A Probabilistic Approach to Pricing a Bundle of Products or Services
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Reviewed work(s):
Published by: American Marketing Association
Stable URL: http://www.jstor.org/stable/3172693

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The authors propose a probabilistic approach to optimally price a bundle of products or services that maximizes seller’s profits. Their focus is on situations in which consumer decision making is on the basis of multiple criteria. For model development and empirical investigation they consider a season ticket bundle for a series of entertainment performances such as sports games and music/dance concerts. In this case, they assume consumer purchase decisions to be a function of two independent resource dimensions, namely, available time to attend performances and reservation price per performance. Using this information, the model suggests the optimal prices of the bundle and/or components (individual performances), and corresponding maximum profits under three alternative strategies: (a) pure components (each performance is priced and offered separately), (b) pure bundling (the performances are priced and offered only as a bundle), and (c) mixed bundling (both the bundle and the individual performances are priced and offered separately). They apply their model to price a planned series of music/dance performances. Results indicate that a mixed bundling strategy is more profitable than pure components or pure bundling strategies provided the relative prices of the bundle and components are carefully chosen. Limitations and possible extensions of the model are discussed.

A Probabilistic Approach to Pricing a Bundle of Products or Services

Bundling, the strategy of marketing two or more products and/or services as a “package” at a special price (Guiltinan 1987), is a pervasive practice in the marketing of products and services. Examples include vacation packages, assortments of food products (e.g., cookies), season tickets for entertainment performances, computer hardware and software combinations, and meal specials in restaurants.

In these situations, in general, a seller faces three alternative strategies to offer her or his products or services (Schmalensee 1984):

1. Pure components: The seller prices and offers the component products/services as separate items, not as bundles.
2. Pure bundling: The seller prices and offers the component products/services only as a bundle and not as individual items.
3. Mixed bundling: The bundle as well as the individual component products/services are priced and offered separately.

To evaluate the desirability of these alternative strategies, the seller must address the following four questions:

- What are the optimal prices of the bundle and/or components under each strategy?
- Correspondingly, what are the levels of profits?
- Which portion(s) of the potential market is (are) attracted under each strategy?
- How sensitive are profits to variations in price levels?

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Journal of Marketing Research
Vol. XXX (November 1993), 494–508
Addressing the four questions simultaneously is crucial for a manager to understand the stakes that he or she faces by following any of the three strategies. Each strategy taps different portions of the market and therefore impacts the degree of uncertainty in revenues. No one strategy may always be the best. As we discuss later, with myopic pricing, mixed bundling strategy may be less effective than pure components or pure bundling strategy. We therefore believe that an approach integrating the four questions is likely to be quite useful to practitioners making bundling decisions.

Interestingly, literature on bundling in marketing and economics does not focus on suggesting specific approaches to integratively answer these questions (Rao 1992). Most articles published on bundling primarily provide theoretical rationales for the bundling concept and project several contexts that are aptly suitable for bundling. The suggested rationales for bundling include price segmentation (Adams and Yellen 1976; Dansby and Conrad 1984; Hanson 1987; Stigler 1968), price discrimination (Adams and Yellen 1976; Burstein 1960; Paroush and Peles 1981; Schmalensee 1982, 1984; Stigler 1968), product range restriction (Eppen, Hanson, and Martin 1991), reduction in classification/processing costs (Kenney and Klein 1983), scope economies (Baumol, Panzar, and Willig 1988; Eppen, Hanson, and Martin 1991), consumers’ search economies (Adams and Yellen 1976), and risk reduction (Hayes 1987; Liebowitz 1983). Guiltinan (1987) has provided a normative framework for bundling that elegantly integrates most of these theoretical rationales.

In advocating the motivations for bundling, some of the contexts addressed are block booking of movies (Stigler 1968), add-ons for automobiles (Eppen, Hanson, and Martin 1991), classification of diamonds (Kenney and Klein 1983), and multiproduct bundling of items that satisfy different needs (Gaeth et al. 1991). In contrast to bundling, Wilson, Weiss, and John (1990) have explained situations and conditions that justify “unbundling.”

Though the aforementioned articles establish the importance and richness of the bundling concept, none provide an optimization approach to practitioners to answer the practical questions that we raised earlier. Two articles that build on Adams and Yellen (1976) and evaluate alternative bundling strategies are McAfee, McMillan, and Whinston (1989) and Schmalensee (1984). McAfee, McMillan, and Whinston have demonstrated that (mixed) bundling is an optimal strategy provided that the reservation prices for the various products/services are independently distributed in the population of consumers. Schmalensee observed that a mixed bundling strategy generally yields higher profits than either pure bundling or pure components strategy. Neither suggests any explicit approach to determine the optimal prices and profits under the three bundling strategies.

In marketing, only two articles (Goldberg, Green, and Wind 1984; Hanson and Martin 1990) have suggested specific approaches for designing and pricing bundles. Of these articles,

(a) Goldberg, Green, and Wind (1984) have suggested a conjoint analysis-based approach to develop bundles for hotel amenities. Their approach is appropriate for designing bundles composed of a core product with optional add-ons such as automobiles and computers. Though they used conjoint analysis to capture attribute level trade-offs to facilitate optimal bundle design, they did not extend their approach to applications in which the objective is profit maximization with a focus on both prices and costs. Their approach does not provide (1) the scenarios of optimal prices and profits under the three alternative bundling strategies, (2) the portions of the market that are attracted under each strategy, and (3) the sensitivity of profits to changes in price levels. As argued earlier, these issues are crucial to a seller and necessitate an optimization approach that provides answers to these questions.

Furthermore, as pointed by Kohli and Mahajan (1991, p. 348), “current conjoint simulators do not extend to (1) assessing the profitability of a new product, and (2) identifying a price that maximizes profit from a new product. The principal reason is that a profit criterion necessitates estimates of fixed and variable costs for every feasible product . . . . Practically, as suggested by Green and Krieger (1989), multi-attribute cost functions are difficult to estimate reliably. . . . [This] has been a major impediment in the development of conjoint models using a profit objective.” Challenge remains for researchers to develop conjoint analysis-based approaches that answer the four bundling questions raised earlier.

(b) Hanson and Martin’s (1990) mixed integer linear programming approach is useful in identifying the right bundle (from a number of predetermined bundles) for each of several segments in a market and the optimal price of each bundle. However, even their approach does not integratively answer the bundling questions posed earlier.

