Optimal Organization: Centralization, Decentralization or Delegation?

Sergei Severinov


Abstract

The paper addresses the issue of optimal organization of production. I compare three organizational forms: centralization (one agent produces different inputs), decentralization (each of two agents produces a different input and contracts directly with the principal), and delegation (two agents produce different inputs, the principal contracts only with one of them). The key issue in the comparison of these organizational forms is whether having more information about costs of production hurts or benefits an agent. I demonstrate that the answer to this question depends on the degree of substitutability/complementarity between the inputs. I also address the issue of collusion between agents in the decentralized organization and characterize conditions under which a stake of collusion exists.

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Fuqua School of Business, Duke University, 1 Towerview Drive, Durham, NC 27708, USA, and Department of Economics, University of Wisconsin-Madison; email: sseverin@duke.edu.

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1 Introduction

One of the central issues in the theory of organizations is how information should be distributed, exchanged, and processed within an organization. Clearly, providing an answer to this question is important for the design of optimal organizational structures. The relevant literature has explored two different approaches to address this issue. The first focuses on the cost of information processing, while the second studies incentive problems generated by the asymmetry of information between different parties in an organization.

This paper attempts to contribute to the second strand of literature. It studies an environment where the principal has to implement a project which requires allocating several tasks to subordinates (or, alternatively, procuring several inputs from providers) who have private information regarding the costs of performing these tasks (producing the inputs). The principal has to determine which organizational structure is optimal and design the contracts with subordinates/providers in an optimal way. A number of questions naturally arise in this context. Should several tasks (production of different inputs) be centralized in the hands of a single agent (supplier), or should those tasks (production of inputs) be allocated across a number of them? Should the agents be organized in a hierarchy or not, and should the amount of communication between them be restricted?1

To address these issues, I examine three organizational forms in the context of a production process requiring two inputs. In a centralized single-agent organization one agent supplies both inputs. In a decentralized two-agent organization each of the two agents supplies a different input. Finally, under delegation two agents supply different inputs, but the principal contracts with one of them and delegates to her the task of contracting with the second agent. The crucial difference between these organizational forms lies in their informational structure. In the single-agent organization, the agent has private information about production costs of both inputs, in the two-agent mechanism each agent knows only the cost of one input, while in the delegated mechanism the primary contractor serves as an informational intermediary passing the subcontractor’s cost information to the principal. Consequently, the relative profitability of these mechanisms depends on the interaction between these two pieces of information.

Intuitively, the value of information to the agent(s) might be either subadditive or super-

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1For example, a city council can hire a single contractor for a municipal project, split the work between several firms, or allow the primary contractor to subcontract some of the work to others. A firm may train its employees as specialists in a certain type of tasks, so that several employees typically work on a project. Alternatively, employees may be trained as generalists who can perform different types of tasks and handle all the work on some projects.

Similar issues arise in a variety of other contexts, including procurement, outsourcing, and regulation. Particularly, while developing a new defense system, the DoD may decide to procure its components from the same manufacturer or from different ones. Similarly, the government may allow the existence of a multi-product monopoly, or break it up into several firms, as in the AT&T case. In more recent examples of deregulation in the electric power industry, the regulators are called to determine whether a public utility producing the bulk of power could also maintain the control over the transmission grid, or the latter should be under control of a separate entity.
additive. In the subadditive case, the value of using two pieces of information together, as in the single-agent and delegated mechanisms, is lower than the sum of the values of each piece of information used independently, as in the two agent mechanism. In the superadditive case, the ordering goes in the opposite way.\(^2\) Since the principal’s interests are opposite to the agent(s)’ interests, the principal prefers informational centralization if the value of information is subadditive, and informational decentralization if the value of information is superadditive.

The main insight of this paper is that the degree of complementarity or substitutability between the inputs determines whether the value of information is sub- or superadditive. Precisely, under complementary or small degree of substitutability (to be defined below), the value of information is typically (i.e. in the absence of large asymmetries between the two inputs) subadditive, and it is superadditive when the degree of substitutability is sufficiently large.

To understand why this is so, consider the value of information in a single-agent mechanism. When the cost of an input is low, the agent earns a rent on this information. The value of this rent is equal to the surplus obtained by misrepresenting this cost as high, and is therefore proportional to the quantity of this input delivered under high cost.

Now consider the effect of misrepresenting a low cost of one input on the value of information about the second input. First, simple incentive compatibility implies that the quantity of the first input must be decreasing in its cost. Second, the optimal quantity of the second input may be increasing in the quantity of the first input uniformly in the cost of the second input, as under complementarity, or decreasing in it, as under substitutability. So, in the first case, misrepresenting the cost of one input causes the quantity of the second input to go down, and the associated informational rent to decrease. In the second case, it has the opposite effect.

Thus, the reported cost of one input affects the value of information about the other. We will refer to this as an ‘internalization factor,’ because a single agent internalizes this effect on her total payoff. In contrast, in the two-agent mechanism each agent exploits the value of her information independently taking the other agent’s cost as given, and this effect is not internalized.\(^3\) Therefore, under complementarity (substitutability) ‘internalization’ factor tends to make the value of information subadditive (superadditive).

The comparison between the single-agent and two-agent mechanisms is also affected by another factor - the difference in the structure of incentive constraints. In the single-agent mechanism, the agent can manipulate both pieces of information, i.e. she can misrepresent production costs of both goods at the same time. So, a larger set of incentive constraints has to be satisfied there. We

\(^2\)Put otherwise, the main issue is whether from an agent’s point of view the knowledge of another piece of information increases the value of the first piece of information or decreases it. In economic literature one can find examples of situations where more information either unambiguously hurts or benefits the informed party. For example, in Stackelberg oligopoly game information about a competitor’s action - i.e. her quantity choice - hurts a firm.

\(^3\)This intuition is similar to one explaining why a firm earns more profits in Cournot competition than in Stackelberg competition where it learns the competitor’s quantity before making its own quantity choice.
refer to this as an ‘extra deviation factor.’ This factor makes each piece of information more valuable when the second piece is also known. So it tends to make information superadditive.

To summarize, whether the value of information is sub- or superadditive and hence which organizational structure is optimal depends on the relative strength of the ‘internalization’ and the ‘extra deviation’ factors. The single-agent mechanism dominates the two-agent one under complementarity, because the ‘internalization’ factor favoring a single-agent mechanism is especially potent in this case. The principal is also able to leverage the effect of the ‘internalization factor’ under separability and small degrees of substitutability, and design a mechanism where the value of information is subadditive. In this mechanism the optimal quantity of one input is increasing in the quantity of the other input, so the efficient ordering is reversed. But since the degree of substitutability is low, the associated efficiency losses are small. So, the single-agent mechanism is also optimal in this case.

Nevertheless, the ‘extra deviation’ factor can overturn these results even under complementarity when there is a strong asymmetry between inputs, i.e. when the marginal product of one input decreases much more rapidly than the marginal product of the other input and the probability distributions of input costs are also asymmetric. In this case, it becomes very attractive for a single agent to make a ‘horizontal’ deviation and misrepresent a combination of low and high cost as high and low respectively. We show by example (see Proposition 2) that this ‘extra deviation’ factor can make the two-agent mechanism optimal.

Further, when the degree of substitutability is sufficiently large, it becomes too costly in terms of efficiency losses to design a mechanism where the quantity of one input increases in the quantity of the other input. But under reverse ordering the value of information in the single-agent mechanism becomes superadditive because of the ‘extra deviation’ factor: a low-cost producer of both inputs can obtain most surplus by misrepresenting both input costs as high. This ‘coordinated’ deviation is infeasible in the two-agent mechanism, so the two agent mechanism is optimal.

Another interesting set of issues arises in the context of delegation. A delegation mechanism cannot be more profitable than the two-agent mechanism, and the two are equivalent if the primary contractor could not exploit her position of an informational intermediary to increase her surplus. Thus, the key issue is whether the primary contractor benefits from intermediating the subcontractor’s cost information or simply passes it on to the principal. Potentially, she could benefit from this role in two ways. First, she could try to appropriate some of the subcontractor’s informational rents. Second, she could manipulate the report regarding the subcontractor’s type to increase the rent on her own information.

We consider four delegation structures which differ in the extent of the principal’s contractual abilities. Although the exact conditions under which the two-agent and delegation mechanism are equivalent vary with the contractual framework, the main conclusion remains the same. The primary contractor benefits from her role of an informational intermediary if the quantity of one input has a significant effect on the marginal product of the other input, i.e. if the degree of complementarity
or substitutability between the inputs is sufficiently large. This result is easy to understand. Under these conditions, the quantity of the input produced by the primary contractor and hence her informational rent are sensitive to the subcontractor’s information. Hence, the primary contractor has stronger incentives to manipulate the latter.

The issues of incentives within organizations and optimal organizational structure have been studied by Baron and Besanko (1992), Dana (1993), Gilbert and Riordan (1994), Jansen (1997), Melumad, Mookherjee and Riechelstein (1995), Laffont and Martimort (1997), (1998), Da Rocha and de Frutos (1999) and Mookherjee and Tsumagari (2003). Dana (1993) considers the effect of correlation in the cost structure on the choice between single-agent and two-agent mechanisms under separability of the production function in the two inputs. He shows that the two-agent mechanism is optimal when correlation is sufficiently strong, so that the firm can exploit relative performance evaluation. ‘Informational economies of scope’ discussed by Dana are similar to the effect of ‘internalization factor’ under separability. In contrast, this paper focuses on technological interdependency and its effect on the relative strength of ‘internalization’ and ‘extra deviation’ factors. There is no counterpart to our ‘extra deviation’ factor in Dana. Several authors have studied the issue of optimal organization under perfect complementarity between the inputs. Baron and Besanko (1992) and Gilbert and Riordan (1994) show that a single-agent mechanism is superior, and the optimal mechanism can also be implemented via delegation. However, Jansen (1997) demonstrates that the two-agent mechanism becomes more profitable in the absence of limited liability.4

Perfect complementarity and separability are interesting but quite special cases. Gilbert and Riordan (1994) point out that their analysis ‘...depends on the fixed proportions production technology. This is perhaps questionable even in the electricity example, because optimizing the transmission grid may reduce the need for the new generation capacity...’ i.e. the quality of the grid and the power output appear to be substitutes. On the other hand, a higher quality of the grid means a higher stability of the network and a lower probability of outages. This may allow consumers to substitute out of other forms of energy in favor of electricity. So, the same two inputs may be complements. Other examples with some degree of complementarity or substitutability include express and regular mail, long distance and local telephony, internet and telephone communication, defense systems or municipal projects with multiple components. Mookherjee and Tsumagari (2003) establish results similar to our Proposition 1 and 3 in an environment with continuous type distribution but symmetric homothetic production function. For this reason, a single-agent mechanism always dominates a two-agent under complementarity in their framework, and the situation giving rise to our Proposition 2 does not arise. Furthermore, their definition of complements and substitutes is based on the properties of the optimal contracts, not the parameters of the model as in this paper.

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4 Iossa (1999) studies the optimal regulatory regime in a two-good economy with one-dimensional uncertainty: the producer(s) have superior information about the demand for one of the goods, but not about the other. She reaches a different conclusion that the regulator prefers monopoly (duopoly) when the goods are substitutes (complements).
Therefore, there is also no counterparts to our Proposition 4.

The comparison of the single-agent and two-agent mechanisms provides additional insights regarding the potential for collusion in organizations. Laffont and Martimort (1997) and (1998) have studied this issue in a similar framework under perfect complementarity. They show that potential for collusion exists only under additional restrictions on contracts, such as anonymity. Our results allow to explain why a stake of collusion may not exist then: under complementarity the value of information is typically subadditive, and so the principal prefers informational centralization. At the same time, our results imply that potential for collusion exists when the inputs are substitutes: collusion enables the agents to make a coordinated deviation and report that both costs are high when they are, in fact, low. More generally, we show that a stake of collusion exists when the two-agent mechanism is more profitable than a single-agent one.

Our analysis of delegation extends the results of Melumad, Mookherjee and Riechelstein (1995). We demonstrate that their result on the equivalence of the two-agent and hierarchical mechanisms under input observability does not hold generally when the set of possible costs is discrete.

The analysis of the single-agent mechanism involves solving a screening problem with two-dimensional types. By characterizing the optimal two-dimensional mechanism with a simple distribution of types, but with non-separability between the goods/inputs in the objective function, the paper contributes to the literature on multidimensional mechanism design (see McAfee and McMillan (1988), Armstrong (1996), and Rochet and Choné (1998)). The contribution in this literature that is most closely related to our analysis is Armstrong and Rochet (1999) who provide a complete characterization of the optimal mechanism under separability between the goods, but with arbitrary degree of correlation between the agents’ types.

The rest of the paper is organized as follows. The model is described in section 2. In section 3, I derive the optimal two-agent mechanism and study the single-agent mechanism. In section 4 I consider the complementarity case. Section 5 deals with the substitutability case. In section 6 the results are illustrated via a number of examples. Section 7 studies delegation. In section 8 the issue of collusion is addressed.

2 Model

A central entity, or principal needs to procure two different goods or inputs. The principal’s benefit is measured by the production/benefit function \( v(q_1, q_2) \), where \( q_1 (q_2) \) is the quantity of the first (second) input. I assume that \( v(\ldots) \) is increasing in both arguments, twice continuously differentiable, and concave. The cross-partial derivative \( v_{12}(\ldots) \) has a constant sign over the relevant domain. We will say that the inputs are complements (substitutes) if \( v_{12}(\ldots) \geq 0 \) (\( v_{12}(\ldots) < 0 \)). To ensure that the optimal quantities are positive, I impose Inada boundary condition: \( \lim_{q_1 \to 0} v_1(q_1, q_2) = \infty \), \( \forall q_2 > 0 \). This condition is dropped when I consider specific examples.
We will compare the performance of the following three organizational forms illustrated in Figure 1: centralized (one agent produces both inputs), decentralized (each input is produced by a different agent), or hierarchical delegation (the principal contracts only with the supplier of one input, who in turn contracts with the supplier of the second input).5

In each organizational form, the principal offers contracts to the agent(s) who may either accept or reject it. If the contract(s) have been accepted, the agent(s) produce and deliver the goods (inputs) to the principal and get paid according to the contract(s). Additional stages involving the contracting between the primary contractor and the subcontractor in the delegation mechanism are described in section 7.

The principal is risk-neutral, and attempts to maximize her expected benefit net of the expected payments for the inputs. The agents(s) are risk-neutral and accept the contract after privately learning their production cost(s). An agent’s reservation utility level is normalized to zero. An agent cannot produce the good which she is not assigned to. The marginal costs of production are constant. The levels of production costs are independently distributed across goods and across agents. Specifically, it is common knowledge that the marginal cost of good \( i \) is low \((c_L)\) with probability \( p_i \), and is high\((c_H)\) with the complementary probability, where \( c_H > c_L > 0 \). Let \( \Delta = c_H - c_L \). Since production function \( v(.,.) \) can be arbitrarily asymmetric, the assumption that the distributions of input costs have a common support is equivalent to a less restrictive ‘common ratio’ assumption \( \frac{c_1^L}{c_1^H} = \frac{c_2^L}{c_2^H} \) from which ‘common supports’ can be obtained by simple renormalization of units. Independence of distributions is assumed in order to abstract from factors on the cost side. The case of correlated marginal costs is explored in Dana (1993) and Jansen (1997).

