The Value of Information and Optimal Organization.

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Abstract

The paper addresses the issue of optimal organization of production. I compare three organizational forms: centralization (one agent produces different inputs), decentralization (each of the two agents produces a different input and contracts directly with the principal), and delegation (two agents produce different inputs, the principal contracts with one of them only). The optimal organizational form depends on the degree of complementarity/substitutability between the inputs in the final use. The degree of complementarity/substitutability also determines whether delegation is payoff-equivalent to the two-agent mechanism from the point of view of the principal. In the context of delegation, I consider which of the two agents should serve as the primary contractor. I also address the issue of collusion between the agents in a decentralized organization and characterize the conditions under which a stake of collusion exists.

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1 Introduction

One of the central issues in the theory of organizations is how information should be distributed, exchanged, and processed within an organization. Clearly, answering this question is important for the design of optimal organizational structures. The relevant literature has explored two different approaches in addressing this issue. The first approach focuses on the cost of information processing, while the second approach involves studying incentive problems generated by the asymmetry of information between different parties in an organization.

This paper contributes to the second strand of literature. It studies an environment where the principal has to implement a project which requires allocating several tasks to subordinates (or, alternatively, procuring several inputs from providers) who have private information regarding the costs of performing these tasks (producing the inputs). The principal has to determine which organizational structure is optimal and design the contracts with subordinates/providers in an optimal way. A number of questions naturally arise in this context. Should several tasks (production of different inputs) be centralized in the hands of a single agent (supplier), or should those tasks (production of inputs) be allocated across a number of them? Should the agents be organized in a hierarchy or not, and should the amount of communication between them be restricted? For example, a city council can hire a single contractor for a municipal project, split the work between several firms, or allow the primary contractor to subcontract some work to others. A firm may train its employees as specialists in certain types of tasks, so that several employees typically work on a project. Alternatively, employees may be trained as generalists who can perform tasks of different types and handle all the work on some projects. Similar issues arise in a variety of other contexts, including procurement, outsourcing, and regulation.¹

To address these issues, I examine three organizational forms in the context of a production process requiring two inputs. In a centralized single-agent organization one agent supplies both inputs. In a decentralized two-agent organization each of the two agents supplies a different input. Finally, under delegation two agents supply different inputs, but the principal contracts with one of them and delegates to her the task of contracting with the second agent. The crucial difference between these organizational forms lies in their informational structure. In the single-agent organization, the agent has private information about production costs of both inputs, in the two-agent mechanism each agent knows only the cost of one input, while in the delegated mechanism the primary contractor serves as an informational intermediary passing the subcontractor’s cost infor-

¹ Particularly, while developing a new defense system, the Department of Defense has to decide whether to procure all its components from the same manufacturer or from different ones. The government may allow the existence of a multi-product monopoly, or break it up into several firms, as in the AT&T case. In more recent examples of deregulation in the electric power industry, the regulators were called to determine whether a public utility producing the bulk of power could also maintain the control over the transmission grid, or the latter should be controlled by a separate entity.
information to the principal. Consequently, the relative profitability of these mechanisms depends on the interaction between these two pieces of information.

Intuitively, the value of information to the agent(s) might be either subadditive or superadditive. In the subadditive case, the value of two pieces of information together, as in the single-agent and delegated mechanisms, is lower than the sum of the values of each piece of information used independently, as in the two agent mechanism. In the superadditive case, the ordering goes in the opposite way. Put simply, the main issue is whether from an agent’s point of view the knowledge of another piece of information increases the value of the first piece of information or decreases it.\(^2\) Since the principal’s interests are the opposite of the agent(s)’ interests, the principal prefers informational centralization if the value of information is subadditive for the agents. Conversely, the principal prefers informational decentralization if the value of information is superadditive.

The main insight of this paper is that the degree of complementarity or substitutability between the inputs\(^3\) determines whether the value of information is sub- or superadditive. Precisely, under complementary or small degree of substitutability the value of information is subadditive, provided that the two inputs are not too asymmetric in the final use, and it is superadditive when the degree of substitutability is sufficiently large.

To understand why this is so, consider the value of information in a single-agent mechanism. When the cost of an input is low, the agent earns a rent on this information. The value of this rent is equal to the surplus obtained by misrepresenting this cost as high, and is therefore proportional to the quantity of this input delivered under high cost.

Now consider the effect of misrepresenting a low cost of one input on the value of information about the second input. First, incentive compatibility requires that quantity of an input to be decreasing in its cost. Second, in an efficient ordering, under complementarity (substitutability) the optimal quantity of the second input is increasing (decreasing) in the quantity of the first input. So, under complementarity, misrepresenting the cost of one input upwards causes the quantity of the second input to go down, and therefore reduces the informational rent on the second piece of information. Under substitutability, such misrepresentation has the opposite effect because the optimal quantity of an input is increasing in the cost of the second input.

Thus, the reported cost of one input affects the value of information about the cost of the other input. We will refer to this as an ‘internalization factor,’ because a single agent internalizes this effect on her total payoff. In contrast, in the two-agent mechanism each agent exploits the value of her information independently taking the other agent’s strategy as given, and this effect is not internalized. Therefore, under complementarity (substitutability) the ‘internalization’ factor tends

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\(^2\)In economic literature one can find examples of situations where more information either hurts or benefits the informed party. For example, in Stackelberg oligopoly game information about a competitor’s action - i.e. her quantity choice- hurts a firm.

\(^3\)Complementarity and substitutability between the inputs are defined in the next section on the basis of the sign of the cross-partial derivative of the principal’s benefit function.
to make the value of information subadditive (superadditive).

The other factor affecting the relative performance of the single-agent and two-agent mechanisms is the difference in the structure of incentive constraints. In contrast to the two-agent mechanism, a single agent can manipulate both pieces of information, i.e. she can misrepresent production costs of both goods at the same time. So, a larger set of incentive constraints has to be satisfied in the single-agent mechanism. We refer to this as an ‘extra deviation’ factor. This factor makes each piece of information more valuable when the second piece is also known. Hence, it tends to make information superadditive.

To summarize, whether the value of information is sub- or superadditive and hence which organizational structure is optimal depends on the relative strength of the ‘internalization’ and the ‘extra deviation’ factors. The single-agent mechanism typically dominates the two-agent one under complementarity, because the ‘internalization’ factor favoring the single-agent mechanism is especially potent in this case. The principal is also able to leverage the effect of the ‘internalization factor’ and design a mechanism in which the value of information is subadditive under separability and even under a small degree of substitutability. In the latter case, the mechanism involves additional efficiency losses, as the quantity of one input is set to be increasing in the quantity of the other input - the opposite of the efficient orderings. But since the degree of substitutability is low, these efficiency losses are small, and the single-agent mechanism still dominates the two-agent one.

Nevertheless, the ‘extra deviation’ factor can overturn the ranking of organizational forms under complementarity when there is a strong asymmetry between inputs and the change in quantity of one input affects the marginal product of this input to a lesser degree than the marginal product of the other input. In this case, it becomes very attractive for a single agent to make a joint misrepresentation of the combination of low and high costs as high and low respectively. Proposition 2 derives the conditions under which this ‘extra deviation’ factor makes the two-agent mechanism more profitable for the principal.

Further, when the degree of substitutability is sufficiently large, it becomes too costly in terms of efficiency losses to use a mechanism in which the quantity of one input increases in the quantity of the other input. But when the ordering of quantities is reversed, the value of information in the single-agent mechanism becomes superadditive because of the ‘extra deviation’ factor: a low-cost producer of both inputs can obtain most surplus by misrepresenting both input costs as high. This ‘coordinated’ deviation is infeasible in the two-agent mechanism, so the two agent mechanism is optimal.

Another interesting set of issues arises in the context of delegation. A delegation mechanism cannot be more profitable than the two-agent mechanism, and the two are equivalent if the primary contractor could not exploit her position of an informational intermediary to increase her surplus. Thus, the key issue is whether the primary contractor benefits from intermediating the subcontractor’s cost information or simply passes it on to the principal. Potentially, she could benefit from
this role in two ways. First, she could try to appropriate some of the subcontractor’s informational rents. Second, she could manipulate the report regarding the subcontractor’s type to increase the rent on her own information.

I consider four delegation structures which differ in the extent of the principal’s contractual abilities. Although the exact conditions under which the two-agent and delegation mechanisms are equivalent vary with the contractual framework, the main conclusion remains the same. The primary contractor benefits from her role of an informational intermediary if the quantity of one input has a significant effect on the marginal product of the other input, i.e. if the degree of complementarity or substitutability between the inputs is sufficiently large. To understand these result, note that under these conditions the quantity of the input produced by the primary contractor and hence her informational rent are sensitive to the subcontractor’s information. Hence, the primary contractor has stronger incentives to manipulate the latter.

In the context of delegation I also consider the issue of the choice of the primary contractor. Given the asymmetry of the agents’ marginal products and cost distributions in my model, this question is natural. I identify the conditions determining whom of the two agents the principal should nominate as the primary contractor. To the best of my knowledge, this issue has never been addressed in the literature before.

The issues of incentives in organizations and optimal organizational structure have been studied by a number of authors.\(^4\) Baron and Besanko (1992), Gilbert and Riordan (1994), Da Rocha and de Frutos (1999) and Jansen (1999) examine the issue of optimal organization under perfect complementarity between the inputs. Baron and Besanko (1992) and Gilbert and Riordan (1994) show that the single-agent mechanism is superior, and the optimal allocation can also be implemented via delegation.\(^5\) In contrast, Da Rocha and de Frutos (1999) demonstrate that the two-agent mechanism becomes superior under perfect complementarity when the supports of the two cost distributions are sufficiently asymmetric.

Dana (1993) focuses on the effect of correlation in the cost structure under separability of the production function in the two inputs. He shows that the two-agent mechanism is optimal when correlation is sufficiently strong, which allows the firm to exploit relative performance evaluation. Jansen (1999) attains a similar conclusion under perfect complementarity and limited liability assumptions. Demski, Sappington and Spiller (1987) study the effect of cost correlation on a different organizational choice - optimal input supplier switching. ‘Informational economies of scope’ discussed by Dana under separability are similar to the effect of our ‘internalization factor.’ However, this paper focuses on technological interdependency between inputs and its effect on the relative strength

\(^4\) Armstrong and Sappington (2004) provide a comprehensive survey of the literature.

\(^5\) Iossa (1999) studies the optimal regulatory regime in a two-good economy with one-dimensional uncertainty: the producer(s) have superior information about the demand for one of the goods, but not about the other. She reaches a different conclusion that the regulator prefers monopoly (duopoly) when the goods are substitutes (complements). Given the differences in informational assumptions, the model in this paper is not directly comparable to hers.
of ‘internalization’ and ‘extra deviation’ factors.

Perfect complementarity and separability are interesting but quite special cases. Gilbert and Riordan (1994) point out that their analysis ‘...depends on the fixed proportions production technology. This is perhaps questionable even in the electricity example, because optimizing the transmission grid may reduce the need for the new generation capacity...’ i.e. the quality of the grid and the electric power itself appear to be substitutes. On the other hand, a higher quality of the grid means a higher stability of the network and a lower probability of outages. This may allow consumers to use more electricity and rely less on other forms of energy. So, the same two inputs may be complements. Other examples with some degree of complementarity or substitutability include express and regular mail, long distance and local telephony, internet and telephone communication, defense systems or municipal projects with multiple components.

Mookherjee and Tsumagari (2004) study a model with a homothetic benefit function of the principal and a continuous type distribution. They show that the single-agent organization dominates under complementarity when the input costs are identically exponentially distributed, while the two-agent organization performs better under substitutability. Thus, their results are similar to Propositions 1 and 3 in this paper. The difference between their paper and this one boils down to two aspects of the model which, in turn, generate two substantive differences in results. First, the assumption of homotheticity of the benefit function implies a stable relationship between the marginal products of the two inputs which guarantees that non-local incentive constraints are never binding in Mookherjee and Tsumagari (2004). In contrast, I allow for an arbitrary benefit function. This leads me to show that, when the benefit function is sufficiently asymmetric, the ‘extra deviation’ factor becomes effective under complementarity via binding horizontal incentive constraints, and as a result the two-agent mechanism becomes optimal (see Proposition 2).

Second, the definitions of substitutes (complements) in Mookhorjee and Tsumagari (2004) are based on the properties of the optimal two-agent (single-agent) mechanism and, thus, do not refer directly to the parameters of the model. In contrast, I define complements and substitutes on the basis of the sign of the cross-partial derivative of the principal's benefit function. Then I show that a single-agent mechanism is optimal under a small degree of substitutability (see Proposition 4). However, it would be impossible to classify this case in Mookherjee and Tsumagari (2004), as it satisfies both their definition of substitutability (the optimal quantity of an input in the two-agent mechanism is increasing in the cost of the other input) and their definition of complementarity (the optimal quantity of an input in the single-agent mechanism is decreasing in the cost of the other input).

The comparison of the single-agent and two-agent mechanisms provides additional insights regarding the potential for collusion in organizations. Laffont and Martimort (1997) and (1998) have studied this issue in a similar framework under perfect complementarity. They have shown that the potential for collusion exists only under additional restrictions on contracts, such as anonymity. Our
results allow to explain why a stake of collusion does not exist without such restrictions: under complementarity the value of information is typically subadditive, and so the principal prefers informational centralization. Thus, the principal would actually benefit if the agents could collude in the two-agent mechanism and coordinate their strategies to maximize their joint profits. More generally, I show that a stake of collusion always exists under substitutability. Under complementarity it exists if the two-agent mechanism is optimal (e.g. under the conditions of Proposition 2).

Our analysis of delegation contracting in hierarchial mechanisms is related to the work of Melumad, Mookherjee and Riechelstein (1995). Of the four delegation mechanisms that we consider, two ($H_1$ and $H_{D_1}^{ep}$) were first studied by these authors, while the other two ($H_D$ and $H_{1_1}^{ep}$) have not been considered previously. Melumad, Mookherjee and Riechelstein (1995) establish that the delegation mechanism $H_1$ in which the primary contractor reports her cost to the principal before communicating with the subcontractor and only has to break-even in the interim- is equivalent to the two-agent mechanism in the case of continuous distribution of types. Interestingly, we show that such equivalence does not hold when the set of types is finite. Intuitively, this is due to the fact that in the continuous type case incentive constraints which involve the primary contractor misrepresenting both her own and the subcontractor’s costs hold if the incentive constraints involving a misrepresentation of only one of the two costs are satisfied, whereas this is not true in the discrete case under large degrees of substitutability and complementarity (for more detailed explanation, see footnote 10 on page 18). In particular, under these conditions in our model the binding deviation involves the primary contractor reporting her own low cost as high and claiming that the subcontractor’s cost is low, irrespective of the latter’s true level, whereas the deviations involving only misrepresentation of the primary contractor’s cost is non-binding.

As for the hierarchy $H_{D_1}^{ep}$ - in which the primary contractor does not accept the contract offered by the principal before contracting with the subcontractor and which is equivalent to hierarchy $H_1$ in Melumad, Mookherjee and Riechelstein (1995)- the added value of the analysis in this paper consists in deriving the exact conditions -in particular, small degree of complementarity- when $H_{D_1}^{ep}$ attains the performance of the two-agent mechanism.

The two new hierarchies introduced in this paper $H_D$ and $H_{1_1}^{ep}$ capture alternative and realistic scenarios of contracting. In $H_D$, the primary contractor, first, accepts the principal’s contract without reporting her cost. She then contracts with the subcontractor and reports both costs to the principal, but does not have an option to withdraw from the contract after learning the subcontractor’s cost. In contrast, in $H_{1_1}^{ep}$ the primary contractor first reports her cost to the principal, but can withdraw from the contract at a later stage after receiving the subcontractor’s cost report.

The analysis of the single-agent mechanism involves solving a screening problem with two-dimensional type distributed over a discrete domain, and an arbitrary benefit function of the prin-

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6The principal would then offer them an allocation profile equivalent to the single-agent mechanism rather than a less profitable two-agent one.
cipal. By characterizing the optimal mechanism in this case and identifying the conditions under which the ‘extra deviation’ factor is effective and hence non-local incentive constraints bind, the paper contributes to the literature on multidimensional mechanism design (see Matthews and Moore (1987), McAfee and McMillan (1988), Armstrong (1996), Rochet and Choné (1998)). The paper in this literature that is most closely related is Armstrong and Roché (1999) who provide a complete characterization of the optimal screening mechanism with two-dimensional agent’s type under separability between the goods, but with an arbitrary degree of correlation between the parameters of the agent’s type. These authors show that different patterns of binding incentive constraints can arise depending on the nature of correlation. Thus our analysis complements theirs as we characterize the optimal two-dimensional screening mechanism for an arbitrary degree of complementarity or substitutability between the goods but with independently distributed type parameters.

On a more technical side, the contribution of this paper lies in demonstrating how the homotopy technique can be applied to compare the performance of different organizational forms. Specifically, I connect the sets of the first-order conditions characterizing the optimal mechanisms in different organizational forms homotopically, i.e. via a continuous transformation, and use this transformation to compute the difference between the principal’s expected payoffs in the two organizations. I believe that this technique can be applied more broadly in the analysis of organizational and contractual problems.

The rest of the paper is organized as follows. In section 2 I present the model and characterize optimal mechanisms. In section 3 I consider the complementarity case. Section 4 deals with the substitutability case. Section 5 studies delegation. In section 6 the issue of collusion is addressed. In section 7 the results are illustrated via a number of examples. The proofs are relegated to an appendix.

2 Model.

A central entity, or principal needs to procure two different goods or inputs. The principal’s benefit is measured by the production/benefit function $v(q_1, q_2)$, where $q_i$ is the quantity of input $i$, for $i \in \{1, 2\}$. I assume that $v(\ldots)$ is increasing in both arguments, twice continuously differentiable, and concave. The cross-partial derivative $v_{12}(\ldots)$ has a constant sign over the relevant domain. We will say that the inputs are complements (substitutes) if $v_{12}(\ldots) \geq 0$ ($v_{12}(\ldots) < 0$). To ensure that the optimal quantities are positive, I impose Inada boundary condition: $\lim_{q_i \to 0} v_1(q_1, q_2) = \infty$, for all $q_2 > 0$. This condition is dropped when I consider specific examples.

I will compare the performance of three organizational forms illustrated in Figure 1: centralized organization (one agent produces both inputs), decentralized organization (each input is produced by a different agent), and delegation mechanism where the agents are organized in a hierarchy and the principal contracts only with the supplier of one input, who in turn contracts with
In each organizational form, the principal offers contract(s) to the agent(s) who may either accept or reject the offer. If the contract(s) are accepted, the agent(s) produce and deliver the goods (inputs) to the principal and get paid according to the contract(s). Additional stages involving the contracting between the primary contractor and the subcontractor in the delegation mechanism are described in section 7.

The principal maximizes her expected benefit net of the expected payments for the inputs. The agents(s) are risk-neutral and decide whether to accept the contract after privately learning their production cost(s). An agent’s reservation utility level is normalized to zero. An agent cannot produce the good which she is not assigned to. The marginal costs of production are constant and are independently distributed across goods and across agents. Specifically, it is common knowledge that the marginal cost of good $i$ is low ($c_L^i$) with probability $p_i$, and is high ($c_H^i$) with the complementary probability, where $c_H^i > c_L^i > 0$. Let $\Delta = c_H - c_L$. Since the benefit/production function $v(,)$. can be arbitrarily asymmetric, the assumption that the distributions of input costs have a common support is equivalent to a less restrictive ‘common ratio’ assumption $\frac{c_L^1}{c_H^1} = \frac{c_L^2}{c_H^2}$ from which ‘common supports’ can be obtained by simple renormalization of units. Independence of distributions is assumed in order to abstract from factors on the cost side. The case of correlated marginal costs is explored in Dana (1993) and Jansen (1999).

Let us now describe the contracts offered by the principal in the single-agent and the two-agent mechanisms. By the Revelation Principle (see e.g. Baron (1989)), we can restrict attention to direct mechanisms where the agent(s) announce her (their) costs truthfully. A direct mechanism is a mapping from the set of possible cost types $\{c_L, c_H\} \times \{c_L, c_H\}$ (or states of the world) into the set of quantities and transfers: $R^2 \times R^2$ (in the two-agent mechanism), or $R^2 \times R$ (in a single-agent mechanism). The four possible states of the world are denoted by $LL, LH, HL$ and $HH$. In this notation, the first (second) letter indicates the marginal cost of the first (second) good.