Our purpose is to propose a probabilistic approach that enables a seller to determine optimal prices of a bundle and/or its component products/services under pure components, pure bundling, and mixed bundling strategies. The model also estimates the corresponding maximum levels of profits under each strategy. Because the seller knows the relative stakes, he or she can choose the desired strategy judiciously. We highlight the portions of the potential market that are likely to purchase different offerings under alternative strategies. The model can indicate the sources of gains and losses in revenue if the seller were to change strategies or price levels. For example, we identify the portion of the market likely to buy individual items under pure components strategy, but not likely to buy the bundle under pure bundling (representing a loss in revenue). We accommodate differences in consumer preferences across product/service components at an individual consumer’s level as well as the heterogeneity across individuals constituting the potential market.

We attempt to make another important contribution.
Bundling literature so far has addressed problems considering essentially one underlying dimension of consumers’ decision making, namely, reservation prices. There are several instances in which the decision is a function of several important dimensions, all of which may not be captured by the reservation price. For example, a person’s decision to go to a series of music performances is not only a function of the monetary dimension (i.e., whether the price of the ticket is less than or equal to her or his reservation price) but also the likelihood that the person has the ability and willingness to spare the time to attend alternative performances. Similarly, in the case of tie-in sales of computer systems and stationery, a consumer’s decision is not only a function of the reservation price per unit but also the probable consumption rates of the tied good, which is likely to vary across consumers. In such cases, unless the prescribed model captures these multiple dimensions, the analysis may possess serious limitations. By bringing in the notion of multiple dimensions (such as reservation prices and time to attend performances), we capture not only the consumers’ tastes for product-related attributes but also constraints that determine their (consumers’) eventual choice.

The remainder of this article is organized as follows: The next section highlights the conceptual and analytical underpinnings of our model; we then apply our model to pricing a series of music/dance performances that an ethnic group is planning to organize in a medium-sized city in the southwest; analysis of results follows; and the last section provides a discussion of the approach and suggests possible directions for future research in bundling.

Our empirical results indicate that mixed bundling strategy is more profitable than the other two strategies. The results are consistent with the theoretical arguments of Schmalensee (1984). More importantly, if the relative prices of the bundle and its components are carefully chosen, mixed bundling can effectively price-discriminate among frequent and occasional consumers and increase total profits.

MODEL DEVELOPMENT

Conceptual Underpinnings

To illustrate our approach, we consider the entertainment industry, which frequently makes bundling decisions at the end-consumer level. An avid follower of football games or music performances is often “lured” by deals in the form of season tickets. The bundle in this case is a series of performances organized over a certain time horizon. Purchase of a season ticket implies a tie-in to the package of performances (assuming that tickets are not easily transferable). The temporal dispersion of performances adds uncertainty to the consumers’ ability to spare the time to attend any performance. Each consumer is also likely to have different intensities of preference for the alternative performances in the package.

Given a set of $n$ performances, the seller faces a crucial and interesting decision problem of pricing the single (performancewise) ticket and/or the season ticket, which maximizes profits. The absolute and relative values of the two types of tickets are crucial determinants of the seller’s profits. We focus on those performances held in auditoriums or stadiums as distinct from home entertainment. The following analysis considers a series of music/dance performances for better clarity of presentation.

We propose that a prospective viewer’s decision to attend a performance is related to two key independent resources—time and money. Vogel (1990) has highlighted the central role of these two dimensions in the entertainment industry. An individual is expected to attend a particular performance only if he or she has the willingness and ability to spend (a) the time to attend that performance and (b) the money to purchase the ticket.

For a series of performances, we capture the time dimension by the likely number of performances an individual can attend from that series. Factors such as work/recreation engagements, possibility of and preferences for alternative types of entertainment, and unforeseen circumstances create uncertainty with regard to an individual’s ability and willingness to spare the time to attend various performances planned over a period of time. In view of these factors, the individual can, up front, only visualize a certain probability to attend a particular performance. In other words, the individual’s ability to attend a specific performance is a random event/outcome (that depends on the circumstances that unfold prior to the performance). For a series of planned performances, this performancewise probability gets translated into a likely number of performances an individual can attend, which is a random variable representing the time resources. The estimate on this dimension is not constrained by the monetary aspect of the decision, namely, the price of the ticket.

The monetary dimension, which is independent of the time dimension (to be discussed following), is captured by an individual’s reservation price. This is the maximum price the individual is willing and able to pay for a particular type of performance assuming that time is not a constraint on the decision process. The relative attractiveness of a performance for an individual is likely to be reflected in her or his reservation price. We recognize that even among a string of seemingly related performances, an individual’s reservation prices may be

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1 Though bundling does bear certain similarities with quantity discount, it is different in several significant ways. A quantity discount is a price reduction for buyers who buy large volumes of the same product or service (Kotler 1991). On the other hand, a bundle is an assortment of different products and/or services. Bundling can in a way fulfill a consumer’s variety-seeking needs. Moreover, from the supply side, the incentive for quantity discount arises from scale economies whereas bundling yields scope economies.
different across different subcategories of performances. Our model is developed accordingly.

We assume that the two variables—the likely number of performances an individual can attend and her or his reservation prices—are independent of each other. It may be argued that people with high incomes may have a high cost of time and less sensitivity to price (implying higher reservation prices), whereas people with low incomes may be more thrifty (implying lower reservation prices) and have a low cost of time. In other words, an argument can be made that there is an inverse relationship between the two variables. However, it is to be recognized that though the earning member of a high-income family may have a high cost of time, her or his family members may not find it too costly to spare the time. It is also possible that people with lower wage rates may have to work for longer hours and may not have adequate time to spare. We therefore believe that any systematic negative relationship between individuals’ reservation prices and the number of performances that they expect to attend (considering only time-related considerations) is likely to be small. One might also highlight certain situations in which a positive relationship between the dimensions is possible. We discuss implications of such situations to our approach and possible modifications to the model in the “Discussion” section.

Our model is appropriate for situations in which there is no significant relationship between the two dimensions.

To articulate the alternative bundling strategies, we must extend the individual level responses on (a) the likely number of performances to attend and (b) reservation prices to the market level for all individuals constituting the potential market. This extension yields three distributions:

- \( f(z) \), the probability density function (pdf) of the likely number of performances potential consumers can attend, which gives the proportion of the potential market that expects to spare the time to attend different numbers of performances, assuming that price is not a constraint;
- \( g(p_i) \), the pdf of performance-wise reservation price, which represents the proportion of consumers willing to pay different prices for their tickets assuming that they are willing to spare the time to attend;
- \( h(p_n) \), the pdf of a derived mean reservation price, which is obtained by averaging the reservation prices of each individual across performances.

Though an individual’s reservation price may change across different performances, we assume at the market level the reservation price distributions are the same across these performances. The basis for this assumption is that the market is comprised of a heterogenous mix of individuals. In simple terms, though some individuals may have low reservation prices for some performances and high reservation prices for others, there are other individuals who may have a reverse pattern of reservation prices for those performances. This assumption enables us to pool the reservation price information and have the same distribution \( g(p_i) \) to represent each performance.

