Investment Tournaments:  
When Should a Rational Agent Put All Eggs in One Basket?*

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Abstract

We introduce and study a class of decision problems which we call “investment tournaments.” The problems of this class involve gradual allocation of investments among several alternatives whose values are subject to exogenous shocks. The payoff to the decision-maker is determined by the final values of the alternatives. We show that in a broad range of situations it is optimal for the decision-maker to allocate all resources to the most promising alternative, hence diversification is suboptimal. In particular this is the case when the decision-maker’s payoff is given by the weighted sum of the final values of alternatives or their convex transforms. In tournaments for a promotion the agents would rationally choose to put forth more effort in the early stage of the tournament in a bid to capture a larger share of employer’s investment, such as mentoring.

Keywords: tournaments, investment, promotion

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1 Introduction

This paper introduces and studies a class of decision problems which we call *investment tournaments*. In an investment tournament, the decision-maker and nature choose actions in a number of periods, sequentially or simultaneously. At each decision node, the decision-maker selects the level of investment in each of several alternatives available to her, with the value of each alternative increasing in the investment that it receives. The values of alternatives also change due to random shocks (actions of nature). The payoff to the decision-maker is determined by the final values of the alternatives.

Problems of this kind have received very little previous attention. In fact, this paper is the first to use the term investment tournament to denote this class of problems. Yet investment tournaments are ubiquitous, and come in a variety of forms. Career choice can be viewed as an investment tournament. A person deliberating whether to major in accounting or engineering, as two alternatives, will at first take some courses in each field. Taking courses can be viewed as investment into these alternatives. From an ex-post perspective the courses taken in accounting may not be useful for someone who eventually chooses engineering, but the student would only select a major after trying several field. The question for the student, as a decision-maker, would be how many courses to take in each discipline before making the final choice of a major. Similarly, contests for promotion between the employees in firms and organizations can also be viewed as investments tournaments, since the firms and organizations make substantial investments in the human capital of their employees, in particular, by providing training, coaching and mentoring.

Another example of investment tournaments is the process of new product development. Suppose that several prototypes are being developed simultaneously. The firm, as a decision-maker, has to allocate investment dollars among alternative prototypes before the performance (i.e. the value) of prototypes becomes known. The expected performance of a prototype is

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1Our basic investment tournament model highlights the aspects of the career choice stemming from the choice of investments into learning alternative professions, before the information regarding which one of them fits a person best, or has the highest value, is revealed. Obviously, the complexity of educational choices can not be fully captured by a simple investment tournament model. A person may take courses in various fields for reasons outside of our model, e.g. in order to learn what career suits him best or in order to satisfy intellectual curiosity. Yet, we believe that our model captures important aspects of students’ career choices.
increasing in the amount of resources committed to it. If the final product combines the features of several prototypes, then the profits of the firm would depend on the realization of several alternatives. On the other hand, if the new product is based exclusively on the best prototype, then the firm’s profits would depend only on the realization of the highest value alternative.\footnote{It is worth noting that many real-life decision problems share features of both investment tournaments and optimal search. For instance, both investment tournaments and search models are relevant for understanding different aspects of R&D investment decisions. For an example of applications of optimal search results to R&D experimentation, see Dahan and Mendelson (2001) who study optimal prototyping strategy. They investigate the optimal number of prototypes and the optimal combination between parallel and sequential prototyping. Note that in the context of optimal search literature, the amount of resources invested in each prototype is assumed to be exogenously fixed and all prototypes are assumed equally promising. In contrast, investment tournament model considers an environment where a firm can adjust its investment level into each prototype depending on preliminary (noisy) evaluations of the potential of each prototype. Thus, investment tournament model isolates an aspect of the optimal prototyping problem that has not been previously investigated.}

Below, we develop a model of investment tournaments capturing these and similar examples, and characterize the optimal investment strategy. The simplest model of an investment tournament has three periods. In the first and third periods the values of each alternative increase as a result of random shocks, while in the second period the decision-maker chooses the level of investment into each alternative by dividing a fixed budget among them. The final value of each alternative, realized in the third period, is equal to the sum of the first and the third period shocks plus the investment received by that alternative in the second period. The payoff to the decision-maker is determined by the final values of the alternatives.

The class of investment tournaments analyzed in this paper is far more general than a simple three period model. In Section 2 we consider investment tournaments with an arbitrary finite number of periods in which the decision-maker and the nature either alternate in taking actions or act simultaneously, and with the value function given by the weighted sum of the final values of the alternatives. A multi-period model captures the possibility that the decision-maker learns over time about the value of each alternative and uses this information to adjust the share of investment into each alternative. Proposition 1 characterizes the optimal investment strategy in such multi-period setting. It calls for investing all resources into the leading alternative which has the highest current value, at every decision node. Thus,
the optimal strategy magnifies the advantage of the leading alternative. Due to random shocks, the leading alternative at one decision node need not remain in the lead at the next decision node. We can interpret Proposition 1 as saying that the optimal strategy is invariably to put all the eggs in the “favorite basket,” although the “favorite basket” may change over time.

Proposition 1 has a simple corollary showing that the strategy of investing only in one favorite alternative in each period remains optimal when the decision-maker’s payoff depends only on the final value of the largest alternative, which fits the example of a student’s choice of a major. This particular setting is a generalization of the auction environment introduced in Schwarz and Sonin (2001).

Proposition 2 establishes that our main result continues to hold if the payoff to the decision-maker is a sum of convex transforms of the final values of all the alternatives.

Section 2.1 generalizes the benchmark model of investment tournaments further by allowing the returns to investment into an alternative to be decreasing. In this case, more than one alternative may receive positive investment at some decision nodes. Even though the optimal strategy in this case is less extreme than “putting all eggs in one basket,” it is still optimal to significantly favor the leading alternative (even if the lead is very small). This can explain why a student selecting between two majors may rationally devote substantially more effort to a major that seems slightly more promising. A very small amount of new information can reverse the order of the alternatives causing a student to dramatically change the amount of time (s)he invests in each alternative. Thus, a seemingly irrational jumping back and forth between majors may be consistent with expected utility maximization. Similarly, a firm working on several prototypes of a new product would always invest substantially more in the development of the prototype that, at the current moment, appears just slightly more promising than other prototypes.