Let us now describe the contracts offered by the principal. First, consider single-agent and two-agent mechanisms. By the Revelation Principle (see e.g. Baron (1989)), we can restrict attention to direct mechanisms where the agent(s) announce her (their) costs truthfully. A direct mechanism is a mapping from the set of possible cost types \( \{c_L, c_H\} \times \{c_L, c_H\} \) (or states of the world) into the set of quantities and transfers: \( R^2 \times R^2 \) (in the two-agent mechanism), or \( R^2 \times R \) (in a single-agent

5There is a number of reasons why the principal may want or have to procure all supply of a particular input from one source. The most common of them is the presence of fixed costs. If large fixed costs in the form of R&D, investment in equipment, infrastructure and training, etc., have to be sunk by each producer of the good before she learns her production costs, then having more than one supplier could be prohibitively expensive. Alternatively, the principal’s commitment to purchase all supply of an input from a particular agent may be required to alleviate potential hold-up problem and induce this agent to invest.

Consider, for example, the development of a new defense system. In the initial stage of procurement, the government normally considers bids from a number of firms. However, only one supplier of each major element is ultimately chosen. Moreover, the final price is usually determined after the contracts have already been awarded. According to Rogerson (1989), “economies of scale together with very small production runs render it economically infeasible to have two or more firms build fully functioning production lines... The prices for all production runs may be left to be determined by future negotiations. Transaction costs together with constantly evolving technological requirements are thought to render long-term contracts infeasible.”
mechanism). The four possible states of the world are denoted by $LL, LH, HL$ and $HH$. In this notation, the first (second) letter indicates the marginal cost of the first (second) good.

Let $q^t = (q^t_{LL}, q^t_{LH}, q^t_{HL}, q^t_{HH})$ denote the vector of quantities of the good $i \in \{1, 2\}$ assigned in the two-agent mechanism. By convention, the first letter in the subscript refers to the marginal cost of good $i$. For example, in the state $LH$ the mechanism assigns quantities $q^2_{LH}$ and $q^1_{HH}$. Similarly, $g^t = (g^t_{LL}, g^t_{LH}, g^t_{HL}, g^t_{HH})$ denotes the vector of quantities of good $i$ assigned in a single-agent mechanism. Let $t^t_{KJ}$ denote the transfer to the agent who produces good $i$ in the two-agent mechanism, if she announces cost $c_K$ and the other agent announces cost $c_J$ ($K, J \in \{L, H\}$). A two-agent mechanism has to satisfy the following incentive and individual rationality constraints:

\[
\begin{align*}
IC^t(L) & : (t^t_{LL} - c_L q^t_{LL}) p_j + (t^t_{LH} - c_L q^t_{LH})(1 - p_j) \geq (t^t_{HL} - c_L q^t_{HL}) p_j + (t^t_{HH} - c_L q^t_{HH})(1 - p_j) \\
IC^t(H) & : (t^t_{HL} - c_H q^t_{HL}) p_j + (t^t_{HH} - c_H q^t_{HH})(1 - p_j) \geq (t^t_{LL} - c_H q^t_{LL}) p_j + (t^t_{LH} - c_H q^t_{LH})(1 - p_j) \\
IR^t(L) & : (t^t_{LL} - c_L q^t_{LL}) p_j + (t^t_{LH} - c_L q^t_{LH})(1 - p_j) \geq 0 \\
IR^t(H) & : (t^t_{HL} - c_H q^t_{HL}) p_j + (t^t_{HH} - c_H q^t_{HH})(1 - p_j) \geq 0.
\end{align*}
\]

Since the principal and the agents are risk-neutral, Bayesian and dominant strategy mechanisms are equivalent, and so the interim incentive and individual rationality constraints can without loss of generality be replaced by the corresponding ex-post constraints. (For details, see Mookherjee and Riechelstein (1992)).

Consider now a single-agent mechanism where the agent produces both goods and can report any possible cost combination. Let $T^t_{KJ}$ denote the transfer to the agent who announces costs $(c_K, c_J)$, where $K, J \in \{H, L\}$. Then, the mechanism has to satisfy the following incentive and individual rationality constraints for all $K, J, U, V \in \{L, H\}$:

\[
\begin{align*}
IC(KJ - UV) & : T^t_{KJ} - c_K g^t_{KJ} - c_J g^2_{JK} \geq T^t_{UV} - c_U g^1_{UV} - c_V g^2_{UV} \\
IR(KJ) & : T^t_{KJ} - c_K g^t_{KJ} - c_J g^2_{JK} \geq 0.
\end{align*}
\]

The structures of incentive constraints in the two-agent and single-agent mechanisms are illustrated in Figure 2. The downward incentive constraint $IC(LL - HH)$, as well as the ‘horizontal’ incentive constraints $IC(LH - HL)$ and $IC(HL - LH)$ in the single-agent mechanism have no counterparts in a two-agent mechanism, because agents choose their reports independently in the two-agent mechanism. When any of these constraints are binding, they reduce the profitability of the single-agent mechanism, so the ‘extra deviation’ factor is effective. On the other hand, constraints $IC(LL - HL)$, $IC(LL - LH)$ and $IC(LL - HH)$ in the single-agent mechanism are mutually exclusive, and so the principal can ensure that all these three constraints hold by paying the agent a single informational rent in the state $LL$. This is a manifestation of the ‘internalization’ factor.

## 3 Optimal Mechanisms

First, consider the optimal two-agent mechanism. Essentially, it consists of two submechanisms, one for each agent. In each of them the individual rationality constraint of the high-cost type
and the incentive constraint of the low-cost type are binding. Technological interdependence, i.e.
substitutability or complementarity in the production function, causes the optimal quantity assigned
to one agent to depend on the cost type of the other agent, but has no effect on the set of binding
constraints.

**Lemma 1** The optimal two-agent mechanism is unique. The optimal quantities are determined by
the following first-order conditions:

\[ v_1(q_{1L}^1, q_{2L}^1) = v_2(q_{1L}^2, q_{2L}^2) = c_L \]  
\[ v_1(q_{1H}^1, q_{2H}^1) = v_2(q_{1H}^2, q_{2H}^2) = c_H + \frac{p_1}{1-p_1} \]  
\[ v_2(q_{1L}^1, q_{2L}^1) = c_H + \frac{p_2}{1-p_2} \]  
\[ v_1(q_{1H}^2, q_{2H}^2) = c_H + \frac{p_1}{1-p_1} \]  
\[ v_2(q_{1H}^2, q_{2H}^2) = c_H + \frac{p_2}{1-p_2} \]

If \( v_{12} \geq 0 \), \( q_{1L}^1 > \max\{q_{1L}^1, q_{1H}^1\} \) and \( \min\{q_{1L}^1, q_{1H}^1\} > q_{1H}^1 \)
If \( v_{12} \leq 0 \), \( q_{1L}^2 > \max\{q_{1L}^2, q_{1H}^2\} \) and \( \min\{q_{1L}^2, q_{1H}^2\} > q_{1H}^2 \)

The principal pays the following transfers to the agents: \( t_{1K}^H = c_H q_{1L}^1, t_{1K}^L = c_L q_{1L}^1 + \Delta q_{1H}^1 \) for
\( K \in \{L, H\} \). Thus, the sum of the agents’ expected informational rents equals:

\[ EIR(2) = \Delta (p_1 p_2 (q_{1L}^1 + q_{2L}^1) + p_1 (1-p_2) q_{1H}^1 + (1-p_1) p_2 q_{2H}^1) \]

**Proof:** See the Appendix.

To decrease the agents’ informational rents, the principal distorts all quantity allocations in
the two-agent mechanism, except \( q_{1L}^1 \) and \( q_{1H}^1 \), downwards relative to the first-best. The quantities
\( q_{1L}^1 \) are set at the first-best level (no distortion ‘at the top’), while \( q_{1H}^1 \) is set above (below) the
first-best level when the two inputs are substitutes (complements).

The optimal single-agent mechanism will be analyzed separately under complementarity
and substitutability in the following two sections.

## 4 Complementarity

In this section I compare the profitability of the single-agent and two-agent mechanisms under
complementarity. The following result does not require characterizing the optimal single-agent
mechanism:

**Proposition 1** Suppose that the inputs are complementary \( (v_{12} \geq 0) \). Single-agent mechanism
is more profitable for the principal than a two-agent mechanism if

\[ \max \left\{ \frac{|v_{12}(q_1, q_2)|}{v_{11}(q_1, q_2)}, \frac{|v_{12}(q_1, q_2)|}{v_{22}(q_1, q_2)} \right\} \leq 1 \ \forall q_1, q_2 \geq 0. \]
Proof: See the appendix.

Under the stated condition, the value of information is subadditive and the principal can implement the allocation profile from the optimal two-agent mechanism via a single-agent mechanism with lower expected informational rent. Specifically, she can pay a lower informational rent in the state \( LL \) and the same informational rents in the states \( LH \) and \( HL \). It is relatively straightforward to see the latter: in states \( HH \), \( LH \) and \( HL \) the principal could satisfy all incentive constraints by paying the agent the sum of transfers that she pays in the same state in the two-agent mechanism.

So, consider state \( LL \). Let us show how the ‘internalization’ factor works there. In the two-agent mechanism each agent can independently misrepresent her cost as high, so the principal needs to pay total informational rent \( \Delta(q_{HL}^1 + q_{HL}^2) \) to the agents. In the single-agent mechanism, the agent can deviate by misrepresenting her cost of the first good, the second good, or both goods. The latter deviation is least attractive under complementarity because \( q_{HL}^1 > q_{HH}^2 \). If the agent misrepresents only the cost of the first good as high, then she earns surplus equal to \( \Delta q_{HL}^1 \) on her information regarding the first good, but her surplus on information regarding the cost of the second good is now at most \( \Delta q_{HH}^2 \). Similarly, if the agent misrepresents only the cost of the second good, her surplus is equal to \( \Delta(q_{HL}^2 + q_{HH}^1) \). So, the principal needs to pay informational rent equal to \( \Delta \max\{q_{HL}^1 + q_{HH}^2, q_{HL}^2 + q_{HH}^1\} \) in state \( LL \). Thus, knowledge of both costs hurts the agent. In other words, the value of information is subadditive.

The condition stated in the proposition requires that the complementarity effect be not too large. If we use the ratio \( \frac{\max\{|v_{12}(\cdot,\cdot)|,|v_{23}(\cdot,\cdot)|\}}{\max\{|v_{11}(\cdot,\cdot)|,|v_{22}(\cdot,\cdot)|\}} \) as a measure of the degree of complementarity, then the proposition requires the degree of complementarity to be less than 1. This condition ensures that the allocation profile from the two-agent mechanism satisfies ‘horizontal’ constraints \( IC(LH - HL) \) and \( IC(HL - LH) \) in the single-agent mechanism.

However, ‘horizontal’ incentive constraints \( IC(HL - LH) \) and \( IC(LH - HL) \) become binding when the degree of complementarity becomes sufficiently large. Then to satisfy these constraints the principal has to distort the quantity allocations further away from the efficient level, and/or pay a higher informational rent to the agent. As a result, under certain parameter values the two-agent mechanism becomes more profitable than a single-agent mechanism. Precisely, we have.

**Proposition 2** Consider quadratic benefit function \( v(q_1, q_2) = A + a(q_1 + q_2) - \frac{b_1}{2} q_1^2 - \frac{b_2}{2} q_2^2 + dq_1 q_2, \) such that the inputs are complementary i.e. \( d > 0 \), and \( b_2^2 > b_1 b_2 - d^2 > 0, d \geq 2b_2, \frac{v_1}{1-p_1} < p_2 < \frac{1-p_1}{4} \). Then the two-agent mechanism is more profitable for the principal.

Proof: See the appendix.

The proof of the Proposition shows that, if the degree of complementarity is sufficiently large \( (d > 2b_2) \), then the horizontal constraint \( IC(HL - LH) \) is binding in the single-agent mechanism. This ‘extra deviation’ factor leads to an increase in the agent’s informational rent, and makes
the single-agent mechanism less profitable than a two-agent one. Binding ‘horizontal’ incentive constraint can also cause two-agent mechanism to outperform single-agent mechanism under perfect complementarity, as shown by Da Rocha and de Frutos (1999).

5 Substitutability

Compared to the complementarity case, there are several differences in the nature and the strength of the ‘internalization’ and ‘extra deviation’ factors under substitutability. First, the ‘extra deviation’ factor now manifests itself through the downward incentive constraint \( IC(LL - HH) \) which can become binding in the optimal single-agent mechanism. In other words, having two pieces of information could be beneficial for the agent because she can misrepresent both low costs as high. In this case ‘internalization’ factor will no longer enhance the profitability of the single-agent mechanism.

To see this, note that under substitutability it is efficient to set \( g_{iL} < g_{iH} \) for \( i \in \{1, 2\} \). If the principal assigns an allocation profile obeying this ordering in the single-agent mechanism, then \( IC(LL - HH) \) becomes binding and the agent obtains informational rent \( \Delta(g_{1H} + g_{2H}) \) in the state \( LL \). Yet, if the same quantity allocation profile is implemented via a two-agent mechanism, in state \( LL \) the principal has to pay a lower aggregate rent of \( \Delta(g_{1L} + g_{2L}) \) to the agents. So, the two-agent will be optimal.

Still, the potential to exploit the ‘internalization’ factor can make it optimal for the principal to choose an allocation profile satisfying \( g_{iL} > g_{iH} \) for \( i \in \{1, 2\} \). We are going to show that this is optimal when the degree of substitutability is small. Then the ‘extra deviation’ factor will not be effective and the value of information will be subadditive in the single-agent mechanism. However, this will not directly imply that single-agent mechanism is optimal, because the quantity allocation profile in the two-agent mechanism will be more efficient, as it satisfies \( q_{iL} > q_{iH} \) for \( i \in \{1, 2\} \) (see Lemma 1). Then the optimal organizational form will be determined by the relative strength of the ‘internalization’ factors and the higher efficiency of the two-agent mechanism. We will use the ‘homotopy’ technique to measure this tradeoff.

The following lemma formalizes this discussion and allows to make an important simplification in the analysis. Consider relaxed single-agent program \( RP(1) \) in which the principal’s problem is solved subject only to the downwards incentive constraints \( IC(LL - LH), IC(LL - HL), IC(LL - HH), IC(LH - HH), \) and \( IC(HL - HH) \), and individual rationality constraint \( IR(HH) \).