We make this assumption for model development under pure components and mixed bundling strategies only. There are likely to be several applications for which this assumption may not be valid. For example, for a series of concerts by performers of different caliber there could be a significant and systematic difference in reservation prices across performances even at the market level. For such situations, a generalized approach relaxing this assumption is suggested in Appendix A.

The mean reservation price distribution \( (h(p_n)) \) captures the transfer of consumer surplus (the difference between an individual’s reservation price for a product and its actual price) and demonstrates the motivation for bundling. To illustrate, consider an individual whose reservation prices for two performances A and B are $10 and $20, respectively (with a mean reservation price of $15) and who is willing and able to spare the time to attend both performances. If there is no season ticket and the price of the ticket per performance is $19, then this person is likely to attend performance B only. In contrast, if only a season ticket priced at $29 is offered, the mean price per performance is $14.50. We expect the individual to purchase the season ticket because her or his mean reservation price ($15) for the combination of A and B is greater than mean price per performance. The season ticket bundle thus helps to transfer the individual’s surplus from the more attractive performance to the less attractive one and enhances seller’s revenue. In this example, the gross utility for this viewer increases from $20 to $30.

Besides the symbols relating to the probability density functions mentioned earlier, we use the following symbols:

\[
M = \text{market size} \\
n = \text{number of performances} \\
P_s = \text{optimal price of a single ticket (to be determined)} \\
P_b = \text{optimal price of the season ticket (to be determined)} \\
E_i = \text{expenditure to organize the } i^{th} \text{ performance}
\]

We commence the discussion of the rationale of consumer decision making under alternative bundling strategies by considering the pure components strategy, under which only the single (performance-wise) tickets are sold. The proportion of the potential market having the time to attend exactly \( i \) out of \( n \) performances is\(^2\)

\[
Pr(i) = \int_{i+1}^{n+1} f(z) dz.
\]

\(^2\)It could be argued that the limits of integration be \( i - 1/2 \) and \( i + 1/2 \) on the grounds that rounding off to the nearest integer is appropriate to determine the proportion of individuals expecting to spare the time for \( i \) performances. However our goodness-of-fit analysis reveals that the limits \( i \) and \( i + 1 \) represent the data better. We have accordingly used these limits.
If $P_i$ is the price per performance, then the proportion of these individuals willing to pay this price for a particular performance, assuming they expect to spare the time to attend the performance, is given by

$$Pr(P_i) = \int_{P_i}^{\infty} g(p)dp.$$  

(2)

Therefore, for viewers who expect to spare the time to attend $i$ performances, the likely number of single tickets sold across all $n$ performances is given by

$$iM\left\{\int_{P_i}^{\infty} f(z)dz\right\}\left\{\int_{P_i}^{\infty} g(p)dp\right\}.$$  

(3)

It is important to recognize that this number is the expectation of single tickets sold at the segment level (that is, from the segment that expects to spare the time to attend $i$ performances). An individual in this segment drawn at random is expected to buy $\left\{i * \int_{P_i}^{\infty} g(p)dp\right\}$ tickets (rather than either $i$ tickets or none at all). Under the pure bundling strategy only the season tickets are offered. Let the bundle price that fetches the maximum profits be $P_b$. An individual who expects to attend $i$ performances is likely to buy the season ticket bundle if the mean price per performance ($P_b/i$) is less than or equal to her/his mean reservation price $P_m$. The bundle is more attractive for those segments of the market willing to spend the time to attend a larger number of performances than others.

Under mixed bundling strategy, both the season and single (performance-wise) tickets are on sale. In this case, the seller is free to price the tickets differently than under the earlier two strategies. Because both the season and single tickets are on sale, the seller has to ensure that the price of one of the two types of tickets is not so low as to drastically cannibalize the sales of the other and reduce profits. An individual who expects to have the time to attend $i$ performances is likely to prefer a season ticket priced at $P_b$ over several single tickets priced at $P_i$, each if ($P_b/i$) is less than $P_i$ and the mean reservation price $P_m$. The relative prices of the season and single tickets are crucial to minimize the harmful effects of cannibalization on profits. The prices have to be simultaneously, not sequentially, optimized. This issue is discussed further later on in this article.

**Analytical Underpinnings**

In this section we propose the functional forms of the three distributions with probability density functions $f(z)$, $g(p)$, and $h(p_m)$ mentioned earlier and the rationale for our choice. Subsequently we provide the mathematical development of our model separately for pure components, pure bundling, and mixed bundling strategies.

The probability density functions that we assume for the three distributions are those of the Weibull distribution. A random variable $X$ that has a Weibull distribution (Johnson and Kotz 1970) has the probability density function

$$f(X) = ca^{-1}[(X - e_o)/a]^{c-1} \cdot \exp\{-(X - e_o)/a\}, \hspace{1em} e_o < X.$$  

The cumulative distribution function is

$$F(X) = 1 - \exp\{-(X - e_o)/a\}.$$  

(4)

Parameters $c$, $a$, and $e_o$ determine the shape, spread, and location respectively of the distribution.

The motivations for using the Weibull distribution are the following:

- a) The distributions that we consider, such as performance-wise reservation price distribution, may not be symmetric. Choosing the Weibull distribution provides the flexibility to accommodate a wide range of distributions of different skewness and kurtosis. For example, parameter values of $c = 1$ and $e_o = 0$ give a conventional decaying exponential distribution. Parameter value of $c = 3.6$ provides a good approximation of the normal distribution; and
- b) It can be algebraically manipulated easily.

In the following paragraphs we provide the mathematical development of the model for pure components, pure bundling and mixed bundling strategies. The location, spread and shape parameters of $f(z)$, $g(p)$, and $h(p_m)$ are $(a, e_o, c)$, $(a_1, e_{o1}, c_1)$, and $(a_2, e_{o2}, c_2)$ respectively.

**Pure components.** As mentioned earlier, this represents the strategy under which only single (performance-wise) tickets are offered. If $P_i$ is the price per performance, the profits from holding $n$ performances are

$$\pi = \int_{1}^{n+1} \int_{P_{i}}^{P_{n+1}} z f(z)(c_1/a_1)(P_i - e_{o1})/a_1^{c-1} \cdot \exp\{-(P_i - e_{o1})/a_1\}\ dP_i \ dz - \sum_{j=1}^{n} E_j.$$  

As $z$ and $p_i$ are independent

$$\pi = \left\{\int_{1}^{n+1} \int_{P_{i}}^{P_{n+1}} z f(z)dz\right\} \cdot \exp\{-(P_i - e_{o1})/a_1\}\ dP_i \ - \sum_{j=1}^{n} E_j.$$  

(5)
For maximum profits, \( \pi \), the optimal price equation\(^3\) is

\[
P_r(P_e - e_{ao})^{n-1} = a_i / c_i.
\]

(8)

If reservation prices can vary from zero to a maximum value, then \( e_{ao} = 0 \). In that case, equation 8 simplifies to yield the optimal single (performancewise) ticket price

\[
P_r = a_i / c_i^{1/(n-i)}.
\]

(9)

Knowing the estimates of parameters \( a_i \) and \( c_i \) of the performancewise reservation price distribution \( g(p_r) \), we can determine \( P_r \) from equation 9 which, when substituted in equation 7 gives the maximum profits for the pure components case. (We explain parameter estimation in a later section.)