Section 3 applies investment tournaments to personnel economics. It builds upon the literature on incentive aspects of tournaments. The study

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3This case is related to optimal search problem, as both are maximal problems. However, neither problem is a special case of the other. Optimal search literature studies the optimal strategy for investment in information acquisition, when there is a cost of obtaining information about a particular alternative. In contrast, in investment tournament problem the information about the value of each alternative is available at no cost to the decision-maker, but the issue is how to allocate the investment between alternatives.
of incentive tournaments was pioneered by Lazear and Rosen (1981) and has been extensively developed in the literature.\footnote{See, for example, Bhattacharya and Guasch (1988), Bull et al. (1987), Ehrenberg and Bognannnon (1990), Eriksson (1999), Ferrall (1996), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Taylor (1995).} In an incentive tournament workers (who play the role of alternatives in our terminology) are competing for a prize (usually a promotion). The performance (value) of each worker depends on his effort and luck. The best performing worker wins a promotion. The workers are motivated to exert costly effort because performance, and hence the probability of winning the tournament, increases in effort.

Within the incentive tournament framework, a firm’s profits depend primarily on the average performance of workers. The value of the worker who wins the tournament has no special significance, aside from her contribution to the total output. However, in many contexts, firm-specific capital of a worker who wins the tournament has special significance. For instance, in an up or out tournament, where the winners are promoted, and the losers are laid off, the losing contenders’ investments into firm-specific human capital are wasted from the ex-post perspective.\footnote{Galanter and Palay (1991) and Rebitzer and Taylor (2007) provide an analysis of the organizational structure of law firms highlighting the role of up-or-out promotion contests, or tournaments.}

We consider a setting where the workers accumulate firm-specific human capital over the course of a promotion contest. The value of a worker’s human capital at the end of the contest depends on a number of factors, including the investment into the worker’s human capital undertaken by the firm during the contest. Proposition 4 shows that, other things being equal, the effort exerted by the worker at the beginning of the tournament increases as the investment component of a tournament gets larger.

In the context of an up or out contest for a partnership, we show that the greater is the role of mentoring by the senior partners in formation of firm-specific human capital,\footnote{Mentoring is a scarce resource that can take the form of meeting important clients or being assigned to more creative and complex projects.} the harder the associates will work in the beginning of the tournament in a bid to get ahead and reap a larger share of mentoring resources.

Several contributions in the literature on promotion tournaments, in particular (Barut and Kovenock (1998), Krishna and Morgan (1998), Moldovanu and Sela (2001)), examine the effect of the design of tournament prizes on
the efforts of the contestants and provide recipes for the optimal design of such prizes. The focus of this paper is different. We concentrate on understanding the investment behavior by the other party in these contests—the decision-maker, who organizes the tournaments and benefits from the value generated by the contestants. Hence, this paper puts an emphasis not on the structure of prizes for the contestants, but on the value, or output, generated by the contestants for the firm. Another important feature, distinguishing our approach from most contributions in the literature on tournaments, is its dynamic nature, as we consider investment decisions made dynamically throughout multiple periods.

The rest of the paper is organized as follows. Section 2 introduces and solves the model of investment tournaments under different specifications of the decision-maker’s value and cost functions. Section 3 combines investment tournament model with the model of incentive tournaments and applies the results to personnel economics. Section 4 concludes. The proofs are relegated to an Appendix.

2 Model and Main Results

In this section, we formulate and solve a model of an investment tournament. To start, consider a $T$-period decision problem. Each period the decision-maker chooses how to distribute an amount $B$ of investment resources among $N$ alternatives.\footnote{For simplicity of exposition, we assume that the amount of resources available for investment is the same in each period. Results and proofs do not change qualitatively if the investment budget was changing across periods deterministically or randomly.} Let $b_{ti}$ denote the nonnegative amount of resources invested into alternative $i$ at period $t$. Then the decision-maker’s action in period $t$ can be represented by a vector $b_t = (b_{t1}, b_{t2}, ..., b_{tN})$ lying in the feasible action space $A = \{b \in \mathbb{R}^N : b_i \geq 0, \sum_{i=1}^{N} b_i = B\}$.

In each period the nature draws a random shock to each alternative from an atomless distribution $F(\cdot)$ with support $[0, \infty)$. The shock to the value of alternative $i$ in period $t$ is denoted by $s_{ti}$. The shocks are independent across alternatives and across time. The action of nature at time $t$ is denoted by vector $s_t = (s_{t1}, s_{t2}, ..., s_{tN})$.

Here, we assume that the decision-maker and nature act simultaneously, but the problem would be identical if they took alternating turns, with the decision-maker being the first to move in each period. The value of alternative
i at the terminal node is given by $V_i = \sum_{t=1}^{T} s_{it} + \sum_{t=1}^{T} b_{it}$. That is, the final value of alternative $i$ is the sum of all shocks and all investments into it. The winner of a tournament is the alternative with the highest value at the end of period $T$.

Let the history at time $t$ be denoted by $h_t = (s_1, b_1, s_2, b_2, ..., s_{t-1}, b_{t-1})$. The payoff-relevant information contained in a history $h_t$ can be summarized by a vector of the current values of alternatives $V_t = V_t(h_t) = (V_{t1}, V_{t2}, ..., V_{tN})$, where $V_{ti} = \sum_{t' = 1}^{t-1} s_{t',i} + \sum_{t' = 1}^{t-1} b_{t',i}$.\footnote{The value at the terminal node is $V_i \equiv V_{(T+1)i}$.} We will say that an alternative $i$ is a favorite in period $t$ if $V_{ti} \geq V_{tj}$ for all $j \in \{1, ..., n\}$. Naturally, there may be more than one favorite. The winner of an investment tournament is the favorite at the terminal node.\footnote{If there is more than one favorite at the terminal node, then an arbitrary tie-breaking rule can be used to determine the winner.}

Let us also define the extended value of alternative $i$ at time $t$, $\tilde{V}_{ti}$, as its value at the end of period $t$ after the decision-maker has allocated the period-$t$ budget but not including the period-$t$ shock, i.e. $\tilde{V}_{ti} \equiv V_{ti} + b_{ti}$. Thus, the values of alternatives at period $t$ contain all relevant information about the history prior to $t$, and extended values of alternatives at period $t$ contain all relevant information regarding the history prior to period $t$ and the decision-maker’s action at $t$. If the decision-maker follows some strategy $\sigma$, her expected payoff at time $t$ can be expressed as a function of $\sigma$ and the current values of alternatives $\Pi(V_{t1}, V_{t2}, ..., V_{tN}, \sigma)$ or $\tilde{\Pi}(\tilde{V}_{t1}, ..., \tilde{V}_{tN}, \sigma)$.