Let $q^i = (q_{LL}^i, q_{LH}^i, q_{HL}^i, q_{HH}^i)$ denote the vector of quantities of good $i \in \{1, 2\}$ assigned

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7There is a number of reasons why the principal may want or have to procure all supply of a particular input from one source. The most common of them is the presence of fixed costs. If large fixed costs in the form of R&D, investment in equipment, infrastructure and training, etc., have to be sunk by each producer of the good before she learns her production costs, then having more than one supplier could be prohibitively expensive. Alternatively, the principal’s commitment to purchase all supply of an input from a particular agent may be required to alleviate potential hold-up problem and induce this agent to make the necessary investment in physical or human capital, or to perform R&D.

Consider, for example, the development of a new defense system. In the initial stage of procurement, the government normally considers bids from a number of firms. However, only one supplier of each major part is ultimately chosen. Moreover, the final price is usually determined after the contracts have already been awarded. According to Rogerson (1989), “economies of scale together with very small production runs render it economically infeasible to have two or more firms build fully functioning production lines... The prices for all production runs may be left to be determined by future negotiations. Transaction costs together with constantly evolving technological requirements are thought to render long-term contracts infeasible.”

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in the two-agent mechanism. By convention, the first letter in the subscript refers to the marginal
cost of good \(i\). For example, in the state \(LH\) the mechanism assigns quantities \(q^1_{HL}\) and \(q^2_{HL}\). Let
t_\{K,J\} denote the transfer to the agent producing good \(i\), in the case when she announces cost \(c_K\) and
the other agent announces cost \(c_J\) \((K, J \in \{L, H\})\). The two-agent mechanism has to satisfy the
following interim incentive and individual rationality constraints for each \(i, j \in \{1, 2\}, i \neq j\):

\[
IC^i(L) : (t_{LL}^i - c_Lq_{LL}^i)p_j + (t_{HL} - c_Lq_{HL}^i)(1 - p_j) \geq (t_{HL}^i - c_Lq_{HL}^i)p_j + (t_{HH} - c_Lq_{HH}^i)(1 - p_j)
\]

\[
IC^j(H) : (t_{HL}^j - c_Hq_{HL}^j)p_j + (t_{HH} - c_Hq_{HH}^j)(1 - p_j) \geq (t_{LL}^j - c_Hq_{LL}^j)p_j + (t_{HH} - c_Hq_{HH}^j)(1 - p_j)
\]

\[
IR^i(L) : (t_{LL}^i - c_Lq_{LL}^i)p_j + (t_{HL} - c_Lq_{HL}^i)(1 - p_j) \geq 0
\]

\[
IR^j(H) : (t_{HL}^j - c_Hq_{HL}^j)p_j + (t_{HH} - c_Hq_{HH}^j)(1 - p_j) \geq 0.
\]

Consider now a single-agent mechanism. Let \(g^i = (g_{LL}^i, g_{HL}^i, g_{HH}^i, g_{HL}^i)\) denote the vector
of quantities of good \(i\) assigned in this mechanism, and \(T_{K,J}\) denote the transfer to the agent who
announces costs \((c_K, c_J)\), where \(K, J \in \{H, L\}\). The single-agent mechanism has to satisfy the
following incentive and individual rationality constraints for all \(K, J, U, V \in \{L, H\}\):

\[
IC(KJ - UV) : T_{K,J} - c_Kg_{K,J}^1 - c_Jg_{K,J}^2 \geq T_{UV} - c_Ug_{UV} - c_Vg_{UV}^2\]

\[
IR(KJ) : T_{K,J} - c_Kg_{K,J}^1 - c_Jg_{K,J}^2 \geq 0.
\]

The structures of incentive constraints in the two-agent and single-agent mechanisms are
depicted in Figure 2. The downward incentive constraint \(IC(LL - HH)\), as well as the ‘horizontal’
incentive constraints \(IC(LH - HL)\) and \(IC(HL - LH)\) in the single-agent mechanism have no
counterparts in a two-agent mechanism, because agents choose their reports independently in the
latter one. When any one of these constraints is binding, it reduces the profitability of the single-
agent mechanism, i.e. the ‘extra deviation’ factor is effective. On the other hand, constraints
\(IC(LL - HL), IC(LL - LH)\) and \(IC(LL - HH)\) in the single-agent mechanism are mutually
exclusive, and so the principal can ensure that all these three constraints hold by paying the agent
a single informational rent in state \(LL\). This is a manifestation of the ‘internalization’ factor.

Let us now consider the optimal mechanisms maximizing the principals’ expected profits.
The properties of the optimal single-agent mechanism turn out to be quite sensitive to the degree
of complementarity and substitutability. Therefore, the optimal single-agent mechanism is characterized
separately in the case of complementarity (see the proof of Proposition 2) and substitutability
(see Lemma 2).

In the remainder of this section we consider the optimal two-agent mechanism. Essentially,
it consists of two submechanisms, one for each agent. In each of them the individual rational-
ity constraint of the high-cost type and the incentive constraint of the low-cost type are binding.
Technological interdependence, i.e. substitutability or complementarity in the production function,
causes the optimal quantity assigned to one of the agents to depend on the cost type of the other
agent, but has no effect on the set of binding constraints.

**Lemma 1** The optimal two-agent mechanism is unique. The optimal quantities are determined by
the following first-order conditions:

\[ v_1(q^1_{LL}, q^2_{LL}) = v_2(q^1_{LL}, q^2_{LL}) = v_1(q^1_{HL}, q^2_{LL}) = v_2(q^1_{HL}, q^2_{LL}) = c_L \]  \hfill (1)

\[ v_1(q^1_{HL}, q^2_{HH}) = c_H + \Delta \frac{p_1}{1 - p_1} \]  \hfill (2)

\[ v_2(q^1_{HL}, q^2_{HH}) = c_H + \Delta \frac{p_2}{1 - p_2} \]  \hfill (3)

\[ v_1(q^1_{HH}, q^2_{HH}) = c_H + \Delta \frac{p_1}{1 - p_1} \]  \hfill (4)

\[ v_2(q^1_{HH}, q^2_{HH}) = c_H + \Delta \frac{p_2}{1 - p_2} \]  \hfill (5)

The optimal quantity of an input is decreasing in its cost. Further, under complementarity \((v_{12} \geq 0)\), the optimal quantity of an input is decreasing in the cost of the other input, i.e.

\[ q^i_{LL} > \max\{q^i_{HL}, q^i_{HH}\} \text{ and } \min\{q^i_{LL}, q^i_{HH}\} > q^i_{HL}. \]

Under substitutability \((v_{12} < 0)\), the optimal quantity of an input is increasing in the cost of the other input, i.e.

\[ q^i_{HL} > \max\{q^i_{LL}, q^i_{HH}\} \text{ and } \min\{q^i_{LL}, q^i_{HH}\} > q^i_{HL}. \]

The transfers are given by: \(t^i_{HK} = c_H q^i_{HK}, t^i_{LK} = c_L q^i_{LK} + \Delta^i_{HK} \text{ for } K \in \{L, H\}.\)

**Proof:** See the Appendix.

To reduce the agents’ informational rents, the principal sets all quantity allocations in the two-agent mechanism, except \(q^i_{LL}\) and \(q^i_{LH}\), below the first-best. The quantities \(q^i_{LL}\) are set at the first-best level (no distortion ‘at the top’), while \(q^i_{LH}\) is set above (below) the first-best level when the two inputs are substitutes (complements).

### 3 Complementarity.

In this section I compare the profitability of the single-agent and two-agent mechanisms under complementarity. The outcome of this comparison depends on the degree of complementarity. The following Proposition does not require characterizing the optimal single-agent mechanism.

**Proposition 1** Suppose that the inputs are complementary, i.e. \(v_{12}(\ldots) \geq 0). Then the single-agent mechanism is more profitable for the principal than the two-agent mechanism if \(\left| \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} \right| \leq 1\) for all \(i \in \{1, 2\}\) and \(q_1, q_2 \geq 0\).

**Proof:** See the appendix.

Under the conditions of the Proposition, the value of information is subadditive and the principal can implement the allocation profile \((q^i_{LL}, q^i_{HL}, q^i_{HH}, q^i_{LH}), i \in \{1, 2\}\), from the optimal two-agent mechanism via a single-agent mechanism requiring lower expected payments. Specifically, in states \(HH, LH\) and \(HL\) the payments in the single-agent mechanism can be set equal to the sum of payments in the two-agent mechanism, while in the state \(LL\) -where both costs are low- a lower payment can be made in the single-agent mechanism due to the ‘internalization factor.’
To see this, note that in state LL the principal needs to pay total informational rent $\Delta(q_{HL} + q_{HL}^2)$ in the two-agent mechanism, because each agent can independently misrepresent her cost as high. If the same allocation profile is assigned in the single-agent mechanism, then the agent can deviate by misrepresenting only the cost of the $i$-th input, $i \in \{1, 2\}$, or the costs of both inputs. The latter deviation is least attractive because under complementarity the optimal quantity of one input is decreasing in the cost of the other input and, in particular, $q_{HL} > q_{HH}^1$ by Lemma 1.

If in state LL the agent misrepresents only the cost of the first input, then she earns a rent (surplus) equal to $\Delta q_{HL}^1$ on her information regarding the first good, but her rent on the information regarding the cost of the second good will be at most $\Delta q_{HL}^2$. Similarly, if the agent misrepresents only the cost of the second good, her surplus will be equal to $\Delta(q_{HL}^1 + q_{HH}^1)$. So, in state LL in the single-agent mechanism the principal needs to pay informational rent equal to $\Delta \max\{q_{HL}^1 + q_{HH}^2, q_{HL}^2 + q_{HH}^1\}$ which is less than the informational rent $\Delta(q_{HL}^1 + q_{HL}^2)$ paid in the two-agent mechanism. Thus, the value of information is subadditive.

Finally, the condition that the degree of complementarity, measured by

$$\max\left\{\frac{v_{12}(q_1, q_2)}{|v_{11}(q_1, q_2)|}, \frac{v_{12}(q_1, q_2)}{|v_{22}(q_1, q_2)|}\right\},$$

be less than 1 ensures that our single-agent mechanism satisfies the ‘horizontal’ incentive constraints $IC(LH - HL)$ and $IC(HL - LH)$.

When the degree of complementarity is sufficiently large and there is a strong asymmetry between the inputs so that an increase in the quantity of one input affects the marginal product of this input to a lesser degree that the marginal product of the other input, then one of the ‘horizontal’ incentive constraints becomes binding, and the result of Proposition 1 may be reversed due to this ‘extra deviation’ factor. Sufficient conditions for this to occur are given below.

**Proposition 2** Suppose that for $i, j \in \{1, 2\}, i \neq j$, we have:

(i) $v_{12}p_i(1 - p_j) + v_{ii}(p_i(1 - p_j) + p_j) \leq 0$,

(ii) $v_{12}(1 - p_j)(2p_i + p_j(1 - p_i)) + v_{jj}(1 - p_j)^2p_i + v_{ii}(p_i(1 - p_j) + p_j)) > 0$.

Then the two-agent mechanism is more profitable for the principal.

**Proof:** See the appendix.

Condition (ii) of Proposition 2 in combination with the fact that $v(.)$ is concave imply that $v_{12}(.) > \frac{|v_{11}(.)|}{1 - p_j}$ (this is formally shown in Step 0 of the proof in the Appendix), so that the degree of complementarity is greater than $\frac{1}{1 - p_j}$. To understand the implications of this condition for the single-agent mechanism, suppose that $i = 2$ and consider the states $HH$ and $LH$. The analysis of the first-order conditions characterizing the quantity allocations in these states shows that the optimal quantity of the first input increases by a smaller increment than the optimal quantity of the second input as we move from state $HH$ to state $LH$, i.e. as the marginal cost of the first input goes down while the cost of the second input remains high. Consequently, the incentive constraint $IC(HL - LH)$ becomes binding, as it gets more attractive for the agent in state $HL$ to misrepresent both costs and report $LH$. By doing so, the agent sustains a small loss on the first input, because
her true cost of producing this input is high, but she earns a large informational rent on her low cost of the second input.

This ‘extra deviation’ factor makes the two-agent mechanism more profitable, because the non-local incentive constraint $IC(\mathcal{H}L - \mathcal{L}H)$ does not have to hold in the two-agent mechanism. The set of binding incentive constraints in the optimal single-agent mechanism under the conditions of Proposition 2 consist of $IC(\mathcal{L}L - \mathcal{H}H)$, $IC(\mathcal{H}L - \mathcal{L}H)$, and $IC(\mathcal{L}H - \mathcal{H}H)$ (see Figure 3). Condition (ii) of the Proposition is needed to ensure that the upwards incentive constraints are not binding, which could happen if the degree of complementarity is too large.

As an example, consider quadratic utility function $A + a(q_1 + q_2) - \frac{b_1}{2} q_1^2 - \frac{b_2}{2} q_2^2 + q_1 q_2$ where $A, a, b_1, b_2$ are positive constants s.t. $b_1 b_2 > 1$, so that $v_{12}(\cdot)$ is normalized to 1. Then assumptions (i) and (ii) hold, in particular, when $b_2 = \frac{p_2(1-p_1)}{p_2(1-p_1) + \epsilon_2}$ and $b_1 = \frac{p_2(1-p_1) + \epsilon_1}{p_2(1-p_1)} - \epsilon_1$, where both $\epsilon_1$ and $\epsilon_2$ are positive numbers satisfying $\frac{p_2(1-p_1) + \epsilon_1}{p_2(1-p_1)} > \frac{\epsilon_1}{\epsilon_2} > \frac{p_2(1-p_1) + \epsilon_2}{p_2(1-p_1)}$.

A related result by Da Rocha and de Frutos (1999) shows that the two-agent mechanism can outperform the single-agent mechanism under perfect complementarity. These authors emphasize the asymmetry of the supports of the cost distributions (i.e. $\frac{c_{H} - c_{L}}{c_{H} - c_{L}}$ is sufficiently larger than 1) as an explanation. Yet, as our analysis indicates, strong complementarity in their production function must also play a role in their result. Indeed, Proposition 1 implies that, for any value of $\frac{c_{H} - c_{L}}{c_{H} - c_{L}}$, the single-agent mechanism remains optimal when the degree of complementarity is sufficiently small.\(^8\) Conversely, performing renormalization it is easy to show that the result of Da Rocha and de Frutos (1999) also holds when the cost distributions have a common support, and the production/benefit function is given by $\min\{\frac{q_1}{r_1}, \frac{q_2}{r_2}\}$ when $\frac{q_1}{r_1}$ is large enough. This condition is similar to the one if Proposition 2. (It is not identical because of the non-differentiability of the Leontieff production function at the corner points.)

## 4 Substitutability

Compared to the complementarity, there are several differences in the nature and strength of the ‘internalization’ and ‘extra deviation’ factors under substitutability. The main reason for this is that, under substitutability, efficiency requires the quantity of one input to increase in the cost of the other input. In particular, it is efficient to set $\frac{q_{i,H}}{q_{i,L}} > \frac{q_{i,L}}{q_{i,L}}$ for $i \in \{1, 2\}$ in the single-agent mechanism. If the quantity profile in the single-agent mechanism satisfies this ordering, then the ‘extra deviation’ factor manifests itself in the form of binding incentive constraint $IC(\mathcal{L}L - \mathcal{H}H)$, i.e. the most profitable deviation for the agent with two low costs is to report that they are both high.

\(^8\)As pointed out in Section 2, all the results of our paper hold if we replace the common support assumption with the ‘common ratio’ assumption $\frac{1}{r_H} = \frac{1}{r_H}$. In turn, an arbitrary large $\frac{1}{r_H} - \frac{1}{r_H}$ is consistent with the ‘common ratio’ assumption.
In the two-agent mechanism, the principal does not need to be concerned about this deviation because the agents could not coordinate their strategies. So, if $IC(LL - HH)$ is binding in the single-agent mechanism, the value of information is superadditive: the agent with low costs in production of both inputs can get an informational rent which is higher than the sum of informational rents of the two agents with the same costs. Then the two-agent mechanism would dominate.

Still, the potential to exploit the ‘internalization’ factor can make it optimal for the principal to violate the efficient ordering in the single-agent mechanism and implement a profile of quantities decreasing in the cost of the other input, in particular, by setting $g_{HL}^i > g_{HH}^i$ for $i \in \{1, 2\}$. Then in the single-agent mechanism the value of information will be subadditive, as the principal will pay a lower informational rent than in the two-agent mechanism with the same quantity assignment. This, however, will be achieved at the cost of productive distortions. In contrast, by Lemma 1, the quantity profile in the two-agent mechanism is increasing in the cost of the other input, which it more efficient. In this case, the optimal organizational form is determined by the tradeoff between lower informational rents in the single-agent mechanism and higher efficiency of the two-agent mechanism.

We use the ‘homotopy’ technique to determine which factor dominates.

Similarly to the complementarity case, the results ultimately depend on the degree of substitutability measured by $\frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)}$ and $\frac{v_{12}(g_1, g_2)}{v_{22}(g_1, g_2)}$. These ratios provide an appropriate measure of the degree of substitutability because they determine the optimal relative rate at which the two quantities change in response to a change in the cost of one of the inputs. Specifically, if $\frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)}$ and $\frac{v_{12}(g_1, g_2)}{v_{22}(g_1, g_2)}$ are high, then a change in the cost of one input causes the optimal quantity of the second input to change by a relatively large amount compared to the change in the quantity of the first input. Intuitively, in this case the degree of substitutability between the inputs is high.

As we demonstrate below, the single-agent mechanism is optimal when the degree of substitutability is low, while the two-agent mechanism dominates under a high degree of substitutability. Additionally, the larger is the probability that at least one input is produced at a high cost, the lower is the threshold degree of substitutability required for the two-agent mechanism to be optimal.

Our analysis relies on the following Lemma which provides a useful simplification in dealing with the single-agent mechanism. Consider relaxed single-agent program $RP(1)$ in which the principal’s problem is solved subject only to the downwards incentive constraints $IC(LL - LH)$, $IC(LL - HL)$, $IC(LH - HH)$, and $IC(HL - HH)$, and the individual rationality constraint $IR(HH)$.

**Lemma 2** Under substitutability, the relaxed program $RP(1)$ has a unique solution. There exist $\alpha_1, \alpha_2 \in [0, 1]$, $\alpha_1 + \alpha_2 \leq 1$ such that the quantity profile in this solution is characterized by the
following first-order conditions:

\[ v_1(g_{LL}, g_{LL}^2) = v_2(g_{LL}, g_{LL}^2) = v_1(g_{HH}, g_{HH}^2) = v_2(g_{HH}, g_{HH}^2) = c_L \] (6)

\[ v_1(g_{HH}, g_{HH}^2) = c_H + \Delta \cdot \frac{p_1}{1 - p_1} \cdot \alpha_1 \] (7)

\[ v_2(g_{HH}, g_{HH}^2) = c_H + \Delta \cdot \frac{p_2}{1 - p_2} \cdot \alpha_2 \] (8)

\[ v_1(g_{HH}, g_{HH}^2) = c_H + \Delta \cdot \frac{p_1}{1 - p_1} - \alpha_1 p_2 \] (9)

\[ v_2(g_{HH}, g_{HH}^2) = c_H + \Delta \cdot \frac{p_2}{1 - p_2} - \alpha_2 p_1 \] (10)

Transfers in the solution to \( RP(1) \) satisfy: \( T_{HH} = c_H(g_{HH}^1 + g_{HH}^2) \), \( T_{HL} = c_L g_{LL}^1 + c_H g_{HL}^2 + \Delta g_{HH}^1 \), \( T_{HL} = c_L g_{LL}^2 + c_H g_{HL}^1 + \Delta g_{HH}^2 \), \( T_{LL} = c_L g_{LL}^1 + c_H g_{HL}^2 + \Delta \max\{g_{HL}^1 + g_{HH}^2, g_{HL}^2 + g_{HH}^1, g_{HH}^1 + g_{HH}^2\} \).