**Pure bundling.** Under this strategy, the seller offers only season tickets. The appropriate distribution to use is the mean reservation price distribution. If \( P_s \) is the price of a season ticket (to be determined), and \( X \) and \( Y \) are the random variables denoting the number of performances a person can attend and the person's mean reservation price, respectively, then the proportion of the market willing to buy the season ticket is

\[
P_s = \sum_{i=1}^{n} \text{Prob}(X = i) \text{Prob}(Y \geq [P_s/i])
\]

(10)

\[
P_s = \sum_{i=1}^{n} \left[ \exp\left(-((i+1)e_{ao})/a_i^2\right) - \exp\left(-((ie_{ao})/a_i^2)\right) \right] \left[ \exp\left(-((P_s/i) - e_{ao})/a_i^2\right) \right].
\]

(11)

If \( \pi \) denotes the profit as before,

\[
\pi = P_r \cdot M \cdot P_s - \sum_{j=1}^{n} E_j
\]

(12)

For maximum profits, the optimal price equation\(^4\) is

\[
\left[ 1 - P_s c_i \left(1/[ia_i^2]\right)^2 \left(P_s - ie_{ao}\right)^{i-1} \right] = 0
\]

(13)

With the estimates of parameters \((a, c)\) of time distribution and \((a_2, c_2)\) of mean reservation price distribution, we can numerically determine the optimal season ticket price \( P_s \) from equation 13 and the corresponding maximum profits from equation 12.

**Mixed bundling.** The seller adopting the mixed bundling strategy offers both single (performancewise) tickets and season tickets at prices \( P_s \) and \( P_r \) apiece respectively. As explained earlier, an individual expecting to attend \( i \) performances is likely to choose the season ticket over several single (performancewise) tickets if \((P_s/i) < P_r\), and if the mean reservation price exceeds \((P_s/i)\). On the other hand, even if \((P_s/i) < P_r\), if the mean reservation price is less than or equal to \((P_s/i)\), then this person is expected to purchase several single tickets. If, however, \((P_s/i) = P_r\), then the individual can exclude the option of purchasing the season ticket and purchase several single tickets on the basis of available time and reservation prices exceeding the single (performancewise) ticket prices. Let \( X, Y, \) and \( Z \) be the random variables representing the number of performances an individual can attend, the person's mean and performance-wise reservation prices respectively. Then at the aggregate (market) level, the objective function is

\[
\pi = z^* \cdot M \cdot P_r \cdot \left[ \exp\left(-((P_r - e_{ao})/a_i)\right)^n \right] - \sum_{j=1}^{n} E_j.
\]

For maximum profits, \( \pi_{\text{max}} \),

\[
\frac{d\pi}{dP_r} = 0 = z^* \cdot M \cdot P_r \cdot \left[ \exp\left(-((P_r - e_{ao})/a_i)\right)^n \right] (-c_i)
\]

\[
\cdot \left[ (P_r - e_{ao})/a_i \right]^{n-1}(1/a_i)
\]

\[
+ z^* \cdot M \cdot \left[ \exp\left(-((P_r - e_{ao})/a_i)\right)^n \right]
\]

If \( P_r < \infty \), then \( \left[ \exp\left(-((P_r - e_{ao})/a_i)\right)^n \right] \neq 0 \).

By simplification,

\[
P_r(P_r - e_{ao})^{n-1} = a_i / c_i
\]

\[\text{subject to: } \sum_{i=1}^{n} \text{Prob}(X = i) \text{Prob}(Y \geq [P_s/i]) \]

\[
\left[ 1 - P_s c_i \left(1/[ia_i^2]\right)^2 \left(P_s - ie_{ao}\right)^{i-1} \right] = 0
\]

\[\text{or: } \sum_{i=1}^{n} \text{Prob}(X = i) \text{Prob}(Y \geq [P_s/i]) \]

\[
\left[ 1 - P_s c_i \left(1/[ia_i^2]\right)^2 \left(P_s - ie_{ao}\right)^{i-1} \right] = 0
\]

This equation can be solved numerically to yield the optimal bundle price \( P_s \).

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\(^3\)Let the mean number of performances for which consumers are willing and to spare the time be \( z^* = \int f(z) dz \).

\(^4\)For maximum profits,

\[
\frac{d\pi}{dP_s} = 0 = MP_r + MP_s \frac{dP_r}{dP_s},
\]

That is,

\[
P_r + P_s \frac{dP_r}{dP_s} = 0
\]

\[
\frac{dP_r}{dP_s} = \sum_{i=1}^{n} \text{Prob}(X = i) \exp\left(-((P_s/i) - e_{ao})/a_i^2\right)
\]

\[
\cdot \left( -c_i \right) \left( ((P_s/i) - e_{ao})/a_i^2 \right)^{i-1}\left(1/[ia_i^2]\right).
\]

Therefore, the optimal price equation is

\[
\sum_{i=1}^{n} \text{Prob}(X = i) \text{Prob}(Y \geq [P_s/i]) \]

\[
\left[ [1 - P_s c_i \left(1/[ia_i^2]\right)^2 \left(P_s - ie_{ao}\right)^{i-1} \right] = 0
\]
(14) \[ \text{maximize } \pi = \sum_{i=1}^{n} \text{Prob}(X = i) \]
\[ \cdot M \{ m_i \text{Prob}(\text{Prob}(Y > (P_s/i)) P_b) \]
\[ + \text{Prob}(Y \leq (P_s/i)) \text{Prob}(Z > P_s) \]
\[ \cdot |Y < (P_s/i)| i P_s \]
\[ + (1 - m_i) \text{Prob}(Z = P_s) i P_b - \sum_{j=1}^{n} E_j \]

such that \( m_i = 1 \) if \( (P_s/i) < P_s \)
\[ = 0 \text{ otherwise.} \]

In other words,

(15) \[ \text{maximize } \pi = \sum_{i=1}^{n} \text{Prob}(X = i) \]
\[ \cdot M \{ m_i \exp[-((P_s - e_{0i})/a_2)^{a_1}] P_b \]
\[ + (1 - \exp[-((P_s - e_{0i})/a_2)^{a_1}]) P_b \]
\[ \cdot \exp[-((P_s - e_{0i})/a_1)^{a_2}] P_s \]
\[ + (1 - m_i) \exp[-((P_s - e_{0i})/a_1)^{a_2}] P_s \]
\[ - \sum_{j=1}^{n} E_j \]

subject to choice rule 15. This choice rule, contingent on the values of \( P_b \) and \( P_s \) that are yet to be determined, necessitates that we resort to a numerical search to identify the simultaneous optimal values of \( P_s \) and \( P_b \).

