0.23 (-0.39, 0.73)</td>
<td>0.52 (-0.10, 1.28)</td>
<td>0.25 (-0.25, 0.91)</td>
</tr>
<tr>
<td>15</td>
<td>0.33 (-0.35, 0.80)</td>
<td>0.51 (-0.09, 1.38)</td>
<td>0.16 (-0.41, 0.91)</td>
</tr>
</tbody>
</table>

Table VI reports the optimal allocation across different investment horizons. Portfolio weights are presented for the EC-VAR specification (Panel A) and the alternative growth-rates VAR model (Panel B). The latter ignores the implications of cointegration between asset cash flows and consumption. Numbers in parentheses are the lower and upper bounds of the corresponding 95% bootstrap confidence intervals.
Table VII
Term Structure of Return Volatilities with Parameter Uncertainty

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.214</td>
<td>0.239</td>
<td>0.022</td>
</tr>
<tr>
<td>2</td>
<td>0.206</td>
<td>0.227</td>
<td>0.027</td>
</tr>
<tr>
<td>5</td>
<td>0.191</td>
<td>0.204</td>
<td>0.033</td>
</tr>
<tr>
<td>10</td>
<td>0.173</td>
<td>0.184</td>
<td>0.036</td>
</tr>
<tr>
<td>15</td>
<td>0.158</td>
<td>0.173</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Panel B: Growth-rates VAR

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.210</td>
<td>0.234</td>
<td>0.024</td>
</tr>
<tr>
<td>2</td>
<td>0.208</td>
<td>0.235</td>
<td>0.030</td>
</tr>
<tr>
<td>5</td>
<td>0.205</td>
<td>0.248</td>
<td>0.039</td>
</tr>
<tr>
<td>10</td>
<td>0.203</td>
<td>0.270</td>
<td>0.045</td>
</tr>
<tr>
<td>15</td>
<td>0.202</td>
<td>0.288</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Table VII reports standard deviations of the predictive distribution of multi-period returns. The term profile of asset risks is presented for the EC-VAR specification (Panel A) and the alternative growth-rates VAR model (Panel B). Panel A is constructed imposing a tight normal prior on the cointegration vector, centered around one.
Table VIII
Optimal Allocation Strategy with Parameter Uncertainty

Panel A: Error-Correction VAR

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.23</td>
<td>0.47</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>-0.21</td>
<td>0.50</td>
<td>0.71</td>
</tr>
<tr>
<td>5</td>
<td>-0.19</td>
<td>0.57</td>
<td>0.62</td>
</tr>
<tr>
<td>10</td>
<td>-0.14</td>
<td>0.64</td>
<td>0.50</td>
</tr>
<tr>
<td>15</td>
<td>-0.10</td>
<td>0.69</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Panel B: Growth-rates VAR

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.09</td>
<td>0.43</td>
<td>0.66</td>
</tr>
<tr>
<td>2</td>
<td>-0.05</td>
<td>0.37</td>
<td>0.66</td>
</tr>
<tr>
<td>5</td>
<td>0.06</td>
<td>0.30</td>
<td>0.63</td>
</tr>
<tr>
<td>10</td>
<td>0.15</td>
<td>0.24</td>
<td>0.62</td>
</tr>
<tr>
<td>15</td>
<td>0.18</td>
<td>0.21</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table VIII reports the optimal allocation of Bayesian-type investors with different holding periods. Portfolio weights are presented for the EC-VAR specification (Panel A) and the alternative growth-rates VAR model (Panel B). Panel A is constructed imposing a tight normal prior on the cointegration vector, centered around one.
Table IX reports the optimal allocation of investors who rely on the EC-VAR specification and incorporate parameter uncertainty. Four panels present portfolio weights for different levels of investors’ risk aversion and their confidence that value and growth firms’ dividends exhibit a unit cointegration with aggregate consumption.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
<th>Growth</th>
<th>Value</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="RA = 5, Tight Prior" /></td>
<td><img src="image" alt="RA = 5, Loose Prior" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.23</td>
<td>0.47</td>
<td>0.75</td>
<td>-0.14</td>
<td>0.46</td>
<td>0.68</td>
</tr>
<tr>
<td>2</td>
<td>-0.21</td>
<td>0.50</td>
<td>0.71</td>
<td>-0.11</td>
<td>0.47</td>
<td>0.64</td>
</tr>
<tr>
<td>5</td>
<td>-0.19</td>
<td>0.57</td>
<td>0.62</td>
<td>-0.05</td>
<td>0.49</td>
<td>0.56</td>
</tr>
<tr>
<td>10</td>
<td>-0.14</td>
<td>0.64</td>
<td>0.50</td>
<td>0.04</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
<td>15</td>
<td>-0.10</td>
<td>0.69</td>
<td>0.41</td>
<td>0.11</td>
<td>0.50</td>
<td>0.39</td>
</tr>
<tr>
<td><img src="image" alt="RA = 10, Tight Prior" /></td>
<td><img src="image" alt="RA = 10, Loose Prior" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.11</td>
<td>0.22</td>
<td>0.88</td>
<td>-0.08</td>
<td>0.23</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>-0.10</td>
<td>0.25</td>
<td>0.85</td>
<td>-0.07</td>
<td>0.24</td>
<td>0.83</td>
</tr>
<tr>
<td>5</td>
<td>-0.09</td>
<td>0.32</td>
<td>0.77</td>
<td>-0.05</td>
<td>0.25</td>
<td>0.79</td>
</tr>
<tr>
<td>10</td>
<td>-0.06</td>
<td>0.37</td>
<td>0.69</td>
<td>0.00</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>15</td>
<td>-0.01</td>
<td>0.37</td>
<td>0.64</td>
<td>0.03</td>
<td>0.24</td>
<td>0.73</td>
</tr>
</tbody>
</table>
Figure 1 plots the logarithm of aggregate dividends to consumption ratio (Panel (a)), and the log ratio of per-share dividends to aggregate consumption (Panel (b)). Dividend series represent cash flows of the aggregate stock market portfolio. The data are real and span the period from 1954 to 2003.
Figure 2. Predictability of Asset Returns

Figure 2 plots the profile of return $R^2$'s implied by the EC-VAR and the standard VAR specification. The latter ignores the implications of cointegration between asset cash flows and consumption. Thick and dash-dotted lines correspond to value and growth stocks respectively.
Figure 3. Unconditional Correlations of Asset Returns

Figure 3 plots the term profile of unconditional correlations of value and growth returns. Thick line corresponds to the EC-VAR model, thin line presents correlations implied by the growth-rates VAR specification.
Figure 4. Portfolio Weights

Figure 4 displays time-series of optimal portfolio holdings of growth and value stocks for buy-and-hold investors with the ten-year investment horizons. Thick black line corresponds to weights implied by the EC-VAR model, thin red line is derived from the alternative growth-rates VAR specification. Both allocations incorporate parameter uncertainty. The prior of the cointegration parameter in the EC-VAR specification is “loose” and centered around 1, the level of risk aversion is 10.