---

6Da Rocha and de Frutos (1999) emphasize the asymmetry of the supports of the cost distributions as an explanation of this result. Yet, the complementarity of the production function also plays a crucial role. Indeed, as explained above, all the results of our paper hold if we replace common support assumption with the ‘common ratio’ assumption \( c_{1H} / c_{1L} = c_{2H} / c_{2L} \). Then, by Proposition 1 for any value of \( c_{1H} / c_{1L} = c_{2H} / c_{2L} \) single-agent mechanism remains optimal when the degree of complementarity is sufficiently small. Under separability, single-agent mechanism is optimal for an arbitrary value of this ratio. Conversely, it is easy to check that the result of Da Rocha and de Frutos (1999) holds under ‘common ratio’ assumption, so it also holds in our ‘common support’ case with benefit function \( v(q_1, q_2) = \min\{q_1 / r_1, q_2 / r_2\} \) when \( r_1 / r_2 \) is large enough.
Lemma 2 Under substitutability, Relaxed program \( RP(1) \) has a unique solution which has the following properties. \( IR(HH) \) is binding, and the agent earns informational rent \( \Delta g_{HH}^i \) in state \( LH \), \( \Delta g_{HH}^2 \) in state \( HL \), and \( \Delta \max \{ g_{HL}^1 + g_{HH}^2, g_{HH}^1 + g_{HL}^2 \} \) in state \( LL \). The optimal quantity allocation is characterized by the following first-order conditions for some \( \alpha \geq 0 \) and \( \gamma \geq 0 \) s.t. \( \alpha + \gamma \leq 1 \):

\[
\begin{align*}
  v_1(g_{LL}^1, g_{LL}^1) = & \quad v_2(g_{LL}^1, g_{LL}^2)v_1(g_{LL}^1, g_{LL}^2) = v_2(g_{LL}^1, g_{LL}^2) = c_L, \\
  v_1(g_{HL}^1, g_{HL}^2) = & \quad c_H + \Delta \frac{p_1}{1 - p_1} \alpha, \\
  v_2(g_{HL}^1, g_{HL}^2) = & \quad c_H + \Delta \frac{p_2}{1 - p_2} \gamma, \\
  v_1(g_{HH}^1, g_{HH}^2) = & \quad c_H + \Delta \frac{p_1}{1 - p_1} \frac{1 - \alpha p_2}{1 - p_2}, \\
  v_2(g_{HH}^1, g_{HH}^2) = & \quad c_H + \Delta \frac{p_2}{1 - p_2} \frac{1 - \gamma p_1}{1 - p_1}.
\end{align*}
\]

If the solution to the relaxed program is such that \( g_{HL}^i > g_{HH}^i \) for \( i \in \{1, 2\} \) then \( \alpha + \gamma = 1 \), and the solution to the relaxed program is the optimal single-agent mechanism. \( \alpha > 0 \) (\( \gamma > 0 \)) only if \( g_{HL}^1 + g_{HH}^2 \geq (\leq) g_{HH}^1 + g_{HL}^2 \).

If \( g_{HH}^i \geq g_{HL}^i \) for some \( i \in \{1, 2\} \), then the two-agent mechanism is optimal.

Proof: See the appendix.

Lemma 2 implies that, instead of deriving the optimal single-agent mechanism, it is sufficient to consider the solution to \( RP(1) \). Specifically, we can adopt the following strategy to find the optimal organizational form. At first, solve \( RP(1) \) and check whether \( g_{HH}^i \geq g_{HL}^i \) for some \( i \in \{1, 2\} \), i.e. whether \( IC(LL - HH) \) is binding. If this is so, then the two-agent mechanism dominates. On the other hand, if \( g_{HL}^i > g_{HH}^i \) for both \( i \in \{1, 2\} \), then the solution to \( RP(1) \) is the optimal single-agent mechanism, and its profitability has to be compared with that of the optimal two-agent mechanism.

The result of this analysis and hence the optimal organizational form depend on the degree of substitutability. The latter will be measured by the ratios \( \frac{v_2(g_{1i}, g_{2i})}{v_1(g_{1i}, g_{2i})} \) and \( \frac{v_2(g_{1i}, g_{2i})}{v_2(g_{1i}, g_{2i})} \). These ratios provide an appropriate measure of the degree of substitutability because they determine how the optimal quantity of one input changes in response to a change in the quantity of the other input.

Our results can be described as follows. Single-agent (two-agent) mechanism is optimal when the degree of substitutability is small (large). Additionally, the higher is the probability that at least one input is produced at high cost, the lower is the threshold degree of substitutability required for the two-agent mechanism to be optimal. The latter result holds because, when either \( p_1 \) or \( p_2 \) is low, the firm incurs high efficiency losses if it attempts to exploit the ‘internalization’ factor and set \( g_{HL}^i > g_{HH}^i \). In the remainder of this section we will establish these results formally.

At first, let us focus on the conditions under which the two-agent mechanism is optimal. Let \( g_1, g_2 \) solve \( v_1(g_1, g_2) = c_H + \Delta \frac{p_1}{1 - p_1} \frac{1 - \alpha p_2}{1 - p_2} \) and \( v_2(g_1, g_2) = c_L \). Also, let \( g_1, g_2 \) solve \( v_1(g_1, g_2) = c_L \) and \( v_2(g_1, g_2) = c_H + \Delta \frac{p_2}{1 - p_1} \frac{1 - \gamma p_1}{1 - p_1} \). Then we have:
Proposition 3 Two-agent mechanism is optimal if for all \( \alpha \in (0, 1) \) either (i) \( \frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)} \geq \frac{p_2\alpha}{(1 - p_2) + (1 - \alpha)p_1p_2} \) \( \forall g_1 \in [\bar{g}_1, \underline{g}_1], g_2 \in [\bar{g}_2, \underline{g}_2]; \) or (ii) \( \frac{v_{22}(g_1, g_2)}{v_{22}(g_1, g_2)} \geq \frac{p_2(1 - \alpha)}{(1 - p_2)(1 - \alpha)} \) \( \forall g_1 \in [\bar{g}_1, \underline{g}_1], g_2 \in [\bar{g}_2, \underline{g}_2]. \)

Proof: See the appendix.

Corollary 1 For all \( p_1, p_2 < 1 \), there exists \( \hat{r} < 1 \) s.t. two-agent mechanism is optimal if
\[
\min \left\{ \frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)}, \frac{v_{12}(g_1, g_2)}{v_{22}(g_1, g_2)} \right\} \geq \hat{r} \quad \forall g_1 \in [\bar{g}_1, \underline{g}_1], g_2 \in [\bar{g}_2, \underline{g}_2].
\]

Proof: Combining (i) and (ii) in Proposition 3 we obtain that for any \( p_1, p_2 < 1 \):
\[
\frac{p_2\alpha}{(1 - p_2) + (1 - \alpha)p_1p_2} \leq \frac{p_1(1 - \alpha)}{(1 - p_1)(1 - p_2) + p_1p_2(1 - (1 - \alpha)p_1)(1 - \alpha)p_2} < 1
\]
So, for all \( \alpha \in [0, 1] \) the right-hand side of either (i) or (ii) is less than 1. Q.E.D.

Corollary 2 Suppose that there exist \( k_1 > 0, k_2 > 0 \) and \( r > 0 \) s.t. for some \( i \in \{1, 2\} \)
\[
\min \left\{ \frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)}, \frac{v_{12}(g_1, g_2)}{v_{22}(g_1, g_2)} \right\} > r \quad \forall g_1 \in [\bar{g}_1, \underline{g}_1], g_2 \in [\bar{g}_2, \underline{g}_2]
\]
where \( \bar{l}_i, \bar{I}_i \) satisfy \( v_1(\bar{l}_i, \bar{I}_i) = c_H + \Delta k_1 \), \( v_2(\bar{l}_i, \bar{I}_i) = c_L \), and \( \bar{I}_i, \bar{l}_i \) satisfy
\[
v_1(\bar{I}_i, \bar{l}_i) = c_L, \quad v_2(\bar{I}_i, \bar{l}_i) = c_H + \Delta k_2.
\]

Then the two-agent mechanism is optimal if \( p_i < \frac{r}{1 + r} \).

Proposition 3 and its Corollaries show that the two-agent mechanism is optimal when the degree of substitutability between the inputs is sufficiently large, and it is likely that at least one input is produced at a high cost. To understand the latter result, note that the ‘internalization’ factor is more powerful when the state LL where this factor applies is more likely, i.e. when both \( p_1 \) and \( p_2 \) are high. Therefore, the threshold degree of substitutability is increasing in \( p_1 \) and \( p_2 \).

Obviously, the single-agent mechanism would be optimal under the opposite conditions: a low degree of substitutability and a high likelihood that both goods are produced at a low cost.

Following the strategy outlined in the discussion of Lemma 2, we show that under these conditions \( g^i_{HL} > g^i_{HH} \) for \( i \in \{1, 2\} \) in the optimal single-agent mechanism. Then we directly compare the principal’s profits in the optimal single-agent and two-agent mechanisms using the homotopy technique developed in the appendix. The results are stated in the following Proposition.

Proposition 4 Suppose that \( v_{12}(q_1, q_2) \leq 0 \) for all \( q_1, q_2 > 0 \), and there exist \( \underline{K} \) and \( \overline{K} \) s.t. \( \underline{K} < v_{11}(q_1, q_2)/v_{12}(q_1, q_2) < \overline{K} \) for all \( q_1 \in [\bar{q}_1, \underline{q}_1] \) and \( q_2 \in [\bar{q}_2, \underline{q}_2] \). Then for all \( p_1, p_2 \in (0, 1) \) there exists \( \omega(p_1, p_2) > 0 \) increasing in \( p_1, p_2 \) such that the single-agent mechanism is optimal if
\[
\max \left\{ \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)}, \frac{v_{12}(q_1, q_2)}{v_{22}(q_1, q_2)} \right\} < \omega(p_1, p_2) \quad \forall q_1 \in [\bar{q}_1, \underline{q}_1] \text{ and } q_2 \in [\bar{q}_2, \underline{q}_2].
\]

Proof: See the appendix.

Thus, when the degree of substitutability is small and/or both \( p_1 \) and \( p_2 \) are sufficiently high, the effect of the ‘internalization’ factor outweighs the efficiency losses from distorting the quantity allocation profile and setting \( g^i_{HL} > g^i_{HH} \) for \( i \in \{1, 2\} \) in the single-agent mechanism.

---

7 A simple method of proof based on the comparison of the informational rents is no longer applicable here, because the optimal quantities are ordered differently in the single-agent and two-agent mechanisms.
6 Examples

The following examples illustrate our results. When specific functional forms are considered, sufficient conditions of Propositions 1-4 often translate into simple restrictions on the parameters.

**Example 1. Constant Elasticity of Substitution:** $v(q_1, q_2) = (\alpha_1 q_1^\rho + \alpha_2 q_2^\rho)^{\frac{m}{\rho}}$ where $\rho < 1, 0 < m < 1$. Note that the inputs are substitutes (complements) if $\rho > m$ ($\rho < m$), and $\frac{\partial^2 v}{\partial q_1 \partial q_2} = \frac{\rho - m}{(1 - m)_{\frac{m}{\rho} + (1 - \rho)_{\frac{m}{\rho}}} \left(\frac{\alpha_1}{\alpha_2}\right)}$ where $i, j \in \{1, 2\}$.

When $q_1, q_2$ satisfy $v_i(q_1, q_2) = c_i$ for $i \in \{1, 2\}$ and $c_i \in [c_L, c_H + \frac{p_i}{(1 - p_i)(1 - p_j)}]$ we have:

$\frac{\partial^2 v}{\partial q_1 \partial q_2} = \frac{\rho - m}{(1 - m)(c_i, c_j/c_i, c_j)} \frac{\alpha_i}{\alpha_j}.$

Under complementarity ($\rho < m$), $\frac{\partial^2 v}{\partial q_1 \partial q_2} < 1$ uniformly over $R^2_+$ if $|m - \rho|$ is sufficiently small. In this case, by Proposition 1 the single-agent mechanism is optimal.

Under substitutability ($\rho > m$), Propositions 3 and 4 imply that the two-agent mechanism is optimal if $\rho - m$ is large enough, or $p_1$ and $p_2$ are small, while the single-agent mechanism is optimal if $\rho - m$ is sufficiently small, and $p_1, p_2$ are sufficiently large.

**Example 2. Quadratic:** $v(q_1, q_2) = A + a(q_1 + q_2) - \frac{b_i}{2} q_i^2 - \frac{b_j}{2} q_j^2 + dq_1 q_2$, where $a, b_i > 0$, $d^2 < b_1 b_2$. In the substitutability case ($d < 0$), by Proposition 3 the two-agent mechanism is optimal if $\frac{-d}{b_i} > \frac{p_i}{1 - p_i}$ for some $i \in \{1, 2\}$. By Proposition 4, the single-agent mechanism is optimal if both $\frac{-d}{b_1}$ and $\frac{-d}{b_2}$ are small, and $p_1$ and $p_2$ are sufficiently large.

In the complementarity case ($d > 0$), by Proposition 1 the single-agent mechanism is optimal if $d \leq \min\{b_1, b_2\}$. However, the two-agent mechanism is optimal under the conditions stated in Proposition 2. Figure 3 illustrates the regions of optimality of the two-agent and single-agent mechanisms in the symmetric case where $b_1 = b_2 = b$.

We can also use this example to illustrate possible corner solutions which so far have been ruled out by the Inada conditions. Suppose $b_1 = b_2 = b$ and $p_1 = p_2 = p$ and consider the optimal single-agent mechanism. Then under complementarity $g_{HL} > g_{HH} = 0$ if $c_H + \Delta \left(\frac{p}{1 - p} + \frac{p^2}{2(1 - p^2)}\right) < d < c_L + \frac{b}{b - c} \Delta \left(1 + \frac{p}{2(1 - p)}\right)^{2/3}$. Under substitutability, $g_{HH} > g_{HL} = 0$ if $c_L + \frac{b}{b - c} \Delta < d < c_H + \Delta \left(\frac{p}{1 - p} + \frac{p^2}{2(1 - p^2)}\right)^{2/3}$.

Under complementarity, the first-order conditions in Lemmas 1 and 2 can be used to show that the set of parameter values under which $g_{HL} > g_{HH} = 0$ is strictly larger than the set of parameters under which $q_{HL} > q_{HH} = 0$ in the two-agent mechanism. This observation is interesting in the context of a more general result due to Armstrong (1996) (see also Rochet and Choné (1998)) establishing the non-emptiness of the set of types who do not trade is the optimal non-linear pricing mechanism with multidimensional types.

Our observation that in a single-agent mechanism under substitutability $g_{HH} > g_{HL} = 0$ for some parameter values, suggests that with a continuous type distribution the set of types who are assigned zero quantity of at least one good could be disconnected and need not include the origin.

**Example 3. Perfect Substitutes:** $v(q_1 + q_2)$. In this case, $\frac{\partial^2 v}{\partial q_1 \partial q_2} \equiv 1$. Therefore, by
Proposition 3 the two-agent mechanism is optimal.

7 Delegation

In this section I consider another form or organization - delegation, or hierarchy. Specifically, production is performed by two agents who contribute different inputs, but the principal directly contracts only with one of them (the primary contractor) and delegates to her the task of contracting with the other agent (the subcontractor)(see figure 1). Delegation is common in the allocation of tasks within an organization, in procurement and in construction industry. In large corporations senior managers usually delegate some supervising authority and responsibilities to middle managers.