First, we characterize the decision-maker’s optimal strategy in the tournament when her payoff is equal to a weighted sum of the terminal values of all alternatives, with higher-ranked alternatives assigned larger weights. That is, the decision-maker’s payoff is given by $\sum_{k=1}^{N} \lambda_k V_{r(k)}$ where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N \geq 0$, and $V_{r(k)}$ denotes the final value of the alternative ranked $k$-th at the end of the tournament; $r(1)$ and $r(N)$ are the alternatives with the lowest and the highest final values respectively. Thus, the weight of an alternative increases in its rank. The following proposition characterizes the optimal strategy in this case.

**Proposition 1** Suppose that the decision-maker’s payoff is given by $\sum_{k=1}^{N} \lambda_k V_{r(k)}$ where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N \geq 0$, and $r(k)$ denotes the alternative with the $k$-th highest final value.

The following strategy is optimal in the investment tournament: in period $t \in \{1, ..., T\}$ the decision-maker allocates all investment resources to an alternative that is a favorite in this period. If there is more than one favorite
alternative in period \( t \), then all resources in period \( t \) are allocated to one of the favorites. The remaining \( N - 1 \) alternatives receive zero amount of investment in period \( t \).

Note that for \( \lambda_1 = \lambda_2 = \lambda_3 \ldots = \lambda_N \) any investment strategy is optimal. However, if at least one inequality is strict, then the optimal strategy calls for investing all resources into a favorite alternative.

This formulation of the decision-maker’s value function with \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N \) is appropriate for a promotion tournament within a firm which makes investments in the human capital of its employees, when the promotion is based on the employees’ past performance, and the employees are given increased responsibilities depending on their performance in the tournament.

An important special case of this set-up is where \( \lambda_1 = 1, \lambda_2 = \ldots = \lambda_N = 0 \). The following corollary of Proposition 1 applies in this case.

**Corollary 1** Suppose that the decision-maker’s payoff is equal to the value of the winning alternative \( \max \{ V_1, ..., V_N \} \). Then the decision-maker’s optimal strategy in period \( t \in \{ 1, ..., T \} \) is to invest all resources into one favorite alternative in period \( t \).

The Corollary is important since in a number of environments the decision-maker cares only about the realization of the highest value alternative. For example, this is so in the promotion tournament with up-or-out rule, under which only the winner gets promoted and stays with the firm. Similarly, this case applies when a student is choosing a major, or a firm is selecting a new product among several prototypes, or a person is choosing a business or life partner among several candidates.

On the other hand, when the promotion is merely a prize and does not entail increased responsibility, all workers stay with the firm whether promoted or not (as in medical doctors’ practices), and a worker’s value reflects how well she has learned her trade or profession, then each worker’s human capital contributes to the firm’s profits in the same way. So, it is more appropriate to model the firm’s final payoff as \( \sum_{k=1}^{N} \mu(V_k) \) where \( \mu(\cdot) \) is increasing and convex. The latter assumptions on \( \mu(\cdot) \) reflect the accelerating nature of learning one’s profession.

Surprisingly, even in this case it is optimal for the firm, as the decision-maker, to invest all resources in any period in the favorite alternative (a worker with the highest human capital) in that period. Indeed, we have:
Proposition 2 Suppose that the decision-maker’s payoff is equal to \( \sum_{k=1}^{N} \mu(V_k) \) where \( \mu'(\cdot) > 0 \) and \( \mu''(\cdot) > 0 \). Then the decision-maker’s optimal strategy is to invest all resources in period \( t \in \{1, \ldots, T\} \) into one favorite alternative at period \( t \).

Note that Proposition 2 is not a special case of Proposition 1, because here the value of an alternative does not depend on its rank.

The results of Propositions 1 and 2 hold if the decision-maker and nature take turns making their moves. The proofs remains virtually unchanged.

2.1 Tournaments with Decreasing Returns

So far, we have assumed that the decision-maker has a fixed investment budget \( B \) in each period. In this section, we consider a different setting in which the decision-maker can choose to invest any amount in each period, but the returns to investment decrease in the total amount of investment into all alternatives. Specifically, suppose that the decision-maker’s action space in period \( t \) is given by

\[
A_t = \{b_t : b_{ti} \geq 0 \text{ for all } i = 1 \ldots N\}.
\]

The cost of investment in period \( t \) is measured by an increasing and convex function \( \sum_{t=1}^{T} C(\sum_{i=1}^{N} b_{ti}) \), reflecting that the returns to investment are decreasing in the total amount of investment in a given period. The decision-maker’s payoff is equal to the final value of the winning alternative.

This model is natural when the investment involves the decision-maker’s time or effort, since the returns to time, or effort, are typically decreasing in the total amount of it. For example, consider a student deliberating the choice between accounting and engineering majors. One extra course in accounting brings a student one course closer to completing the accounting major, regardless of whether it is a second or fifth course in accounting. That is, an increase in the value of the accounting alternative from taking one course in accounting is the same, regardless of the number of accounting courses that a student takes in a given semester. At the same time, the student’s aggregate effort cost of attaining a certain performance level in a given semester typically depends on the total number of courses that the student takes in this semester, rather than on the distribution of courses by field.
Generalizing the proof of Proposition 1, we can show that in this case there exists an optimal strategy \( b_{ti} (V_{t1}...V_{tN}) \) that depends only on the current values of the alternatives. The following Proposition shows that it remains optimal in any time period to allocate all resources, or effort, to the favorite alternative.

**Proposition 3** Suppose that the action space at each decision node is 

\[
A = \{ b \in \mathbb{R}^N : b_{ti} \geq 0 \}
\]

and the decision-maker’s payoff is given by

\[
\max \{ V_1, V_2...V_N \} - \sum_{t=1}^{T} C(B_t),
\]

where \( C'(\cdot) > 0 \) and \( C''(\cdot) > 0 \) and \( B_t = \sum_{i=1}^{N} b_{ti} \). Then,

(i) an optimal strategy requires that at each decision node only one favorite alternative receives positive investment;

(ii) the amount of optimal investment in period \( t \) is increasing in the value of the favorite alternative at decision node \( t \).

This proposition confirms the robustness of our main result establishing the optimality of investing only in one favorite alternative in every time period. However, it does not imply, for example, that the student will never switch between majors. On the contrary, we could observe complete switching if the job-market situation changes and one profession becomes more attractive than the other. The latter change would constitute an act of nature in our model.