**ESTIMATION OF PARAMETERS**

The parameters we need to estimate to use our model are those of the probability density functions \( f(x) \), \( g(p_s) \), and \( h(p_m) \) that are assumed to have Weibull distributions.

The Weibull distribution, represented by equations 4 and 5, has three parameters, namely, \( a \), \( c \), and \( e_i \). For distributions that take values zero, \( e_i \) is zero (Johnson and Kotz 1970). If the values commence from a certain nonzero level, \( e_i \) is taken as the minimum of the set of random variables from \( s \) observations \( X_1, \ldots, X_i, \ldots, X_r \), relating to the subject distribution where \( X_k \) is the response from the \( k^{th} \) respondent and \( s \) is the sample size. Using the probability density function of a Weibull distribution (mentioned earlier), and knowing \( e_i = 0 \) (i.e., distribution varies from zero at the lower end), the maximum likelihood estimates (Johnson and Kotz 1970) for \( a \) and \( c \) are given by

(17) \[ \hat{a} = \left\{ s^{-1} \sum_{i=1}^{s} X_i^{1/a_c} \right\}^{1/a_c} \]

**EMPIRICAL INVESTIGATION**

The model we propose can be applied to actual situations in which consumers’ bundle purchase decisions are on the basis of multiple criteria. We demonstrate application of our model in the context of a series of music/dance performances to be organized by an Asian Indian classical music/dance association in a medium-sized city in the southwest. We need to emphasize that we could have as well applied our model to numerous other contexts, such as symphony orchestra performances, movies, and sports games with very little change in our approach.

The relevant population for our study is comprised of nearly 1000 individuals, excluding students, belonging to the Asian Indian community living in this southwestern city. Their annual household income is typically higher than $36,000. We mailed questionnaires and self-addressed business reply envelopes to 360 respondents on the basis of a list supplied by an organization representing the Asian Indian community. We avoided any explicit references to our pricing model to reduce bias in feedback on reservation prices. The respondents were also informed that ten music/dance performances of uniformly high standard were scheduled to be performed at regular monthly intervals.

We divided the ten performances into five groups of two performances each on the basis of the type of performance. Respondents in our pretest concurred with our basis of distinguishing between the alternative performances. Specific information on time and venue of these performances was provided to enable respondents to answer questions on the number of performances they were likely to spare the time to attend and the money they were willing to pay as precisely as possible.

The first important question (see Appendix B) related to the respondents’ ability and willingness to spare the time to attend the performances. We highlighted to the respondents the uncertainty that exists on this dimension.

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1Indian classical music consists of two major types: (a) **Hindustani music**, which has its roots in Northern India and includes well known musicians such as Pandit Ravi Shankar, and (b) **Carnatic music**, which has a South Indian base and is popularized by stalwarts such as M. S. Subbulakshmi. Indian classical dance is of several types such as Bharatanatyam, Kathak, Kuchipudi, and Odissi. The five categories that we considered are Hindustani music—vocal and instrumental, Carnatic music—vocal and instrumental, and classical dance.
tion. On the basis of their past experience of their willingness and ability to spare the time for performances similar to the ones planned, their expected obligations and future time commitments, respondents were asked to indicate the probable number of performances they thought they could attend in the planned series. They were specifically asked not to be influenced by the price of the tickets in providing their response to this question.

Respondents were also asked the maximum prices they were willing to pay per person per performance for each of the five performance types, assuming they had the time to attend. We stated the exact categories of performances and listed the exemplar musicians/dancers under each category. They were free to choose different prices (including zero) for different performances to accommodate their heterogeneity in preferences across performances.

It is to be recognized that certain performances may not appeal to the tastes of certain respondents irrespective of the caliber of the performers. In other words, even isolating the influence of price of the ticket, the consideration set for each respondent is probably a subset of the total number of performances. Therefore, under the question on reservation prices, we also sought for each type of performance whether respondents would be willing to attend excluding the price factor. On the basis of each individual’s response to this part of the question, we had to shrink the preceding response, namely, the probable number of performances the individual can attend (on the basis of time related considerations alone). For example, suppose an individual expects a priori that he or she can spare the time to attend all ten performances for the broad category planned. However, considering the specific menu of performances, if this person believes he or she is not willing to attend four out of ten performances even if they are free of charge (purely because of lack of taste for those subcategories of performances), then the likely number of performances that this individual will attend on the basis of time considerations alone is shrunk from ten to six.

We also asked for the number of persons likely to accompany the respondents for each type of performance. Our purpose was to check whether the reservation prices were independent of the number of persons accompanying them. To appreciate the need for this check, contrast individuals who are accompanied by several family members with others who go with fewer people. If the ticket prices are perceived to be a sizable portion of the families’ entertainment budget, the reservation prices may exhibit a systematic negative relationship with the number of accompanying individuals. In that case, we would have to weigh the reservation price information suitably to estimate the appropriate market level distribution.

Respondents who had no interest whatsoever in Indian classical music/dance were requested to return the questionnaires without filling them out. Using this information, we could estimate the potential market size given the population of the ethnic group. The questionnaire did not seek the identity of the respondents to facilitate more honest feedback.

In all, 119 questionnaires were received representing a response rate of about 33%. Of these, 31 questionnaires were returned blank (consistent with our instructions) leaving 88 useful responses.

The following questions may now be raised to facilitate data analysis and interpretation:

- Do we satisfy the assumptions underlying our model and data?
- How well do the parameter estimates fit the data?
- What are the answers to the four questions raised in the introduction?

Validity of Assumptions

Table 1 summarizes the results of our tests to validate the assumptions underlying the applicability of the model in the context of the Asian Indian music/dance performances.

Our analysis requires there be no significant non-response bias in our sample. Consistent with this assumption, we did not observe significant differences between earlier and later respondents on (a) the probable number of performances they could attend on the basis of time-related considerations alone ($p = .36$), and (b) reservation prices ($p = .81$). This gives us some confidence that the responses are representative of the sample we contacted.