Hierarchical delegation with asymmetric information was studied by Melumad, Mookherjee and Riechelstein (1995), Baron and Besanko (1992), Gilbert and Riordan (1994), and Laffont and Martimort (1998). This literature points out that the advantages of delegation include an economy of communication costs achieved by shifting some of the contracting tasks from the principal to one of the subordinates. On the other hand, delegation leads to a loss of control by the principal which may negatively effect the incentives within hierarchies. The same point is made by McAfee and McMillan (1995) in the context of a model where intermediate layers of supervision separate the principal from the agent engaged in production. Riordan and Sappington (1987) show that the principal’s decision whether to delegate both stages of the production process to the agent or only one of them depends on whether the costs at the two stages are positively or negatively correlated.

The goal of this section is to compare the profitability of the delegation mechanism vis-a-vis the two-agent and the single-agent mechanisms. To focus on the issue of the loss of control in contracting, we make the same assumptions regarding input observability as in the single-agent and two-agent cases: we assume that under delegation the principal can monitor the quantity of input supplied by each agent. We will consider two different contractual set-ups referred to as delegation hierarchies \( H_1 \) and \( H_2 \). The following sequence of moves characterizes hierarchy \( H_1 \):

1. The principal offers the contract to the primary contractor.
2. The primary contractor decides whether to accept or reject the contract. If she rejects, the game ends and all players obtain their reservation payoffs. If the primary contractor accepts the contract, then the game proceeds through the following steps:
3. The primary contractor reports her cost type to the principal.
4. The primary contractor offers a contract to the subcontractor. If the subcontractor rejects it, then the game ends and all players obtain their reservation payoffs.
5. If the subcontractor accepts, she reports her cost type to the primary contractor, who then reports it to the principal.
6. Both contractors produce their inputs, the final output is delivered to the principal, and the transfers take place according to the two contracts.
$H_1$ is most profitable for the principal among all delegation structures with the same observability assumptions, because the principal has the broadest possible contracting abilities there. In particular, the principal signs a contract with the primary contractor and obtains her cost report before the latter communicates with the subcontractor. This feature has two important consequences.

First, only the interim, rather than ex post, individual rationality constraints of both agents need to be satisfied. This distinction is irrelevant for the subcontractor, because her situation falls into the class of cases for which Mookherjee and Riechelstein (1992) has established the equivalence of Bayesian and dominant strategy implementation. However, there is a significant difference between the two types of constraints as far as the primary contractor is concerned. If interim constraints have to hold, then the principal can structure her contract in such a way that the primary contractor obtains a negative payoff for one realization of the subcontractor’s cost, and a positive payoff for a different realization of the subcontractor’s cost. On the other hand, the primary contractor’s ex post individual rationality constraint has to be satisfied if she can withdraw from the contract after receiving a report from the subcontractor. We demonstrate below that the former regime makes eliciting information easier for the principal. So, her profits in the delegation mechanism with interim IR constraints are higher.

Further, in $H_1$ the primary contractor’s decision whether to report her true cost or not cannot be contingent on the subcontractor’s cost. This reduces the set of feasible deviations by the primary contractor, which benefits the principal. To highlight this factor, we consider an alternative contracting arrangement - hierarchy $H_2$ - where the primary contractor does not make a cost report to the principal before communicating with the subcontractor. Formally, the sequence of steps in $H_2$ is the same as in $H_1$, except that Step 3 is eliminated, and in Step 5 the primary contractor reports both costs to the principal.

Thus, $H_2$ allows to save some communication costs. However, the set of feasible deviations for the primary contractor is larger in $H_2$ because she may decide to misrepresent her cost for one realization of the subcontractor’s cost, but not for the other realization. As we demonstrate in Proposition 6, this has real consequences: in some cases, the hierarchy $H_1$ is strictly more profitable for the principal than the hierarchy $H_2$.

A few comments are in order regarding the choice of the primary contractor. In our model, agents 1 and 2 can be asymmetric with respect to their cost distribution as well as their marginal products. Intuitively, we allow one agent to be more productive than the other. As shown below, the nature of these asymmetries is an important determinant affecting the optimal choice of the primary contractor. In some cases, a hierarchial mechanism can attain the performance of a two-agent mechanism only if a certain agent is chosen as a primary contractor. However, the choice of the primary contractor may be determined by factors outside our model, such as specialized knowledge about potential suppliers of other inputs. For example, the primary contractor in a construction
project has to be well-informed about the market for suppliers of necessary materials and services, as well as the reputations of different firms in this market. These factors may then require the ‘wrong’ agent (from the technological and cost structure point of view) to be chosen as a primary contractor.

To address this issue, we will always start by considering agent 1 as a primary contractor, and then explore whether switching the roles could affect the performance of the delegated mechanism.

At first, let us establish some preliminary results. Since agents 1 and 2 are risk-neutral, the informed principal problem does not arise in contracting between them. So, without loss of generality, we will suppose that the primary contractor with cost \( c_K, K \in \{L, H\} \), reveals her type to the subcontractor and offers her a menu contract consisting of two quantity/transfer pairs.

It is easy to establish that the two-agent mechanism is at least as profitable for the principal as \( H_1 \) or \( H_2 \). To see this, fix a quantity schedule \( \{h^1_{LL}, h^1_{HL}, h^1_{HH}, h^2_{HH}\} \) in a hierarchial mechanism where 1 stands for the primary contractor, and 2 stands for the subcontractor, and let \( (T_{HH}, T_{HL}, T_{LL}) \) be the vector of transfers from the principal to the primary contractor. The interim participation constraint of the primary contractor with cost \( c_H \) can be satisfied only if \((1-p_2) T_{HH} + p_2 T_{HL} (1-p_2)\) is sufficient to cover the expected production costs in states \( HH \) and \( HL \) and the subcontractor’s informational rent \( \Delta h^2_{HH} \) in \( HL \). So, in \( H_1 \) and \( H_2 \) we have:

\[
(1-p_2) T_{HH} + p_2 T_{HL} \geq (1-p_2) c_H (h^1_{HH} + h^2_{HH}) + p_2 (c_H h^1_{HL} + c_L h^2_{HL} + \Delta h^2_{HH})
\]  

(12)

Similarly, \((1-p_2) T_{HL} + p_2 T_{LL}\) must cover: (i) production costs in states \( LH \) and \( LL \), (ii) subcontractor’s informational rent \( \Delta h^2_{HL} \) in state \( LL \), (iii) primary contractor’s expected informational rent of at least \( \Delta ((1-p_2) h^1_{HH} + p_2 h^1_{HL})\). So, in both \( H_1 \) and \( H_2 \) we have:

\[
(1-p_2) T_{HL} + p_2 T_{LL} \geq (1-p_2) (c_L h^2_{HL} + c_H h^2_{HL} + \Delta h^2_{HL}) + p_2 (c_L (h^1_{LL} + h^2_{LL}) + \Delta (h^2_{HL} + h^1_{HL}))
\]  

(13)

Combining (12) and (13), we conclude that the expected informational rent that the principal pays in either hierarchy \( H_1 \) or \( H_2 \) is no less than \( \Delta (p_1 p_2 (h^1_{HL} + h^2_{HL}) + p_1 (1-p_2) h^1_{HH}) + (1-p_1) p_2 h^2_{HH} \) which is the same as the expected informational rent paid by the principal in the optimal two-agent mechanism. Therefore, the principal’s expected cost of implementing a quantity schedule via delegation is at least as large as via the two-agent mechanism.

So, the principal’s payoff from either \( H_1 \) and \( H_2 \) is no greater than her payoff from the two-agent mechanism. In Propositions 5 and 6 we characterize the conditions under which hierarchies \( H_1 \) and \( H_2 \) attain the same profitability as the two-agent mechanism.

**Proposition 5** If agent 1 (2) serves as a primary contractor, then \( H_1 \) attains the same performance as the two-agent mechanism if \( \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} \leq \frac{1}{1-p_2} \left( \frac{v_{12}(q_1, q_2)}{v_{22}(q_1, q_2)} \right) \forall q_1, q_2 \in \mathbb{R}_+^2 \). Conversely, the hierarchy \( H_1 \) with agent 1 (2) as a primary contractor is strictly less profitable for the principal if \( \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} > \frac{1}{1-p_2} \left( \frac{v_{12}(q_1, q_2)}{v_{22}(q_1, q_2)} \right) \forall q_1, q_2 \in \mathbb{R}_+^2 \).

Under complementarity the principal obtains the same payoff in \( H_1 \) as in the two-agent mechanism if either agent can serve as the primary contractor.

---

\(^8\)Otherwise, (12) implies that the primary contractor can earn a higher expected profit by simply reporting \( H \) in the first stage and then implementing \( (h^1_{HH}, h^2_{HH}) \) in state \( LH \) and \( (h^1_{HL}, h^2_{HL}) \) in state \( LL \).
Under substitutability, \( H_1 \) is strictly more profitable for the principal than the single-agent mechanism, if \( IC(LL - HH) \) is binding in the latter.

**Proof:** see the appendix.

Proposition 5 establishes that \( H_1 \) is equivalent to the two-agent mechanism when the cross-effects in production are not too large, i.e. the marginal product and hence the optimal quantity of one input is not too sensitive to the quantity of the other input.

To understand this result, note that (12) and (13) imply that each agent obtains at least as much surplus from private information regarding her own cost as in the two-agent mechanism. So, \( H_1 \) is equivalent to the two-agent mechanism only if the primary contractor cannot exploit her role as an informational intermediary to earn additional surplus, and passes on the information from the subcontractor to the principal without manipulating it. Manipulating this information could be profitable for the primary contractor for two reasons: (a) she could appropriate part of the informational rents that the principal intends for the subcontractor; (b) she could extract more surplus from her own information.

In hierarchy \( H_1 \), option (a) is infeasible because the primary contractor has to report her cost type before communicating with the subcontractor. Given the primary contractor’s report, the informational rents on the subcontractor’s information can be appropriated only by the subcontractor. At the same time, option (b) is feasible. It becomes important when the report regarding the subcontractor’s cost has a significant effect on the quantity assigned to the primary contractor, which is exactly when the degree of complementarity or substitutability between the inputs is sufficiently large.

Specifically, suppose that the inputs are complementary and the principal offers a contract with (12) and (13) holding as equalities. Consider the following deviation: the primary contractor misrepresents her low cost as high in the first stage, and then always reports that the subcontractor’s cost is low. Then, the primary contractor has to pay \( c_H q_{LH}^2 \) to the subcontractor, with a net loss of at least \( \Delta(q_{LH}^2 - q_{HH}^2) \). However, the surplus obtained by the primary contractor on information about her own cost increases from \( \Delta(q_{LH}^2 p_2 + q_{HH}^2(1 - p_2)) \) to \( \Delta q_{LL}^2 \). In the proof of Proposition 5, I show that this increase outweighs the extra payment to the subcontractor when the degree of complementarity is sufficiently large. This is so because a report that the subcontractor’s cost is low rather than high causes a larger increase in the quantity supplied by the primary contractor, and hence in her informational rent, than in the quantity supplied by the subcontractor, and hence the extra payment to her. But then the original contract is not incentive compatible, and so \( H_1 \) cannot attain the same performance as the two-agent mechanism.

Similarly, under substitutability the primary contractor’s informational rent increases in the subcontractor’s cost. So, the primary contractor has an incentive to exaggerate the subcontractor cost. The principal can offset this incentive to a certain extent by imposing a penalty on the primary contractor when the latter reports that both costs are high. Yet, this penalty cannot be too large,
because otherwise the primary contractor will misrepresent her own high cost as low. So the incentive to exaggerate the subcontractor’s cost cannot be mitigated when the degree of substitutability is large.

Next, consider hierarchy $H2$. There, the primary contractor has a wider set of possible deviations, and can try to appropriate some of the subcontractor’s informational rents. In particular, under complementarity she will have an incentive to misrepresent the state $LH$ as $HH$ in order to lower the informational rent that she pays to the subcontractor in state $LL$. Thus, $H2$ attains the same performance as the two-agent mechanism under more restrictive conditions than $H2$.

**Proposition 6** If agent 1 (2) serves as a primary contractor, then $H2$ attains the same performance as the two-agent mechanism if $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} \leq \frac{1-p_1}{1-p_2} \left( \frac{v_{12}(q_1,q_2)}{v_{22}(q_1,q_2)} \right) \forall q_1, q_2 \in \mathbb{R}^2$. Conversely, the hierarchy $H2$ with agent 1 (2) as a primary contractor is strictly less profitable for the principal if $\left| \frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} \right| > \frac{1-p_1}{1-p_2} \left( \frac{v_{12}(q_1,q_2)}{v_{22}(q_1,q_2)} \right) \forall q_1, q_2 \in \mathbb{R}^2$.

If either agent can serve as a primary contractor, then $H2$ attains the same performance as the two-agent mechanism if $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} \leq \frac{1-p_2}{1-p_1} \left( \frac{v_{12}(q_1,q_2)}{v_{22}(q_1,q_2)} \right) \forall q_1, q_2 \in \mathbb{R}^2$.

**Proof:** see the appendix.

Finally, let us consider the case where the primary contractor could refuse to deliver the inputs and opt out of the contract after receiving the subcontractor’s report. Then the individual rationality constraint of the primary contractor has to hold ex post. Accordingly, let $H1^{ep}$ ($H2^{ep}$) be a modification of hierarchy $H1$ ($H2$) obtained by adding agent 1’s option to withdraw immediately after stage 5. We have:

**Proposition 7** Under substitutability, both $H1^{ep}$ and $H2^{ep}$ are strictly less profitable for the principal than the two-agent mechanism.

Under complementarity, $H1^{ep}$ attains the same performance as the two-agent mechanism, if $H1$ attains this performance. When agent 1 serves as a primary contractor, then $H2^{ep}$ attains the same performance as the two-agent mechanism if $H2$ attains the same performance and, additionally, $\frac{v_{12}(q_1,q_2)}{v_{22}(q_1,q_2)} \leq \frac{1-p_2}{p_2} \forall q_1, q_2$, $p_2$ is sufficiently small, $p_1$ is large.

In $H1^{ep}$ and $H2^{ep}$, the principal does not have the ability to distribute the expected payments to the primary contractor across the states of the world in an arbitrary way. This restricts her ability to mitigate the primary contractor’s incentives to manipulate the subcontractor’s information, and so $H1^{ep}$ and $H2^{ep}$ attain the same performance as the two-agent mechanism under more restrictive conditions than $H1$ and $H2$ respectively.

### 8 Collusion

The results of the previous sections can be used to address the issue of collusion in organizations. Laffont and Martimort (1997) and Laffont and Martimort (1998)) (LM in the sequel) analyze this
issue in a virtually identical framework, and therefore it will be natural to compare our results to theirs. The potential for collusion arises in the two-agent mechanism if the agents can communicate and adopt a joint reporting strategy. However, collusion will not necessarily generate additional payoff for them and hence a loss for the principal. In the terminology of LM, a stake of collusion may not exist.

Obviously, the agents can achieve the highest joint payoff from collusion if they can overcome the asymmetric information and bargaining problem between them and act as a single entity. We call this situation perfect collusion. Clearly, if there is no stake of perfect collusion, then there is also no stake of collusion which is less than perfect, i.e. when some bargaining frictions exist between the agents.