3 Bilateral Investment in Promotion Tournaments

In this section we consider a situation where, unlike in the previous sections, competing alternatives are themselves players in the tournament game, and can take actions affecting their values. In order to emphasize the active role played by the alternatives, we will from now on refer to them as “contenders.”

As an example, consider a tournament for a promotion among associates in a law partnership or a consulting firm. The decision-maker is a senior
partner or a management committee of the firm. This decision-maker selects the levels of investment into contenders’ firm-specific human capital. This may involve dividing scarce mentoring resources among contenders, assigning them to more or less high-profile projects, etc. At the same time, the contenders also choose the amount of effort or investment in their own human capital to increase their own value. For simplicity, we assume that the tournament is up-or-out, and so the investments into firm-specific human capital of associates who are not promoted are wasted from the ex-post perspective. However, as shown above, the results of the previous sections hold under a range of specifications and value functions, as long as the investment in the winning alternative is at least as useful as the investment into a losing alternative. Similar extensions would also apply to the results presented below.

Our goal in this section is to characterize the outcome of this tournament where the investment incentives of the decision-maker and the contenders interact. We model this situation as follows. In every period starting from \( t = 1 \) the contenders select nonnegative efforts which they invest in acquiring firm-specific human capital. Contender \( i \)'s effort in period \( t \) is denoted by \( e_{ti} \). Her cost of effort \( e_{ti} \) in period \( t \) is given by \( g(e_{ti}) > 0 \), where \( g'(e_{ti}) > 0 \), \( g''(e_{ti}) > 0 \) for all \( e_{ti} \geq 0 \). Also, in every period starting from \( t = 1 \) the decision-maker selects the levels of investment into each contender. Her action space is \( A^d = \{ b_t \in R^N : b_{ti} \geq 0 \sum_{i=1}^N b_{ti} = B \} \), where \( b_{ti} \) denotes the decision-maker’s investment in contender \( i \) in period \( t \).

We continue to assume that in each period the nature independently draws a random shock to the value of each contender from an atomless distribution \( F(\cdot) \) over nonnegative support. A random shock to the value of contender \( i \) in period \( t \) is denoted by \( s_{ti} \).

Nature takes an action in period zero. In subsequent periods the nature, the decision-maker and the contenders move simultaneously. Thus, the value of contender \( i \) in period \( t \in \{1, ..., T\} \), \( V^t_i \), is a sum of her/his own investments, the investments by the decision-maker and random shocks up to period \( t \). That is, \( V^t_i = \sum_{t'=0}^t s_{t'i} + \sum_{t'=1}^t b_{t'i} + \sum_{t'=1}^t e_{t'i} \). The terminal value of contender \( i \) in the last period \( T \) is equal to \( V_i = V^T_i \). The history of the game after period \( t \in \{1, ..., T\} \) is summarized by the vector of the contenders’ values \((V^t_1, ..., V^t_N)\).

\(^{10}\)Galanter and Palay (1991) provide a detailed account of the role of tournaments in large law firms in the U.S. On this topic, see also Rebitzer and Taylor (2007).
The information structure of the tournament is as follows. In any period the decision-maker knows the value of each contender in the previous period. The contenders do not observe random shocks or investments by the decision-maker. So, in every period each contender’s information set contains only his or her effort levels in the previous periods.

The contender with the highest final value wins the tournament. The decision-maker’s payoff is the value of the tournament winner \( \max\{V_1, ..., V_N\} \). The payoff of contender \( i \), who loses the tournament, is equal to the negative of the sum of the costs of effort that (s)he invested across all time periods, i.e. \(- \sum_{t=1}^{T} g(e_{ti})\). If contender \( i \) wins the tournament her/his payoff is \( R - \sum_{t=1}^{T} g(e_{ti}) \), where \( R \) can be interpreted as the rent associated with winning a promotion.

Thus, a promotion tournament of this section combines an investment tournament introduced in the previous section, where the only players are the decision-maker and the nature, with the elements of an incentive tournament where the players are the contenders and the nature (see Lazear and Rosen (1981)). The following result characterizes symmetric Nash equilibria in this promotion tournament game. The symmetry restriction only requires that all alternatives are treated symmetrically by the decision-maker.

**Proposition 4** Every symmetric pure strategy Nash equilibrium of a promotion tournament has the following properties:

(i) in every period, the decision-maker chooses to invest all resources into one of the favorite contenders;

(ii) the effort of each contender is decreasing over time.

Observe that the investment strategy of the decision-maker is qualitatively similar to her strategy in the tournament without contenders’ investment. This is so because the contenders cannot condition their behavior on the decision-maker’s actions.

On the other hand, the contenders do have an incentive to influence the actions of the decision-maker. So, the contenders invest more effort into improving their position at the early stages of the tournament, because early effort can attract investments from the decision-maker putting a promising employee on a “fast track” for a promotion. Consequently, all contenders will put forth more effort at the early stages of the tournament in order to become a leader.

Note that we have assumed that contender \( i \)’s effort cost is separable across periods. If her cost of effort was equal to \( Q(\sum_{t=1}^{T} e_{ti}) \), where \( Q'(\cdot) > 0 \)
, $Q''(\cdot) > 0$, then the decision-maker would invest all resources into the favorite contender, and each contender would invest all effort in the first period. This is so because contenders receive no additional information after the first period. Consequently, they would shift all effort to the first period in a bid to receive more mentoring (investment) from the decision-maker.

The results of this section are consistent with the phenomena of a “rat race” that young professionals presumably have to endure and “fast track” that they aspire for. In particular, we should expect that the first-year graduate students would work harder than the second-year students, and the first-year associates in a law firm would put in longer hours than the later-year associates. Anecdotal evidence regarding career development appears to confirm that. \(^{11}\)

\section{Concluding Remarks}

In this paper we have analyzed investment tournaments, which we have modelled both as a decision-problems, and as a game. We have shown that in every period the decision-maker will optimally allocate all her investment resources to a single alternative, or a contender. This result holds robustly under a variety of specifications and assumptions.

In many tournaments, a contender with a small lead tends to enjoy substantially better chances of winning the tournament than other contenders. This paper provides an explanation for why an early leader may be favored by a fully rational decision-maker.

