

Specialization and Diversification of Venture Capital Funds: The Effect of VC Effort

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ABSTRACT

Current research concerning the choice between diversification and specialization of venture capital (VC) funds typically does not consider how the VC's effort and subsequent knowledge spillovers within different portfolios might influence venture performance. We develop a model that analyzes VC effort when there is the potential for cross-sectional and/or serial knowledge spillover across projects in a VC's set of projects. The model generates two sets of implications concerning VC effort and performance. First, VC post-investment effort is a non-monotonic function of performance shocks, especially for diversified VCs. Second, greater cross-sectional and serial knowledge spillovers improve the relative performance of specialization over diversification, while the manner in which the number of decision stages in a project affects the benefits of specialization and diversification depends on the degree of knowledge spillovers.

Venture capitalists stimulate innovation in a wide variety of industrial and geographic settings (Gompers and Lerner, 2001), with some VCs investing in highly specialized portfolios of projects and others making more diversified investments (see Gupta and Sapienza, 1992; Kaplan and Stromberg, 2004; Table 1 reports examples).¹ Studies in the venture capital literature suggest multiple reasons for differences in portfolio breadth, including risk aversion, fund history, deal size, investment stage, structural leverage, corporate focus, and characteristics of top management teams (Hall and Tu, 2003; Patzelt, Knyphausen-Aufseß, and Fischer, 2009; Wasserman, 2008). However, these explanations typically treat the choice as a portfolio selection problem without considering how the VC's effort and subsequent knowledge spillovers within different portfolios might influence venture performance. This paper develops a model that argues that the potential benefits of post-investment effort by VCs, together with cross-sectional and serial knowledge spillovers across projects, influence the choice of specialized and diversified VC projects.

***** **Table 1** about here *****

Our model builds on the fact that VCs often exert substantial managerial effort after they invest in ventures. VCs are more active in managing entrepreneurial ventures than many other financial intermediaries, such as banks and mutual funds. VCs conduct governance activities that require active effort such as monitoring, consulting, and supporting entrepreneurs, as well as influencing board composition and CEO appointment and replacement (Agarwal, Echambadi, Franco, and Sarkar, 2004). Several studies show that such VC effort is valuable: Hellman and Puri (2002) demonstrate that entrepreneurs financed by VCs tend to have more professional management and bring products to market more quickly; Fitza, Matusik, and Mosakowski (2008) find that VCs' management skills account for significant variations of entrepreneurial performance; other studies

¹ VC refers to both “venture capital” and “venture capitalist”; we refer to VCs' investments as ventures and projects.

show that the success of entrepreneurial ventures tends to increase with VC experience (Sorensen, 2007; Kaplan and Schoar, 2005; Gompers, Kovner, Lerner, and Scharfstein, 2008; Hochberg, Ljungqvist, and Lu, 2007). Hence, compared with asset managers such as mutual funds and hedge funds, VCs place more emphasis on choosing how much effort to exert after their investment in order to help improve the performance of entrepreneurial ventures.

A meaningful model of VC organization must address such VC effort. We develop a model that draws on ideas from real options theory to demonstrate how the benefits of post-investment effort shape a VC's choice between specialization and diversification of investment portfolios when there is potential for knowledge spillovers across projects and over time. Within the real options framework, specialization purchases knowledge spillovers between similar projects, while diversification purchases a flexibility option across different types of projects.

The following intuition underlies our model about why the choice between specialization and diversification is relevant in the organization of a VC fund that can undertake post-investment effort. Specialization allows knowledge about an industry and/or geographic area to accumulate and spill across projects in a portfolio as a VC exerts effort for the projects, but the VC may lose the opportunity to exercise effort because an entire portfolio of similar projects may fail when problems arise in an industrial or geographic sector. Diversification offers higher probability that some projects will survive, in which case the VC can bring its effort to surviving investments but cannot generate knowledge spillovers across projects. This intuition differs from the portfolio argument of corporate diversification, which emphasizes risk aversion – instead, the choice between specialization and diversification in our model stems from differences in benefits when a VC undertakes post-investment effort such as monitoring and engaging with entrepreneurs. Traditional portfolio analysis ignores the need for VC effort.

Our model generates implications about how VC effort and knowledge spillovers affect project choice, liquidation, and the relative performance of specialization and diversification. First, a basic two-period model builds on a logic in which the VC determines at the beginning of period one whether to engage in two similar projects or two different projects based on probability distributions of performance shocks and knowledge spillovers, then decides whether to continue and how much effort to assign to projects as new information about the state of the world emerges after the investment decision. We find that post-investment effort of a VC for an entrepreneurial venture is a discontinuous non-monotonic function of the expected performance of the venture.