We have \( \alpha_i > 0 \) only if \( g_{HH}^i + g_{HL}^i \in \arg\max\{g_{HL}^1 + g_{HH}^2, g_{HL}^2 + g_{HH}^1, g_{HH}^1 + g_{HH}^2\} \).

Either \( g_{HL}^i > g_{HH}^i \) for \( i \in \{1, 2\} \) or \( g_{HL}^i \geq g_{HH}^i \) for \( i \in \{1, 2\} \). If \( g_{HL}^i > g_{HH}^i \) for \( i \in \{1, 2\} \), then \( \alpha_1 + \alpha_2 = 1 \) and the solution to \( RP(1) \) is the optimal single-agent mechanism.

If \( g_{HH}^i \geq g_{HL}^i \) for \( i \in \{1, 2\} \), then the two-agent mechanism is optimal.

**Proof:** See the Appendix.

Lemma 2 allows to adopt the following strategy of determining the optimal organizational form. First, solve \( RP(1) \) and check whether its solution is such that \( IC(LL - HH) \) is binding, i.e. if \( g_{HH}^i \geq g_{HL}^i \) for \( i \in \{1, 2\} \). If this is so, then the two-agent mechanism dominates. On the other hand, if \( g_{HL}^i > g_{HH}^i \) for \( i \in \{1, 2\} \), then the solution to \( RP(1) \) is the optimal single-agent mechanism, and its profitability has to be compared with that of the optimal two-agent mechanism.

Applying this strategy we, first, focus on the conditions under which the two-agent mechanism is optimal. Let \( \mathbf{g}_1, \mathbf{g}_2 \) solve \( v_1(g_1, g_2) = c_H + \Delta \cdot \frac{p_1}{1 - p_1} \cdot \alpha_1 \) and \( v_2(g_1, g_2) = c_L \). Also, let \( \overline{g}_1, \overline{g}_2 \) solve \( v_1(\overline{g}_1, \overline{g}_2) = c_L \) and \( v_2(\overline{g}_1, \overline{g}_2) = c_H + \Delta \cdot \frac{p_2}{1 - p_2} \cdot \alpha_2 \). Then we have:

**Proposition 3** The two-agent mechanism is optimal if for any \( \alpha_1 \in (0, 1) \) either (i) \( \frac{v_2(g_1, g_2)}{v_1(g_1, g_2)} \geq \frac{p_2 \alpha_1}{(1 - p_2) + (1 - \alpha_1)p_2} \) for all \((g_1, g_2) \in [\mathbf{g}_1, \mathbf{g}_2] \times [\overline{g}_2, \overline{g}_2] \); or (ii) \( \frac{v_2(g_1, g_2)}{v_1(g_1, g_2)} \geq \frac{p_1 (1 - \alpha_1)}{(1 - p_1) + \alpha_1 p_1} \) for all \((g_1, g_2) \in [\mathbf{g}_1, \mathbf{g}_1] \times [\mathbf{g}_2, \overline{g}_2] \).

**Proof:** See the appendix.

According to Proposition 3, the two-agent mechanism is optimal whenever the degree of substitutability between the inputs is sufficiently high. In this case, the distortion of the quantity profile (compared to the first-best) needed to neutralize the ‘extra deviation’ factor in the single-agent mechanism becomes too large. The principal then sets \( g_{HH}^i > g_{HL}^i \) in the single-agent, making the constraint \( IC(LL - HH) \) binding, and so the two-agent mechanism becomes more profitable. Furthermore, the threshold degree of substitutability increases in both \( p_1 \) and \( p_2 \). This is so because the state \( LL \) is more likely when both \( p_1 \) and \( p_2 \) are high, and the ‘internalization’ factor makes the single-agent mechanism more profitable for the principal precisely in state \( LL \).
The following corollary shows that the degree of substitutability required for the two-agent mechanism to be optimal is less than 1.

Corollary 1 For all \( p_1, p_2 < 1 \), there exists \( r < 1 \) s.t. the two-agent mechanism is optimal if

\[
\min \left\{ \frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)}, \frac{v_{12}(g_1, g_2)}{v_{22}(g_1, g_2)} \right\} \geq r \text{ for all } (g_1, g_2) \in [g_1, \bar{g}_1] \times [g_2, \bar{g}_2].
\]

Proof: See the Appendix.

Next, suppose that the degree of substitutability is low. Then in the single-agent mechanism the principal exploits the internalization factor by making the quantity of one input to be decreasing in the cost of the other input - much like under complementarity. The optimal mechanism is then characterized in Lemma 2. However, in the two-agent mechanism we have the opposite ordering: the optimal quantity of one input is increasing in the cost of the other input.

Since the optimal quantities are ordered differently in the single-agent and two-agent mechanisms, a simple method of proof based on the comparison of informational rents is not applicable. Instead, to compare the principal’s expected profits in the optimal single-agent and two-agent mechanisms we use the homotopy technique developed in the Appendix. The following Proposition describes the result of this comparison.

Proposition 4 Suppose that there exist \( K \) and \( \bar{K} \), \( 0 < K \leq \bar{K} < \infty \), s.t. \( K < \frac{v_{11}(q_1, q_2)}{v_{22}(q_1, q_2)} < \bar{K} \) for all \((q_1, q_2) \in [q_1, \bar{q}_1] \times [q_2, \bar{q}_2]\). Then for all \( p_1, p_2 \in (0, 1) \) there exist \( \omega_i, i \in \{1, 2\} \), increasing in \( p_j, i \neq j \), such that the single-agent mechanism is optimal if \( \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} < \omega_i \) for all \((q_1, q_2) \in [q_1, \bar{q}_1] \times [q_2, \bar{q}_2]\) and \( i \in \{1, 2\} \).

Proof: See the Appendix.

The Proposition holds because the effect of the ‘internalization’ factor outweighs the efficiency losses from distorting the quantity profile in the single-agent mechanism when the degree of substitutability is low and both \( p_1 \) and \( p_2 \) are sufficiently high. The first condition guarantees that the negative effect on welfare of making the quantity profile decreasing in the cost of the other input (in particular, setting \( g_{1i}^{HL} > g_{1i}^{HH} \) for \( i \in \{1, 2\} \)) is not too large, while the second condition implies that the state \( LL \), where the internalization factor works, occurs with a high likelihood.

5 Delegation

In this section I consider another form or organization - delegation, with agents organized in a hierarchy. Specifically, production is performed by two agents who contribute different inputs, but the principal directly contracts only with one of them (the primary contractor) and delegates to her the task of contracting with the other agent, the subcontractor (see figure 1). Delegation is common in the allocation of tasks within an organization, in procurement and in construction industry. In large corporations senior managers typically delegate some supervisory functions and authority to middle managers.
Hierarchical delegation with asymmetric information was studied by a number of authors, in particular, Melumad, Mookherjee and Riechelstein (1995), Baron and Besanko (1992), Gilbert and Riordan (1994), Laffont and Martimort (1998). This literature points out that the advantages of delegation include an economy of communication costs achieved by shifting some of the contracting tasks from the principal to one of the subordinates. On the other hand, delegation leads to a loss of control by the principal which may negatively affect the incentives within hierarchies. The last point is made by McAfee and McMillan (1995) in the context of a model where intermediate layers of supervision separate the principal from the agent engaged in production. Riordan and Sappington (1987) show that the principal’s decision whether to delegate both stages of the production process to the agent or only one stage depends on whether the costs at the two stages are positively or negatively correlated.

The goal of this section is two-fold. The first goal is to compare the profitability of the delegation mechanism vis-a-vis the two-agent and the single-agent mechanisms in several contractual environments. The second goal is to examine the issue of the optimal choice of the primary contractor. In our model, agents 1 and 2 can have different cost distributions and different productivities, as reflected in the asymmetry of the production function. It is natural to ask whether these asymmetries imply differential performance by agents 1 and 2 in the role of a primary contractor. Thus, I will examine to whom of the two agents the principal should delegate the authority of contracting with the second agent.

To make legitimate comparisons across organizational forms, I make the same assumptions regarding input observability as in the single-agent and two-agent organizations studied in the previous sections. Specifically, I assume that under delegation the principal can monitor the quantity of input supplied by each agent. I will consider four different contractual set-ups referred to as delegation hierarchies \( H_1 \), \( H_D \), \( H_{1p} \) and \( H_{Dp} \). The following sequence of moves characterizes hierarchy \( H_1 \) (named so by Melumad, Mookherjee and Riechelstein (1995)):

1. The principal offers the contract to the primary contractor.
2. The primary contractor decides whether to accept or reject the contract. If she rejects, the game ends and all players obtain their reservation payoffs. If the primary contractor accepts the contract, then the game proceeds through the following stages.
3. The primary contractor reports her cost type to the principal.
4. The primary contractor offers a contract to the subcontractor. If the subcontractor rejects it, then the game ends and all players obtain their reservation payoffs.
5. If the subcontractor accepts, she reports her cost to the primary contractor, who then reports it to the principal.

Typically in this literature, communication costs are not modeled explicitly. Rather, they are assumed to be increasing in the amount of information transmitted between the parties and the number of rounds of communication. I will use this approach to interpret the results of this section.
6. Both contractors produce their inputs, the final output is delivered to the principal, and the transfers take place according to the two contracts.

The hierarchy $H_1$ is the most profitable for the principal among all delegation hierarchies with the same observability assumptions, because it endows the principal with broadest possible contracting abilities. In particular, the principal signs a contract with the primary contractor and receives her cost report before the latter communicates with the subcontractor. Therefore, $H_1$ serves as a natural benchmark establishing what is attainable in a delegation mechanism. This hierarchy provides a good representation of contractual schemes in construction industry where the customer, first, hires a primary contractor and obtains a cost estimate from her. The primary contractor is then typically given the authority to subcontract other providers whose costs are ex-ante uncertain.

By the Revelation Principle, the two-agent mechanism is at least as profitable for the principal as $H_1$. The question is then whether the principal can achieve the same expected profits in $H_1$ as in the two-agent mechanism, and how $H_1$ compares to the single-agent mechanism. An answer to these questions is provided in the following Proposition. Before presenting it, let us introduce the following piece of notation. Recall that the quantity schedule in the optimal two-agent mechanism is denoted by $\{q_{iLL}, q_{iLH}, q_{iHL}, q_{iHH}\}_{i=1}^{\aleph=2}$. Let $q_i = \max\{q_{iLL}, q_{iLH}\}$ and $q_i = \min\{q_{iHL}, q_{iHH}\}$.

**Proposition 5** If agent $i \in \{1, 2\}$ serves as the primary contractor, then the principal obtains the same payoff in $H_1$ as in the two-agent mechanism if $\left|\frac{v_{i12}(q_1, q_2)}{v_{ii}(q_1, q_2)}\right| \leq 1 - \frac{1}{1 - p_i}, \ i \neq j$, for all $(q_1, q_2) \in [q_1, q_1] \times [q_2, q_2]$. Conversely, the hierarchy $H_1$ with agent $i \in \{1, 2\}$ as the primary contractor is strictly less profitable for the principal than the two-agent mechanism if $\left|\frac{v_{i12}(q_1, q_2)}{v_{ii}(q_1, q_2)}\right| > 1 - \frac{1}{1 - p_i}, \ i \neq j$, for all $(q_1, q_2) \in [q_1, q_1] \times [q_2, q_2]$.

If either agent can serve as the primary contractor, then the principal obtains the same payoff in $H_1$ as in the two-agent mechanism in the following cases: (i) under complementarity; (ii) under substitutability, if $v_{iii}(.) \geq 0, v_{iij} \leq 0$ and $v_{ijj} \geq 0$ for some $i \neq j$ and all $(q_1, q_2) \in [q_1, q_1] \times [q_2, q_2]$.

$H_1$ is strictly more profitable for the principal than the single-agent mechanism if $IC(LL - HH)$ is binding in the latter.

**Proof:** see the Appendix.

According to Proposition 5, if only one of the agents can serve as the primary contractor, then $H_1$ is equivalent to the two-agent mechanism when the interdependence between the inputs in their final use is not too large, i.e. the marginal benefit/product of one input is not too sensitive to the quantity of the other input.

To understand this result, note that $H_1$ is equivalent to the two-agent mechanism only if the principal can implement the quantity profile from the optimal two-agent mechanism via $H_1$ at the same expected cost. It is easy to see that in $H_1$ each agent obtains at least as much surplus from private information regarding her own cost as in the two-agent mechanism with the same
quantity profile. So, $H_1$ can only attain the same level of profitability as the two-agent mechanism if the primary contractor cannot exploit her role as an informational intermediary to earn additional surplus, and simply passes on the information from the subcontractor to the principal without manipulating it. Manipulating this information could be profitable for the primary contractor for two reasons: (a) she could appropriate part of the informational rent that the principal intends for the subcontractor; (b) she could extract more surplus from her own information.

In hierarchy $H_1$, option (a) is infeasible because the primary contractor has to report her cost type before communicating with the subcontractor. Given the primary contractor’s report, the informational rents on the subcontractor’s information can be appropriated only by the subcontractor. At the same time, option (b) is feasible. It becomes significant when the report regarding the subcontractor’s cost has a large effect on the quantity assigned to the primary contractor, which is exactly when the degree of complementarity or substitutability between the inputs is sufficiently large.

Specifically, suppose that the inputs are complementary and consider the following deviation: the primary contractor misrepresents her low cost as high in the first stage, and then always reports that the subcontractor’s cost is low, i.e. in states $LH$ and $LL$ the primary contractor reports state $HL$. Then, in states $LH$ and $LL$ the primary contractor has to pay $c_H q_{LH}^2$ to the subcontractor, with a net loss of $\Delta(q_{LH}^2 - q_{HH}^2)$. However, the expected surplus obtained by the primary contractor on the information about her own cost increases from $\Delta(q_{HL}^1 p_2 + q_{HH}^1 (1 - p_2))$ to $\Delta q_{HL}^1$. In the proof of Proposition 5, I show that this increase outweighs the extra payment to the subcontractor when the degree of complementarity is sufficiently large. This is so because a report that the subcontractor’s cost is low rather than high causes a larger increase in the quantity supplied by the primary contractor, and hence in her informational rent, than in the quantity supplied by the subcontractor, and hence the extra payment to her. That is, $(q_{HL}^1 - q_{HH}^1)(1 - p_2) > q_{LH}^2 - q_{HH}^2$. Then in $H_1$ the principal has to pay a larger informational rent to the primary contractor than in the two-agent mechanism.

Under substitutability, the primary contractor with a low cost has a strong incentive to announce that both costs are high, irrespective of the subcontractor’s cost. This incentive is similar to the ‘extra deviation’ factor and binding incentive constraint $IC(LL - HH)$ in the single-agent mechanism. The principal can offset this incentive to a certain extent by imposing a penalty on the primary contractor when the latter reports that both costs are high. Yet, this penalty cannot be too large, because otherwise the primary contractor will misrepresent her own high cost as low. As a result, the primary contractor’s incentive to overstate her cost cannot be mitigated when the degree of substitutability is high, and again the principal has to pay a higher informational rent in $H_1$.10

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10 Melumad, Mookherjee and Riechelstein (1995) study $H_1$ in a model with a continuous distribution of types. They show that $H_1$ always attains the performance of the two-agent mechanism. The difference in results can be explained as follows. In our discrete model, a minimal deviation in any direction has a finite size equal to the corresponding cost difference. So, a minimal deviation involving a misrepresentation of both costs has a strictly larger
If the principal can choose either agent to serve as the primary contractor, then under complementarity she can always do so in such a way that $H_1$ attains the same performance as the two-agent mechanism. This is so because only one of the agents, when serving as the primary contractor, can have an incentive to always report her cost as high and the subcontractor’s cost as low.\textsuperscript{11} Under substitutability the ability of the principal to choose either agent to serve as the primary contractor guarantees that $H_1$ is equivalent to the two-agent mechanism only under additional restrictions on the signs of the third-order derivatives of the benefit function, because both agents could potentially have an incentive to overstate both costs.

Finally, the hierarchy $H_1$ performs better than the single-agent mechanism if $IC(LL - HH)$ is binding in the latter (which could only happen under substitutability), because in this case the principal can implement the optimal single-agent quantity profile at a lower cost via $H_1$.

The hierarchy $H_1$ has two important properties which affect its performance. First, in $H_1$ the primary contractor’s decision whether to report her true cost or not cannot be contingent on the subcontractor’s cost. This reduces the set of feasible deviations by the primary contractor and benefits the principal. Second, only the interim, rather than ex post, individual rationality constraints of both agents need to be satisfied in $H_1$. There is a significant difference between these two types of constraints, in particular, as far as the primary contractor is concerned. With the interim constraints, the principal structures her contract with the primary contractor in such a way that the primary contractor with a high cost obtains a negative payoff for one realization of the subcontractor’s cost, and a positive payoff for a different realization of the subcontractor’s cost. However, this would be impossible if the primary contractor’s ex post individual rationality constraint had to be satisfied, as would be the case, for example, if the primary contractor could withdraw from the contract after receiving the subcontractor’s cost report.

To understand the significance of these two effects, we will consider three alternative contractual arrangements. First, consider hierarchy $H_D$ in which the primary contractor does not make a cost report to the principal before communicating with the subcontractor. Formally, the sequence of steps in $H_D$ is the same as in $H_1$, except that stage 3 is eliminated, and in stage 5 the primary contractor reports both costs to the principal. Since the primary contractor accepts the contract with the principal before interacting with the subcontractor, only the interim participation constraints size than a minimal deviation which involves misrepresenting only one cost. This size difference makes the former deviation more attractive than any of the latter in some cases. In contrast, in the continuous type model a minimal deviation in each direction is infinitely small. Therefore, ensuring that incentive constraints hold along each cost dimension separately also ensures that incentive constraints involving a misrepresentation of both costs also hold.

On a more technical level, it is well known that an agent’s informational rent in the continuous multidimensional type model can be computed by integrating the agent’s utility along any direction from the lowest type (the so-called path-independence; see Armstrong (1996), Krishna and Maenner (2001) or Jehiel, Moldovanu and Stacchetti (1999)), whereas this is certainly not true in the discrete type model as incentive constraints do not bind in some directions.\textsuperscript{11} Mathematically, examination of the first-order conditions (1) -(5) in Lemma 1 shows that, if $(q_{hL} - q_{hH})(1 - p_2) > q_{hL} - q_{hH}$, then $(q_{hL} - q_{hH})(1 - p_1) \leq q_{hL} - q_{hH}$. 

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straints of the primary contractor have to hold. Hierarchy $H_D$ appears to be a good representation of contracting in the defense industry where marginal costs of production are not learned until significant fixed costs have been incurred, production lines have been built and supplier relationships have been established.

In $H_D$ the primary contractor has a larger set of possible deviations than in $H_1$, as she may decide to misrepresent her cost for one realization of the subcontractor’s cost, but not for the other realization. Consequently, the primary contractor can try to appropriate some of the informational rents intended by the principal for the subcontractor. In particular, under complementarity the primary contractor will have an incentive to misrepresent the state $LH$ as $HH$ in order to reduce the informational rent that she pays to the subcontractor in state $LL$ which will be reported truthfully. As a result, $H_D$ attains the performance of the two-agent mechanism under more restrictive conditions than $H_1$. Precisely, we have:

**Proposition 6** If agent $i$, $i \in \{1, 2\}$, serves as the primary contractor, then $H_D$ attains the same performance as the two-agent mechanism if

\[
\left| \frac{v_{12}(q_1, q_2)}{v_{j1}(q_1, q_2)} \right| \leq \frac{1-p_i}{1-p_j}, \quad j \neq i, \quad \text{for all } (q_1, q_2) \in [q_1, \bar{q}_1] \times [q_2, \bar{q}_2].
\]

Conversely, the hierarchy $H_D$ with agent $i$, $i \in \{1, 2\}$, as the primary contractor is strictly less profitable for the principal if

\[
\left| \frac{v_{12}(q_1, q_2)}{v_{j1}(q_1, q_2)} \right| > \frac{1-p_i}{1-p_j}, \quad j \neq i, \quad \text{for all } (q_1, q_2) \in [q_1, \bar{q}_1] \times [q_2, \bar{q}_2].
\]

If either agent can serve as the primary contractor, then $H_D$ attains the same performance as the two-agent mechanism if

\[
\left| \frac{v_{i1}(q_1, q_2)}{v_{j1}(q_1, q_2)} \right| \leq \frac{1}{1-p_j}, \quad \text{for each } i \in \{1, 2\} \quad \text{and } j \neq i, \quad \text{and all } (q_1, q_2) \in [q_1, \bar{q}_1] \times [q_2, \bar{q}_2].
\]

**Proof:** see the Appendix.

Comparison of Propositions 5 and 6 shows that the additional deviations available to the primary contractor in hierarchy $H_D$ have real consequences, and in some cases the hierarchy $H_1$ is strictly more profitable for the principal than $H_D$. Specifically, under intermediate degrees of complementarity the principal in $H_D$ has to leave a higher informational rent to the primary contractor to prevent the latter from exaggerating her cost in state $LH$ (without a misreport in state $LL$). This deviation -unavailable in $H_1$- allows the primary contractor to reduce the informational rent which she pays to the subcontractor in state $LL$. Similarly, under intermediate degrees of substitutability implementation in $H_D$ becomes more costly for the principal because she has to prevent the primary contractor from exaggerating her cost only in state $LL$, with state $LH$ announced truthfully. Such deviation is also unavailable in $H_1$.