Applications of investment tournaments are not limited to career development and product design areas, and stretch beyond the purely economic domain. Indeed, investment tournament is a metaphor for many decisions involving choice between several alternatives. For instance, investment tour-

\(^{11}\) Landers et al. (1996) offer an explanation of rat race based on the adverse selection, and provide evidence that associates in law firms log in an inefficiently high number of hours. Holmstrom (1999) predicts that effort would be relatively high early in a worker’s career due to signaling. Andersson (2002) considers a model in which the wage history of a worker is observable by a hiring firm, but the ability and employment contract of a job applicant are not observable. He shows that effort early in a worker’s career is distorted upward because the first-period wage will be used as an ability signal by the employer who may hire a worker in the second period. This paper generates similar predictions but in a very different setting. In particular, we do not rely on asymmetric information or bounded rationality.
naments may help to explain why people tend to date one person at a time. Dating amounts to spending time with a potential partner. This can be viewed as an investment into the relationship-specific value of a particular match. By Proposition 4, even if it is highly uncertain which partner will be ultimately preferred, it is still optimal to invest disproportionately into the most promising alternative. Proposition 4 predicts that the effort invested into a relationship by competing contenders is largest at the early stages of the relationship.

Of course, the investment tournament model does not reflect all the complexity of a dating problem. The limitations of investment tournament model in application to dating suggest a few promising directions for future work. It will be interesting to extend investment tournament model to two-sided matching, where both sides of the market can make investments into individual contenders on the other side of the market. Also, investment tournament model neglects the search aspect of dating. In this and in many other contexts optimal search models and investment tournament models are complementary—each highlights one important aspect of choice while neglecting some others. The models of search focus on information acquisition, assuming away a possibility of investment into improving the quality of a match. In contrast, the investment tournament model assumes away a possibility of strategic information acquisition while explicitly modeling investments into relationships. Combining search and matching models with investment tournament models would open directions in research. We intend to pursue these directions in future work.

Finally, our analysis of promotion tournaments is restricted to the situations where the contestants do not observe the performance (values) of other contestants at the interim stages where they make additional effort decisions. It will be useful to relax this observation and consider how enhances observability affects their behavior throughout the tournament. We plan to address this issue in future research. Intuition suggests that our main qualitative results should survive.
References


Proof of Proposition 1.
First, we formulate and prove a useful Lemma which allows us to focus on Markov strategies.

**Lemma 1** There exists a Markov optimal strategy $\sigma$.

**Proof.** The proof is by backwards induction. Consider the last period $T$. An optimal strategy in this period, $\sigma^*_T$, prescribes such allocation of budget $B$ between $N$ alternatives that maximizes the expected value of the objective. That is, $\sigma^*_T$ is a solution to the following problem:

$$\max_{(b_{T1},...,b_{TN} \geq 0; \sum_{i=1}^n b_{Ti} = B)} E_F \times \ldots \times F \max\{V_{T1} + b_{T1} + s_{T1}, ..., V_{TN} + b_{TN} + s_{TN}\}. \quad (1)$$

Note that the expectation is taken with respect to the vector $(s_{T1}, ..., s_{TN})$. Since the objective is continuous in $(b_{T1}, ..., b_{TN})$ and the feasible domain for $(b_{T1}, ..., b_{TN})$ is compact, this maximization problem has a solution - an optimal strategy $\sigma^*_T$. Clearly, this solution depends only on $(V_{T1}, ..., V_{TN})$, i.e. $\sigma^*_T$ is Markov.

By Berge’s maximum Theorem, the value of (1) is a continuous function of $(V_{T1}, ..., V_{TN})$ which we denote by $W^T(V_{T1}, ..., V_{TN})$.

Proceeding to period $T - 1$, we can use a similar method to show that the optimal strategy for this period, $\sigma^*_{T-1}$, is Markov. Indeed, $\sigma^*_{T-1}$ prescribes an allocation of budget $B$ between $N$ alternatives, $(b_{(T-1)1},...,b_{(T-1)N})$, to maximize the expected value of the following objective:

$$E_F \times \ldots \times F W^T(V_{(T-1)1} + b_{(T-1)1} + s_{(T-1)1}, ..., V_{(T-1)N} + b_{(T-1)N} + s_{(T-1)N})$$

This objective is also continuous in $(b_{(T-1)1}, ..., b_{(T-1)N})$ and the feasible domain for $(b_{(T-1)1}, ..., b_{(T-1)N})$ is compact, so the maximization problem has a solution - an optimal strategy $\sigma^*_{T-1}$ which depends only on $(V_{(T-1)1}, ..., V_{(T-1)N})$, i.e. $\sigma^*_{T-1}$ is Markov. Proceeding backwards through all periods to the start of the game, we can establish that an optimal strategy $(\sigma^*_1, ..., \sigma^*_T)$ is Markov. ■

With Lemma 1 in hand, we proceed to prove the Proposition in two steps. Step 1 shows that in every period, an optimal strategy requires all investment to be allocated to one alternative. Step 2 shows that the alternative that receives all investment in some period $t$ is a favorite in that period.
Step 1. Let $\sigma^*$ be an optimal Markov strategy. Recall that $V_t = (V_{t1}, ..., V_{tN})$ ($\tilde{V}_t = (\tilde{V}_{t1}, ..., \tilde{V}_{tN})$) stands for the vector of values of alternatives (extended alternatives) at period $t$, and $V_i$ stands for the terminal value of alternative $i$ at the end of the tournament. Note that $V_i = \tilde{V}_t + s_{it} + \sum_{\tau=t+1}^{T} (b_{i\tau} + s_{i\tau})$.

Given the information available at period $t$ and given the decision-maker’s Markov strategy $\sigma^*$, we can write $s_{it} + \sum_{\tau=t+1}^{T} (b_{i\tau} + s_{i\tau}) = \eta_{it}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$. That is, $\eta_{it}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$ is the change in the value of alternative $i$ between period $t$, (after investment $b_{it}$ has been made) and its terminal value, given that the decision-maker uses the strategy $\sigma^*$ and the profile of random shocks in periods $t, ..., T$ is given by $(s_t, ..., s_T)$. Note that $\eta_{it}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$ depends on the whole vector $\tilde{V}_t$ of extended alternatives in period $t$ and the profile of random shocks $(s_t, ..., s_T)$, because Markov strategy $\sigma^*$ determining investment decisions depends on $\tilde{V}_t$ for all $r \in \{t, ..., T\}$. For brevity, we will sometimes write $\eta_{it}(\tilde{V}_t, \sigma^*)$ omitting the dependence on $(s_t, ..., s_T)$, but this dependence is implicitly understood. Then, the terminal value of alternative $i$, as a function of the information available at period $t$, the strategy and the shocks’ profile, can be written as $V_i(\tilde{V}_t, \sigma^*, s_t, ..., s_T) = \tilde{V}_t + \eta_{it}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$.