Second, multi-period models find that the degree of cross-sectional and serial knowledge spillovers and the number of investment decision stages individually and jointly influence the relative performance specialization and diversification. When knowledge spillovers are low, diversified VCs tend to participate in earlier-stage ventures than specialized VCs. In turn, cross-sectional and serial knowledge spillovers have substituting impact on the relative performance of specialized over diversified VCs: Each type of knowledge spillover reduces the positive impact of the other on the relative performance of specialization.

Our approach has conceptual and practical contributions. We extend the real options (Trigeorgis, 1993; Amram and Kulatilaka, 1999) and knowledge spillover views (Franco and Filson, 2006; Agarwal, Audretsch, and Sarkar, 2008) of strategic entrepreneurship by explaining key elements of the heterogeneous structure and performance of VC funds. This, in turn, helps identify desirable organizational choices of VC funds.

LITERATURE

One can view the distinction between diversified and specialized VC funds as a choice between generalist and specialist strategies, which arises in several literatures. Population ecology

(Hannan and Freeman, 1977) investigates which organizational forms prosper in response to environmental change, for instance, while the knowledge-based view of the firm (Kogut and Zander, 1992) analyzes the tradeoff between specialized and general knowledge. Rather than reviewing all perspectives that discuss specialization and diversification, we will focus on work that underlies the structure of our model, particularly studies of VC decisions, VC performance, and real options.

VC decisions and performance

Before turning to our model, it is useful to outline approaches to VC decisions and performance in the literature. Several papers have described decision points that VCs typically consider. Field studies by Tyebjee and Bruno (1984) and De Clercq, Fried, Lehtonen, and Sapienza (2006) decompose the process into pre-investment activities (origination, screening, evaluation, and structuring), post-investment activities (monitoring and advising), and exit. Several larger sample and theoretical papers have investigated VC decision making. Key choices that arise in this literature include industry and geographic diversification, controlling risk via portfolio diversification versus sharing information across specialized portfolios, the extent of assistance that the VCs provide to the founding entrepreneurs, choice of investment stages, corporate/non-corporate choices, deal size, and portfolio size (Gupta and Sapienza, 1992; Norton and Tenenbaum, 1993; Elango, Fried, Hisrich, and Polonchek, 1995; Hall and Tu, 2003; Kannianen and Keuschnigg, 2003; Bernile, Cumming, and Lyandres, 2007; Patzelt *et al.*, 2009). The summary point is that VCs often need to make decisions about diversification and effort.

Studies of VC performance have found mixed results. Some studies show similar results of VC and other investment activities (Moskowitz and Vissing-Jorgensen, 2002; Kaplan and Schoar, 2005), others find lower returns for VC investments (Hwang, Quigley, and Woodward, 2005;

Cochrane, 2005), and still others find superior returns for VCs (Brander, Amit, and Antweiler, 2002; Ljungqvist and Richardson, 2003; Jones and Rhodes-Kropf, 2003). In a model comparing VC portfolios, meanwhile, Gompers *et al.* (2008) find that specialized VCs do better than diversified ones. Overall, VC performance has substantial variation.

We clarify our understanding of the sources of variation in decisions and performance by developing a model that draws on real options arguments to demonstrate how decisions about fund specialization or diversification can influence VC performance. The VCs' decision choices and post-investment effort in the model are similar to call options; diversified portfolios also have a flexibility option. The option values within our model change with cross-sectional and serial knowledge spillovers, marginal benefits of post-investment effort, and investment uncertainty.

Real options and flexibility

The real options approach can help describe the investment behavior of VC funds. Hurry, Miller, and Bowman (1992) show that real options logic helps explain the behaviors of Japanese high-technology VC investments. Trigeorgis (1993) broadens real options arguments to investigate financial flexibility. The core conclusion of this work is that real options arguments provide a useful lens through which to consider the benefits and limits of flexibility in compositions of VC portfolios. We will extend this idea into VCs' flexibility in post-investment stages by considering VC effort and knowledge spillovers across projects, which existing real options models do not address.

Real options arguments highlight the benefits of flexibility (Amram and Kulatilaka, 1999). A flexibility option arises when a VC can liquidate badly performing projects and move its efforts to other projects. Within the real options framework, we can regard specialization as the purchase of knowledge spillovers between similar projects, while diversification is the purchase of a flexibility

option across different types of projects. Table 2 summarizes how our model reflects and extends ideas from real options arguments and the knowledge spillover view of entrepreneurship.

***** **Table 2 about here** *****

SETTING

Our model builds on the idea that a specialized VC acquires sector-specific experience that can improve the value of its investments, but that it also exposes itself to sectoral shocks that may destroy the value of the effort before the projects in its portfolio reach successful completion. We introduce assumptions and concepts that formalize this idea. We begin with a benchmark model that we extend in later sections. The following assumptions and concepts underlie the model.