When does a stake of perfect collusion exists? We can provide an answer to this question using the comparison of the single-agent and two-agent mechanisms. Specifically, suppose that the allocation profile from the optimal two-agent mechanism is assigned in the single-agent mechanism. Then a stake of collusion exists if such mechanism is not incentive compatible, i.e. in some state(s) of the world the single agent has an incentive to misrepresent the costs of both inputs which is not feasible in the two-agent mechanism without collusion. On the contrary, there is no stake of collusion if the mechanism remains incentive compatible. This observation gives rise to the following proposition:

**Proposition 8**  
A stake of perfect collusion exists in each of the following two cases:
(i) the two-agent mechanism is more profitable for the principal than the single-agent one,
(ii) under substitutability.

A stake of perfect collusion does not exist when the inputs are complementary and
\[
\max \left\{ \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)}, \frac{v_{12}(q_1, q_2)}{v_{22}(q_1, q_2)} \right\} \leq 1 \forall q_1, q_2.
\]

**Proof:** Suppose that the two-agent mechanism dominates the single-agent one. Let M2 be the optimal two-agent mechanism with quantities and transfers given by \( (q_{1KJ}, t_{1KJ}, q_{2KJ}, t_{2KJ}) \) \( (K, J \in \{L, H\}) \). Suppose that the principal offers to a single agent a mechanism M1 which assigns the same quantities as in M2 and transfers equal to the sum of transfers in M2. Formally,
\[
g_{iKJ} = q_{iKJ} \quad \forall i, K, J \quad \text{and} \quad T_{KJ} = t_{1KJ} + t_{2KJ}.
\]
M1 cannot be incentive compatible, because otherwise single-agent mechanism would be at least as profitable as the optimal two-agent mechanism, contradicting our original assumption. When the two agents collude perfectly, they act as a single agent, and so the colluding pair will make the same announcements in M2 as a single agent in M1. Thus, M2 is not incentive compatible under perfect collusion, so a stake of collusion exists.

Under substitutability, the allocation profile in the optimal two-agent mechanism satisfies \( q_{iHH} > q_{iHL} \), so the colluding agents would deviate by reporting HH in state LL.

Consider now the complementarity case. Then the allocation profile in the optimal two-agent mechanism satisfies \( q_{iHH} > q_{iHL} \). So, if this allocation profile is assigned in a single-agent
mechanism, \( IC(LL - HH) \) will hold. Furthermore, under the stated condition both ‘horizontal’
constraint will also hold in the single-agent mechanism. Q.E.D.

Since LM focus on the case of perfect complementarity where a stake of collusion does not
exist, they had to impose additional restrictions on the set of feasible mechanisms. Specifically, they
require the principal to offer an anonymous contract so that both agents get the same transfer in
each state of the world. The anonymity generates a stake of collusion proportional to \( q_{HL} - q_{HH}. \)
However, this stake of collusion disappears under substitutability and also when the benefit function
is additively separable.

LM (1998) demonstrate that the principal can avoid the cost of preventing collusion in
an anonymous mechanism through delegation. Their delegation mechanism (equivalent to our \( H_2 \)
hierarchy) is more profitable for the principal than a two-agent mechanism with collusion. Yet,
without anonymity restriction this result is not always true. In particular, suppose that inputs
are complementary and agent 1 is a primary contractor. Propositions 6 and 8 imply that if
\( \max \left\{ \frac{v_{13}(q_1, q_2)}{|v_{11}(q_1, q_2)|}, \frac{v_{12}(q_1, q_2)}{|v_{22}(q_1, q_2)|} \right\} \leq 1 \) and \( \frac{v_{12}(q_1, q_2)}{|v_{11}(q_1, q_2)|} \geq \frac{1-p_1}{1-p_2} \), then there is no stake of collusion, and
\( H_2 \) is strictly less profitable than a two-agent mechanism.

9 Appendix

The following easy to prove properties of concave functions will be useful below.

**Property 1.** Let \( v(.,.) \) be a twice-continuously differentiable, increasing, concave function. Consider
d, c ∈ (0, ∞) s.t. \( v_1(q_1, q_2) = c \) and \( v_2(q_1, q_2) = d \). Then \( \frac{dq_1}{dc} < 0 \) and \( \frac{dq_2}{dc} < 0. \)

**Property 2:** Suppose that \( v_1(q_1, q_2) = c_1 \) and \( v_2(q_1, q_2) = c_2 \) for some \( c_1, c_2 > 0. \) Then:

\[
\frac{dq_1}{dc_1} \leq \frac{dq_2}{dc_1} \text{ if } |v_{22}(q_1, q_2)| \geq |v_{12}(q_1, q_2)| \quad \text{and} \quad \frac{dq_2}{dc_2} \leq \frac{dq_2}{dc_1} \text{ if } |v_{11}(q_1, q_2)| \geq |v_{12}(q_1, q_2)|
\]

**Proof of lemma 1:** The optimal two-agent mechanism is derived by solving the following problem
subject to the constraints \( IR^i(H) \) and \( IC^i(L) \) for \( i \in \{1, 2\} \):

\[
\max_{q_{JK}, q_{JK}, t_{JK}, t_{JK}, J,K \in \{1, 2\}}\quad p_1p_2[v(q_{1L}, q_{2L}^2) - t_{1L}^L - t_{2L}^L] + p_1(1-p_2)[v(q_{1H}, q_{2H}^2) - t_{1L}^H - t_{2H}^H] + (1-p_1)p_2[v(q_{1L}, q_{2L}^2) - t_{1L}^H - t_{2L}^H] + (1-p_1)(1-p_2)[v(q_{1H}, q_{2H}^2) - t_{1H}^H - t_{2H}^H]
\]

Since \( v(.,.) \) is concave, this problem has a unique solution. The quantities are characterized by the
first-order conditions in the lemma. Applying Property 1, we obtain desired ordering. The transfers
are chosen so that \( IR^i(H) \) and \( IC^i(L) \) are binding. Since \( q_{1L}^L > q_{1L}^H \) and \( q_{1H}^L > q_{1H}^H \), the incentive
constraint of the high-cost agent is not binding, and the expected informational rent \( EIR(2) \) is given
by the expression in the statement of the lemma. Q.E.D.

**Proof of Proposition 1:** Consider the following single-agent mechanism. In state of the world
\( KJ \), assign the same quantity allocation \( (q_{KJ}^L, q_{KJ}^H) \) as in the optimal two-agent mechanism, and
the transfer from the following list: \( T_{HH} = c_H(q_{1H}^1 + q_{2H}^2) \), \( T_{LL} = c_Lq_{1L}^1 + c_Hq_{HL}^2 + \Delta q_{HH}^1 \), \( T_{HL} = c_Hq_{1H}^1 + c_Lq_{2L}^2 + \Delta q_{HH}^2 \), \( T_{LL} = c_L(q_{1L}^1 + q_{2L}^2) + \Delta \max\{q_{HH}^1 + q_{HH}^2, q_{HL}^2 + q_{HH}^1\} \).

This mechanism is more profitable for the principal than the optimal two-agent mechanism, because her outlay is the same in all states of the world except \( LL \), in which her outlay is lower by \( \Delta \min\{q_{HL}^1 - q_{1H}^1, q_{HL}^2 - q_{2H}^2\} > 0 \) than in the two-agent mechanism. The proposed mechanism satisfies all individual rationality constraint. Let us show that this mechanism is incentive compatible. Clearly, it satisfies all downwards incentive constraints \( IC(LL - HL), IC(LL - LH), IC(LL - HH), IC(LH - HH) \). It is easy to see that all upwards constraints hold.

Now consider the ‘horizontal’ incentive constraints \( IC(LH - HL) \) and \( IC(HL - LH) \). Since \( IC(LH - HH) \) and \( IC(HL - HH) \) are binding, \( IC(LH - HL) \) holds if

\[ q_{2L}^2 - q_{1L}^1 \geq q_{1H}^2 - q_{1H}^1 \]  

Similarly, \( IC(HL - LH) \) holds if \( q_{1L}^1 - q_{1H}^1 \geq q_{1L}^2 - q_{1H}^2 \). To see that (14) holds note that by (1)-(5), \( v_2(q_{HL}^1, q_{HL}^2) < v_2(q_{1H}^1, q_{1H}^2) \) and \( v_1(q_{HL}^1, q_{HL}^2) = v_1(q_{1H}^1, q_{1H}^2) \). Since \( |v_{11}(q_1, q_2)| \geq v_{12}(q_1, q_2) \), Property 2, implies that (14) holds. Similarly, the first-order conditions in Lemma 1, the assumption that \( |v_{22}(q_1, q_2)| \geq v_1(q_1, q_2) \) and Property 2 imply that \( IC(HL - LH) \) holds.

Finally, since \( IC(LH - HL) \) and \( IC(HL - LH) \) hold and \( q_{LL}^2 > q_{LL}^1 \), it follows that the upwards constraints \( IC(HL - LL), IC(HH - LL) \), and \( IC(HH - LL) \) also hold.

**Proof of Proposition 2**: The proof consists of 3 steps. First, I characterize the optimal single-agent mechanism. Armstrong and Rochét (1999) solve this problem under separability between the goods, but for an arbitrary degree of correlation between the types. Second, I derive a method to compare the profitability of the single-agent and two-agent mechanisms. Third, this method is applied to show that two-agent mechanism is more profitable for specific functional forms.

**Step 1.** Consider the firm’s profit maximization problem with the following constraints imposed explicitly: \( IR(HH), IC(LL - HL), IC(LL - LH), IC(LL - HH), IC(LH - HH), IC(HL - HH), IC(LH - HL), IC(HL - LH) \). The Lagrangian associated with this problem is:

\[
\max \mathcal{L} = p_1p_2 (v(g_{1L}^1, g_{2L}^2) - T_{LL}) + p_2(1 - p_2) (v(g_{1H}^1, g_{2H}^2) - T_{HL}) + (1 - p_1)p_2 (v(g_{1L}^1, g_{2H}^2) - T_{HL}) + (1 - p_1)(1 - p_2) (v(g_{1H}^2, g_{2H}^2) - T_{HH}) + \\
\lambda_{HL} (T_{LL} - c_L(g_{1L}^1 + g_{2L}^2) - T_{LL} + c_L(g_{2L}^1 + g_{2L}^2)) + \lambda_{HL} (T_{LL} - c_L(g_{1L}^1 + g_{2L}^2) - T_{HL} + c_L(g_{1H}^1 + g_{2H}^2)) + \\
\lambda_{HH} (T_{LL} - c_L(g_{1L}^1 + g_{2L}^2) - T_{HH} + c_L(g_{2L}^1 + g_{2L}^2)) + \delta_{HH} (T_{HH} - c_L g_{HH}^1 - c_H g_{HH}^2) + \\
\mu (T_{HH} - c_L g_{HH}^1 - c_H g_{HH}^2 - T_{HH} + c_L g_{HH}^1 + c_H g_{HH}^2) + \kappa (T_{HH} - c_L g_{HH}^1 - c_H g_{HH}^2 - T_{HH} + c_L g_{HH}^1 + c_H g_{HH}^2) + \eta (T_{HH} - c_H (g_{HH}^1 + g_{HH}^2))
\]

\( (15) \)
The first-order conditions with respect to transfers are:

\[ T_{LL} : p_1p_2 = \lambda_{LH} + \lambda_{HL} + \lambda_{HH} \]  
\[ T_{LH} : p_1(1 - p_2) = \delta_{LH}^1 - \lambda_{LH} - \mu + \kappa \]  
\[ T_{HL} : (1 - p_1)p_2 = \delta_{LH}^2 - \lambda_{HL} + \mu - \kappa \]  
\[ T_{HH} : (1 - p_1)(1 - p_2) = \lambda_{HH} - \delta_{LH}^1 - \delta_{LH}^2 + \eta \]  

(16)-(19) imply that \( \eta = 1 \), \( \delta_{LH}^1 + \delta_{LH}^2 = p_1(1 - p_2) + p_2 \), which can be used to simplify the other first-order conditions as follows:

\[ v_1(g_{LL}^1, g_{LL}^2) = v_2(g_{LL}^1, g_{LL}^2) = c_L \]  
\[ v_1(g_{LH}^1, g_{LH}^2) = c_L - \Delta \frac{\mu}{p_1(1 - p_2)} \]  
\[ v_2(g_{LH}^1, g_{LH}^2) = c_L - \Delta \frac{\kappa}{p_2(1 - p_1)} \]  
\[ v_1(g_{LH}^1, g_{HH}^2) = \frac{\delta_{LH}^1 - \lambda_{LH} - \mu - \kappa}{p_1(1 - p_2)} c_H + \frac{\lambda_{HL} + \kappa}{p_1(1 - p_2)} \Delta = c_H + \frac{\lambda_{HL} + \kappa}{p_1(1 - p_2)} \Delta \]  
\[ v_2(g_{LH}^1, g_{HH}^2) = \frac{\delta_{LH}^1 - \lambda_{LH} - \mu + \kappa}{p_1(1 - p_2)} c_H + \frac{\lambda_{HL} + \mu - \kappa}{p_1(1 - p_2)} \Delta \]  
\[ v_1(g_{HH}^1, g_{HH}^2) = c_H + \frac{p_1 \Delta}{1 - p_1} + \frac{\lambda_{HL} + \lambda_{HH} + \mu + \kappa}{(1 - p_1)(1 - p_2)} \Delta \]  
\[ v_2(g_{HH}^1, g_{HH}^2) = c_H + \frac{p_2 \Delta}{1 - p_2} + \frac{\lambda_{HL} + \lambda_{HH} - \mu + \kappa}{(1 - p_1)(1 - p_2)} \Delta \]

To characterize the solution it is important to determine which constraints are binding. Obviously, IR(HH) must be binding, because otherwise \( T_{HH} \) could be lowered without violating other constraints. So, \( T_{HH} = c_H(g_{HH}^1 + g_{HH}^2) \). Further, at least one of IC(HL − HH) and IC(LH − HH) must be binding, because otherwise the principal could decrease both \( T_{LH} \) and \( T_{HL} \) by the same amount without violating other incentive constraints.

We will consider two possible cases: (i) Both ‘horizontal’ constraint IC(LH − HL) and IC(HL − LH) are non-binding, (ii) at least one of these two constraints is binding.

In case (i) \( \kappa = \mu = 0 \). Then it is easy to establish the following ordering: \( g_{HH}^1 < \min\{g_{HL}^1, g_{LH}^1\} \). So, IC(LH − HH) is non-binding, and \( \lambda_{HH} = 0 \).

At the same time, both IC(HL − HH) and IC(LH − HH), and one or both of IC(LL − LH) and IC(LL − HL) must be binding. If IC(LL − LH) is non-binding, then \( \lambda_{LL} = 0 \), \( \lambda_{HL} = p_1p_2 \) and \( g_{HL}^1 + g_{HH}^1 > g_{HL}^1 + g_{HH}^1 \). If IC(LL − HL) is non-binding, then \( \lambda_{LL} = 0 \), \( \lambda_{HL} = p_1p_2 \) and \( g_{HL}^1 + g_{HH}^1 < g_{HL}^1 + g_{HH}^1 \). If both IC(LL − LH) and IC(LH − HH) are binding, then \( \lambda_{HH} + \lambda_{HL} \) must be such that \( g_{HL}^1 + g_{HH}^2 = g_{HL}^2 + g_{HH}^1 \). It is easy to check that in either case IC(\( HHH \)), IC(HL − LL) and IC(HH − LL) hold.