Further, let $R_{t,r}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$ denote the rank function which is equal to 1 if the terminal value of alternative $i$, $V_i$, is $r$-th highest among the terminal values of all $N$ alternatives $(V_1, ..., V_N)$, and is equal to zero otherwise. Note that the $R_{t,r}(\cdot)$ depends on the vector of values of extended alternatives $\tilde{V}_t$ at time $t$ and the profile of random shocks in periods $t, ..., T$ because, given $\sigma^*$, their combination determines the terminal values of all alternatives.

Suppose that at period $t$, $\sigma^*$ prescribes positive investments $b_{ij}$ and $b_{ik}$ into alternatives $j$ and $k$. Let $\delta \in (-\min\{b_{jk}, b_{ij}\}, \min\{b_{jk}, b_{ij}\})$ be a small reallocation of investment between alternatives $j$ and $k$ on top of what is prescribed by strategy $\sigma^*$. We will make this reallocation without changing the future allocation rule, so we still use $\tilde{V}_t$ as an argument of $\eta_{it}(\cdot)$ and $R_{t,r}(\cdot)$ for all $i$ and $r \in \{1, ..., N\}$, so that, after this modification, the strategy $\sigma^*$ prescribes the same actions in periods $t+1, ..., T$ as without this modification.

Define a vector of perturbed terminal values $V^f(i(\delta), k(-\delta))(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$ as follows:

$$V^f(i(\delta), k(-\delta))(\tilde{V}_t, \sigma^*, s_t, ..., s_T) \equiv$$

$$(V_i(\tilde{V}_t, \sigma^*, s_t, ..., s_T), ..., V_j(\tilde{V}_t, \sigma^*, s_t, ..., s_T) + \delta, ..., V_k(\tilde{V}_t, \sigma^*, s_t, ..., s_T) - \delta, ..., V_N(\tilde{V}_t, \sigma^*, s_t, ..., s_T)).$$
That is, all entries of the vector $\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$, except the $j$-th and $k$-th, are the same as in the vector $\mathbf{V}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$, while the $j$-th ($k$-th) entry of $\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$ is equal to the $j$-th ($k$-th) entry of $\mathbf{V}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)$ plus (minus) $\delta$.

Then for any $\delta \in (-\min\{b_{tk}, b_{tj}\}, \min\{b_{tk}, b_{tj}\})$, the decision-maker’s expected value function in period $t$ is equal to

\begin{align*}
E_{(s_t, ..., s_T)} & \sum_{i=1, ..., N, i \notin \{j,k\}} \sum_{r=1, ..., N} \lambda_r R_{i,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T))V_i(\tilde{V}_t, \sigma^*, s_t, ..., s_T) \\
+ & E_{(s_t, ..., s_T)} \sum_{r=1, ..., N} \lambda_r R_{j,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T))V_j(\tilde{V}_t, \sigma^*, s_t, ..., s_T) + \delta \\
+ & E_{(s_t, ..., s_T)} \sum_{r=1, ..., N} \lambda_r R_{k,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T))V_k(\tilde{V}_t, \sigma^*, s_t, ..., s_T) - \delta
\end{align*}

(2)

Consider (2) as a function of $\delta$. Since $b_{tj} > 0$ and $b_{tk} > 0$ are optimal investments, $\delta = 0$ is an interior optimum of (2). Therefore, the first derivative of (2) with respect to $\delta$ must be equal to zero at $\delta = 0$ while the second derivative must be nonnegative. In the rest of the proof, we show that this is not the case, thereby establishing a contradiction with our original hypothesis that $b_{tj} > 0$ and $b_{tk} > 0$. Indeed, the first derivative of (2) with respect to $\delta$ is equal to:

\begin{align*}
E_{(s_t, ..., s_T)} & \sum_{r=1, ..., N} \lambda_r R_{j,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)) \\
-E_{(s_t, ..., s_T)} & \sum_{r=1, ..., N} \lambda_r R_{k,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T)) \\
+ & E_{(s_t, ..., s_T)} \left( \sum_{i=1}^{N} \sum_{r=1, ..., N} \lambda_r \frac{\partial R_{i,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T))}{\partial V_j} V_i(\tilde{V}_t, \sigma^*, s_t, ..., s_T) \right) \\
- & E_{(s_t, ..., s_T)} \left( \sum_{i=1}^{N} \sum_{r=1, ..., N} \lambda_r \frac{\partial R_{i,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T))}{\partial V_k} V_i(\tilde{V}_t, \sigma^*, s_t, ..., s_T) \right)
\end{align*}

(3)

Next, note that the third and fourth terms in (3) are equal to zero. To see this note that for all $r \in \{1, ..., N\}$, we have:

\begin{equation}
E_{(s_t, ..., s_T)} \left( \sum_{i=1}^{N} \frac{\partial R_{i,r}(\mathbf{V}^{f(j,k)(-\delta)}(\tilde{V}_t, \sigma^*, s_t, ..., s_T))}{\partial V_j} V_i(\tilde{V}_t, \sigma^*, s_t, ..., s_T) \right) = 0
\end{equation}

(4)
To see why (4) is equal to zero, note that \( \sum_{i=1}^{N} R_{i,r}(\mathbf{V}) = 1 \) for all \( r \) and all \( \mathbf{V} \), because some alternative is always ranked \( r \)-th. So, 
\[
\frac{\partial R_{j,r}(\mathbf{V}, \delta)}{\partial \mathbf{V}_j} = - \sum_{i' \in \{1, \ldots, N\}, i' \neq j} \frac{\partial R_{i',r}(\mathbf{V}, \delta)}{\partial \mathbf{V}_j}.
\]
Furthermore, if \( \frac{\partial R_{i',r}(\mathbf{V}, \delta)}{\partial \mathbf{V}_j} \neq 0 \) for some \( i' \in \{1, \ldots, N\}, i' \neq j \) and some \((\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T)\), then we must have \( V_{i'}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T) = V_j(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T) + \delta \), since the derivative of the rank function \( R_{i',r}(.) \) with respect to the terminal value of the \( j \)-th alternative is not equal to zero only if the terminal value of alternatives \( i' \) and \( j \) are equal to each other. This establishes that the third term in (3) is equal to zero. An identical argument establishes that the fourth term in (3) is also equal to zero.