Investment Opportunities $\{A, A, B\}$: Three projects exist $\{A, A, B\}$. Two are identical (A), while the other is independent (B). The VC can choose two of the three projects. The choice $\{A, A\}$ denotes specialization such as investing in same industrial sector or geographic location; $\{A, B\}$ is diversification. Diversification makes performance of the two investments independent of each other. Specialization has perfectly correlated performance so that the projects succeed or fail together.² We denote the probabilities of not failing as p_1 for the first project and p_2 for the second project. We assume the probability that the first and second project move together becomes a Leontief function: $\min(p_1, p_2)$ for not failing and $1 - \min(p_1, p_2)$ for failing together.³

Value of Projects (V or 0, with binomial distribution at each stage): The value of the project is V , which is an element of a positive value and zero. A project moves to ‘continuation state’ at each stage with probability p (equivalently, the project moves to ‘failure state’ at every stage with

² Specialization and diversification in our model cover both sectoral and geographic strategies (an example of geographic diversification is the *China plus one in Asia* strategy of some international firms, which helps insure against macroeconomic shocks in China); Hull (2009) argues that ventures in the same sector or region have high default correlation. Sorenson and Stuart (2001) emphasize information flow across sectoral and geographic spaces of VC activity; in parallel, we emphasize a VC’s post-investment efforts and knowledge spillovers.

³ One could alternatively set the probability as $\max(p_1, p_2)$; we will consider positive cross-sectional spillovers between probabilities of success with separate parameterization at specialization.

probability $1-p$). Project value becomes zero if the project goes to failure state at a period (i.e., the event with the probability $1-p$ occurs), with no continuation of the project thereafter. If the project goes to continuation state, the project can again move to a continuation or failure state at the next period unless the decision maker liquidates the project. This process continues until the terminal period (our basic model has two periods, while extensions examine more periods), when the project generates value V if it survives and zero otherwise. This approach builds on Fried and Hisrich (1994), Tyebjee and Bruno (1984), and De Clercq *et al.* (2006).

Liquidation value (τV): τV is the salvage value of a project upon liquidation. τ is the exogenously given recovery rate from selling a surviving project. Liquidation can occur during period 1 in our basic model, after a VC observes performance shocks. Failed projects have no liquidation value. Thus, we can regard V as the *no default* value.

Up probability (p), effort (e, u), serial knowledge spillover (ρ): The probability of reaching continuation state ('up probability') is $p(J_i^t + \varepsilon_i^t)$; the probability of reaching failure state ('down probability') is $1-p(J_i^t + \varepsilon_i^t)$. J_i^t and ε_i^t are the effective effort and the performance shock at time t to project i respectively. Effective effort at time t (J_i^t) is a function of two parameters: Committed effort at time t (x_i^t) plus effective effort at time $t-1$ (J_i^{t-1}); i.e., $J_i^t = x_i^t + \rho J_i^{t-1}$, where ρ is the serial knowledge spillover parameter (our basic model sets $\rho = 0$, which we relax in subsequent t in which e_i^t is i 's effort at t , α is cross-sectional knowledge spillover and I is an indicator function. We will describe α and I in detail shortly. We assume $p' > 0$ and $p'' < 0$, which means that greater effort increases the probability of survival ($p' > 0$) but at a decreasing rate ($p'' < 0$); i.e., there are diminishing returns to effort.

Performance shock (ε_i^t): At time t , a performance shock (ε_i^t) occurs to project i and changes the probability function of continuation (p). The performance shock is a random variable, beyond

the control of any player in our model. Performance shocks are perfectly correlated in specialization $\{A, A\}$ and independent in diversification $\{A, B\}$. The shocks address uncontrollable risk that influences project success. In practice, projects with large exposure to systematic factors are similar to $\{A, A\}$ and projects with few systematic factors are similar to $\{A, B\}$. Failure and success occur to projects rather than at the level of industry or geography.

Cross-sectional knowledge spillover (α): Cross-sectional knowledge spillover (α) occurs when a VC selects similar projects. Suppose the VC conducts $\{A, A\}$ under perfect cross-sectional knowledge spillover. If the VC invests one unit of effort in one of the projects, the other project receives the same knowledge due to perfect spillover. Thus, one unit of effort has the impact of two units of effort. Ongoing feedback between projects will magnify the size of cross-sectional spillover in which a first project spills knowledge to a second project, which in turn spills the knowledge to the first project, as long as the projects continue. In general, one unit of committed effort creates a fraction (α) of effective effort in $\{A, A\}$, with $\alpha > 0$ (if $\alpha = 0$, there is no spillover; if $\alpha < 0$, there is negative cross-sectional spillover). We assume $\alpha > 0$ for the $\{A, A\}$ specialized portfolio and $\alpha = 0$ for the $\{A, B\}$ diversified portfolio (i.e., there is no penalty for knowledge dispersion).