To test our assumption that reservation prices across types of performances are not significantly different, we used a repeated measures ANOVA on reservation prices across the five types of performances. It did not indicate significant differences ($p = .18$). This validates our assumption that the types of performances are comparable on reservation prices at the market level even though they may vary at an individual level. We pooled the performance-wise reservation price information to have one distribution representing reservation price per performance for all types of performances. Had this assumption been violated, we would have used the more general procedure suggested in Appendix A.6

Our approach assumes that the expected number of performances individuals can attend on the basis of time-related considerations alone is independent of their reservation prices. In line with this assumption, the correlation between the two variables was not significant ($p = .40$).

Last, we had assumed that reservation prices of individuals are independent of the number of persons ac-

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6As the organizers perceived some uncertainty regarding the availability of certain performers, we could only provide an exemplary list of performers in our questionnaire and not an accurate list of actual performers. If we had the exact list of performers, we would have performed the more appropriate test of checking whether the reservation prices are significantly different across performers rather than across the five types of performances.
companying them to performances. Reservation prices of individuals were not significantly related to the number of their companions ($p = .80$).

**Parameter Estimates and Goodness of Fit**

The parameter estimates of Weibull distributions and chi-square values are given in Table 2. To assess the goodness of fit of the Weibull distributions, we need to evaluate the chi-square values. For a given number of degrees of freedom, a lower chi-square value is indicative of a better fit. The $p$ values represent the probability of the data coming from the assumed distributions. Typically, $p$ values greater than .05 are considered non-significant and adequate to accept the stated distributions. As Table 2 indicates, we had non-significant chi-square values for the Weibull distributions when fitted to data on performance-wise reservation prices ($p > .10$), mean reservation prices ($p > .90$), and the likely number of performances that individuals can attend ($p > .10$). These results suggest that Weibull distributions describe these data very well.

To highlight the shape of the fitted Weibull distributions, we provide their plot in Figure 1. The three distributions have moderate to pronounced skewness. How do Weibull fits compare with normal fits? For our data, the chi-square values from Weibull fits are markedly lower than those from normal fits for all three distributions with probability density functions $f(z)$, $g(p_{\alpha})$, and $h(p_{\beta})$. The Weibull distribution is more appropriate than the normal distribution in this case. It is relevant to note that several studies in bundling such as Schmalensee (1984) assume a normal distribution of reservation prices. Normality assumption may reduce the efficiency of results when the distributions such as ours are skewed. The ability of the Weibull distribution to capture both symmetric and skewed patterns provides a major encouragement for its usage.

We corrected the size of the potential market from 1000 to 750 on the basis of the proportion of respondents indicating they had no interest in the planned series of performances. Drawing from the cultural association's past experience for similar performances, the cost per performance was assumed to be $1,200. The model can, however, accommodate different costs across performances.

**Optimal Prices and Profits**

Applying the model to the data yields the results in Table 3. Mixed bundling is the most attractive of the strategies. The optimal prices under this option are $14 for the single ticket and $55 for the season ticket, yield-

### Table 1

**Validity of Assumptions Underlying the Proposed Model for the Asian Indian Music/Dance Performances**

<table>
<thead>
<tr>
<th>Assumption</th>
<th>$F$-value</th>
<th>$p$</th>
<th>Is assumption valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>- There are no significant differences between earlier and later respondents on number of performances they can attend based on time-related considerations alone</td>
<td>.83</td>
<td>.36</td>
<td>Yes</td>
</tr>
<tr>
<td>- reservation prices (money)</td>
<td>.06</td>
<td>.81</td>
<td>Yes</td>
</tr>
<tr>
<td>- There are no significant differences in reservation prices across types of performances</td>
<td>1.59</td>
<td>.18</td>
<td>Yes</td>
</tr>
<tr>
<td>- Individuals' reservation prices are independent of the number of performances they can attend</td>
<td>.70</td>
<td>.40</td>
<td>Yes</td>
</tr>
<tr>
<td>- Reservation prices of individuals are independent of number of companions</td>
<td>.07</td>
<td>.80</td>
<td>Yes</td>
</tr>
</tbody>
</table>

### Table 2

**Weibull Parameter Estimates for the Asian Indian Music/Dance Performances**

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Weibull Parameter/Estimates</th>
<th>$\text{chi-square}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{a}$</td>
<td>$\hat{e}$</td>
</tr>
<tr>
<td>Performance-wise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reservation Price</td>
<td>15.08</td>
<td>0</td>
</tr>
<tr>
<td>Mean Reservation Price</td>
<td>7.46</td>
<td>6</td>
</tr>
<tr>
<td>Number of Performances to Attend</td>
<td>5.42</td>
<td>0</td>
</tr>
</tbody>
</table>

* $p > .10$

**Table 2 continued**

* $p > .90$
PRICING A BUNDLE OF PRODUCTS OR SERVICES

Figure 1

PROBABILITY DENSITY FUNCTIONS OF ESTIMATED WEIBULL DISTRIBUTIONS

At these price levels, 620 single tickets and 308 season tickets are expected to be sold for the series of performances. The profits are about 32% higher than those under pure components strategy ($10,330 by selling approximately 2233 single tickets at $10 apiece) and about 95% higher than profits from pure bundling ($6,980 through sales of nearly 542 season tickets at $35 per ticket). We carried out the jackknife procedure to estimate the variance of the profit estimates. We provide a 95% confidence interval for the profits estimated under each strategy in Table 3. The confidence intervals do not overlap across the three bundling strategies. There is a clear hierarchy in the relative attractiveness of the three strategies. These provide strong evidence for the superiority of mixed bundling strategy over pure components and pure bundling strategies.

Sources of Gains and Losses in Revenues

Another important issue is the source of revenues from different portions of the markets under each strategy and gains and losses in revenues as we shift from one strategy to another.
Under the pure components strategy, with the optimal single ticket price $P_s$ (equal to $10), we are able to attract all consumers with reservation prices above this price for a given performance provided the individual consumers are willing to spend the time to attend. However, in the process, the organizers are “subsidizing” consumers with higher reservation prices for specific performances, which translates as the “consumer surplus.” Moreover, those potential consumers who expect to spare the time to attend a large number of performances but are not willing or able to pay at the single ticket rate represent another source of opportunity loss under pure components strategy.

Pure bundling typically captures a portion of potential consumers with low mean reservation prices but who expect to attend a large number of performances. The optimal season ticket price $P_b$ is $35. The gains, compared to pure components strategy, come from those consumers who are willing and able to spare the time to attend a large number of performances at less than the single ticket prices under pure components. The gain from each new consumer is equal to the bundle price. The switch from pure components to pure bundling results in two sources of losses in revenue: (a) some consumers under pure components strategy who expect to attend only a few performances and, hence, do not feel inclined to buy the season ticket and (b) consumers who would have attended a large number of performances anyway by buying single tickets and are now reaping the advantage of buying season tickets. In our example, pure bundling emerges as a less attractive strategy than pure components under optimal pricing conditions.