Thus, we conclude that the derivative of (2) with respect to \( \delta \) is equal to the first two terms of (3) which can be rearranged as follows:

\[
E_{(\mathbf{s}_t, \ldots, \mathbf{s}_T)} \left( \sum_{r=1}^{N} (\lambda_r - \lambda_{r+1}) \sum_{m=1}^{r} R_{j,m}(\mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T)) \right) -
E_{(\mathbf{s}_t, \ldots, \mathbf{s}_T)} \left( \sum_{r=1}^{N} (\lambda_r - \lambda_{r+1}) \sum_{m=1}^{r} R_{k,m}(\mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T)) \right)
\]  

(5)

where by convention we set \( \lambda_{N+1} = 0 \). Note that \( \lambda_r - \lambda_{r+1} \geq 0 \) for all \( r \in \{1, \ldots, N\} \).

Thus, to show that the second derivative of (2) is increasing in \( \delta \) and hence to complete the proof, it suffices to show that for all \( r \in \{1, \ldots, N\} \),

\[
E_{(\mathbf{s}_t, \ldots, \mathbf{s}_T)} \sum_{m=1}^{r} R_{j,m}(\mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T)) \text{ is increasing in } \delta 
\]

and

\[
E_{(\mathbf{s}_t, \ldots, \mathbf{s}_T)} \sum_{m=1}^{r} R_{k,m}(\mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T)) \text{ is decreasing in } \delta.
\]

To see the latter let \( \mathbf{V}^{f(j,d),k(-d)} \) denote the \( r \)-th highest entry in the vector of alternatives \( \mathbf{V}^{f(j,d),k(-d)} \). Then note that

\[
E_{(\mathbf{s}_t, \ldots, \mathbf{s}_T)} \sum_{m=1}^{r} R_{j,m}(\mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T))
\]  

\[= P_{\text{prob}}(\mathbf{s}_t, \ldots, \mathbf{s}_T)(\mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T) \geq \mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \delta)) \]  

(6)

The latter is the probability that alternative \( j \) is \( r \)-th highest or higher in the vector of alternatives \( \mathbf{V}^{f(j,d),k(-d)} \). Clearly, this probability is non-decreasing in \( \delta \) because an increase in \( \delta \) increases \( \mathbf{V}^{f(j,d),k(-d)} \), the value of alternative \( j \), lowers \( \mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T) \), the value of alternative \( k \), and leaves the values of all other alternatives unchanged. Similarly, we can show that

\[
E_{(\mathbf{s}_t, \ldots, \mathbf{s}_T)} \sum_{m=1}^{r} R_{k,m}(\mathbf{V}^{f(j,d),k(-d)}(\tilde{\mathbf{V}}_t, \sigma^*, \mathbf{s}_t, \ldots, \mathbf{s}_T)) \text{ is decreasing in } \delta.
\]
Next, we will show that the alternative that receives all investment in any period $t$ must be a favorite. That is, if alternative $i'$ receives all investment in period $t$, then it must be that $i' \in \arg \max_{i \in \{1, \ldots, N\}} V_{ti}$. The proof is by contradiction. So suppose not, i.e., the optimal investment vector in period $t$, $\hat{b}_t^* = (b^*_{t1}, \ldots, b^*_{tN})$, is such that $b^*_{ti} = B$ and $i' \notin \arg \max_{i \in \{1, \ldots, N\}} V_{ti}$ and $b^*_{ti} = 0$ for $i \neq i'$.

There are two cases to consider.

**Case 1.** $V_{tj'} - V_{ti'} < B$ for some $j' \in \arg \max_{i \in \{1, \ldots, N\}} V_{ti}$.

In this case, we have $B > \tilde{V}_{ti'} - \tilde{V}_{tj'} = (V_{ti'} + b^*_{ti'}) - (V_{tj'} + b^*_{tj'}) > 0$. Since the optimal strategy $\sigma^*$ is Markov and the problem is symmetric, the same expected payoff can be attained by making the following alternative investment decisions $\hat{b}_t$ in period $t$: $\hat{b}_{ti'} = V_{tj'} - V_{ti'}$ and $\hat{b}_{tj'} = B - \hat{b}_{ti'}$. That is, we have $\Pi(V_t + \hat{b}_t^*, \sigma^*) = \Pi(V_t + \hat{b}_t, \sigma^*)$. But the argument in the first part of the proof establishes that investment $\hat{b}_t$ is strictly suboptimal, so $\hat{b}_t^*$ cannot be an optimal investment allocation either.

**Case 2.** $V_{tj'} - V_{ti'} \geq B$ for some $j' \in \arg \max_{i \in \{1, \ldots, N\}} V_{ti}$.

In this case, we have $\tilde{V}_{ti'} - \tilde{V}_{tj'} = (V_{ti'} + b^*_{ti'}) - (V_{tj'} + b^*_{tj'}) \leq 0$. Then, consider a reallocation of investment in period $t$ in addition to $\hat{b}_t^*$. Particularly, suppose that in period $t$ alternative $j'$ receives an investment $\delta$, for some $\delta \in (0, B)$, while $i'$ receives investment $B - \delta$. This reallocation $\delta$ is feasible. Further, suppose that in all subsequent periods the decision-maker continues to use the same optimal strategy $\sigma^*$ ignoring this reallocation of investment. Then, using the same computation as in Part 1 of the proof and differentiating with respect to $\delta$, at $\delta = 0$, we conclude that the derivative of the expected payoff with respect to $\delta$ is equal to (5), with $j = j'$ and $k = i'$. From the result of Step 1, it follows that the probability distribution of $V_{j'}$ first-order stochastically dominates the probability distribution of $V_{i'}$. Then the value of (5), where $j = j'$ and $k = i'$, is strictly positive. That is, the reallocation of investment from $i'$ to $j'$ strictly increases the expected payoff to the decision-maker. Hence, $\hat{b}_t^*$ is suboptimal. So the optimal strategy requires allocating all investment to a favorite alternative in every period.

**Proof of Proposition 2.**

We establish this Proposition by modifying the proof of Proposition 1. Specifically, we can use the same notation and the same sequence of steps. Below, we will only explain the steps that require modification compared to the proof of Proposition 1.