Finally, suppose that the primary contractor could opt out of the contract after receiving the subcontractor’s report. Then the individual rationality constraints of the primary contractor have to hold ex post. Accordingly, let $H_D^{ep} (H_1^{ep})$ be a modification of hierarchy $H_1 (H_D)$ obtained by giving the primary contractor an option to withdraw after receiving the subcontractor’s cost report in Stage 5. We then have:

**Proposition 7** Under substitutability, both $H_1^{ep}$ and $H_D^{ep}$ are strictly less profitable for the principal
than the two-agent mechanism.

Under complementarity, we have:

(i) $H_1^{ep}$ attains the same performance as the two-agent mechanism if $H_1$ attains such performance.
(ii) If agent $i \in \{1, 2\}$ serves as the primary contractor, then $H_D^{ep}$ attains the same performance as the two-agent mechanism if $H_D$ attains the same performance and, additionally,
$$\left|\frac{v_{12}(q_1, q_2)}{v_{ij}(q_1, q_2)}\right| \leq \frac{1 - p_j}{p_j}$$
for all $(q_1, q_2) \in [\underline{q}_1, \overline{q}_1] \times [\underline{q}_2, \overline{q}_2]$, $p_j$ is sufficiently small and $p_i$ is sufficiently large.\(^{12}\)

In $H_1^{ep}$ and $H_D^{ep}$, the principal no longer has the freedom to distribute expected payments to the primary contractor across the states of the world in an arbitrary way. This restricts her ability to mitigate the primary contractor’s incentives to manipulate the subcontractor’s information and/or to capture some of the informational rents intended for the subcontractor. Specifically, since in $H_1^{ep}$ and $H_D^{ep}$ the primary contractor has to earn a nonnegative payoff in state $HH$, under substitutability the primary contractor has a stronger incentive to report $HH$ in states $LH$ and $LL$. For this reason, implementation in $H_1^{ep}$ and $H_D^{ep}$ is strictly more costly under substitutability.

Under complementarity, $H_1^{ep}$ performs as well as $H_1$. But in $H_D^{ep}$ the primary contractor has an even stronger incentive to misrepresent her cost in state $LH$ in order to capture a part of the informational rent intended for the primary contractor in state $LL$. So, $H_D^{ep}$ attains the same performance as the two-agent mechanism under more restrictive conditions than either $H_1$ or $H_D$.

Finally, a few words about the choice of the primary contractor are in order. Propositions 5-7 demonstrate that asymmetries in the cross-effects between the two inputs and differences of cost distributions affect the agents’ relative performance as primary contractors. Propositions 5 and 6 show that the principal is better off when the primary contractor is the agent who produces an input that has a smaller effect on the marginal product of the other input and who is more likely to be a high cost producer. Moreover, the principal benefits when she can choose either agent to serve as the primary contractor. Propositions 5-7 demonstrate that in some cases the ability to choose the primary contractor ensures that the principal gets the same profits as in the two-agent mechanism. These results have policy implications for optimal assignment of tasks within hierarchies.

### 6 Collusion

The results of the previous sections can be used to address the issue of collusion in organizations. Laffont and Martimort (1997) and (1998) -LM in the sequel- analyze this issue in a similar framework, and therefore it will be natural to compare our results to theirs. The potential for collusion arises in the two-agent mechanism if the agents can communicate and adopt a joint reporting strategy.

\(^{12}\)In the case of continuous distribution of types, the result that $H_D^{ep}$ does not attain the performance of the two-agent mechanism under substitutability has been established by Melumad, Mookherjee and Riechelstein (1995) who refer to this hierarchy as $H_1'$. So the added value of our analysis of this hierarchy lies in establishing the exact conditions under complementarity when it attains the performance of the two-agent mechanism.
However, collusion will not necessarily generate additional payoff for them and a loss for the principal. In the terminology of LM, a stake of collusion may not exist.

Obviously, the agents can achieve the highest joint payoff from collusion if they can overcome asymmetric information and bargaining problems between themselves and act as a single entity. We call this situation perfect collusion. Clearly, if there is no stake of perfect collusion under some contract offered by the principal, then there is no stake of collusion which is less than perfect, i.e. with some bargaining friction between the agents. The proof of this assertion is immediate: perfectly colluding parties can always imitate the behavior of the parties with a bargaining friction, and will do so if this increases their (i.e. perfectly colluding parties’) joint payoff.

When does a stake of perfect collusion exist? We can provide an answer to this question using the comparison of the single-agent and two-agent mechanisms. Specifically, suppose that the allocation profile from the optimal two-agent mechanism is assigned in the single-agent mechanism. Then a stake of perfect collusion exists if such mechanism is not incentive compatible, i.e. in some state(s) of the world the single agent has an incentive to misrepresent the costs of both inputs which is not feasible in the two-agent mechanism without collusion. On the contrary, there is no stake of collusion if the mechanism remains incentive compatible. This observation gives rise to the following proposition:

**Proposition 8** A stake of perfect collusion exists in each of the following two cases: (i) under substitutability; (i) under complementarity, whenever the two-agent mechanism is more profitable for the principal than the single-agent one.

A stake of perfect collusion does not exist when the inputs are complementary and

\[
\max \left\{ \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)}, \frac{v_{12}(q_1, q_2)}{v_{22}(q_1, q_2)} \right\} \leq 1 \text{ for all } q_1, q_2 \in \mathbb{R}_2^+.
\]

**Proof:** Under substitutability, the allocation profile in the optimal two-agent mechanism satisfies \(q_{HL}^i > q_{HH}^i\), so the colluding agents would deviate by reporting \(HH\) in state \(LL\).

Next, let \(M_2\) be the optimal two-agent mechanism with quantities and transfers \((q_{KJ}, t_{KJ}^1, q_{JK}^2, t_{JK}^2)\), \(K, J \in \{L, H\}\) characterized in Lemma 1. Suppose that the principal offers to a single agent a mechanism \(M_1\) which assigns the same quantities as in \(M_2\) and transfers equal to the sum of transfers in \(M_2\). Formally, \(g_{KJ}^i = q_{KJ}^i \forall i, K, J\) and \(T_{KJ} = t_{KJ}^1 + t_{JK}^2\).

If the two-agent mechanism dominates the single-agent mechanism (either under complementarity or under substitutability), then \(M_1\) cannot be incentive compatible, because otherwise the optimal single-agent mechanism would be at least as profitable as the optimal two-agent mechanism. When the two agents collude perfectly, they act as a single agent. Such agents will use the same joint strategy in \(M_2\) as the single agent would use in \(M_1\). Thus, \(M_2\) is not incentive compatible under perfect collusion, i.e. a stake of perfect collusion exists.

Now suppose that the inputs are complementary and \(\max \left\{ \frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)}, \frac{v_{12}(q_1, q_2)}{v_{22}(q_1, q_2)} \right\} \leq 1\) for all \(q_1, q_2 \in \mathbb{R}_2^+\). Let us show that \(M_1\) is incentive compatible. For this, we only need to show...
that $IC(LL - LH), IC(LH - HL)$ and $IC(HL - LH)$ hold in M1. First, since the quantity profile in M2 is such that $q_{HL}^i > q_{HH}^i, IC(LL - HH)$ holds in M1. Finally, applying the condition max $\left\{ \frac{v_{12}(q_1, q_2)}{|v_{11}(q_1, q_2)|}, \frac{v_{12}(q_1, q_2)}{|v_{22}(q_1, q_2)|} \right\} \leq 1$ for all $q_1, q_2 \in \mathbb{R}_+^2$ to the first-order conditions in Lemma 1 characterizing the quantity profile in M2, it is easy to see that both $IC(LH - HL)$ and $IC(HL - LH)$ also hold in M1. Q.E.D.

LM focus on the case of perfect complementarity. But the argument of Proposition 8 can be applied in this case to show that a stake of perfect collusion does not exist. This explains why LM had to impose additional restrictions on the set of feasible mechanisms to generate a stake of collusion. Specifically, they require the principal to offer an anonymous contract so that both agents get the same transfer in each state of the world. The anonymity generates a stake of collusion proportional to $q_{HL} - q_{HH}$. However, this stake of collusion disappears under substitutability and also when the benefit function is additively separable.

LM (1998) demonstrate that the principal can avoid the cost of preventing collusion in an anonymous mechanism through delegation. Their delegation mechanism (equivalent to our $H_D$ hierarchy) is more profitable for the principal than a two-agent mechanism with collusion. Yet, without anonymity restriction this result is not always true. In particular, suppose that inputs are complementary and agent 1 is the primary contractor. Propositions 6 and 8 imply that if max $\left\{ \frac{v_{12}(q_1, q_2)}{|v_{11}(q_1, q_2)|}, \frac{v_{12}(q_1, q_2)}{|v_{22}(q_1, q_2)|} \right\} \leq 1$ and $\frac{v_{12}(q_1, q_2)}{|v_{11}(q_1, q_2)|} > \frac{1 - p_i}{1 - p_j}$, then there is no stake of collusion, and $H_D$ is strictly less profitable than the two-agent mechanism.

7 Examples

The following examples illustrate our results. When specific functional forms are considered, sufficient conditions of Propositions 1-4 often translate into simple restrictions on the parameters.

**Example 1. Constant Elasticity of Substitution:** $v(q_1, q_2) = (\beta_1 q_1^\rho \cdot \beta_2 q_2^\rho)^\frac{1}{\rho}$ where $\rho < 1, 0 < m < 1$. The inputs are substitutes (complements) if $\rho > m (\rho < m)$, and $\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} = \frac{\rho - m}{(1 - m)(\beta_1 q_1^\rho \cdot \beta_2 q_2^\rho)^\frac{\rho}{\rho - 1}}$ for $i, j \in \{1, 2\}$.

If $q_1, q_2$ satisfy $v_i(q_1, q_2) = c_i$ for $i \in \{1, 2\}$ and $c_i \in [c_L, c_H + \frac{p_i}{(1 - p_j)(1 - p_j)^{\frac{1}{\rho}}}]$, $\frac{c_i}{q_2} = \left( \frac{c_i}{\beta_2} \right)^\frac{1}{\rho - 1}$, and so $\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} = \frac{(1 - m)(c_i / \beta_i)}{(1 - m)(c_i / \beta_i) + (1 - \rho) \frac{c_i}{q_2}}$. Thus, $\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)}$ is proportional to $\rho - m$, and its absolute value is increasing in $\beta_i$ and decreasing in $\beta_j$.

Hence, under complementarity ($\rho < m$), $-\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} < 1$ for $i \in \{1, 2\}$ if $m - \rho$ is sufficiently small. In this case, by Proposition 1 the single-agent mechanism is optimal. Further, for fixed $m$ and $\rho$, the conditions of Proposition 2 hold if $\frac{\beta_i}{\beta_j}$ is sufficiently large. In this case, the two-agent mechanism is optimal.

Under substitutability ($\rho > m$), Propositions 3 and 4 imply that the two-agent mechanism is optimal if $\rho - m$ is large enough and $p_1$ and $p_2$ are small, while the single-agent mechanism is
optimal if \( \rho - m \) is small and either \( p_1 \) or \( p_2 \) is sufficiently large.

Further, Propositions 5-7 imply that the delegation hierarchies considered in Section 5 perform better when \( |m - \rho| \) is small, and the principal prefers to use agent \( i \) rather than agent \( j \) as the primary contractor if \( \frac{d}{p_i} \) is large.

**Example 2. Quadratic:** \( v(q_1, q_2) = A + a(q_1 + q_2) - \frac{b_1}{2} q_1^2 - \frac{b_2}{2} q_2^2 + dq_1q_2 \), where \( a, b_i > 0 \), \( d^2 < b_1 b_2 \).

Under complementarity \((d > 0)\), by Proposition 1 the single-agent mechanism is optimal if \( d \leq \min\{b_1, b_2\} \). The conditions on the parameters which ensure that the two-agent mechanism is optimal due to binding horizontal constraints are discussed after the statement of Proposition 2.

Under substitutability \((d < 0)\), by Proposition 3 the two-agent mechanism is optimal if

\[
-d \frac{b_i}{p_i} > \frac{p_i}{1-p_i} \quad \text{for some } i \in \{1, 2\}. 
\]

By Proposition 4, the single-agent mechanism is optimal if both \(-d \frac{b}{\min\{b_1, b_2\}}\) is small, and \( p_1 \) and \( p_2 \) are sufficiently large. Figure 4 illustrates the regions of optimality of the two-agent and single-agent mechanisms in the symmetric case \( b_1 = b_2 = b \).

**Example 3. Perfect Substitutes:** \( v(q_1 + q_2) \). In this case, \( \frac{v_i(q_1, q_2)}{v(q_1 + q_2)} = 1 \) for \( i \in \{1, 2\} \). Therefore, by Corollary 1 the two-agent mechanism is optimal.

## 8 Conclusions.

In this paper, I have studied optimal organization of production in an environment where agents have private information about the cost of producing their inputs. The optimality of centralizing production in the hands of a single agent or decentralizing it between two agents depends on whether the value of cost information is sub- or superadditive for the agent(s) which in turn depends on whether the inputs are complementary or substitutable in their final use. Under complementarity or low degrees of substitutability centralization is optimal, unless the production function is highly asymmetric so that the quantity of one of the inputs affects the marginal benefit of the other input in a significant way. In such case, decentralization is optimal even under complementarity. Decentralization is also optimal when the degree of substitutability is large.

The degree of substitutability/complementarity between the inputs also affects the performance of delegation mechanisms. When it is large, then the interdependence between quantities of different inputs is also large in the optimal mechanism. This allows the primary contractor to benefit from her position of an informational intermediary either by increasing the informational rent that she obtains on her cost information or by appropriating part of informational rent intended for the subcontractor. The paper also highlights which of the two asymmetric agents should be chosen as the primary contractor to maximize the performance of a hierarchy. The latter set of results has policy implications for optimal allocation of supervisory functions and assignment of tasks within organizations.
9 Appendix

The following simple properties of concave functions will be useful below. They are established by differentiation.

Property 1. Let \( v(\cdot, \cdot) \) be a twice-continuously differentiable, increasing, concave function, and suppose that \( v_1(q_1, q_2) = c_1 \) and \( v_2(q_1, q_2) = c_2 \) for some \( c_1, c_2 \in (0, \infty) \). Then \( \frac{dv_1}{dx_1} = v_{22} < 0 \), \( \frac{dv_2}{dx_2} = v_{11} < 0 \), \( \frac{dv_2}{dx_1} = \frac{dv_1}{dx_2} = -v_{12} \).

Property 2: Suppose that \( v_1(q_1, q_2) = c_1 \) and \( v_2(q_1, q_2) = c_2 \) for some \( c_1, c_2 > 0 \). Then:

\[
\begin{align*}
\frac{dq_1}{dc_1} &< \frac{dq_2}{dc_1} \quad \text{if} \quad |v_{22}(q_1, q_2)| < |v_{12}(q_1, q_2)|, \\
\frac{dq_1}{dc_2} &< \frac{dq_2}{dc_2} \quad \text{and} \quad \frac{dq_2}{dc_2} < \frac{dq_2}{dc_1} \quad \text{if} \quad |v_{11}(q_1, q_2)| > |v_{12}(q_1, q_2)|
\end{align*}
\]

Property 3: Let \( v_1(q_1(t), q_2(t)) = c_1 + a_1 t \) and \( v_2(q_1(t), q_2(t)) = c_2 + a_2 t \) for \( t \in [0, 1] \), and \( c_i \geq 0 \) and \( a_i \) s.t. \( c_i + a_i \geq 0 \). Then:

\[
(q_1(1) - q_1(0))a_1 + (q_2(1) - q_2(0))a_2 \leq 0
\]

Proof of Lemma 1: The optimal two-agent mechanism is derived by solving the following problem subject to individual rationality constraints \( IR^i(H) \) and incentive constraints \( IC^i(L) \) for \( i \in \{1, 2\} \):

\[
\max_{q_1^{i,k}, q_2^{i,k}, t_i^{i,k}, j,k \in \{1,2\}} p_1 p_2 (v(q_{LL}^1, q_{LL}^2) - t_{LL}^1 - t_{LL}^2) + p_1 (1 - p_2) (v(q_{LH}^1, q_{HL}^2) - t_{LH}^1 - t_{LH}^2) + (1 - p_1) p_2 (v(q_{HL}^1, q_{LH}^2) - t_{HL}^1 - t_{HL}^2) + (1 - p_1) (1 - p_2) (v(q_{HH}^1, q_{HH}^2) - t_{HH}^1 - t_{HH}^2)
\]

Since \( v(\cdot, \cdot) \) is concave, this problem has a unique solution. The optimal quantities are characterized by the first-order conditions in the statement of the Lemma. Applying Property 1, we obtain the specified ordering. The transfers are chosen so that \( IR^i(H) \) and \( IC^i(L) \) are binding. Since \( q_{LL}^i > q_{HL}^i \) and \( q_{LL}^i > q_{HH}^i \), the incentive constraint of the high-cost agent is not binding. QED.

Proof of Proposition 1: Consider the following single-agent mechanism. In any state of the world \( KJ \), assign the same quantity allocation \( (q_{KJ}^L, q_{KJ}^H) \) as in the optimal two-agent mechanism, and the corresponding transfer from the following list: \( T_{HH} = c_H (q_{HH}^1 + q_{HH}^2), T_{LH} = c_L q_{HL}^1 + c_H q_{HL}^2 + \Delta q_{HH}^1, T_{HL} = c_H q_{HL}^1 + c_L q_{HL}^2 + \Delta q_{HH}^2, T_{LL} = c_L (q_{LL}^1 + q_{LL}^2) + \Delta \max\{q_{HL}^1 + q_{HL}^2 + q_{HL}^1 + q_{HL}^2\} \).

This mechanism is more profitable for the principal than the optimal two-agent mechanism, because her total payment is the same in all states of the world except \( LL \), where her total payment is lower than in the two-agent mechanism by \( \Delta \min\{q_{HL}^1 - q_{HH}^2, q_{LL}^2 - q_{HH}^1\} > 0 \). This mechanism satisfies all individual rationality constraints. Let us show that it is incentive compatible. Clearly, it satisfies all downwards incentive constraints \( IC(LL - HL), IC(LL - LH), IC(LH - HH), IC(HL - HH), IC(LL - HH) \). In particular, the latter holds because \( q_{HH}^1 < q_{HH}^2 \). The upwards constraints \( IC(HL - LL), IC(LH - LL), \) and \( IC(HH - LL) \) hold because \( q_{KL}^1 > q_{KL}^2 \) by Lemma 1.
Finally, consider the ‘horizontal’ incentive constraints $IC(LH - HL)$ and $IC(HL - LH)$. Since $IC(LH - HH)$ and $IC(HL - HH)$ are binding, $IC(LH - HL)$ holds if

$$q_{1L}^2 - q_{1L}^1 \geq q_{1H}^2 - q_{1H}^1$$

(11)

Similarly, $IC(HL - LH)$ holds if $q_{1H}^2 - q_{1H}^1 \geq q_{1L}^2 - q_{1L}^1$. To see that (11) holds note that by (1)-5, $v_2(q_{1L}^1, q_{1L}^2) < v_2(q_{1H}^1, q_{1H}^2)$ and $v_1(q_{1L}^1, q_{1L}^2) = v_1(q_{1H}^1, q_{1H}^2)$. Since $|v_{11}(q_1, q_2)| \geq v_{12}(q_1, q_2)$, Property 2, implies that (11) holds. Similarly, the first-order conditions in Lemma 1, the assumption that $|v_{22}(q_1, q_2)| \geq v_{12}(q_1, q_2)$ and Property 2 imply that $IC(HL - LH)$ holds.