The power of mixed bundling lies in its ability to act as a price discrimination device. We have two “products,” the season ticket and single ticket priced at $55 and $14, respectively. It is interesting to note that these prices are different from those under pure components ($10 for the single ticket) and pure bundling ($35 for the season ticket) strategies. Availability of both types of tickets enables sellers to resort to price segmentation. Potential consumers who expect to attend four or more performances and whose mean reservation prices are higher than the mean performance prices are likely to prefer the season ticket at $55 each over several single tickets. Others are likely to buy single tickets at $14 per performance (if the price is less than or equal to their reservation price for the performances for which they end up having the time). At these price levels, the seller’s profits are the highest. The extent of harmful cannibalization of the two types of tickets is lower than under any other price combination.

**Sensitivity of Profits to Price Changes — Importance of Simultaneous Optimization**

It is apt to note that mixed bundling should involve a simultaneous optimization of the single (performance-wise) and season ticket prices to maximize profits. If the

---

7What accounts for the differences in prices of the same types of tickets across alternative strategies? The problem in pure bundling, for example, is that prices may have to be kept low enough to attract a sizable proportion of the market that expects to spare the time for a few performances only. Introduction of single tickets under mixed bundling takes care of this category of customers by offering single tickets at a price higher than that under pure components but lower than the average price per performance under pure bundling. With this segment taken care of, those willing to attend a larger number of performances need not be offered the season ticket at $35 any longer (which is why the season ticket price increases to $55).
price of, for example, the season ticket were the same under pure bundling and mixed bundling strategies, the "low-priced" season ticket would have drastically reduced the effectiveness of the single ticket, making mixed bundling irrelevant.

To clarify this point, we refer the reader to Table 3, which gives the profits from mixed bundling when the (a) prices are optimized simultaneously and (b) prices are optimized sequentially, that is, the single ticket price is optimized given the optimal season ticket price is as under pure bundling strategy (i.e., $35). The second case has a sequentially optimized single ticket price of $12 and a profit of $9,920, lower than that under pure components ($10,330). It illustrates that the advantages of mixed bundling can be undermined by myopic pricing. When the single ticket price is as under pure components ($10), the sequentially optimized season ticket price (for ten performances) turns out to be $55, yielding a profit of $11,610, less than the profits under mixed bundling with simultaneous optimization of single and season ticket prices. In this case also the effectiveness of mixed bundling strategy is reduced.

These figures underline the importance of simultaneous optimization of single and season ticket prices. In mixed bundling, the single ticket "skims" the segment that did not want to buy the season ticket because of time constraints and achieves price discrimination.

**DISCUSSION**

Our primary objective has been to propose an approach to integratively answer four issues (across pure components, pure bundling, and mixed bundling strategies) that are crucial to the seller: (a) optimal prices of bundles and/or their components, (b) corresponding profit levels, (c) sources of revenues as well as gains and losses in revenue if the seller changes from one bundling strategy to another, and (d) price sensitivity of profits. We have addressed a buyer’s purchase decision contingent on meeting two criteria, such as reservation prices for alternative performances and available time to attend performances in the context of entertainment performances. We have empirically tested our approach using an actual bundling problem. The approach can be extended to situations involving more than two criteria as well. We believe our model represents a refinement over other alternatives thus far that have modeled the problem considering only one relevant dimension, such as reservation prices. Though a unidimensional approach may be adequate for several situations, it is likely to be inadequate for problems such as the one considered here.

Our empirical results are consistent with Schmalensee (1984), who maintains mixed bundling is better than pure bundling and pure components. However, the extent of advantage is determined by the ingenuity of fixing the price of the bundle relative to that of the components. Suboptimal pricing leads to extensive cannibalization of one "product" by another and may make the idea of mixed bundling irrelevant. Mixed bundling can be an effective price discrimination device only when the prices are optimized simultaneously.

**Limitations and Future Research Directions**

Our approach does not consider certain relevant situations that form the limitations of this article and provide the opportunity for further research. For example, we restricted our attention to a monopolist organizing the series of performances. The model does not have competitors who may strategically interact with the pricing decisions of the focal seller. Bundle pricing with multiple criteria in the presence of closely competing alternatives (such as another series of Asian Indian music performances staged in parallel by another association) is likely to be an interesting extension.

Our choice of a distributional approach facilitated the analytical development of the results. Distributions representing the market may occasionally be multimodal, requiring certain modifications in the proposed approach. If trade-offs between the multiple dimensions are relevant and important, conjoint analysis-based approaches can be quite attractive. Challenges remain for researchers to develop and expand alternative approaches such as conjoint-based techniques to answer the four bundling questions addressed here.

In modeling the time dimension, we contended that individuals may not be able to spare the time to attend even for the performances they like the most. The primary reason is that the individuals may not foresee, up front, factors such as inclement weather, work pressures, and unknown, alternative entertainment programs that may constrain them from sparing the time to attend the offered performances. A counter argument is that the probability of attending alternative performances is influenced purely by the attractiveness of the performance. However, for contexts such as ours, an individual faces a nonzero probability that he or she cannot attend even if the performance is perceived to be highly attractive. It may be more appropriate to model the attractiveness of the performance as a moderating factor that motivates the individual to manage the external uncertainties to an extent and correspondingly change the individual’s probability to spare the time to attend.

For a series of entertainment performances such as the one we consider, an important exception in which lack of appeal has an overriding influence on allocated time is for performances for which an individual has no motivation to spare the time even when admission is free. Our approach accommodates this adjustment (as discussed earlier). An alternative approach to model the time dimension is to consider an individual level probability of sparing the time to attend alternative performances as a composite of the probability in the manner we have defined, together with another component that is monotonically related to attractiveness of the performances for the individual. This requires explicitly modeling a positive relationship between the time dimension and reservation prices because the latter is clearly a measure of
the attractiveness of alternative performances for the individual. As discussed earlier, in certain situations one may also expect a negative relationship between reservation prices and allocated time. To model situations in which the correlations (positive or negative) between dimensions is important and sizable, bundling approaches need to be developed that consider these relationships using multivariate distributions.

For certain situations in which individuals have no uncertainty in allocating time for performances that they find the most appealing, our approach will underestimate the profits and number of single tickets sold and overstate the number of season tickets sold. The advantage of mixed bundling strategy over pure components strategy is likely to be rather limited under such circumstances.