First, we note that there exists a Markov optimal strategy. The proof is identical to that of Lemma 1. Further, we proceed by contradiction as in the proof of Proposition 1. Specifically, we assume that in period $t$ an
optimal strategy prescribes positive investments $b_{tj}$ and $b_{tk}$ into alternatives $j$ and $k$, and consider a small perturbation $\delta \in (-\min\{b_{tk}, b_{tj}\}, \min\{b_{tk}, b_{tj}\})$ reallocating investments between alternatives $j$ and $k$ on top of what is prescribed by the optimal strategy. This reallocation is made without changing the future allocation rule. Then the decision-maker’s objective is given by the following counterpart of expression (2):

$$
E(s_t, \ldots, s_T) \sum_{i=1, \ldots, N, i \notin \{j, k\}} \mu(V_i(\tilde{V}_t, \sigma^*, s_t, \ldots, s_T)) + E(s_t, \ldots, s_T) \left( \mu(V_j(\tilde{V}_t, \sigma^*, s_t, \ldots, s_T) + \delta) + \mu(V_k(\tilde{V}_t, \sigma^*, s_t, \ldots, s_T) - \delta) \right)
$$

(7)

The first-order derivative of the objective with respect to $\delta$ is equal to:

$$
E(s_t, \ldots, s_T) \left( \mu'(V_j(\tilde{V}_t, \sigma^*, s_t, \ldots, s_T) + \delta) - \mu'(V_k(\tilde{V}_t, \sigma^*, s_t, \ldots, s_T) - \delta) \right)
$$

(8)

while the second-order derivative is given by:

$$
E(s_t, \ldots, s_T) \left( \mu''(V_j(\tilde{V}_t, \sigma^*, s_t, \ldots, s_T) + \delta) + \mu''(V_k(\tilde{V}_t, \sigma^*, s_t, \ldots, s_T) - \delta) \right) > 0
$$

(9)

Since (9) is positive, $\delta = 0$ cannot be an optimal choice. This establishes that it cannot be optimal to make positive investments in two different alternatives in any period $t$.

To prove that an optimal strategy requires allocating all investment to a favorite alternative in every period, we can use the same argument that was used to establish a similar assertion in Proposition 1, with the only difference that in the current case the proof has to refer to (8), instead of (5). Q.E.D.

**Proof of Proposition 3**

The proof of the first part of the Proposition can be provided along the same lines as its counterpart in Proposition 1. That it, we suppose that at period $t$, the optimal strategy prescribes positive investments $b_{tj}$ and $b_{tk}$ into alternatives $j$ and $k$. On top of that and without changing either the total amount of investment in period $t$ or the future allocation rule, consider a small reallocation of investment $\delta \in (-\min\{b_{tk}, b_{tj}\}, \min\{b_{tk}, b_{tj}\})$ between alternatives $j$ and $k$ in period $t$. So, after this modification, our strategy prescribes the same actions in periods $t+1, \ldots, T$, as without this modification.
Then, since $\delta = 0$ is optimal by assumption, the first-order condition (3) still has to hold, and the second-order derivative of the objective, i.e. the derivative of (3), with respect to $\delta$ must be nonpositive. Repeating the same steps as in the proof of Proposition 1, we can establish a contradiction by showing that this second-order derivative is, in fact, strictly positive.

To prove the second part of the Proposition, note the following. Suppose that $i$ is the favorite alternative that receives all investment under the optimal strategy in period $t$. Then, we have:

$$\frac{\partial \tilde{\Pi}(\tilde{V}_{t1}, \ldots, \tilde{V}_{tN}, \sigma)}{\partial \tilde{V}_{ti}} = \frac{dC(B_t)}{dB_t} \quad (10)$$

Further, differentiating the decision-maker’s value function, we obtain:

$$\frac{\partial \tilde{\Pi}(\tilde{V}_{t1}, \ldots, \tilde{V}_{tN}, \sigma)}{\partial \tilde{V}_{ti}} = \tilde{P}_{ti}(\tilde{V}_t, \sigma) \quad (11)$$

Recall that $\tilde{P}_{ti}(\tilde{V}_t, \sigma)$ stands for the probability that alternative $i$ wins the tournament conditional on information at period-t information, i.e., $\tilde{V}_t$ and strategy $\sigma$.

Combining Equations (10) and (11) yields:

$$\tilde{P}_{ti}(\tilde{V}_t, \sigma) = \frac{dC(B_t)}{dB_t} \quad (12)$$

Further, since in each period the decision-maker invests all resources into one alternative, the probability that the favorite alternative $i$ wins is increasing in the value of the favorite, i.e.

$$\frac{d\tilde{P}_{ti}(\tilde{V}_t, \sigma)}{d\tilde{V}_{ti}} > 0 \quad (13)$$

Taking into account that $C'(\cdot) > 0$ and $C''(\cdot) > 0$ and combining equations (12) and (13) completes the proof.

Proof of Proposition 4.

First, note that a symmetric Nash equilibrium exists. Indeed, if the decision-maker’s strategy is symmetric across the contenders, then every contender has the same expectation regarding the values of the decision-maker’s
investments and random shocks at every period. So, every contender has the same best-response investment function. Further, given that each contender uses the same investment function, it is indeed optimal for the decision-maker to use a strategy which is symmetric across the contenders. The fixed point of these best response functions exists by standard argument and constitutes a symmetric Nash equilibrium.

Next, let \((e^*_1, \ldots, e^*_T)\) be the equilibrium effort investment profile for each of the contenders. Given this profile and given the distribution of shocks, the decision-maker’s problem is exactly the same as in the benchmark model studied in Section 2. The only modification here is that, independently of the decision-maker’s investment, the value of contender \(i\) now changes by the amount \(e^*_t + s_{ti}\) in each period rather than by \(s_{ti}\). So part (i) - the optimality of investing all resources in one contender in each period- follows directly from Proposition 1.

To establish part (ii), again consider the optimal effort profile \((e^*_1, \ldots, e^*_T)\) of each contender. Suppose that we have \(e^*_t \geq e^*_{t'}\) for some \(t > t'\). Then, consider some contender \(i\). Recall that the decision-maker puts all the resources into a favorite alternative in each period. Therefore, by switching effort levels in periods \(t\) and \(t'\), contender \(i\) will keep her overall costs constant. At the same time, this modification will increase the probability that \(i\) receives the decision-maker’s investment in period \(i\) and hence in all later periods. So, this deviation is strictly profitable for contender \(i\). Q.E.D.