**Proof of Proposition 2:**

The proof consists of 7 steps. Step 0 is preliminary. In steps 1-4, I characterize the optimal single-agent mechanism. In step 5, I derive a method allowing to compare the profitability of the single-agent and two-agent mechanisms. Finally, in step 6 this method is employed to show that the two-agent mechanism is more profitable than the single-agent mechanism under the conditions of the Proposition.

**Step 0.** Consider Conditions (i) and (ii) of the Proposition and let $i = 2$ and $j = 1$. In the case $i = 1, j = 2$ the proof is symmetric. Condition (ii) can be rewritten as

$$(v_{11}(1 - p_1) + v_{22})p_1(1 - p_2) + \left(2v_{12} + \frac{v_{22}}{1 - p_1} + v_{11}(1 - p_1)\right)p_2(1 - p_1) > 0.$$  

Note that

$$\left(2v_{12} + \frac{v_{22}}{1 - p_1} + v_{11}(1 - p_1)\right) < 0.$$  

Indeed, consider $\frac{v_{22}(1)}{1 - p_1} + v_{11}(1 - p_1)$ as a function of $p_1$. If $|v_{22}(.)| < |v_{11}(.)|$, then it reaches a unique maximum equal to $-2\sqrt{v_{11}v_{22}}$ at $p_1 = 1 - \frac{v_{22}(1)}{v_{11}(1)}$. If $|v_{22}(.)| \geq |v_{11}(.)|$, then it reaches a unique maximum equal to $v_{11} + v_{22}$ at $p_1 = 1$. But $-2\sqrt{v_{11}v_{22}} + 2v_{12} < 0$ and $v_{11} + v_{22} + 2v_{12} < 0$ because $v$ is concave.

Thus, Condition (ii) requires that $v_{11}(1 - p_1) + v_{22} > 0$.

**Step 1. Single-agent problem.** We start by considering a relaxed profit-maximization problem of the firm subject to $IR(HH)$, the individual rationality constraint of the type $HH$, and the downwards and horizontal incentive constraints $IC(LL - HL), IC(LL - LH), IC(LH - HH), IC(LH - LH), IC(HL - HH), IC(HL - LH)$. In step 4 we show that this problem, in fact, characterizes the optimal single-agent mechanism. The Lagrangian associated with this problem is:

$$\max L = p_1p_2(v(g_{1L}, g_{2L}) - T_{LL}) + p_1(1 - p_2) (v(g_{1H}, g_{2H}) - T_{HL})$$

$$+ (1 - p_1)p_2 (v(g_{1L}, g_{2L}) - T_{HL}) + (1 - p_1)(1 - p_2) (v(g_{1H}, g_{2H}) - T_{HH}) +$$

$$\lambda_{LH} (T_{LL} - c_L(g_{1L}^1 + g_{2L}^1) - T_{LH} + c_L(g_{1L}^2 + g_{2L}^2)) + \lambda_{HL} (T_{LL} - c_L(g_{1L}^2 + g_{2L}^2)) +$$

$$+ \lambda_{HH} (T_{LL} - c_L(g_{1L}^1 + g_{2L}^1) - T_{HH} + c_L(g_{1L}^2 + g_{2L}^2)) + \delta_{HH} (T_{LL} - c_H(g_{1L}^1 - c_Hg_{1L}^1) - T_{HH} + c_Hg_{1L}^2 + c_Hg_{2L}^2) +$$

$$+ \kappa (T_{LL} - c_Lg_{1L}^1 - c_Hg_{2L}^1 - T_{HH} + c_Hg_{1L}^2 + c_Lg_{2L}^2) +$$

$$\eta (T_{HH} - c_H(g_{1H}^1 + g_{2H}^1)) = 0$$

(12)

The Lagrange multipliers $\lambda_{LH}, \lambda_{HL}, \lambda_{HH}, \mu, \kappa, \delta_{HH}, \delta_{HH}$ and $\eta$ are nonnegative and satisfy the complementary slackness conditions, i.e. $\eta (T_{HH} - c_H(g_{1H}^1 + g_{2H}^1)) = 0$ and similarly for the other
constraints. The first-order conditions with respect to transfers are:

\[
T_{LL} : p_1p_2 = \lambda_{HH} + \lambda_{HL} + \lambda_{HH}
\]

\[
T_{LH} : p_1(1 - p_2) = \delta_{HH} - \lambda_{HH} - \mu + \kappa
\]

\[
T_{HL} : (1 - p_1)p_2 = \delta_{HH} - \lambda_{HL} + \mu - \kappa
\]

\[
T_{HH} : (1 - p_1)(1 - p_2) = \eta - \lambda_{HH} - \delta_{HH} - \delta_{HH}
\]

The equations (13)-(16) imply that \( \eta = 1 \) and \( \lambda_{HH} + \delta_{HH} + \delta_{HH} = p_1(1 - p_2) + p_2 \), which can be used to simplify the other first-order conditions as follows:

\[
v_1(g_{LL}, g_{HL}) = v_2(g_{LL}, g_{HL}) = -v_1(g_{LL}, g_{HL}) = c_L
\]

\[
v_1(g_{LH}, g_{HL}) = c_L - \frac{\mu}{p_1(1 - p_2)} \Delta
\]

\[
v_2(g_{LH}, g_{HL}) = c_L - \frac{\kappa}{p_2(1 - p_1)} \Delta
\]

\[
v_1(g_{HH}, g_{HL}) = \frac{\delta_{HH} - \lambda_{HH} + \mu - \kappa}{(1 - p_1)p_2} c_H + \frac{\lambda_{HL} + \kappa}{(1 - p_1)p_2} \Delta = c_H + \frac{\lambda_{HL} + \kappa}{(1 - p_1)p_2} \Delta
\]

\[
v_2(g_{HH}, g_{HL}) = \frac{\delta_{HH} - \lambda_{HH} + \mu - \kappa}{p_1(1 - p_2)} c_H + \frac{\lambda_{HL} + \mu}{p_1(1 - p_2)} \Delta
\]

\[
v_1(g_{HH}, g_{HH}) = c_H + \frac{p_1 \Delta}{1 - p_1} + \frac{\lambda_{HL} + \lambda_{HH} + \mu - \kappa}{(1 - p_1)(1 - p_2)} \Delta
\]

\[
v_2(g_{HH}, g_{HH}) = c_H + \frac{p_2 \Delta}{1 - p_2} + \frac{\lambda_{HL} + \lambda_{HH} - \mu + \kappa}{(1 - p_1)(1 - p_2)} \Delta
\]

**Step 2.** In this step, we determine which constraints are binding in the relaxed problem (12) and compute the values of the multipliers. Obviously, the constraint \( IR(HH) \) must be binding, because otherwise \( T_{HH} \) could be lowered without violating any other constraint. So, \( T_{HH} = c_H(g_{HH} + g_{HH}) \). Further, at least one of \( IC(HL - HH) \) and \( IC(LH - HH) \) must be binding, because otherwise the principal could lower \( T_{LH} \) and \( T_{HL} \) by the same amount without violating any other incentive constraints.

Next, let us establish that \( IC(HL - LH) \) is binding in the optimal mechanism. The proof is by contradiction. So suppose that \( IC(HL - LH) \) is non-binding. This has two immediate implications. First, \( \mu = 0 \). Second, \( IC(HL - HH) \) must be binding because otherwise \( T_{HL} \) can be reduced by some positive amount without violating any constraints of problem (12). Therefore, the informational rent (net payoff) of type \( HL \) who reports truthfully must be equal to \( \Delta g_{HH}^2 \). Hence, non-binding \( IC(HL - LH) \) implies that the following inequality holds:

\[
\Delta g_{HH}^2 > T_{LH} - c_H g_{HL}^1 - c_L g_{HL}^2.
\]

In turn, since \( IC(LH - HH) \) holds, the informational rent of type \( LH \) who reports truthfully must be at least \( \Delta g_{HH}^1 \). So, \( T_{LH} - c_H g_{HL}^1 - c_L g_{HL}^2 \geq \Delta (g_{HH}^1 - g_{HH}^1 + g_{HH}^2) \).

Hence, to arrive at a contradiction and thereby to establish that \( IC(HL - LH) \) must be binding, it is sufficient to show that \( g_{HH}^1 - g_{HH}^1 < g_{HH}^1 - g_{HH}^2 \). To establish this, note that by (18), (22) and (14), \( v_1(g_{HH}, g_{HH}) - v_1(g_{HH}, g_{HH}) \leq -\Delta \). On the other hand, \( \mu = 0 \) and, by (13), \( \lambda_{HH} \leq p_1p_2 \).
Therefore, $(21)$ and $(23)$ imply that $v_2(g^1_{HL}, g^2_{HL}) - v_2(g^1_{HH}, g^2_{HH}) \leq 0$. Applying Property 2 to these two inequalities and employing the fact that $|v_{11}(g_1, g_2)| > |v_{12}(g_1, g_2)| > |v_{22}(g_1, g_2)|$, we obtain that $g^1_{HL} - g^1_{HH} < g^2_{HL} - g^2_{HH}$, i.e. $IC(\text{HL} - \text{HH})$ must be binding.

Next, suppose that constraints $IC(\text{HL} - \text{LL})$, $IC(\text{HL} - \text{HH})$, and $IC(\text{LL} - \text{HH})$ are non-binding in the optimal mechanism. Then the corresponding multipliers $\kappa, \lambda_{HL}, \lambda_{HH}$ and $\delta^2_{HH}$ are equal to zero and, by $(13)$ and $(15)$, $\lambda_{HL} = p_1p_2$ and $\mu = p_2$. This set of multipliers determines the profile of quantity allocations according to the first-order conditions $(17)$- $(23)$. To confirm that these quantity allocations constitute a unique solution to problem 12, we need to show that $IC(\text{HL} - \text{LL})$, $IC(\text{HL} - \text{HH})$, and $IC(\text{LL} - \text{HH})$ are, indeed, non-binding when $\kappa = \lambda_{HL} = \lambda_{HH} = \delta^2_{HH} = 0$, $\lambda_{HL} = p_1p_2$, and $\mu = p_2$. The uniqueness would follow because the objective of the Problem 12 is concave and all its constraints are linear.

First, consider $IC(\text{HL} - \text{HH})$. By $(19)$, $(20)$, $(22)$ and $(23)$, we have: $v_1(g^1_{HL}, g^2_{HL}) = c_H + \frac{p_1}{1-p_1} < c_H + \frac{p_1}{1-p_1} + \frac{p_2}{p_1(1-p_2)} = v_1(g^1_{HH}, g^2_{HH})$ and $v_2(g^1_{HL}, g^2_{HL}) = c_L < c_H = v_2(g^1_{HH}, g^2_{HH})$. So, by Property 1, $g^1_{HL} > g^1_{HH}$. Hence, $IC(\text{HL} - \text{HH})$ is non-binding.

Next, consider $IC(\text{HL} - \text{LL})$. Observe that binding $IC(\text{HL} - \text{HH})$ and non-binding $IC(\text{HL} - \text{LL})$ are equivalent to the following two conditions respectively:

\[ T_{HL} - c_H g^1_{HL} - c_L g^2_{HL} = T_{HL} - c_H g^1_{HL} - c_L g^2_{HL} \quad (24) \]

\[ T_{HL} - c_L g^1_{HL} - c_H g^2_{HL} > T_{HL} - c_L g^1_{HL} - c_H g^2_{HL} \quad (25) \]

Above, we have established that $(24)$ must hold. So, $(25)$ holds and hence $IC(\text{HL} - \text{LL})$ is non-binding if and only if $g^1_{HL} - g^1_{HH} + g^2_{HL} - g^2_{HH} > 0$. To establish this inequality, consider the following system:

\[ v_1(g^1(t), g^2(t)) = c_H + \frac{p_1}{1-p_1} - \Delta \left( 1 + \frac{p_1}{p_1(1-p_2)} + \frac{p_2}{p_1(1-p_2)} \right) \quad (26) \]

\[ v_2(g^1(t), g^2(t)) = c_L + \Delta \left( 1 + \frac{p_2}{p_1(1-p_2)} \right) \quad (27) \]

Comparing $(26)$ and $(27)$ to the first-order conditions $(18)$-$(21)$, observe that $g^1_{HL} = g^1(1), g^1_{HH} = g^2(1), g^1_{HL} = g^1(0), \text{and } g^2_{HL} = g^2(0)$. Hence, $g^1_{HL} - g^1_{HH} + g^2_{HL} - g^2_{HH} = \int_0^1 \frac{dg^1(t)}{dt} - \frac{dg^2(t)}{dt} \, dt$. Totally differentiating $(26)$ and $(27)$ with respect to $t$ and solving for $\frac{dg^1(t)}{dt}$ and $\frac{dg^2(t)}{dt}$, we obtain:

\[ \frac{dg^1(t)}{dt} = \frac{dg^2(t)}{dt} = \Delta \left( -v_{22} \left( 1 + \frac{p_1}{1-p_1} + \frac{p_2}{p_1(1-p_2)} \right) - v_{11} \left( 1 + \frac{p_2}{p_1(1-p_2)} \right) - v_{12} \left( 2 + \frac{p_1}{1-p_1} + \frac{2p_2}{p_1(1-p_2)} \right) \right) \]

\[ \frac{v_{11}v_{22} - v_{12}^2}{v_{11}v_{22} - v_{12}^2} \leq \Delta \left( -v_{11} - v_{22} + 2v_{12} \right) \quad \text{for } v_{22}v_{11} - v_{12}^2 \text{ is positive because } v(.) \text{ is concave. } \text{So, consider its numerator. First, } -v_{11} - v_{22} + 2v_{12}, \text{ also by concavity of } v(.). \text{ Further, } -v_{11} - \frac{v_{22}}{1-p_1} > 0. \text{ This is so because, as shown in Step 0, Condition (ii) implies that } -\frac{v_{22}}{1-p_1} < v_{12}, \text{ but at the same time } v_{22}v_{11} - v_{12}^2 > 0 \text{ by concavity of } v(.). \text{ Further, in Step 0 we have also shown that } -\frac{v_{22}}{1-p_1} > v_{11}(1-p_1) - 2v_{12} > 0, \text{ So, } g^1_{HL} - g^1_{HH} + g^2_{HL} - g^2_{HH} > 0, \text{ as required.} \]

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Next, observe that since $IC(LL – HL)$ holds, $IC(LL – LH)$ is non-binding if
\[ T_{LH} - c_L g_{LH}^1 - c_L g_{HL}^2 < T_{HL} - c_L g_{HL}^1 - c_L g_{LH}^2 \]  
(29)

Given that (24) holds, (29) is equivalent to $g_{LH}^1 < g_{HL}^1$. Equations (26) and (27) yield:
\[ g_{LH}^1 - g_{HL}^1 = \int_0^1 \frac{dg^1(t)}{dt} dt = -v_{12} \left( 1 + \frac{p_1}{1 - p_1} + \frac{p_2}{p_1(1 - p_1)} \right) \frac{v_{11}v_{22} - v_{12}^2}{v_{11}v_{22} - v_{12}^2} dt \]

The integrand of this expression is negative for all $t$ because $-\frac{v_{12}}{1 - p_1} < v_{12}$, which implies that $g_{LH}^1 - g_{HL}^1 < 0$.

Next, let us show that $IC(HL – HH)$ is non-binding. Since $IC(HL – LH)$ is binding, $IC(HL – HH)$ is non-binding if $T_{LH} - c_H g_{LH}^1 - c_H g_{HL}^2 > \Delta g_{HH}^2$. In turn, since $IC(LH – HL)$ is non-binding, $IC(LH – HH)$ must be binding because otherwise $T_{LH}$ could be decreased by some positive amount. So $T_{LH} - c_L g_{LH}^1 - c_H g_{HL}^2 = \Delta g_{HH}^1$. Using this expression to substitute $T_{LH}$ from the previous inequality, we conclude that $IC(HL – HH)$ is non-binding if $g_{HH}^2 - g_{LH}^1 > g_{HH}^2 - g_{HL}^1$.

To show this, consider the following system:
\[ v_1(g^1(s), g^2(s)) = c_H + \Delta \frac{p_1}{1 - p_1} + \frac{p_2}{(1 - p_1)(1 - p_2)} - s\Delta \left( \frac{1}{1 - p_1} + \frac{p_2}{p_1(1 - p_1)(1 - p_2)} \right) \]  
(30)
\[ v_2(g^1(s), g^2(s)) = c_H + s\Delta \frac{p_2}{p_1(1 - p_2)} \]  
(31)

Note that $g_{LH}^1 = g^1(1)$, $g_{HL}^2 = g^2(1)$, $g_{HH}^1 = g^1(0)$, and $g_{HH}^2 = g^2(0)$. Hence, $g_{HH}^2 - g_{LH}^1 = (g_{HH}^2 - g_{HH}^1) = \int_0^1 \frac{dg^2(s)}{ds} ds$. Totally differentiating (30) and (31) with respect to $s$, we obtain:
\[ \frac{dg^2(s)}{ds} - \frac{dg^1(s)}{ds} = \Delta \frac{2p_2(1 - p_1) + v_{12} + v_{11}p_2(1 - p_1) + v_{22}(p_2(1 - p_1) + p_1)}{p_1(1 - p_1)(1 - p_2)(v_{11}v_{22} - v_{12}^2)} \]  
(32)

The denominator of (32) is positive by concavity of $v(.)$. Let us now focus on its numerator. First, note that Condition (ii) of the Proposition can be rewritten as follows, with $i = 2$ in the current case:
\[ (1 - p_1)(v_{12}p_2(1 - p_1) + p_1) + v_{11}p_2(1 - p_1) + v_{22}(p_2(1 - p_1) + p_1)) > 0 \]  
(33)

Since $v_{12}p_2(1 - p_1) + v_{22}(p_2(1 - p_1) + p_1) < 0$ by Condition (i) of the Proposition, it follows that
\[ (v_{12}(p_2(1 - p_1) + p_1) + v_{11}p_2(1 - p_1)) > 0. \]  
(34)

In combination (33) and (34) imply that the numerator of (32) is positive. So, $g_{HH}^2 - g_{LH}^1 > g_{HH}^2 - g_{HL}^1$, as required.

**Step 4.** In this step, we show that the solution to the relaxed problem (12) satisfies the omitted ‘upwards’ incentive constraints $IC(HH – LH), IC(HH – HL), IC(HH – LL), IC(LH – LL)$, and hence describes the optimal mechanism.

Since $IC(LH – HH)$ is binding, $IC(HH – LH)$ holds if $g_{LH}^1 \geq g_{HH}^1$. Note that $g_{LH}^1 - g_{HH}^1 = \int_0^1 \frac{dg^1(s)}{ds} ds$ where $g^1(s)$ is a solution to (30) and (31). Differentiation yields $\frac{dg^1(s)}{ds} = \Delta - \frac{p_2(1 - p_1)v_{12} - v_{12} + p_2(1 - p_1) + p_1}{p_1(1 - p_1)(1 - p_2)(v_{11}v_{22} - v_{12}^2)}$ which is positive by Condition (i) of the Proposition.
To show that $IC(HH - HL)$ holds, note that since $IC(HL - LH)$ and $IC(LH - HH)$ are binding, we have $T_{HL} = c_L g_{HL}^1 + c_H g_{HL}^1 + \Delta(g_{HL}^2 - g_{HL}^1 + g_{HL}^1)$. So, $IC(HH - HL)$ holds if $g_{HL}^1 + g_{HL}^2 - g_{HL}^1 - g_{HL}^2 \geq 0$. This inequality holds because, as shown in Step 3, $g_{HL}^1 + g_{HL}^2 - g_{HL}^1 - g_{HL}^2 > 0$ and $g_{HL} > g_{HL}^1$.