If this allocation satisfies IC(LH − HL) and IC(HL − LH), then it is optimal. However, if one of these constraints fails, then at least one of IC(LH − HL) or IC(HL − LH) will be binding in the optimal mechanism. So, let us consider case (ii). There are three possibilities depending on which of IC(LH − HL) and IC(HL − LH) is binding. Suppose that IC(LH − HL) is binding,
while $IC(HL - LH)$ is not. Then $\mu = 0$, and it is easy to check that $g_{HL}^2 > g_{HH}^2$, which implies that $IC(LL - HH)$ is non-binding. Similarly, if $IC(HL - LH)$ is binding and $IC(LL - LH)$ is not, then $\kappa = 0$ and $\lambda_{HH} = 0$.

If both $IC(LH - HL)$ and $IC(HL - LH)$ are binding, then $-p_2 \leq \mu - \kappa \leq p_1$. Let us show that $IC(LL - HH)$ remains non-binding i.e., either $q_{HL}^1 > q_{HH}^1$ or $q_{HL}^2 > q_{HH}^2$. At least one of $IC(LH - HH)$ and $IC(HL - HH)$ must be binding. So, suppose $IC(LH - HH)$ is. Simple manipulation of incentive constraints shows that $q_{HL}^1 - q_{HL}^2 = q_{HH}^1 - q_{HH}^2$. But then we must have $q_{HL}^2 > q_{HH}^2$, because at least one $q_{HL}^1 > q_{HH}^1$ or $q_{HL}^2 > q_{HH}^2$ must be true. But if only the former is true, but not the latter, then the previous inequality fails.

**Step 2.** To compare the profitability of the single-agent and two-agent mechanisms, we connect the principal’s maximization problems in the single-agent and the two-agent cases homotopically i.e., via a continuous transformation.

**Homotopy construction.** For $t \in [0, 1]$, define $V(t)$ as follows:

$$V(t) = \max_h (v(h_{LL}^1, h_{LL}^2) - c_L(h_{LL}^1 + h_{LL}^2)) \cdot p_1 p_2$$

$$+ \left( v(h_{HL}^1, h_{HL}^2) - \left( c_L - \frac{\Delta \mu (1 - t)}{p_1 (1 - p_2)} \right) h_{HL}^1 \right) - \left[ c_H + \Delta \left( \frac{p_2 t}{1 - p_2} + \frac{(\lambda_{HL} + \mu)(1 - t)}{p_1 (1 - p_2)} \right) \right] h_{HL}^1$$

$$+ \left( v(h_{HH}^1, h_{HH}^2) - \left[ c_H + \Delta \left( \frac{p_1 t}{1 - p_1} + \frac{(\lambda_{HL} + \mu - \kappa)(1 - t)}{(1 - p_1)(1 - p_2)} \right) \right] h_{HH}^1 \right) (1 - p_1) (1 - p_2)$$

(27)

For fixed $t \in [0, 1]$, the unique solution $h^*(t) = (h_{LL}^1(t), h_{LL}^2(t), h_{HL}^1(t), h_{HL}^2(t), h_{HH}^1(t), h_{HH}^2(t))$, $(i \in \{1, 2\})$ to the above maximization problem is characterized by the following first-order conditions.

$$v_1(h_{LL}^1(t), h_{LL}^2(t)) = v_2(h_{LL}^1(t), h_{LL}^2(t)) = c_L$$

(28)

$$v_1(h_{HL}^1(t), h_{HL}^2(t)) = c_L - \frac{\Delta \mu (1 - t)}{p_1 (1 - p_2)}$$

(29)

$$v_2(h_{HL}^1(t), h_{HL}^2(t)) = c_H + \Delta \frac{p_2 t}{1 - p_2} + \frac{(\lambda_{HL} + \mu)(1 - t)}{p_1 (1 - p_2)}$$

(30)

$$v_2(h_{HL}^1(t), h_{HL}^2(t)) = c_L - \frac{\Delta \kappa (1 - t)}{p_2 (1 - p_1)}$$

(31)

$$v_1(h_{HL}^1(t), h_{HL}^2(t)) = c_H + \Delta \frac{p_1 t}{1 - p_1} + \frac{(\lambda_{HL} + \kappa)(1 - t)}{p_2 (1 - p_1)}$$

(32)

$$v_1(h_{HH}^1(t), h_{HH}^2(t)) = c_H + \Delta \left( \frac{p_1}{1 - p_1} + \frac{(\lambda_{HL} + \mu - \kappa)(1 - t)}{(1 - p_1)(1 - p_2)} \right)$$

(33)

$$v_2(h_{HH}^1(t), h_{HH}^2(t)) = c_H + \Delta \left( \frac{p_2}{1 - p_2} + \frac{(\lambda_{HL} - \mu + \kappa)(1 - t)}{(1 - p_1)(1 - p_2)} \right)$$

(34)

Note that $h^*(0) \equiv g^*$, $h^*(1) \equiv q^*$, and $V(0)$ ($V(1)$) is equal to the expected profit from the optimal
Proof of lemma 2: The Lagrangian of \( RP(1) \) is given by (15) with \( \mu \) and \( \kappa \) set to zero. Uniqueness
of the solution follows from strict concavity of $v(.,.)$ and linearity of the constraints. To obtain (8)-(11), set $\alpha \equiv \frac{\lambda p_1}{p_1 p_2}$, $\gamma \equiv \frac{\lambda p_2}{p_1 p_2}$, and use these together with (16) to simplify (23)-(26).

Using a standard optimality argument one can show that both $IC(LH-HH)$ and $IC(HL-HH)$ must be binding. Since $\lambda_{HL} + \lambda_{HH} + \lambda_{LH} = p_1 p_2$, at least one of $IC(LL-HL)$, $IC(LL-LH)$, $IC(LL-HH)$ is binding, so the agent’s informational rent in state $LL$ is as stated in the lemma.

First, suppose that $IC(LL-HH)$ is non-binding. Let us show that in this case both $IC(LL-LH)$ and $IC(LL-HL)$ are binding. If $IC(LL-LH)$ is non-binding, then $\gamma = 0$ and $\alpha = 1$. So, by (8) and (10), $v_1(g_{1H}, g_{1H}^2) = v_1(g_{1H}^2, g_{1H}^2)$, while by (7) and (11) $v_2(g_{1H}, g_{1H}^2) < v_2(g_{1H}, g_{1H}^2)$. Then, $v_1(.) < 0$ and Property 1 imply that $g_{1H}^1 > g_{1H}^1$, so $IC(LL-HH)$ is binding. Contradiction. Similarly, we can show that $IC(LL-HL)$ must be binding.

Thus, if $IC(LL-HH)$ is non-binding, then $g_{1H}^1 + g_{1H}^1 = g_{2H}^2 + g_{1H}^1$, and $g_{1H} > g_{1H}^1$ for $i \in \{1, 2\}$. Applying Properties 1 and 2 to (7)-(11), we can establish that $g_{1H}^1 > g_{1H}^1 > g_{1H}^1$. Then it is easy to verify that all the remaining incentive constraints hold. So, the solution to the relaxed program is the optimal single-agent mechanism.

Now, suppose that the quantity allocation profile solving $RP(1)$ is such that $g_{1H}^1 \geq g_{1H}^1$ for both $i \in \{1, 2\}$, so that $IC(LL-HH)$ is binding. The principal could implement the same profile via a two-agent mechanism where in all states of the world except $LL$ the sum of transfers to the agents is equal to the transfer to the agent in the solution to the relaxed program. In state $LL$, agents 1 and 2 have to be paid informational rents equal to $\Delta g_{1H}$ and $\Delta g_{2H}$ respectively. The sum of these rents is (weakly) smaller than the informational rent $\Delta(g_{1H} + g_{2H})$ paid in $RP(1)$. Since the principal’s profits from $RP(1)$ is greater than her profits from the optimal single-agent mechanism, the two-agent mechanism dominates the single-agent one. The dominance is strict, because by lemma 1, the quantity allocation profile in the unique optimal two-agent mechanism differs from the solution to $RP(1)$. The latter follows from the difference in the first-order conditions characterizing the optimal quantities and, in particular, from the fact that $\gamma + \alpha \leq 1$. QED.

**Proof of Proposition 3:** By lemma 2 it is sufficient to show that $IC(LL-HH)$ is binding in the optimal single-agent mechanism, i.e. $g_{1H}^1 \geq g_{1H}^1$ for some $i \in \{1, 2\}$. The proof is by contradiction. So, suppose otherwise. Then the quantity allocations in the optimal single-agent mechanism satisfy (7)-(11) with $\gamma = 1 - \alpha$, and $g_{1H} + g_{2H} = g_{1H}^1 + g_{2H}^1$. For all $s \in [0, 1]$, define $g^1(s, \alpha)$ as follows:

$$
v_1(g^1(s, \alpha), g^2(s, \alpha)) = c_H + \frac{\alpha p_1}{1 - p_1} + s \Delta \frac{p_1}{1 - p_1} \frac{1 - \alpha}{1 - p_2}
$$

$$
v_2(g^1(s, \alpha), g^2(s, \alpha)) = c_L + s \Delta \left(1 + \frac{p_2}{1 - p_2} \frac{1 - \alpha}{1 - p_1}\right)
$$

Comparing (39) and (40) to (7)-(11), observe that $g^1(0, \alpha) = g_{1H}^1$, $g^2(0, \alpha) = g_{2H}^2$, while $g^1(1, \alpha) = g_{1H}^1$, $g^2(1, \alpha) = g_{2H}^2$. Differentiating (39) and (40) with respect to $s$, we get:

$$
\frac{dg^1(s, \alpha)}{ds} = \Delta \frac{v_2(g^1(s, \alpha), g^2(s, \alpha)) p_1}{v_1(g^1(s, \alpha), g^2(s, \alpha)) v_2(g^1(s, \alpha), g^2(s, \alpha)) - v_1^2(g^1(s, \alpha), g^2(s, \alpha))}
$$

25
Since \( v(\cdot, \cdot) \) is concave, we conclude that 
\[
\frac{d^g v^i(s, \alpha)}{ds} \geq 0 \forall s \in [0, 1] \text{ and hence } g_{HL} \geq g_{HL} \text{ if } \frac{v_{12}(g^i(t, \alpha), g^2(t, \alpha))}{v_{11}(g^i(t, \alpha), g^2(t, \alpha))} \geq \frac{p_{10}}{(1 - p_{10}) + \alpha p_{10}}.
\]
Similarly, \( g_{HH} \geq g_{HL} \) if the following inequality holds for \( g_1, g_2 \) on the relevant domain: 
\[
\frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)} \geq \frac{p_{20}}{(1 - p_{20}) + \alpha p_{20}}.
\]

**Proof of Proposition 4:** The proposition will be proved in a sequence of steps.

**Step 1.** When \( \omega \) is sufficiently small, then the optimal single-agent mechanism is given by the solution to the relaxed program in lemma 2.

To establish this, by lemma 2 it suffices to show that the solution to RP(1) is such that 
\( g^1_{HL} > g^1_{HH} \) for some \( i \in \{1, 2\} \). Fix \( \alpha \in [0, 1] \), and suppose that \( g_{HL}(\alpha), g^1_{HL}(\alpha), g^2_{HL}(\alpha) \) solve (7) - (11), while \( g^1(s, \alpha) \) and \( g^2(s, \alpha) \) solve (39) and (40)) for given \( \alpha \). Then \( g^1(0, \alpha) = g^1_{HL}(\alpha), g^2(0, \alpha) = g^2_{HL}(\alpha), g^1(1, \alpha) = g^1_{HH}(\alpha). \)

Equations (8) and (9) imply that \( g^1_{HL}(\alpha) \) is decreasing, and \( g^2_{HL}(\alpha) \) is increasing in \( \alpha \), while (10) and (11) imply that \( g^1_{HH}(\alpha) \) is increasing and \( g^2_{HH}(\alpha) \) is decreasing in \( \alpha \). Therefore, \( g^1_{HL}(\alpha) - g^1_{HH}(\alpha) \) decreasing, and \( g^2_{HL}(\alpha) - g^2_{HH}(\alpha) \) is increasing in \( \alpha \). Note that \( g^1_{HL}(1) - g^1_{HH}(1) < 0 \), while \( g^2_{HL}(0) - g^2_{HH}(0) < 0 \). Therefore, there exists a unique \( \alpha^* \in (0, 1) \) s.t. \( g^1_{HL}(\alpha^*) - g^1_{HH}(\alpha^*) > 0 \).

If \( \omega(p_{1}, p_{2}) < \frac{p_{1}}{(1 - p_{1}) + \alpha p_{1}} \) for all \( (q_1, q_2) \in [\tilde{q}_1, q_1] \times [q_2, \tilde{q}_2], \) then by (41) \( g^1_{HL}(1/2) - g^1_{HH}(1/2) = 0 \). Similarly, if \( \omega(p_{1}, p_{2}) < \frac{p_{2}}{(1 - p_{2}) + \alpha p_{2}} \) for all \( (q_1, q_2) \in [\tilde{q}_1, q_1] \times [q_2, \tilde{q}_2], \) then \( g^2_{HL}(1/2) - g^2_{HH}(1/2) = 0 \). So, if \( \omega(p_{1}, p_{2}) < \frac{p_{1}}{(1 - p_{1}) + \alpha p_{1}} \) for all \( i, j \in \{1, 2\} \), then \( g^i_{HL}(\alpha) > g^i_{HH}(\alpha). \)

**Step 2.** Simplifying (35), we get that the single-agent mechanism is optimal if
\[
\int_0^1 \left( 1 - \frac{1}{\omega}(h^1_{HL}(t, \alpha^*) - h^1_{HL}(t, \alpha^*)) + \alpha^*(h^2_{HL}(t, \alpha^*) - h^2_{HL}(t, \alpha^*)) \right) dt > 0 \tag{42}
\]
where \( h^i_{HL}(t, \alpha^*) \) solve (28)-(34) with \( \alpha^* = \frac{\lambda_{HL}}{\lambda_H}, \kappa = \mu = \lambda_H = 0 \).