We do not capture supply side constraints such as the capacity of the auditorium or stadium that may alter the optimal solutions. Research that considers the capacity constraint may examine several interesting issues such as (a) yield management (that is, offering several price tiers for single/season tickets) with more effective price discrimination than the approach we suggest and (b) optimization with the objective of selling to capacity. Furthermore, on the cost side, though we accommodate differences across “product” offerings, we do not treat costs as variable with the number of units sold. The advantage is akin to scope economies and can be incorporated. Marketing of bundles represents a means of reducing the seller’s risk, especially when the competing alternatives are likely to be strong. The seller might offer special prices for consumers who are willing to purchase tickets well in advance of the performance(s). Future bundling research can optimize bundle prices recognizing their risk reducing capacity.

In situations such as symphony orchestra performances, consumers may have the option of choosing from multiple season ticket bundles offering admission to different assortments of performances. The power of price discrimination demonstrated under the mixed bundling strategy can be further enhanced by offering multiple bundles. The seller’s profits are likely to increase if the prices are optimized correctly. Future research can generalize our approach to incorporate multiple bundles.

Bundling models can also be developed to select viable combinations of products/services. For example, which set of \( n \) performances should the organizers choose to maximize profits? This also relates to an earlier point on the “appeal” of alternative performances. Though researchers such as Hanson and Martin (1990) address similar issues for products, there is opportunity for further development in situations such as ours.

**APPENDIX A**

Relaxing the Assumption on Similarity in Reservation Prices Across Performances

For model development we had assumed that though an individual’s reservation price may vary across performances, at the market level the reservations price distributions are not significantly different. In certain cases, some performances may be perceived by most consumers to be significantly superior or inferior to others. It may be more appropriate to price the single tickets differently from performance to performance. Relaxing our earlier assumption does not have an impact on pure bundling strategy since the mean reservation price distribution can accommodate this variation. The modification has an impact only under pure components and mixed bundling strategies.

For situations in which the market as a whole perceives performances to be widely different, we assume the distributions differ from one another on their location parameter \( (e_s) \) but their patterns (shape and variance) are not significantly different. Let the performance with the lowest reservation price be the first performance and \( \mu_2, \ldots, \mu_n \) be the extent to which the remaining \((n - 1)\) performances are shifted away from the first performance. If we subtract these values from the corresponding reservation prices, we would have distributions that are not significantly different, enabling us to use the same procedure as before with the following modifications.

For the pure components case, equation 7 representing profits from holding the \( n \) performances would change to

\[
\pi = \sum_{i=1}^{n+1} z f(z) dz \cdot \exp\left(-\frac{(P_i - e_s)/a}{r}\right) M \quad P^{**}_i - \sum_{j=1}^{n} E_j \tag{19}
\]

where \( P^{**}_i = P_i + \left(\frac{\sum_{i=2}^{n} \mu_i}{n-1}\right) \). Note that \( P^{**}_i \) is a linear function of \( P_i \). The optimal price of (single) tickets for the first performance will continue to be \( P_i \) as under equation 9. The prices for performances 1 through \( n \) are given by the vector

\[
P_i = P_i[1, 1 + (\mu_2/P_s), \ldots, 1 + (\mu_n/P_s)]^T. \tag{20}
\]

In the case of mixed bundling, an individual’s choice of a season ticket over several single tickets will occur when \((P_s/i) < P^{**}_i \) and \( Y > (P_s/i) \) where \( P^{**}_i \) is computed similar to that under equation 20. As mentioned earlier, the values of \( P_i \) under pure components strategy will differ from that under mixed bundling. Hence, \( P^{**}_i \) in equation 20 will be numerically different from that under mixed bundling. The objective function is

\[
\text{maximize } \pi = \sum_{i=1}^{n} \text{Prob}(X = i) M \cdot [m_i \text{Prob}(Y > (P_s/i)P_s) + \text{Pr}(Y \leq (P_s/i)) \text{Pr}(Z > P_i)] \quad [Y \leq (P_s/i)] \cdot iP^*_i \\
+ (1 - m_i) \text{Prob}(Z \geq P_i) iP^*_i \\
- \sum_{j=1}^{n} E_j \tag{22}
\]
(23) such that \( m_i = 1 \) if \( (P_i/\ell) < P^*_i \)

\[ = 0 \text{ otherwise.} \]

The simultaneous optimal values of \( P_i \) and \( P^*_i \) are determined. The vector of single ticket prices is computed similar to that under equation 21.

APPENDIX B

Summary of the Questionnaire Used for the Survey:

The questionnaire consisted of four parts:

- **Part I**: Six questions on (a) viewing frequency of entertainment performances (all types/Asian Indian category), (b) money spent on tickets, (c) participation of members of household, (d) details of attending 8 specific performances (including 5 Asian Indian music/dance performances).

- **Part II**: One question (with 8 sub-sections) on intensity of preference for eight different types of music/dance performances (including five mainstream American and three Asian Indian types).

- **Part III**: Two questions with a lead paragraph seeking information on the two key questions relating to available time to attend performances and reservation prices.

- **Part IV**: Demographic characteristics.

We reproduce verbatim the two questions mentioned under Part III:

**For questions 8 and 9 assume that 10 professional Indian classical music/dance performances are to be held in … at … Hall, the first Saturday of each month for the next 10 months and musicians/dancers of a high caliber perform on each occasion. Also assume that no other professional Indian classical music/dance performance is being organized by any other association/group during this period.**

8. You may not be able to attend all 10 performances due to time-related constraints. For example, your work commitments may interfere with your leisure time on certain performance days or you may be attracted to a different program. Based on your past experience, expected obligations and future time commitments, indicate the number of performances (out of ten) you may be able to attend. Do not worry about the price of the ticket per performance to answer this question.

Provide your

a) most pessimistic estimate : _______ performances out of 10

b) most likely estimate : _______ performances out of 10

c) most optimistic estimate : _______ performances out of 10

9. Depending on your intensity of your preference and within the overall budget that you may have for entertainment, you may be willing to pay a certain maximum amount for each type of performances indicated below. Assuming that you have the time to attend each of the performances, in-

dicate your desire to attend the performances, the maximum price you are willing to pay if you are inclined to attend them and the number of persons likely to accompany you. You are free to indicate different amounts for different types of performances, if you so desire.

<table>
<thead>
<tr>
<th>Program/ Performer</th>
<th>Are you willing to attend performance (Yes/No)</th>
<th>Maximum price you are willing to pay per person per performance ($)</th>
<th>Number of persons likely to accompany you</th>
</tr>
</thead>
</table>

Additional Points for Data Collection and Coding:

- The exact menu of performances is to be indicated under the first column of the table under question 9.
- Depending on response under the second column of the above table, the responses under question 8 have to adjusted as explained in the text.

REFERENCES


Hanson, Ward (1987), "The Strategic Role of Bundling," working paper, Center for Research in Marketing, University of Chicago.


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