Now, consider $IC(HL - LL)$. Applying Property 1 to two pairs of first-order conditions, the first pair consisting of both equations in (17) and the second pair consisting of (18) and (20), we obtain that $g_{LL}^1 > g_{HL}^1$ and $g_{LL}^2 > g_{HL}^2$. The inequality $g_{LL}^1 > g_{HL}^1$ and the fact that $IC(LL - HL)$ is binding imply that $IC(HL - LL)$ is non-binding.

Similarly, the fact that $IC(LL - HL)$ holds, $IC(LL - HL)$ is binding and $g_{LL}^2 > g_{HL}^2$ imply that $IC(LH - LL)$ holds.

Finally, consider $IC(HH - LL)$. We have shown that $IC(HH - HL)$ and $IC(HL - LL)$ are non-binding, i.e.

$$T_{HH} - c_H g_{HH}^1 - c_H g_{HH}^2 > T_{HL} - c_H g_{HL}^1 - c_H g_{HL}^2 \quad (35)$$

$$T_{HL} - c_H g_{HL}^1 - c_L g_{HL}^2 > T_{LL} - c_L g_{LL}^1 - c_L g_{LL}^2 \quad (36)$$

Combining (35) and (36) with the fact that $g_{LL}^1 > g_{HL}^1$ gives us:

$$T_{HH} - c_H g_{HH}^1 - c_H g_{HH}^2 > T_{LL} - c_H g_{LL}^1 - c_H g_{LL}^2$$

i.e. $IC(HH - LL)$ holds.

**Step 5.** To compare the profitability of the single-agent and two-agent mechanisms, we connect problem 12 characterizing the optimal single-agent mechanism and the firm’s profit maximization problem in the two-agent case via a homotopy, i.e. a continuous transformation.

**Homotopy construction.** For $t \in [0, 1]$, define $V(t)$ as follows:

$$V(t) = \max \left( v(h_{LL}^1, h_{LL}^2) - c_L(h_{LL}^1 + h_{LL}^2) \right) p_1 p_2$$

$$+ \left( v(h_{HL,H}^1, h_{HL,H}^2) - \left( c_L - \frac{\Delta \mu(1 - t)}{p_1(1 - p_2)} \right) h_{HL,H}^1 - \left[ c_H + \Delta \left( \frac{p_2 t}{1 - p_2} + \frac{\lambda_{HL} + \mu(1 - t)}{p_1(1 - p_2)} \right) \right] h_{HL,H}^2 \right) p_1(1 - p_2)$$

$$+ \left( v(h_{HL,H}^1, h_{HL,H}^2) - \left( c_L - \frac{\Delta \kappa(1 - t)}{p_2(1 - p_1)} \right) h_{HL}^1 - \left[ c_H + \Delta \left( \frac{p_1 t}{1 - p_1} + \frac{\lambda_{HL} + \kappa(1 - t)}{p_2(1 - p_1)} \right) \right] h_{HL,H}^2 \right) (1 - p_1) p_2$$

$$+ \left( v(h_{HL,H}^1, h_{HL,H}^2) - \left[ c_H + \Delta \left( \frac{p_1}{1 - p_1} + \frac{\lambda_{HL} - \mu - \kappa(1 - t)}{(1 - p_1)(1 - p_2)} \right) \right] h_{HL,H}^1 \right) (1 - p_1)(1 - p_2)$$

(37)

For fixed $t \in [0, 1]$, the unique solution $h^i(t) \equiv (h_{LL}^1(t), h_{HL,H}^1(t), h_{HL,H}^2(t), h_{HL,H}^1(t)), (i \in \{1, 2\})$ to the
above maximization problem is characterized by the following first-order conditions.

\[ v_1(h_{1L}(t), h_{2L}(t)) = v_2(h_{1L}(t), h_{2L}(t)) = c_L \]  
\[ v_1(h_{1H}(t), h_{2H}(t)) = c_L - \frac{\Delta \mu (1-t)}{p_1(1-p_2)} \]  
\[ v_2(h_{1H}(t), h_{2H}(t)) = c_H + \Delta \left( \frac{p_2 t}{1-p_2} + \lambda_{1H}(1-t) \right) \]  
\[ v_2(h_{1H}(t), h_{2H}(t)) = c_L - \frac{\Delta \kappa (1-t)}{p_2(1-p_1)} \]  
\[ v_1(h_{1H}(t), h_{2H}(t)) = c_H + \Delta \left( \frac{p_1 t}{1-p_1} + (\lambda_{1H} + \kappa)(1-t) \right) \]  
\[ v_1(h_{1H}(t), h_{2H}(t)) = c_H + \Delta \left( \frac{p_1}{1-p_1} + (\lambda_{1H} + \mu - \kappa)(1-t) \right) \]  
\[ v_2(h_{1H}(t), h_{2H}(t)) = c_H + \Delta \left( \frac{p_2}{1-p_2} + (\lambda_{1H} - \mu + \kappa)(1-t) \right) \]  

Note that \( h^i(0) \equiv g^i \), \( h^i(1) \equiv q^i \), and \( V(0) (V(1)) \) is equal to the firm’s expected profit in the optimal single-agent (two-agent) mechanism. Using the envelope theorem to differentiate \( V(t) \) we get:

\[ V(1) - V(0) = \int_0^1 V'(t)dt = \Delta \int_0^1 \mu h_{1H}(t) - \kappa h_{1H}(t) + (\lambda_{1H} + \kappa - p_1 p_2) h_{1H}(t)dt \]
\[ + \Delta \int_0^1 (\lambda_{1H} + \mu - p_1 p_2) h_{2H}(t) + (\lambda_{1H} + \mu - \kappa) h_{1H}(t) + (\lambda_{1H} - \mu + \kappa) h_{2H}(t)dt \]

(45)

**Step 6.** Let us show that \( V(1) > V(0) \). With \( \mu = p_2, \lambda_{1H} = p_1 p_2, \kappa = \lambda_{1H} = 0 \), (45) simplifies to:

\[ V(1) - V(0) = \Delta \int_0^1 p_2(1-p_1)(h_{1H}(t) - h_{1H}(t)) - p_2 (h_{1H}(t) - h_{1H}(t)) dt \]

Observe that \( h_{1H}(t) - h_{1H}(t) = \int_0^1 \frac{\partial h_{1H}(t,s)}{\partial s}ds \) and \( h_{2H}(t) - h_{2H}(t) = \int_0^1 \frac{\partial h_{2H}(t,s)}{\partial s}ds \) where

\[ v_1(h^1(t,s), h^2(t,s)) = c_H + \Delta \left( \frac{p_1}{1-p_1} + \frac{(1-t)p_2}{(1-p_1)(1-p_2)} \right) = \Delta \frac{p_2(1-t)}{p_1(1-p_2)} s \]  
\[ v_2(h^1(t,s), h^2(t,s)) = c_H + \Delta \frac{p_2 t}{1-p_2} + \Delta \frac{p_2}{p_1(1-p_2)}(1-t)s \]

Differentiating (46) and (47), we obtain:

\[ \frac{\partial h^1(t,s)}{\partial s} = -\Delta v_{22}(h^1(t,s), h^2(t,s)) \frac{1}{1-p_1} \left( 1 + \frac{p_2(1-t)}{p_1(1-p_2)} \right) v_{12}(h^1(t,s), h^2(t,s)) = p_2 \]  
\[ \frac{\partial h^2(t,s)}{\partial s} = \Delta v_{11}(h^1(t,s), h^2(t,s)) \frac{1}{1-p_1} \left( 1 + \frac{p_2(1-t)}{p_1(1-p_2)} \right) v_{12}(h^1(t,s), h^2(t,s)) - v_{12}(h^1(t,s), h^2(t,s)) \]

(48)

Consequently, we obtain:

\[ V(1) - V(0) = \Delta p_2 \int_0^1 \left( \frac{v_{12}(.) + v_{22}(.)}{p_1} + \frac{2v_{12}(.) + v_{22}(.) - v_{12}(.)}{p_1(1-p_2)} \right) ds dt \]

(50)
Consider the numerator of the integrand of (50). Step 0 shows that $2v_{i2}(.) + \frac{v_{i2}(.)}{\lambda_2 - \lambda_1} + v_{i1}(.) (1 - p_1) < 0$. Hence, (50) is positive if \( \left( v_{i2}(.) + \frac{v_{i2}(.)}{\lambda_2 - \lambda_1} \right) + \left( 2v_{i2}(.) + \frac{v_{i2}(.)}{\lambda_2 - \lambda_1} + v_{i1}(.) (1 - p_1) \right) > 0 \). In Step 0 we have shown that this inequality is equivalent to Condition (ii) of the Proposition. \textit{Q.E.D.}

**Proof of Lemma 2.** The Lagrangian of $RP(1)$ is given by (12) in the proof of Proposition 2 with $\mu$ and $\kappa$ set equal to zero. Uniqueness of the solution follows from strict concavity of $v(.)$ and linearity of the constraints. The first-order conditions (7)-(10) are obtained from (17)-(23) by setting $\alpha_1 = \frac{\lambda_{HL}}{p_{1p_2}}$, $\alpha_2 = \frac{\lambda_{HH}}{p_{1p_2}}$, and using (13)-(16) to obtain $\eta = 1$, $\lambda_{HH} = p_{1p_2} (1 - \alpha_1 - \alpha_2)$, $\delta_{HH}^1 = p_1 (1 - (1 - \alpha_2) p_2)$, $\delta_{HH}^2 = p_2 (1 - (1 - \alpha_1) p_1)$.

Both $IC(LH - HH)$ and $IC(HL - HH)$ must be binding because otherwise the value of $RP(1)$ can be increased by reducing $T_{LH}$ or $T_{HH}$, respectively. This observation implies that $T_{LH}$ and $T_{HH}$ are as stated in the Lemma. A similar argument establishes that $T_{HH}$ is as stated in the Lemma.

Since by (13), $\lambda_{HL} + \lambda_{HH} + \lambda_{HH} = p_{1p_2}$, we have: (i) $\alpha_1 + \alpha_2 \leq 1$, (ii) at least one of $IC(LL - HL)$, $IC(LL - LH)$, $IC(LL - HH)$ is binding. Combining (ii) with the fact that both $IC(LH - HH)$ and $IC(HL - HH)$ are binding, we conclude that the agent's informational rent in state $LL$ is equal to $\Delta \max \{ g_{HL} + g_{HH}^1, g_{HL}^1 + g_{HH}^2, g_{HL}^2 + g_{HH}^1 \}$, which implies the expression for $T_{LL}$ in the statement of the Lemma. Also, if $g_{HL} + g_{HH}^1 \notin \arg \max \{ g_{HL}^1 + g_{HH}^1, g_{HL} + g_{HH}^1 + g_{HH}^1 + g_{HH}^1 \}$, then $IC(LL - HL)$ is non-binding and hence $\lambda_{HL} = \alpha_1 = 0$. Similarly, $\alpha_2 = 0$ if $g_{HL} + g_{HH}^2 \notin \arg \max \{ g_{HL}^1 + g_{HH}^2, g_{HL}^1 + g_{HH}^1 + g_{HH}^1 \}$.

Suppose that $g_{HL}^2 \geq g_{HH}^2$. Let us show that $g_{HH}^2 \geq g_{HH}^2$. For suppose not, i.e. $g_{HH}^2 < g_{HH}^2$. Then $g_{HH}^2 + g_{HH}^2 > g_{HH}^2 + g_{HH}^2 \geq g_{HH}^1 + g_{HH}^2$, and so $\alpha_1 = 0$ and $\lambda_{HH} = 0$. Hence $\alpha_2 = 1$. But then by (7)-(10), $v_1(g_{HL}^2 g_{HH}^2) < v_2(g_{HL}^2 g_{HH}^2)$ and $v_2(g_{HL}^1 g_{HH}^2) = v_2(g_{HL}^2 g_{HH}^1)$. Applying Property 1 to these two (in)equalities, we obtain $g_{HL}^2 < g_{HH}^2$. Contradiction.

We need to consider two cases: Case A: $g_{HL}^2 > g_{HH}^2$ for $i \in \{1,2\}$; Case B: $g_{HL}^i \leq g_{HH}^i$ for some $i \in \{1,2\}$.

**Case A.** In this case, $\lambda_{HH} = 0$ and $\alpha_1 + \alpha_2 = 1$. Let us show that the omitted incentive constraints $IC(HH - LL)$, $IC(HH - LH)$, $IC(HH - HL)$, $IC(LH - LL)$, $IC(LH - LH)$, $IC(LH - HL)$ and $IC(HL - LH)$ hold. The constraints $IC(HH - LL)$, $IC(HH - LH)$, $IC(HH - HL)$, $IC(LH - LL)$, $IC(HL - LH)$ are ‘upwards.’

Given the expressions for transfers $T_{LL}$, $T_{HH}$, $T_{HL}$ and $T_{HH}$, these constraints hold if $\min \{ g_{HL}^i g_{HH}^i \} \geq g_{HH}^i$, and $g_{HL}^i \geq g_{HH}^i$ for $i \in \{1,2\}$. Since $g_{HL}^i > g_{HH}^i$ for $i \in \{1,2\}$, we only need to show $\min \{ g_{HL}^i, g_{HH}^i \} \geq g_{HH}^i$. To see this, note that by (7)-(10), $v_1(g_{HL}^i, g_{HH}^i) = v_2(g_{HL}^i, g_{HH}^i) < \min \{ v_1(g_{HH}^i, g_{HH}^i), v_2(g_{HH}^i, g_{HH}^i) \}$. Then Property 1 implies that $g_{HL}^i > g_{HH}^i$ for $i \in \{1,2\}$.

Next, consider the horizontal incentive constraints $IC(HL - LH)$ and $IC(LH - HL)$. As shown in the proof of Proposition 2, these constraints hold if $g_{HL}^i - g_{HH}^i \geq g_{HH}^i - g_{HH}^i$ for $i \in \{1,2\}$, $j \neq i$. Above, we have shown that $g_{HL}^i > g_{HH}^i$ for $i \in \{1,2\}$. So we will be done if we can show that
\[g_{HL}^1 + g_{HH}^2 = g_{HL}^2 + g_{HH}^1, \text{ i.e. both } IC(LL - LH) \text{ and } IC(LL - HL) \text{ are binding.} \]

The proof is by contradiction. Suppose without loss of generality that \(IC(LL - LH)\) is non-binding. Since \(IC(LL - HH)\) is also non-binding, we have \(\alpha_2 = 0 \) and \(\alpha_1 = 1\). Consequently, by (7) and (9), \(v_1(g_{HL}^1, g_{HH}^1) = v_1(g_{HL}^2, g_{HH}^2)\), while by (6) and (10) \(v_2(g_{HL}^1, g_{HH}^1) < v_2(g_{HL}^2, g_{HH}^2)\). Then, since \(v_{12}() < 0\), Property 1 implies that \(g_{HH}^1 > g_{HL}^1\), so \(IC(LL - HH)\) is binding. Contradiction. Similarly, we can show that \(IC(LL - HL)\) must be binding.

Finally, consider Case B: \(g_{HH}^i \geq g_{HL}^i\) for \(i \in \{1, 2\}\), so that \(IC(LL - HH)\) is binding. Then the principal can implement the same quantity profile via a two-agent mechanism such that in states \(HL, LH\) and \(HH\) the sum of transfers to the agents is equal to the transfer to the agent in the solution to \(RP(1)\). In state \(LL\) of this two-agent mechanism the agents need to be paid informational rents equal to \(\Delta g_{HL}^1\) and \(\Delta g_{HH}^2\), respectively. The sum of these rents is (weakly) smaller than the informational rent \(\Delta(g_{HL}^1 + g_{HH}^2)\) paid in \(RP(1)\). Since the principal’s profits from \(RP(1)\) is at least weakly greater than her profits from the optimal single-agent mechanism, the two-agent mechanism dominates the single-agent one. The dominance is strict, because the quantity profile solving \(RP(1)\) is different from the quantity profile in the unique optimal two-agent mechanism. This follows because the set of the first-order conditions characterizing the unique optimal two-agent mechanism (see Lemma 1) is different from the set of the first-order conditions characterizing the solution to \(RP(1)\). In particular, the difference of the two solutions is due to the fact that \(\alpha_1 + \alpha_2 \leq 1\). QED.

**Proof of Proposition 3:** By Lemma 2 it is sufficient to show that \(IC(LL - HH)\) is binding in the optimal single-agent mechanism, i.e. \(g_{HL}^i \geq g_{HL}^i\) for \(i \in \{1, 2\}\). The proof is by contradiction. So, suppose otherwise. Then the quantity allocations in the optimal single-agent mechanism satisfy (6)-(10) with \(\alpha_2 = 1 - \alpha_1\), and \(g_{HL}^i + g_{HH}^2 = g_{HL}^1 + g_{HH}^2\). For all \(s \in [0, 1]\), define \(g^i(s, \alpha_1)\) as follows:

\[
v_1(g^1(s, \alpha_1), g^2(s, \alpha_1)) = c_H + \frac{\alpha_1 p_1}{1-p_1} + s\Delta \frac{p_1}{1-p_1} \frac{1 - \alpha_1}{1 - p_2} \tag{51}
\]

\[
v_2(g^1(s, \alpha_1), g^2(s, \alpha_1)) = c_L + s\Delta \left(1 + \frac{p_2}{1-p_2} \frac{1-p_1(1-\alpha_1)}{1-p_1}\right) \tag{52}
\]

Comparing (51) and (52) to (6)-(10), observe that \(g^1(0, \alpha_1) = g_{HL}^1, g^2(0, \alpha_1) = g_{HH}^2\), while \(g^1(1, \alpha_1) = g_{HL}^1, g^2(1, \alpha_1) = g_{HH}^2\). Differentiating (51) and (52) with respect to \(s\), we get:

\[
\frac{dg^1(s, \alpha_1)}{ds} = \Delta \frac{v_{12}(g^1(s, \alpha_1), g^2(s, \alpha_1))}{v_{11}(g^1(s, \alpha_1), g^2(s, \alpha_1)) v_{22}(g^1(s, \alpha_1), g^2(s, \alpha_1)) - v_{12}^2(g^1(s, \alpha_1), g^2(s, \alpha_1))} \tag{53}
\]

Since \(v(.,.)\) is concave, the denominator of (53) is positive. So, \(\frac{dg^1(s, \alpha_1)}{ds} \geq 0 \forall s \in [0, 1]\) and hence \(g_{HL}^1 \geq g_{HL}^1\). Now, for \(\frac{g_{HH}^2(g^1(t, \alpha_1), g^2(t, \alpha_1))}{v_{22}(g^1(t, \alpha_1), g^2(t, \alpha_1))} \geq \frac{p_2 \alpha_1}{p_1 \alpha_1 + (1-p_1) p_2} = \frac{p_1(1-\alpha_1)}{(1-p_1) \alpha_1 + (1-p_1)p_2}.\) Similarly, \(g_{HH}^2 \geq g_{HL}^1\) if the following inequality holds for \(g_1, g_2\) on the relevant domain: \(\frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)} \geq \frac{p_2 \alpha_1}{p_1 \alpha_1 + (1-p_1) p_2}.\) QED.

**Proof of Corollary 1.** Let \(r = \max_{\alpha_1 \in [0, 1]} \min \left\{ \frac{2p_3 \alpha_1}{1-p_3 + (1-\alpha_1) p_3}, \frac{p_1(1-\alpha_1)}{(1-p_3) p_1 + (1-\alpha_1) p_3} \right\} \). By Proposition
3, it is sufficient to show that \( r < 1 \). But this is so, indeed, because we have:

\[
\frac{p_2 \alpha_1}{(1 - p_2) + (1 - \alpha_1)p_2} \leq \frac{p_1(1 - \alpha_1)}{(1 - p_1)(1 - p_2) + p_1p_2(1 - (1 - \alpha_1)p_1)(1 - \alpha_1p_2)} < 1.
\]

The last inequality follows because \( p_1p_2(1 - (1 - \alpha_1)p_1)(1 - \alpha_1p_2) > \alpha_1(1 - \alpha_1)p_1p_2 \).