Differentiating and using the fact that \( \frac{\omega(p_{1}, p_{2})}{\omega(q_{1}, q_{2})} < 1 \), we establish that \( h^i_{HL}(t, \alpha^*) \) is decreasing in \( t \), while \( h^i_{HH}(t, \alpha^*) \) is increasing in \( t \) for \( i \in \{1, 2\}. \)

**Step 3.** Fix \( \alpha^* \in (0, 1) \). For all \( \tilde{t} \in [0, 1], \tilde{\tilde{t}} \in [0, 1] \) s.t. either \( \tilde{t} < 1 \) or \( \tilde{\tilde{t}} < 1 \), there exists \( \omega(\alpha^*, \tilde{t}, \tilde{\tilde{t}}) \) s.t. \( h^1_{HL}(\tilde{t}, \alpha^*) > h^1_{HH}(\tilde{t}, \alpha^*) \) if \( \omega \leq \omega(\alpha^*, \tilde{t}, \tilde{\tilde{t}}). \)

Note that \( h^1_{HL}(\tilde{t}, \alpha^*) \) and \( h^2_{HL}(\tilde{t}, \alpha^*) \) are implicitly defined by \( v^1(h^1_{HL}(\tilde{t}, \alpha^*), h^2_{HL}(\tilde{t}, \alpha^*)) = c_H + \Delta_{21}(\alpha^* + (1 - \alpha^*) \tilde{t}) \) and \( v^2(h^1_{HL}(\tilde{t}, \alpha^*), h^2_{HL}(\tilde{t}, \alpha^*)) = c_L \). Similarly, \( h^1_{HL}(\tilde{\tilde{t}}, \alpha^*) \) and \( h^2_{HL}(\tilde{\tilde{t}}, \alpha^*) \) are implicitly defined by \( v^1(h^1_{HL}(\tilde{\tilde{t}}, \alpha^*), h^2_{HL}(\tilde{\tilde{t}}, \alpha^*)) = c_H + \Delta_{12}(\alpha^* + (1 - \alpha^*) \tilde{\tilde{t}}) \) and \( v^2(h^1_{HL}(\tilde{\tilde{t}}, \alpha^*), h^2_{HL}(\tilde{\tilde{t}}, \alpha^*)) = c_L + \Delta_{21}(\alpha^* + (1 - \alpha^*) \tilde{\tilde{t}}). \)

Let \( \tilde{h}^1_{\tilde{t}} \) solve \( v^2(h^1_{HL}(\tilde{t}, \alpha^*), \tilde{h}^1_{\tilde{t}}) = c_H + \Delta_{12}(\alpha^* + (1 - \alpha^*) \tilde{t}) \). Then we have:

\[
v^1(h_{HL}(\tilde{t}, \alpha^*), \tilde{h}^1_{\tilde{t}}) v^1(h_{HL}(\tilde{\tilde{t}}, \alpha^*), h^2_{HL}(\tilde{\tilde{t}}, \alpha^*)) - \int_{\tilde{h}^1_{\tilde{t}}}^{h^2_{\tilde{t}}(\tilde{t}, \alpha^*)} v^1(h_{HL}(\tilde{t}, \alpha^*), h) dh < v^1(h_{HL}(\tilde{\tilde{t}}, \alpha^*), h^2_{HL}(\tilde{\tilde{t}}, \alpha^*)) - \int_{\tilde{h}^1_{\tilde{t}}}^{h^2_{\tilde{t}}(\tilde{t}, \alpha^*)} v^2(h_{HL}(\tilde{\tilde{t}}, \alpha^*), h) dh.
\]

26
Let \( \omega_1(\alpha^*, i, i) = \frac{p_1(1-\alpha^*)}{(1-p_1(1-\delta)+p_2(1-\delta))}. \) If \( \omega < \omega_1(\alpha^*, i, i) \), then \( v_1(h^1_{HL}(i, \alpha^*), h^2_{H}) \leq c_H + \Delta \frac{p_1}{p_2} h^2_{HL}(1, \alpha^*) \), and hence \( h^2_{HL}(i, \alpha^*) \geq h^1_{HL}(i, \alpha^*). \)

**Step 4.** Similarly to step 3, for all \( \alpha^* \in (0, 1), i \in [0, 1], i \in [0, 1] \) s.t. either \( i < 1 \) or \( i > 1 \), if \( \omega \leq \omega_2(\alpha^*, i, i) \equiv \frac{p_2 \alpha^* (1-p_1(1-\delta)+p_2(1-\delta))}{(1-p_2(1-\delta))}, \) then \( h^2_{HL}(i, \alpha^*) \geq h^1_{HL}(i, \alpha^*). \)

**Step 5.** Since \( v_1(q_1, q_2)/v_2(q_1, q_2) < K \) for \( q_1 > 0, q_2 > 0 \), there exists \( \alpha \) s.t. \( \alpha^* \geq \alpha \), because otherwise we would have \( h^1_{HL}(0, \alpha^*) - h^2_{HL}(0, \alpha^*) > h^2_{HL}(0, \alpha^*) - h^1_{HL}(0, \alpha^*). \) Similarly, since \( v_1(q_1, q_2)/v_2(q_1, q_2) > K \) for \( q_1 > 0, q_2 > 0 \), there exists \( \alpha \) s.t. \( \alpha^* \leq \alpha \).

**Step 6.** If \( \omega < \omega_1(\pi, 1, 2, 1) \), then \( h^1_{HL}(1, \alpha^*) > h^2_{HL}(1, \alpha^*). \) Also, \( h^2_{HL}(1/2, \alpha^*) > h^1_{HL}(1, \alpha^*) \) if \( \omega < \omega_2(\alpha, 1/2, 1) \), and \( h^2_{HL}(1, \alpha^*) > h^1_{HL}(1, \alpha^*) \) if \( \omega \leq \omega_2(\alpha, 1/2, 1) \). So, \( h^1_{HL}(1/2, \alpha^*) - h^2_{HL}(1/2, \alpha^*) > h^1_{HL}(1, \alpha^*) - h^2_{HL}(1, \alpha^*) \) for \( i \in \{1, 2\} \) if \( \omega < \min(\omega_1(\pi, 1, 1/2), \omega_2(\alpha, 1/2, 1), \omega_2(\alpha, 1, 1/2)). \) Since \( h^1_{HL}(t, \alpha^*) - h^2_{HL}(t, \alpha^*) \) is decreasing in \( t \), this implies that \( (42) \) is positive. Finally, note that at each step of the proof the threshold value of \( \omega \) was chosen to be increasing in \( p_1 \) and \( p_2 \). *Q.E.D.*

**Proof of proposition 5:** By (12) and (13), the expected cost of implementing any quantity schedule via a hierarchy is not less than in the two-agent mechanism. Thus, \( H_1 \) (\( H_2 \)) attains the same performance as the two-agent mechanism if and only if the quantity schedule \( \{q^{i}_{HL}, q^{i}_{HH}, q^{i}_{HL}, q^{i}_{HH}\}_{i=1}^{2} \) from the optimal two-agent mechanism can be implemented in \( H_1 \) (\( H_2 \)) with transfers satisfying (12) and (13) as equalities, i.e. there exist \( \lambda \) and \( \delta \) s.t. \( T_{HL} = c_H q^{i}_{HL} + q^{i}_{HH} - p_2 \lambda \) and \( T_{HL} = c_H q^{i}_{HL} c_H + q^{i}_{HH} c_L + \Delta q^{i}_{HH} + (1-p_2) \lambda, T_{HH} = c_L q^{i}_{HL} + c_H q^{i}_{HH} + \Delta q^{i}_{HH} - \delta p_2, \) and \( T_{LL} = c_L q^{i}_{HL} + q^{i}_{HH} + \Delta(q^{i}_{HH} + q^{i}_{HL}) + \delta(1-p_2). \)

By differentiation, we can establish the following **Property 3:** Let \( v_1(q_1(t), q_2(t)) = c_1 + a_1 t \) and \( v_2(q_1(t), q_2(t)) = c_2 + a_2 t \) for \( t \in [0, 1], \) and \( c_i \geq 0 \) and \( a_i \) s.t. \( c_i + a_i \geq 0 \). Then:

\[
(q_1(1) - q_1(0))a_1 + (q_2(1) - q_2(0))a_2 \leq 0
\]

Consider possible deviations by the primary contractor in \( H_1 \). She can deviate either by misrepresenting her true cost in the first stage, or by announcing her true cost, but then misrepresenting the subcontractor’s cost. Both types of deviations involve two ‘adjacent’ states of the world which differ only in the subcontractor’s cost. For the second type of deviation this is so, because if the primary contractor reports her own cost truthfully but misrepresents the subcontractor’s true cost, the subcontractor’s informational rent in the adjacent state of the world is affected.

Consider the incentives constraints associated with these two types of deviations. First, \( IC(LH - HH, LL - HL) \) holds because (12) and (13) hold as equalities. \( IC(HH - LH, HL - LL) \) holds because \( q^{i}_{HH} < q^{i}_{HL} \) and \( q^{i}_{HL} < q^{i}_{LL} \). \( IC(LH - LL, LL - LH) \), \( IC(LH - HL, LL - HH) \), \( IC(HH - HL, HL - HH) \) and \( IC(HH - LL, LH - LL) \) correspond to infeasible deviations requiring the subcontractor’s quantity to increase in her cost. For the remaining constraints we have:

(i) \( IC(LH - HH, LL - HH) \) holds if \( \lambda \geq \Delta(q^{i}_{HH} - q^{i}_{HH}). \)

(ii) \( IC(HH - HH, HL - HH) \) holds if \( \lambda \geq 0. \)
(iii) $IC(\HH - \HL, \HL - \HH)$ holds if $(1 - p_2) \lambda \leq \Delta(q^2_{\HH} - q^1_{\HH})$.

(iv) $IC(\LL - \HL, \LL - \HL)$ holds if $(1 - p_2) \lambda \leq \Delta(q^2_{\LL} - q^1_{\LL} + (1 - p_2)(q^1_{\HH} - q^1_{\LL}))$.

(v) $IC(\LL - \HL, \LL - \HL)$ holds if $\delta \geq \Delta(q^1_{\HH} - q^1_{\LL})$.

(vi) $IC(\HH - \LL, \HH - \LL)$ holds if $\delta(1 - p_2) \leq \Delta(q^1_{\HH} + q^1_{\HH} - q^1_{\HH})$.

(vii) $IC(\HH - \LL, \HH - \LL)$ holds if $\delta(1 - p_2) \leq \Delta((1 - p_2)(q^1_{\HH} - q^1_{\LL}) + q^1_{\HH} - q^1_{\HH})$.

(viii) $IC(\HH - \LL, \HL - \HH)$ holds if $\delta p_2 \geq \Delta(q^1_{\HH} - q^1_{\LL})$.

Complementarity. By (ii), $\lambda \geq 0$. So, (iv) fails if $q^2_{\HH} - q^2_{\HH} < (1 - p_2)(q^1_{\HH} - q^1_{\HH})$ which is so if $\frac{\nu(\cdot)}{v(1, \cdot)} \geq \frac{1}{1 - p_2}$ for all $(q_1, q_2) \in [q^1_{\HH}, q^1_{\HH}] \times [q^2_{\HH}, q^2_{\HH}]$. If $\frac{\nu(\cdot)}{v(1, \cdot)} \leq \frac{1}{1 - p_2}$ $\forall (q_1, q_2)$ on this interval, set $\lambda = 0$. Then (iv) holds. (i) and (iii) hold because $q^1_{\HH} \geq q^1_{\HH}$ and $q^1_{\HH} \geq q^2_{\HH}$. Choose $\delta = \min\{0, \Delta(q^1_{\HH} - q^1_{\HH} + \frac{q^1_{\HH} - q^1_{\HH}}{1 - p_2})\}$. If $\delta = 0$, then it is easy to check that (v)-(vii) hold.

Now suppose that $\delta = \Delta(q^1_{\HH} - q^1_{\HH} + \frac{q^1_{\HH} - q^1_{\HH}}{1 - p_2}) < 0$. Then (vii) holds as an equality. Since $q^2_{\HH} > q^2_{\HH}$, (v) holds. (vi) holds because $q^1_{\HH} > q^1_{\HH}$. (viii) can now be rewritten as:

$$(1 - p_2)(q^1_{\HH} - q^1_{\HH}) + \frac{p_2}{1 - p_2}(1 - p_2)(q^1_{\HH} - q^1_{\HH}) + q^1_{\HH} - q^1_{\HH} \geq 0.$$ 

To show that this inequality holds, let $v_1(1, t), v_2(1, t) = c_L + \frac{\Delta t}{1 - p_2}$ and $v_2(1, t) = c_L + \frac{\Delta t}{1 - p_2}(1 - t)$ for $t \in [0, 1]$. Then, $q^1_{\HH} = q^1(0), q^1_{\HH} = q^1(0), q^1_{\HH} = q^1(1)$. By Property 3 with $a_i = \frac{\Delta}{1 - p_1}$ we have $(q^1_{\HH} - q^1_{\HH})(1 - p_2) + q^1_{\HH} - q^1_{\HH}(1 - p_1) \geq 0$. Since $q^1_{\HH} > q^1_{\HH}$, (vii) holds.

Let us establish that the optimal two-agent allocation can be implemented if the principal chooses the primary contractor appropriately. So, suppose that $IC(\LL - \HL, \LL - \HL)$ fails when agent 1 is the primary contractor, i.e. $q^1_{\HH} = q^1_{\HH} < (1 - p_2)(q^1_{\HH} - q^1_{\HH})$, and consider agent 2 to be the primary contractor. Then the corresponding incentive constraints are given by (i)-(vii) which are obtained from (i)-(viii) by switching the agents’ indices on quantities: 1 to 2 and vice versa. As above, set $\lambda = 0$ so that (iii) is impossible and set $\delta$ in such a way that (vii) holds. Repeating the same steps as before, we can easily confirm that all constraints other than $IC(\LL - \HL, \LL - \HL, \HH - \HL)$.

It remains to check (iv). To see this, combine $(q^1_{\HH} - q^1_{\HH})(1 - p_2) + (q^1_{\HH} - q^1_{\HH})(1 - p_1) \geq 0$ with $q^1_{\HH} - q^1_{\HH} < (1 - p_2)(q^1_{\HH} - q^1_{\HH})$, to get that

$$(q^1_{\HH} - q^1_{\HH})(1 - p_1) < q^1_{\HH} - q^1_{\HH} - (q^2_{\HH} - q^2_{\HH})(1 - p_1) < (1 - p_2)(q^1_{\HH} - q^1_{\HH}).$$ 

So, (iv) holds.

Substitutability. By (i) and (iii), we must have $(q^1_{\HH} - q^1_{\HH})(1 - p_2) \leq q^1_{\HH} - q^1_{\HH}$, which fails if $\frac{\nu(\cdot)}{v(1, \cdot)} > \frac{1}{1 - p_2} \forall (q_1, q_2) \in [q^1_{\HH}, q^1_{\HH}] \times [q^1_{\HH}, q^1_{\HH}]$. If $\frac{\nu(\cdot)}{v(1, \cdot)} > \frac{1}{1 - p_2} \forall (q_1, q_2) \in [q^1_{\HH}, q^1_{\HH}] \times [q^1_{\HH}, q^1_{\HH}]$, then (i)’ and (iii)’ are incompatible when agent 2 becomes the primary contractor.

The two domains do not intersect, so it is possible that each inequality holds on its respective domains.

If $\frac{\nu(\cdot)}{v(1, \cdot)} \leq \frac{1}{1 - p_2} \forall (q_1, q_2) \in [q^1_{\HH}, q^1_{\HH}] \times [q^1_{\HH}, q^1_{\HH}]$, then (i) and (iii) hold if we set $\lambda = \Delta(q^1_{\HH} - q^1_{\HH})$. (ii) and (iv) also hold in this case, the latter because $q^2_{\HH} \geq q^2_{\HH}$.