**Q.E.D.**

**Proof of Proposition 4:** The proposition will be proved in a sequence of steps.

**Step 1.** If \( \omega_i < \frac{p_i}{2(1 - p_1) + p_1p_2}, i \in \{1, 2\}, \) then the optimal single-agent mechanism is given by the solution to the relaxed program \( \text{RP}(1) \) in Lemma 2 with multipliers \( \alpha_1^* \in (0, 1) \) and \( \alpha_2^* = 1 - \alpha_1^* \) such that \( \min \{ \frac{1}{2}, \frac{1 - \alpha_1}{2} \} < \alpha_1^* < \min \{ \frac{3}{4}, 1 - \frac{p_1}{2} \} \).

By Lemma 2, to establish that the optimal single-agent mechanism is given by the solution to \( \text{RP}(1) \) it is sufficient to show that \( g_{HL}^i > g_{HH}^i \) for \( i \in \{1, 2\} \). So, suppose that \( g_{HL}^i(\alpha_1), g_{HH}^i(\alpha_1), g_{HL}^i(\alpha_1) \) solve (6)–(10) for \( \alpha_1 \in [0, 1] \) and \( \alpha_2 = 1 - \alpha_1 \). The first-order conditions (6)-(10) imply that \( g_{HL}^i(\alpha_1) \) and \( g_{HH}^i(\alpha_1) \) are decreasing in \( \alpha_1 \), \( g_{HL}^i(\alpha_1) \) and \( g_{HH}^i(\alpha_1) \) are increasing in \( \alpha_1 \), \( g_{HL}^i(1) < g_{HL}^i(1) \) and \( g_{HL}^i(0) < g_{HH}^i(0) \). Further, using (51)-(53) we obtain that \( g_{HL}^i(0) > g_{HL}^i(0) \) since \( \omega_2 < \frac{p_1}{2(1 - p_1)} \).

Similarly, \( g_{HL}^i(1) > g_{HH}^i(1) \) since \( \omega_1 < \frac{p_2}{2(1 - p_2)} \).

So, \( g_{HL}^i(\alpha_1) - g_{HH}^i(\alpha_1) \) is decreasing in \( \alpha_1 \) on \([0, 1]\) and takes a positive (negative) value at 0 (1). Conversely, \( g_{HL}^i(\alpha_1) - g_{HH}^i(\alpha_1) \) is increasing in \( \alpha_1 \) on \([0, 1]\) and takes a negative (positive) value at 0 (1). Therefore, there exists a unique \( \alpha_1^* \in (0, 1) \) s.t.

\[
g_{HL}^i(\alpha_1^*) - g_{HL}^i(\alpha_1^*) = g_{HL}^i(\alpha_1^*) - g_{HH}^i(\alpha_1^*).
\]

Let us show that \( g_{HL}^i(\alpha_1^*) - g_{HL}^i(\alpha_1^*) > 0 \) for \( i \in \{1, 2\} \). This is so if there is \( \alpha_1 \) such that \( g_{HL}^i(\alpha_1) - g_{HH}^i(\alpha_1) > 0 \) for \( i \in \{1, 2\} \). By (53) and its analogue for \( g^2(\cdot) \), \( g_{HL}^i(1) - g_{HH}^i(1) > 0 \) if \( \frac{p_1}{(1 - p_1)(1 - p_2)} < \frac{p_1}{2(1 - p_1) + p_1p_2} \) for \( i \neq j \) and all \( (q_1, q_2) \in [\bar{q}_1, \bar{q}_1] \times [\bar{q}_2, \bar{q}_2] \). Since \( \omega_i \leq \frac{p_i}{2(1 - p_i) + p_ip_2} \) for \( i \neq j \), the desired result obtains.

Finally, let us establish the bounds on \( \alpha_1^* \). Suppose that \( \alpha_1^* \geq 3/4 \). Then consider the following pair of the first-order conditions:

\[
v_1(\tilde{g}_{HL}^i, \tilde{g}_{HL}^i) = c_h + \frac{p_1\alpha_1}{1 - p_1},
\]

\[
v_2(\tilde{g}_{HL}^i, \tilde{g}_{HL}^i) = c_h + \frac{p_2}{1 - p_2}.
\]

Note that \( \tilde{g}_{HL}^i > g_{HL}^i(\alpha_1^*) \) and, since \( \omega_2 \leq \frac{p_1}{2(1 - p_1) + p_1p_2} \), we also have \( \tilde{g}_{HL}^i < g_{HL}^i(\alpha_1^*) \). Now consider the following pair of the first-order conditions:

\[
v_1(\tilde{g}(\cdot), \tilde{g}(\cdot)) = c_h + \frac{\alpha_1p_1}{1 - p_1} + s\Delta(1 - \alpha_1^*) \frac{p_1}{1 - p_1(1 - p_2)}
\]

\[
v_2(\tilde{g}(\cdot), \tilde{g}(\cdot)) = c_h + \frac{p_2}{2(1 - p_2)} + s\Delta \left( \frac{p_2}{2(1 - p_2)} + \frac{\alpha_1p_1p_2}{1 - p_1(1 - p_2)} \right)
\]

(54)

(55)

Note that \( \tilde{g}(0) = \tilde{g}_{HL}^i \), \( \tilde{g}(\cdot) = g_{HL}^i, \tilde{g}(0) = g_{HH}^i \), \( \tilde{g}(1) = g_{HH}^i(\alpha_1^*) \). Since \( g_{HL}^i(\alpha_1^*) - g_{HL}^i(\alpha_1^*) = g_{HL}^i(\alpha_1^*) - g_{HL}^i(\alpha_1^*) \), we must have \( \tilde{g}_{HL}^i - g_{HL}^i(\alpha_1^*) > \tilde{g}_{HL}^i - g_{HL}^i(\alpha_1^*) \). But differentiating (54) and (55) we obtain that \( \frac{d^2\tilde{g}(\cdot)}{d\alpha^2} < 0 \) only if \( |(1 - \alpha_1^*)|n_22| \geq |\frac{2p_1}{2} \), and so \( \alpha_1^* < 1 - \frac{p_1}{2} \). The case of \( \alpha_1^* < 1/4 \) is handled in a similar way.

**Step 2.** Now, let us apply the homotopy method developed in the proof of Proposition 2. Using Step 5 of that proof and substituting the values \( \lambda_{HL} = \alpha_1^*p_1p_2, \lambda_{HH} = (1 - \alpha_1^*)p_1p_2, \) and
\( \kappa = \mu = \lambda_{HH} = 0 \) into (45), we obtain that the single-agent mechanism is more profitable than the two-agent mechanism if
\[
\int_0^1 (1 - \alpha_1^*) (h_{HH}^1(t, \alpha_1^*) - h_{HH}^i(t, \alpha_1^*)) + \alpha_1^* (h_{HL}^i(t, \alpha_1^*) - h_{HH}^i(t, \alpha_1^*)) \, dt > 0
\]
where \( h_{HL}^i(t, \alpha_1^*), h_{HH}^i(t, \alpha_1^*) \), \( i \in \{1, 2\} \), solve (38)-(44) with \( \lambda_{HL} = \alpha_1^* p_1 p_2, \lambda_{HH} = (1 - \alpha_1^*) p_1 p_2 \), and \( \kappa = \mu = \lambda_{HH} = 0 \).

**Step 3.** Note that \( h_{HL}^i(0, \alpha_1^*) = g_{HL}^i(\alpha_1^*) \) and \( h_{HH}^i(0, \alpha_1^*) = g_{HH}^i(\alpha_1^*) \). Differentiating (38)-(44), we obtain that \( h_{HL}^i(t, \alpha_1^*) \) is decreasing in \( t \) for \( i \in \{1, 2\} \) and \( (1 - \alpha_1^*) h_{HL}^i(t, \alpha_1^*) + \alpha_1^* h_{HH}^i(t, \alpha_1^*) \) is increasing in \( t \). Therefore, (56) holds if \( h_{HL}^i(1/2, \alpha_1^*) > h_{HH}^i(1/2, \alpha_1^*) \) and \( h_{HH}^i(1, \alpha_1^*) \geq h_{HH}^i(1/2, \alpha_1^*) \) for \( i \in \{1, 2\} \).

**Step 4.** For all \( \hat{\ell} \in [0, 1], \tilde{\ell} \in [0, 1] \) s.t. either \( \hat{\ell} < 1 \) or \( \tilde{\ell} < 1 \), \( h_{HL}^i(\hat{\ell}, \alpha_1^*) \geq h_{HL}^i(\tilde{\ell}, \alpha_1^*) \) if \( \omega_2 \leq \omega_2(\alpha_1^*, \hat{\ell}, \tilde{\ell}) \equiv \frac{p_1 (1 - \alpha_1^*) (1 - p_2) (1 - \hat{\ell}) + p_2 (1 - \tilde{\ell})}{(1 - p_1) p_2 (1 - \alpha_1^*) (1 - \hat{\ell})} \).

To see this, note that \( h_{HL}^i(\hat{\ell}, \alpha_1^*) \) and \( h_{HL}^i(\tilde{\ell}, \alpha_1^*) \) are defined by \( v_1(h_{HL}^i(\hat{\ell}, \alpha_1^*), h_{HL}^i(\hat{\ell}, \alpha_1^*)) = c_H + \Delta \frac{p_1 (1 - \alpha_1^*) (1 - \hat{\ell})}{1 - p_1} \) and \( v_2(h_{HL}^i(\hat{\ell}, \alpha_1^*), h_{HL}^i(\tilde{\ell}, \alpha_1^*)) = c_H + \Delta \frac{p_1 (1 - \alpha_1^*) (1 - \tilde{\ell})}{1 - p_1} \). Similarly, \( h_{HL}^i(\hat{\ell}, \alpha_1^*) \) and \( h_{HL}^i(\tilde{\ell}, \alpha_1^*) \) are defined by \( v_1(h_{HL}^i(\hat{\ell}, \alpha_1^*), h_{HL}^i(\tilde{\ell}, \alpha_1^*)) = c_H + \Delta \frac{p_1 (1 - \alpha_1^*) (1 - \tilde{\ell})}{1 - p_1} \) and \( v_2(h_{HL}^i(\hat{\ell}, \alpha_1^*), h_{HL}^i(\tilde{\ell}, \alpha_1^*)) = c_H + \Delta \frac{p_1 (1 - \alpha_1^*) (1 - \tilde{\ell})}{1 - p_1} \).

Let \( h_H^i \) solve \( v_2(h_{HL}^i(\hat{\ell}, \alpha_1^*), h_{HL}^i(\tilde{\ell}, \alpha_1^*)) = c_H + \Delta \frac{p_1 (1 - \alpha_1^*) (1 - \tilde{\ell})}{1 - p_1} \). Then we have:
\[
v_1(h_{HL}^i(\hat{\ell}, \alpha_1^*), h_{HL}^i(\tilde{\ell}, \alpha_1^*)) - \int_{h_{HL}^i(\hat{\ell}, \alpha_1^*)}^{h_{HL}^i(\tilde{\ell}, \alpha_1^*)} v_2(h_{HL}^i(\hat{\ell}, \alpha_1^*), h) \, dh < v_1(h_{HL}^i(\tilde{\ell}, \alpha_1^*), h_{HL}^i(\tilde{\ell}, \alpha_1^*)) - \omega_2 \int_{h_{HL}^i(\hat{\ell}, \alpha_1^*)}^{h_{HL}^i(\tilde{\ell}, \alpha_1^*)} v_2(h_{HL}^i(\hat{\ell}, \alpha_1^*), h) \, dh = c_H + \Delta \left( \frac{p_1 (1 - \alpha_1^*) (1 - \tilde{\ell})}{1 - p_1} \omega_2 \left( \frac{1 + p_2 + \frac{p_1 p_2 (1 - \tilde{\ell})}{(1 - p_1) (1 - p_2)} \right) \right)
\]
Since \( \omega_2 \leq \omega_2(\alpha_1^*, \hat{\ell}, \tilde{\ell}) \equiv \frac{p_1 (1 - \alpha_1^*) (1 - p_2) (1 - \hat{\ell}) + p_2 (1 - \tilde{\ell})}{(1 - p_1) p_2 (1 - \alpha_1^*) (1 - \hat{\ell})} \), we have
\[
v_1(h_{HL}^i(\hat{\ell}, \alpha_1^*), h_{HL}^i(\tilde{\ell}, \alpha_1^*)) \leq c_H + \Delta \frac{p_1 (1 - \alpha_1^*) (1 - \tilde{\ell})}{1 - p_1} \left( 1 + \frac{p_2 (1 - \alpha_1^*) (1 - \tilde{\ell})}{1 - p_1} \right), \text{ and hence } h_{HL}^i(\hat{\ell}, \alpha_1^*) \geq h_{HL}^i(\tilde{\ell}, \alpha_1^*) \).

**Step 5.** Similarly to step 4, for all \( \alpha_1^* \in (0, 1), \hat{\ell} \in [0, 1], \tilde{\ell} \in [0, 1] \) s.t. either \( \hat{\ell} < 1 \) or \( \tilde{\ell} < 1 \), \( h_{HH}^i(\hat{\ell}, \alpha_1^*) \geq h_{HH}^i(\tilde{\ell}, \alpha_1^*) \) if \( \omega \leq \omega_2(\alpha_1^*, \hat{\ell}, \tilde{\ell}) \equiv \frac{p_1 (1 - \alpha_1^*) (1 - p_2) (1 - \hat{\ell}) + p_2 (1 - \tilde{\ell})}{(1 - p_1) p_2 (1 - \alpha_1^*) (1 - \hat{\ell})} \).

**Step 6.** Suppose that \( \omega_i < \min{\omega_i(\alpha_1^*, 1, 1/2), \omega_i(\alpha_1^*, 1/2, 1)} \) for \( i \in \{1, 2\} \). Then by Steps 4 and 5, \( h_{HL}^i(1/2, \alpha_1^*) > h_{HH}^i(1, \alpha_1^*) \) and \( h_{HL}^i(1, \alpha_1^*) > h_{HH}^i(1, \alpha_1^*) \). So, by Step 3, (56) holds.

Finally, note that at each step of the proof the threshold value of \( \omega_i \) was chosen to be increasing in \( p_j, i \neq j \). Q.E.D.

**Proof of Proposition 5:**

First, since agents 1 and 2 are risk-neutral, the informed principal problem does not arise in contracting between them (see Proposition 11 in Maskin and Tirole (1990)). So, without loss of generality, we will suppose that the primary contractor with cost \( c_H, K \in \{L, H\} \), reveals her type to the subcontractor and offers her a menu consisting of two quantity/transfer pairs.

By the Revelation Principle, the expected cost of implementing any quantity schedule via the hierarchy \( H_1 \) is at least as large as in the two-agent mechanism. So, \( H_1 \) attains the same performance as the two-agent mechanism if and only if the quantity schedule \( \{q_{LL}^i, q_{LH}^i, q_{HL}^i, q_{HH}^i\}_{i=1}^2 \) from the
optimal two-agent mechanism can be implemented in $H_1$ at the same expected cost for the principal.

In fact, we need to be more precise here. So, suppose that the principal implements the quantity schedule \{\(q_{1L}^i, q_{1H}^i, q_{2L}^i, q_{2H}^i\), \(i \in \{1, 2\}\), via $H_1$, and, without loss of generality, suppose that agent 1 is the primary contractor. Let \((T_{HH}, T_{HL}, T_{LH}, T_{LL})\) be the vector of transfers from the principal to the primary contractor. The interim IR of the primary contractor with cost $c_H$ holds only if \((1 - p_2)T_{HH} + p_2T_{HL}\) covers the expected production costs in states $HH$ and $HL$ and the subcontractor’s informational rent $\Delta q_{HH}^2$ in state $HL$. So, we must have:

\[
(1 - p_2)T_{HH} + p_2T_{HL} \geq (1 - p_2)c_H(q_{1H}^i + q_{2H}^i) + p_2(c_H q_{1L}^i + c_L q_{2L}^i + \Delta q_{HH}^2)\quad (57)
\]

Similarly, \((1 - p_2)T_{LH} + p_2T_{LL}\) must cover the sum of: (i) the production costs in states $LH$ and $LL$, (ii) the subcontractor’s informational rent $\Delta q_{HL}^2$ in state $LL$, (iii) the primary contractor’s expected informational rent of at least $\Delta((1 - p_2)q_{1H}^i + p_2 q_{1L}^i)$.

So, we must have:

\[
(1 - p_2)T_{LH} + p_2T_{LL} \geq (1 - p_2)(c_L q_{2L}^i + c_H q_{2H}^i + \Delta q_{HL}^2) + p_2(c_L(q_{1L}^i + q_{2L}^i) + \Delta(q_{2L}^2 + q_{1L}^2))\quad (58)
\]

In combination, (57) and (58) imply that the expected informational rent which the principal pays in hierarchy $H_1$ is at least $\Delta((p_1 p_2 q_{1H}^i + q_{1L}^i) + p_i(1 - p_2)q_{1H}^i) + (1 - p_1)p_2 q_{1H}^i$ which is the same as the expected informational rent paid by the principal in the optimal two-agent mechanism (see Lemma 1).

Therefore, hierarchy $H_1$ with agent 1 as a primary contractor attains the same profitability as the two-agent mechanism if the principal can implement \{\(q_{1L}^i, q_{1H}^i, q_{2L}^i, q_{2H}^i\), \(i \in \{1, 2\}\), in $H_1$ with a system of transfers satisfying (57) and (58) as equalities. That is, the transfers in $H_1$ satisfy: $T_{HH} = c_H(q_{1H}^i + q_{2H}^i) - p_2 \lambda$, $T_{HL} = q_{1H}^i c_H + q_{2H}^i c_L + \Delta q_{HL}^2 + (1 - p_2) \lambda$, $T_{LH} = c_L q_{1L}^i + c_H q_{2H}^i + \Delta q_{HL}^2 - \delta p_2$, and $T_{LL} = c_L(q_{1L}^i + q_{2L}^i) + \Delta(q_{2L}^2 + q_{1L}^2) + \delta(1 - p_2)$ for some $\lambda$ and $\delta$.

There are two types of possible deviations by the primary contractor in $H_1$. First, she can deviate by misrepresenting her true cost in Stage 3 and then either misrepresent the subcontractor’s cost or announce it truthfully. Alternatively, she can deviate by only misrepresenting the subcontractor’s cost in Stage 5. Both types of deviations involve two ‘adjacent’ states of the world which differ only in the subcontractor’s cost. For the second type of deviation this is so because, if the primary contractor reports her own cost truthfully but misrepresents the subcontractor’s true cost, then the subcontractor’s informational rent in the adjacent state of the world (which the primary contractor announces non-truthfully) and hence the primary contractor’s expected payoff are affected. So, we write down the incentive constraints as mappings from pairs of adjacent states of the world that differ in the subcontractor’s costs to pairs of states with the same cost of the primary contractor. When agent 1 is the primary contractor, then the two pairs of adjacent states of the world are \(LL, LH\) and \(HL, HH\).
Let us consider the incentives constraints associated with these two types of deviations. First, $IC(LH - HH, LL - HL)$ holds because (57) and (58) hold as equalities. $IC(HH - LH, HL - LL)$ holds because $q_{1HH}^2 < q_{1LL}^1$ and $q_{1HL}^2 < q_{1LL}^1$. $IC(LH - LL, LL - LH), IC(LH - HL, LL - HH), IC(HH - HL, HL - HH)$ and $IC(HH - LL, HL - LH)$ correspond to infeasible deviations requiring the subcontractor’s quantity to increase in her cost. For the remaining constraints the following is established by direct computation:

(i) $IC(LH - HH, LL - HH)$ holds iff $\lambda \geq \Delta(q_{1HH}^1 - q_{1LL}^1)$. 
(ii) $IC(HH - HH, HL - HH)$ holds iff $\lambda \geq 0$. 
(iii) $IC(HH - HL, HL - HL)$ holds iff $(1 - p_2)\lambda \leq \Delta(q_{1HH}^2 - q_{1HL}^2)$. 
(iv) $IC(LH - HL, LL - HL)$ holds iff $(1 - p_2)\lambda \leq \Delta(q_{1HH}^2 - q_{1HL}^2 + (1 - p_2)(q_{1HH}^1 - q_{1HL}^1))$. 
(v) $IC(LH - LH, LL - LH)$ holds iff $\delta \geq \Delta(q_{1HH}^2 - q_{1HL}^2)$. 
(vi) $IC(HH - LL, HL - LL)$ holds iff $\delta(1 - p_2) \leq \Delta(q_{1LL}^2 - q_{1HL}^2 + q_{1LL}^1 - q_{1HL}^1)$. 
(vii) $IC(LH - LL, LL - LL)$ holds iff $\delta(1 - p_2) \leq \Delta((1 - p_2)(q_{1HH}^1 - q_{1HL}^1) + q_{1LL}^1 - q_{1HL}^1)$. 
(viii) $IC(HH - LH, HL - LH)$ holds iff $\delta p_2 \geq \Delta(q_{1HH}^2 - q_{1HL}^2)$. 

Recall that the quantity schedule $\{q_{1LL}^i, q_{1HL}^i, q_{1HH}^i, q_{1LLLL}^i, q_{1HHHL}^i, q_{1HHLL}^i, q_{1LHH}^i, q_{1LHH}^i\}, \ i \in \{1, 2\}$, is characterized by the first-order conditions (1)-(5) in Lemma 1. The arguments in the sequel will rely on these first-order conditions.

**Complementarity.** By (ii), $\lambda \geq 0$. So, (iv) fails if $q_{1HL}^2 - q_{1HH}^2 < (1 - p_2)(q_{1HH}^1 - q_{1HL}^1)$ which is so if $\frac{\Delta_i}{\Delta_{i+1}} \geq \frac{1}{1 - p_2}$ for all $(q_1, q_2) \in [q_{1HH}^1, q_{1HH}^1] \times [q_{1HH}^1, q_{1HH}^1]$. If $\frac{\Delta_i}{\Delta_{i+1}} \leq \frac{1}{1 - p_2}$ then $(q_1, q_2)$ on this interval, set $\lambda = 0$. Then (iv) holds. (i) and (iii) hold because $q_{1HL}^1 \geq q_{1HH}^1$ and $q_{1HL}^2 \geq q_{1HH}^1$. Choose $\delta = \min(0, \Delta(q_{1HH}^1 - q_{1HL}^2 + q_{1HL}^2 - q_{1HL}^1))$. If $\delta = 0$, then it is easy to check that (v)-(viii) hold.

Now suppose that $\delta = \Delta(q_{1HH}^1 - q_{1HL}^2 + q_{1HL}^2 - q_{1HL}^1) < 0$. Then (vii) holds as an equality. Since $q_{1HL}^2 > q_{1HL}^2$, (vii) holds because $q_{1HH}^1 < q_{1HH}^1$. (vii) can now be rewritten as:

$(1 - p_2)(q_{1HL}^1 - q_{1HH}^2) + \frac{\Delta_{i+1}}{\Delta_{i+1}}((1 - p_2)(q_{1HL}^1 - q_{1HL}^1) + q_{1HL}^1 - q_{1HL}^1) \geq 0$. To show that this inequality holds, let $v_1(q_1(t), q_2(t)) = c_L + \frac{\Delta_i}{1 - p_1}t$ and $v_2(q_1(t), q_2(t)) = c_L + \frac{\Delta_{i+1}}{1 - p_1}(1 - t)$ for $t \in [0, 1]$. Then $q_{1HH}^1 = q_1(0), q_{1HL}^1 = q_2(0), q_{1HH}^1 = q_1(1)$ and $q_{1HL}^1 = q_2(1)$. By **Property 3** with $a_i = \frac{\Delta_i}{1 - p_1}$ we have $(q_{1HL}^1 - q_{1HH}^1)(1 - p_2) + (q_{1HL}^1 - q_{1HL}^1)(1 - p_1) \geq 0$. Since $q_{1HL}^2 > q_{1HL}^2$, (vii) holds.

Let us establish that the optimal two-agent allocation can be implemented under complementarity if the principal can choose any agent to be the primary contractor. So, suppose that $IC(LH - HL, LL - HL)$ fails when agent 1 is the primary contractor, i.e. $q_{1HH}^2 < q_{1HH}^2 < (1 - p_2)(q_{1HL}^1 - q_{1HL}^1)$, and consider agent 2 to be the primary contractor. Then the corresponding incentive constraints are given by (i)-(viii) which are obtained from (i)-(viii) by switching the agents’ indices on quantities: 1 to 2 and vice versa. As above, set $\lambda = 0$ so that (iii) binds, and set $\delta$ in such a way that (vii) binds. Repeating the same steps as before, we can easily confirm that all constraints other than $IC(LH - LH, LL - HL)$.

It remains to check (iv)’. To see this, combine $(q_{1HH}^1 - q_{1HL}^2)(1 - p_2) + (q_{1HL}^1 - q_{1HL}^1)(1 - p_1) \geq 0$. 

\[14\] After this deviation the subcontractor receives transfer $c_Hq_{1HL}^2$ in states $HH$ and $HL$.
with \( q_{LL}^2 - q_{HH}^2 < (1 - p_2)(q_{LL}^1 - q_{HH}^1) \), to obtain
\[
(q_{LL}^2 - q_{HH}^2)(1 - p_1) < q_{LL}^2 - q_{HH}^2 - (q_{LL}^2 - q_{HH}^2)(1 - p_1) \leq (1 - p_2)(q_{LL}^1 - q_{HH}^1). \]
So, (iv) holds.

Substitutability. First, suppose that \( \frac{v_{11}(\cdot)}{v_{11}(\cdot)} \leq \frac{1}{1 - p_1} \) \( \forall q_1, q_2 \in [q_{LL}^1, q_{HH}^1] \times [q_{HH}^2, q_{HH}^2] \) and agent 1 is the primary contractor. Then, by Lemma 1, \( (q_{LL}^1 - q_{HH}^1)(1 - p_2) < q_{LL}^2 - q_{HH}^2 \) and \( q_{LL}^2 > q_{HH}^2 \) for \( i \in \{1, 2\} \). So (i)-(iv) hold if we set \( \lambda = (q_{HH}^1 - q_{LL}^1) \).

Next, set \( \delta = \Delta(q_{HH}^1 - q_{LL}^1) \), so that (v) binds and \( \delta \) cannot be decreased further. It is easy to check that (vii) and (viii) hold. (vi) holds if \( q_{LL}^2 - q_{HH}^2 \) and \( q_{LL}^1 - q_{HH}^2 \) for \( \forall q_1, q_2 \in [q_{LL}^1, q_{HH}^1] \times [q_{HH}^2, q_{HH}^2] \) and \( q_{HH}^2 - q_{HH}^1 \). Applying Proof of Proposition 6: In \( H_D \) the primary contractor submits combined cost report after communicating with the subcontractor. So, in contrast to \( H_1 \), she decides whether to misrepresent her own cost or not after learning the subcontractor’s cost. As a result, the set of feasible deviations for the primary contractor in \( H_D \) is larger than in \( H_1 \), and is the same as in the single-agent mechanism. Q.E.D.
But as in $H_1$, a deviation by the primary contractor in one state of the world affects her payoff in the ‘adjacent’ state. Specifically, there are two pairs of adjacent states of the worlds: $(LL, LH) \text{ and } (HL, HH)$. If the primary contractor misrepresents her cost in $LH$, but reports it truthfully in $LL$, then the subcontractor’s informational rent in state $LL$ depends on the primary contractor’s report in $LH$. So, each incentive constraint of this kind must also involve two ‘adjacent’ states of the world. Thus, in addition to (i)-(viii), the following constraints have to hold in $H_2$.

(ix) $IC(HH - LH, HL - HL)$: $(1 - p_2)p_2(\lambda - \delta) \leq \Delta(p_2(q_{HL}^2 - q_{HH}^2) + (1 - p_2)(q_{LL}^2 - q_{HH}^2))$. (feasible only if $q_{HL}^2 \geq q_{HH}^2$)

(x) $IC(HH - LL, LH - HL)$: $(p_2\lambda + (1 - p_2)\delta)(1 - p_2) \leq \Delta(p_2(q_{HL}^2 - q_{HH}^2) + (1 - p_2)(q_{LL}^2 - q_{HL}^2) + q_{LL}^2 - q_{HH}^2)$) (feasible only if $q_{HL}^2 \geq q_{HH}^2$ i.e. under substitutability).

(xi) $IC(HH - LH, HH - HL)$: $(1 - p_2)\lambda + \delta p_2 \geq \Delta(q_{HH}^2 - q_{LH}^2 + q_{HL}^2 - q_{HH}^2)$ (feasible only if $q_{HL}^2 \geq q_{HH}^2$ i.e. under complementarity)

(xii) $IC(HH - LL, LH - LL)$: $(1 - p_2)(\delta - \lambda) \leq \Delta(q_{LL}^2 - q_{LH}^2 + q_{HL}^2 - q_{HH}^2)$ (feasible only if $q_{LL}^2 \geq q_{HH}^2$).

(xiii) $IC(LL - HH, LH - LL)$: $(1 - p_2)(\delta - \lambda) \leq \Delta(q_{HH}^2 - q_{LH}^2)$. (feasible only if $q_{HH}^2 \leq q_{LH}^2$).

(xiv) $IC(LL - HL, LH - LL)$: $(\lambda(1 - p_2) + \delta p_2)(1 - p_2) \leq \Delta((1 - p_2)(q_{HH}^2 - q_{HL}^2) + q_{LL}^2 - q_{HH}^2)$) (feasible only if $q_{HL}^2 \leq q_{HH}^2$) i.e. under substitutability.

(xv) $IC(LL - LH, LL - LH)$: $\lambda p_2 + \delta(1 - p_2) \geq \Delta(q_{HL}^2 - q_{LH}^2 + q_{HH}^2 - q_{HH}^2)$ (feasible only if $q_{HL}^2 \geq q_{HH}^2$) i.e. under substitutability.)

(xvi) $IC(LL - LH, LL - HL)$: $(\delta - \lambda)(1 - p_2) \geq \Delta(q_{HH}^2 - q_{HH}^2)$, (feasible only if $q_{HH}^2 \geq q_{HL}^2$).

The primary contractor could also make two deviations in adjacent states of the world by misrepresenting her cost in the first state, and by misrepresenting the subcontractor’s cost in the second. However, either the corresponding deviation is not feasible (because the quantities increase in costs), or the two incentive constraints corresponding to each of these two deviations hold separately.

Consider (ix)-(xvi). First, eliminate (ix), (xii) and (xv): (ix) is implied by (xvi), (xv) is implied by the combination of (xvi) and (i), and (xii) is implied by (xiii).

**Complementarity.** By (xiii) and (iv), $\delta \leq \Delta\left(\frac{q_{HL}^2 - q_{HH}^2}{1 - p_2} + q_{HH}^2 - q_{LL}^1\right) < \Delta(q_{HH}^2 - q_{HH}^2)$. So, (v) holds if and only if $q_{HL}^2 \geq q_{HH}^2$. To understand when this inequality holds, consider the following two equations: $v_1(q_1(t), q_2(t)) = c_L + \frac{\Delta}{1 - p_2}t$, $v_2(q_1(t), q_2(t)) = c_L + \frac{\Delta}{1 - p_2}(1 - t)$ for $t \in [0, 1]$. Observe that $q_{HL}^2 = q_2(1), q_{HH}^2 = q_2(0)$ and $\frac{dq_2(t)}{dt} = \Delta\frac{v_{11}(q_1(t), q_2(t))v_{22}(q_1(t), q_2(t)) - v_{12}(q_1(t), q_2(t))}{v_{11}(q_1(t), q_2(t))v_{22}(q_1(t), q_2(t)) - v_{12}(q_1(t), q_2(t)))}$. So, $q_{HL}^2 - q_{HL}^1 = \int_0^1 \frac{dq_2(t)}{dt}dt \geq 0$ if the stated condition holds. So, $q_{HL}^2 \geq q_{HH}^2$ if $q_{HL}^2 - q_{HL}^1 = \int_0^1 \frac{dq_2(t)}{dt}dt \geq 0$.

Now let us suppose that $q_{HL}^2 \geq q_{HH}^2$. Then, (xi) is implied by (v) and (xiii). By (xiii) and (xvi), $\delta - \lambda = \frac{q_{HH}^2 - q_{HL}^2}{1 - p_2}$. Using this to substitute $\delta$ out, we obtain that (xiv) and (i)-(iv) hold iff $0 \leq \lambda \leq 2\frac{q_{HH}^2 - q_{HL}^2}{1 - p_2}$. As shown above, the right-hand side of this inequality is non-negative if $\frac{v_{12}(q_1(t), q_2(t))}{v_{11}(q_1(t), q_2(t))} \leq 1$. Further, check that (vi), (vii) hold for any $\lambda \in [0, 2\frac{q_{HH}^2 - q_{HL}^2}{1 - p_2}]$. Now consider (v) and (viii). Since they bound $\delta$ from below, choose the largest possible $\lambda$. (v) holds
since $q_{1H}^i \geq q_{2LL}$. (viii) can be rewritten as $(q_{1H}^i - q_{1H}^i)p_2 + (q_{1H}^i - q_{1H}^i)(1-p_2) + p_2 \frac{q_{2H}^i - q_{2H}^i}{1-p_2} \geq 0$ which holds by Property 3 and $q_{1H}^i \geq q_{2H}^i$.

Suppose now that either agent can be chosen as the primary contractor. Since $q_{1H}^i \geq q_{1H}^i$ for some $i \in \{1,2\}$, $H_D$ attains the same performance as the two-agent mechanism if we can choose $\lambda$ to satisfy (xiv) and (i)-(v). As shown above, this can be done if $0 \leq \frac{q_{1H}^i - q_{1H}^i}{1-p_2} + q_{1H}^i - q_{1H}^i$ for $j \neq i$. This inequality holds if $-\frac{v_{ij}(\lambda)}{v_{ij}(\lambda)} \leq \frac{1}{1-p_2}$ on the appropriate interval.

Substitutability. (i) and (xvi) imply that $\delta \geq \Delta \left(q_{1H}^i - q_{1H}^i + \frac{q_{2H}^i - q_{2H}^i}{1-p_2}\right)$, so (vi) fails if $q_{1H}^i > q_{2LL}^i$. The first-order conditions (1)-(5) imply that $q_{1H}^i \leq q_{2LL}^i (q_{1H}^i > q_{2LL}^i)$ if $\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} \leq \frac{1}{1-p_2} \left(\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} > \frac{1}{1-p_2}\right) \forall (q_1, q_2) \in \min\{q_{1H}^i, q_{1H}^i, q_{1H}^i\}, \max\{q_{1H}^i, q_{LL}^i\}]$.

Suppose that $q_{2LL}^i \geq q_{1H}^i$. In the proof of Proposition 5 we showed that (i)-(iv) are compatible only if $(q_{1H}^i - q_{1H}^i)(1-p_2) \leq q_{1H}^i - q_{1H}^i$ which holds (fails) if $\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} \leq \frac{1}{1-p_2} \left(\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} > \frac{1}{1-p_2}\right)$ on the corresponding interval. (xiii) and (xvi) imply that $(\delta - \lambda) = \frac{q_{2H}^i - q_{2H}^i}{1-p_2}$. Combining this with (i) we conclude that (v) and (vii) hold. Next, substitute $\delta$ out and consider (vi), (vii) and (x). These inequalities restrict $\lambda$ from above. So it is optimal to choose the smallest possible $\lambda$ satisfying (i)-(iv) i.e. $\lambda = \Delta(q_{1H}^i - q_{1H}^i)$. Then (vii) holds, while (vi) and (x) can be rewritten as: $q_{1LL}^i + q_{2LL}^i \geq q_{1H}^i + q_{2H}^i - p_2(q_{1H}^i - q_{1H}^i)$, and $q_{1LL}^i + q_{2LL}^i \geq q_{1H}^i + q_{2H}^i - \frac{p_2(q_{1H}^i - q_{2H}^i)}{1-p_2}$ respectively. Inequality (59) implies that both inequalities hold because $q_{2LL}^i > q_{1H}^i$ and $q_{1LL}^i > q_{1H}^i$.

Now suppose that either agent can serve as the primary contractor. Observe that $q_{1H}^i \leq q_{1LL}^i$ for some $i \in \{1,2\}$. So if $j \neq i$ is assigned to be the primary contractor then, as shown above, conditions (i)-(xvi) hold if $(q_{1H}^i - q_{1H}^i)(1-p_2) \leq q_{1H}^i - q_{1H}^i$. The latter inequality holds (fails) if $\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} \leq \frac{1}{1-p_2} \left(\frac{v_{12}(q_1, q_2)}{v_{11}(q_1, q_2)} > \frac{1}{1-p_2}\right)$ on the corresponding interval. $Q.E.D.$

**Proof of Proposition 7.** The set of incentive constraints in $H_1^{IP}$ is the same as in $H_1$, so an allocation implementable in $H_1^{IP}$ has to satisfy conditions (i)-(viii) in the proof of Proposition 5. Similarly, the set of incentive constraints in $H_2^{IP}$ is the same as in $H_D$. So, an allocation implementable in $H_2^{IP}$ has to satisfy conditions (i)-(viii) in the proof of Proposition 5, and conditions (ix)-(xvi) in the proof of Proposition 6.

Also, observe that the ex-post individual rationality constraints of the primary contractor require that $\lambda = 0$ and $-\Delta q_{1LL}^i/(1-p_2) \leq \delta \leq \Delta q_{1H}^i/p_2$ in $H_1^{IP}$ and $H_2^{IP}$.

Substitutability: An allocation optimal in the two-agent mechanism cannot be implemented in $H_1^{IP}$ or $H_2^{IP}$ because the incentive constraint $IC(LH - HH, LL - HH)$ requires that $\lambda \geq \Delta(q_{1H}^i - q_{1H}^i) > 0$ (see condition (i) in the proof of Proposition 5) which contradicts $\lambda = 0$.

Complementarity. First, consider $H_1^{IP}$. Recall that we had $\lambda = 0$ and $\delta = \min(0, \Delta(q_{1H}^i - q_{1H}^i + \frac{q_{2H}^i - q_{2H}^i}{1-p_2}))$ in the mechanism implemented in $H_1$. If, in fact, $\delta = 0$ in $H_1$, then the mechanism used in $H_1$ is ex-post individually rational. Further, if $\delta = \Delta(q_{1H}^i - q_{1H}^i + \frac{q_{2H}^i - q_{2H}^i}{1-p_2}) < 0$, then $\delta > -\Delta q_{1LL}^i/(1-p_2)$, because $q_{1H}^i \geq q_{1H}^i$ and $q_{2LL}^i \geq q_{2H}^i$, so again ex post individual rationality constraints hold.

Next, consider $H_2^{IP}$. From $\lambda = 0$, (xiii) and (xvi) it follows that $\delta = \frac{q_{2H}^i - q_{2H}^i}{1-p_2}$. It is easy to
check that all ex post IR and incentive constraints, except (v) and (viii), hold with these values of \( \lambda \) and \( \delta \). Conditions (v) and (viii) hold if \( \frac{q^2_{HL} - q^2_{HH}}{p_2} \leq \min \{q^1_{HL} - q^1_{HH}, (q^1_{HL} - q^1_{HH})/p_2\} \). Observe that \( \frac{q^2_{HL} - q^2_{HH}}{1-p_2} \leq (q^1_{LH} - q^1_{HH})/p_2 \) holds (fails) if \( -v_{12}(q_1, q_2) v_{22}(q_1, q_2) \leq \frac{1-p_2}{p_2} (1-\frac{1}{p_2}) \) on the corresponding interval. Furthermore, \( \frac{q^2_{HL} - q^2_{HH}}{p_2} \leq q^1_{HL} - q^1_{HH} \) holds (fails) if \( p_2 \) is sufficiently small (large) and \( p_1 \) is sufficiently large (small).

References


Figure 1: Three Organizational Forms.

Single-agent

\[ v(q_1, q_2) \]

\[ (q_1, c_1); (q_2, c_2) \]

Two-agent

\[ v(q_1, q_2) \]

\[ q_1, c_1 \]

\[ q_2, c_2 \]

Delegation (Hierarchy)

\[ v(q_1, q_2) \]

\[ q_1, c_1 \]

\[ q_2, c_2 \]

Figure 2: Incentive Constraints in the Single-Agent and Two-Agent Mechanisms.

Single Agent

Two Agents

Agent 1

\[ (LL, LH) \]

Agent 2

\[ (LL, HL) \]

\[ (HL, HH) \]

\[ (LL, HH) \]

\[ (HL, HH) \]

Figure 3: Binding Incentive Constraints under the Conditions of Proposition 2.
Figure 4: Regions of optimality with quadratic production function.