Next, set $\delta = \Delta(q^1_{\HH} - q^1_{\HH})$, so that (v) binds and $\delta$ cannot be decreased further. It is easy to check that (vii) and (viii) hold. (vi) holds if $q^1_{\HH} - q^1_{\HH} - q^1_{\HH}(1 - p_2) + q^1_{\HH} - q^1_{\HH} \geq 0$. To establish this inequality, consider the following pair of the first-order conditions: $v_1(1, t), v_2(1, t) = c_L + \frac{\Delta t}{1 - p_2}$ and $v_2(1, t) = c_L + \frac{\Delta t}{1 - p_2}t$, $t \in [0, 1]$. Then, $q^1_{\HH} = q^1(0), q^1_{\HH} = q^1(0), q^1_{\HH} = q^1(1)$ and

\[ lim_{\Delta \to 0} \frac{\nu(\cdot)}{v(1, \cdot)} \leq \frac{1}{1 - p_2} \forall (q_1, q_2) \in [q^1_{\HH}, q^1_{\HH}] \times [q^1_{\HH}, q^1_{\HH}]. \]
Combining (43) with \( \min\{q_{LL}, q_{HH}\} \geq q_{HL} \), we conclude that the desired inequality holds.

Finally, suppose that in the optimal single-agent mechanism \( q_{HH}^{1} \geq q_{HL}^{1} \) i.e. IC(\( LL - HH \)) is binding and the informational rent in state \( LL \) equals \( \Delta(p_{1}q_{HH}^{1} + p_{2}q_{HH}^{2}) \). In \( H_{1} \) assign the same quantity schedule as in optimal single-agent mechanism and the following transfers:
\[
T_{HH}^{1} = c_{H}(q_{HH}^{1} + q_{HH}^{2}), \quad T_{HL} = c_{H}q_{HL}^{1} + c_{L}q_{HL}^{2} + \Delta q_{HH}^{2}, \quad T_{LL} = c_{L}(q_{LL}^{1} + q_{HL}^{2}) + \Delta(q_{HH}^{1} + q_{HL}^{2}).
\]
This mechanism satisfies all incentive constraints in \( H_{1} \), but the principal pays a lower informational rent than in the single-agent mechanism. Q.E.D.

**Proof of Proposition 6:** In \( H_{2} \) the primary contractor submits combined cost report after communicating with the subcontractor. So, in contrast to \( H_{1} \), she decides whether to misrepresent her own cost or not after learning the subcontractor’s cost. As a result, the set of feasible deviations for the primary contractor in \( H_{2} \) is larger than in \( H_{1} \), and is the same as in the single-agent mechanism.

But as in \( H_{1} \), a deviation by the primary contractor in one state of the world affects her payoff in the ‘adjacent’ state. Specifically, there are two pairs of adjacent states of the worlds: (\( LL, LH \)) and (\( HL, HH \)). If the primary contractor misrepresents her cost in \( LH \), but reports it truthfully in \( LL \), then the subcontractor’s informational rent in state \( LL \) depends on the primary contractor’s report in \( LH \). So, each incentive constraint of this kind must also involve two ‘adjacent’ states of the world. Thus, in addition to (i)-(viii), the following constraints have to hold in \( H_{2} \):

(ix) IC(\( HH - LH, HL - HL \)): \( (1 - p_{2})p_{2}(\lambda - \delta) \leq \Delta(p_{2}(q_{HL}^{2} - q_{HH}^{2}) + (1 - p_{2})(q_{LL}^{1} - q_{HH}^{1})) \).

(x) IC(\( HH - LL, HL - HL \)): \( (p_{2}\lambda + (1 - p_{2})\delta)(1 - p_{2}) \leq \Delta(p_{2}(q_{HL}^{1} - q_{HH}^{2}) + (1 - p_{2})(q_{LL}^{1} - q_{HH}^{1} + q_{LL}^{2} - q_{HL}^{2})) \).

(xi) IC(\( HH - HH, HL - LH \)): \( (1 - p_{2})\lambda + \delta p_{2} \geq \Delta(q_{HH}^{1} - q_{HL}^{1} + q_{LL}^{2} - q_{HL}^{2}) \) (feasible only if \( q_{HL}^{1} \geq q_{HH}^{1} \) i.e. under substitutability).

(xii) IC(\( HH - HH, HL - LL \)): \( (1 - p_{2})(\delta - \lambda) \leq \Delta(q_{LL}^{1} - q_{HH}^{1} + q_{HH}^{2} - q_{HL}^{2}) \) (feasible only if \( q_{LL}^{2} \geq q_{HH}^{2} \).

(xiii) IC(\( LH - HH, LL - LL \)): \( (1 - p_{2})(\delta - \lambda) \leq \Delta(q_{HH}^{1} - q_{HL}^{2} - q_{LL}^{2}) \) (feasible only if \( q_{HH}^{1} \leq q_{HL}^{2} \).

(xiv) IC(\( LH - HL, LL - LL \)): \( (\lambda(1 - p_{2}) + \delta p_{2})(1 - p_{2}) \leq \Delta((1 - p_{2})(q_{HL}^{1} - q_{HH}^{1} + q_{LL}^{2} - q_{HL}^{2}) + p_{2}(q_{HL}^{2} - q_{HL}^{1})) \) (feasible only if \( q_{HL}^{1} \leq q_{HL}^{2} \) i.e. under complementarity).

(xv) IC(\( LH - LH, LL - HH \)): \( \lambda p_{2} + \delta(1 - p_{2}) \geq \Delta(q_{HH}^{1} - q_{HL}^{1} + q_{HH}^{2} - q_{HL}^{2}) \) (feasible only if \( q_{HH}^{1} \geq q_{HL}^{2} \).

The primary contractor could also make two deviations in adjacent states of the world by misrepresenting her cost in the first state, and by misrepresenting the subcontractor’s cost in the second. However, either the corresponding deviation is not feasible (because the quantities increase in
Consider (ix)-(xvi). First, eliminate (ix), (xii) and (xv): (ix) is implied by (xvi), (xv) is implied by the combination of (xvi) and (i), and (xii) is implied by (xiii).

Complementarity. By (xiii) and (iv), \( \delta \leq \Delta \left( \frac{\Delta H - \Delta L}{1-p_2} + q_{HH} - q_{HL}^1 \right) < \Delta(q_{HH} - q_{HL}) \). So, (v) fails if \( q_{HL}^2 < q_{HL}^1 \). Observe that \( q_{HL}^1 \geq q_{HL}^2 \) if \( \frac{v_{11}(q_1, q_2)}{\xi_{11}} \leq \frac{1-p_2}{1-p_1} \left( \frac{-v_{11}(q_1, q_2)}{\xi_{11}} > \frac{1-p_2}{1-p_1} \right) \) for \( (q_1, q_2) \in \{ \min\{q_{HL}^1, q_{HL}^1\}, \max\{q_{HL}^1, q_{HL}^1\} \} \times [q_{HL}^2, q_{HL}^1] \). To see this, consider first-order conditions: \( v_1(q_1(t), q_2(t)) = c_L + \frac{\Delta L}{1-p_2} t, \quad v_2(q_1(t), q_2(t)) = c_L + \frac{\Delta L}{1-p_2} (1-t) \) for \( t \in [0,1] \). Then \( q_{HL} = q_2(1), q_{HL}^2 = q_2(0) \) and \( \frac{dq_2(t)}{dt} = \Delta \left( \frac{v_1(q_1(t), q_2(t))}{\xi_{11}} + \frac{v_1(q_1(t), q_2(t))}{\xi_{11}} - \frac{v_1(q_1(t), q_2(t))}{\xi_{11}} - \frac{v_1(q_1(t), q_2(t))}{\xi_{11}} \right) \). So, \( q_{HL}^1 - q_{HL}^2 = \int_0^1 \frac{dq_2(t)}{dt} dt \geq 0 \) if the stated condition holds. Obviously, \( q_{HL}^1 < q_{HL}^2 \) if this condition fails everywhere on this interval.

Now let us suppose that \( q_{HL}^1 \geq q_{HL}^2 \). Then, (xi) is implied by (v) and (xiii). By (xiii) and (xvi), \( \delta - \lambda = \frac{\Delta H - \Delta L}{1-p_2} + q_{HL} - q_{HL}^1 \). As shown in the discussion of \( H_1 \), the right-hand side of this inequality is non-negative if \( \frac{v_{12}(q_1, q_2)}{v_{22}} \leq \frac{1-p_2}{1-p_1} \). Further, check that (vii), (vii) hold \( \forall \lambda \in [0, \frac{\Delta H - \Delta L}{1-p_2} + q_{HL} - q_{HL}^1] \).

Substitutability. (i) and (xvi) imply that \( \delta \leq \Delta \left( q_{HL} - q_{HL}^1 + \frac{\Delta H - \Delta L}{1-p_2} \right) \), so (vi) fails if \( q_{HL}^1 > q_{HL}^2 \). (1)-(5) imply that \( q_{HL}^1 \leq q_{HL}^2 \) if \( \frac{v_{12}(q_1, q_2)}{v_{22}} \leq \frac{1-p_2}{1-p_1} \left( \frac{v_{12}(q_1, q_2)}{v_{22}} > \frac{1-p_2}{1-p_1} \right) \) \( \forall (q_1, q_2) \in \{ \min\{q_{HL}^1, q_{HL}^1\}, \max\{q_{HL}^1, q_{HL}^1\} \} \times [q_{HL}^2, q_{HL}^1] \).

Suppose that \( q_{HL}^1 > q_{HL}^2 \). In the proof of Proposition 5 we showed that (i)-(iv) are compatible only if \( (q_{HL}^1 - q_{HL})^2 = q_{HL}^2 - q_{HL}^1 \) which holds (fails) if \( \frac{v_{12}(q_1, q_2)}{v_{22}} \leq \frac{1-p_2}{1-p_1} \left( \frac{v_{12}(q_1, q_2)}{v_{22}} > \frac{1-p_2}{1-p_1} \right) \) on the corresponding interval. (xiii) and (xvi) imply that \( \delta - \lambda = \frac{\Delta H - \Delta L}{1-p_2} \). Combining this with (i) we conclude that (v) and (vii) hold. Next, substitute \( \delta \) out and consider (vi), (vii) and (x). These inequalities restrict \( \lambda \) from above. So it is optimal to choose the smallest possible \( \lambda \) satisfying (i)-(iv) i.e. \( \lambda = \Delta(q_{HL} - q_{HL}^1) \). Then (vii) holds, while (vi) and (x) can be rewritten as:

\( q_{HL}^1 + q_{HL}^2 \geq q_{HL}^1 + q_{HL}^2 - p_2(q_{HL} - q_{HL}^1) \) and \( q_{HL}^1 + q_{HL}^2 \geq q_{HL}^1 + q_{HL}^2 - \frac{v_{12}(q_1, q_2)}{v_{22}} \) respectively. (33) implies that both inequalities hold because \( q_{HL}^1 > q_{HL}^1 \) and \( q_{HL}^1 > q_{HL}^1 \).

Now suppose that agent 2 is used as a primary contractor. Observe that \( q_{HL} \leq q_{HL}^1 \) if \( \frac{v_{12}(q_1, q_2)}{v_{22}} \leq \frac{1-p_2}{1-p_1} \left( \frac{v_{12}(q_1, q_2)}{v_{22}} > \frac{1-p_2}{1-p_1} \right) \), while \( q_{HL}^2 < q_{HL}^1 ) \) if \( \frac{v_{12}(q_1, q_2)}{v_{22}} \leq \frac{1-p_2}{1-p_1} \left( \frac{v_{12}(q_1, q_2)}{v_{22}} > \frac{1-p_2}{1-p_1} \right) \) on the corresponding interval. Q.E.D.

Proof of Proposition 7: Ex-post individual rationality constraints of the primary contractor require that \( \lambda = 0 \) and \( -\Delta q_{HL}^1/(1-p_2) \leq \delta \leq \Delta q_{HL}^1/p_2 \).

Substitutability: An allocation optimal in the two-agent mechanism cannot be implemented either in \( H_1^{sp} \) or \( H_2^{sp} \) because constraint (i) requires \( \lambda \geq \Delta(q_{HL}^1 - q_{HL}^1) > 0 \) which cannot be satisfied.
Complementarity. First, consider $H_1$. Recall that in $H_1$, $\lambda = 0$ and $\delta = \min\{0, \Delta(q_{HH}^1 - q_{HL}^1 + \frac{q_{22}^1 - q_{21}^1}{1-p_2})\}$. If $\delta = 0$ in $H_1$, then transfers $T_{HH}$ and $T_{HL}$ used in $H_1$ are feasible in $H_1$. Further, if $\delta = \Delta(q_{HH}^1 - q_{HL}^1 + \frac{q_{22}^1 - q_{21}^1}{1-p_2}) < 0$, then $\delta > -\Delta q_{HL}^1/(1-p_2)$, because $q_{HL}^1 \geq q_{HH}^1$ and $q_{LL}^1 \geq q_{HL}^1$, so the ex post individual rationality constraints are satisfied.

In $H_2$, from $\lambda = 0$ it follows that $\delta = \frac{q_{HH}^1 - q_{HL}^1}{1-p_2}$, which satisfies ex post IR and incentive constraints, except (v) and (viii). (v) and (viii) require that $\frac{q_{HL}^1 - q_{HH}^1}{1-p_2} \leq \min\{q_{HL}^1 - q_{HH}^1, (q_{HL}^1 - q_{HH}^1)/p_2\}$. $\frac{q_{HL}^1 - q_{HH}^1}{1-p_2} \leq (q_{HL}^1 - q_{HH}^1)/p_2$ holds (fails) if $-\frac{v_{12}(q_{11},q_{12})}{v_{22}(q_{11},q_{12})} \leq \frac{1-p_2}{p_2}$ ($-\frac{v_{12}(q_{11},q_{12})}{v_{22}(q_{11},q_{12})} > \frac{1-p_2}{p_2}$) on the corresponding interval. Furthermore, $\frac{q_{HL}^1 - q_{HH}^1}{1-p_2} \leq q_{HL}^1 - q_{HH}^1$ holds (fails) if $p_2$ is sufficiently small (large) and $p_1$ is sufficiently large (small).

References


Figure 1: Three Organizational Forms.

- **Single-agent**
  \[ v(q_1, q_2) \]
  \[ (q_1, c_1); (q_2, c_2) \]

- **Two-agent**
  \[ v(q_1, q_2) \]
  \[ q_1, c_1 \]
  \[ q_2, c_2 \]

- **Delegation (Hierarchy)**
  \[ v(q_1, q_2) \]
  \[ q_1, c_1 \]
  \[ q_2, c_2 \]

Figure 2: Incentive Constraints in the Single-Agent and Two-Agent Mechanisms.

- **Single Agent**
  - LL
  - HH

- **Two Agents**
  - **Agent 1**
    - (LL, LH)
  - **Agent 2**
    - (LL, HL)
    - (HL, HH)
    - (LH, HH)

Figure 3: Regions of optimality with quadratic production function.