In sum, our approach specifies that a venture capitalist has two units of effort to commit in each period. The VC can invest in two projects. The VC's effort for one project provides a degree of cross-sectional knowledge spillover to the other project with specialization $\{A, A\}$; there is no cross-sectional knowledge spillover with diversification $\{A, B\}$. We assume that any disutility of VC effort is small compared with potential return from the projects and/or the VC has low alternative opportunity cost for its effort.

The operationalization implies that, without liquidation, a VC's expected return (π) from investing effort in two projects with T return periods is the sum of the expected return of the

projects in the fund, including any knowledge spillovers, such that:

$$\pi = \sum_{i=1}^2 (\prod_{t=1}^T p_i^t (J_i^t + \varepsilon_i^t)) V_i, \text{ in which } J_i^t = e_i^t + I\alpha e_{-i}^t + \rho J_i^{t-1}$$

The ‘ i ’ subscript denotes an entrepreneurial project in a venture capital fund. ‘ t ’ denotes the timing of VC effort, which in our basic case lasts for 2 periods ($T=2$). $p_i^t(\bullet)$ is the probability of success of project i at time t . $\{e_i^t, e_{-i}^t\}$ are the VC efforts to project i and the other project at time t . ‘ T ’ is an indicator function taking on a value of 1 if two projects are similar and 0 if they are not; I is 1 for $\{A, A\}$ and 0 for $\{A, B\}$. ‘ α ’ is the cross-sectional knowledge spillover parameter. ε_i^t denotes that a performance shock occurs at time t to a project i , affecting the probability of i ’s success. ‘ V_i ’ is the value of project i without failure. We assume $p_i^t(\bullet) = p(\bullet)$ and $V_i = V$ for all i and t .

Our multi-period model parallels the process of VC behavior and corresponding entrepreneurial value in other research. Fried and Hisrich (1994) decompose the decision-making process of the VC into proposal, origination, specific screening, general screening, 1st and 2nd evaluations, closing, and funding. We model key parts of this sequence. First, we formulate how a VC makes investment and effort decisions after receiving new information. Second, we allow premature exit from a project by considering the option to liquidate a project with predetermined exercise price (τV). This corresponds to selling or closing a project at each decision stage of Fried and Hisrich while receiving some proportion (τ) of the final value (V). Our setting is similar to Tyebjee and Bruno (1984) and De Clercq *et al.* (2006), who decompose the process into pre-investment activities (origination, screening, evaluation, and structuring), post-investment activities (monitoring and advising), and exit. The latter two papers also allow the possibility of post-investment activities and premature exit, which we model by allowing a VC to liquidate a project at the salvage value at any continuation state and to evaluate projects in order to determine investment and efforts when new information arrives.

ANALYSIS

We start with a two-period decision model with the following assumptions: (1) projects can last up to 2 periods following the initial decision about portfolio construction ($T = 2$); (2) no shocks occur ($\varepsilon = 0$); (3) there is no serial or cross-sectional knowledge spillover ($\rho = 0$ and $\alpha = 0$); (4) the VC has two units of effort at each period. We assume that the recovery rate $\tau < p(1)^2$, so that a VC prefers continuing a project with one unit of effort rather than liquidating.⁴ Lemma 1 reports calculations that derive the performance of specialized and diversified VCs on these benchmark assumptions. A VC will determine specialization over diversification if the expected return from specialization is higher or vice versa. Appendix A contains the proofs.

Lemma 1 (Benchmark model: No knowledge spillovers or performance shocks): Let us denote $E(\pi_{AA})$ and $E(\pi_{AB})$ as the expected profits under specialization and diversification. Then, $E(\pi_{AA}) = \max\{2\tau, p(2)^2 + \tau, 2p(1)^2\}V$ and $E(\pi_{AB}) = \max\{2\tau, p(2)^2 + \tau, 2(p(1)^3 + p(1)(1-p(1))p(2))\}V$. $E(\pi_{AB}) \geq E(\pi_{AA})$, i.e., diversification weakly dominates specialization.

Corollary 1 (Diversification versus specialization): The larger the marginal impact that rearranging VC effort ($p(2)-p(1)$) has on expected profits, the larger the domination of diversification. The larger the liquidation value, the more similar the performance of diversification and specialization.

In the benchmark model with neither performance shocks nor knowledge spillovers, diversification weakly dominates specialization. This happens because a diversified VC owns the option to rearrange its managerial efforts (*flexibility option*). The flexibility option arises when one project survives but the other fails; a VC can invest its two units of efforts in the surviving project. A specialized VC does not own this flexibility option because its projects always survive or fail together. The value of the flexibility option increases if rearranging efforts ($p(2)-p(1)$) creates a large effect. As we can denote the exercise of flexibility option with $p(2)-p(1)$, the larger its impact to the performance of diversified VC, the more the diversification dominates the specialization.

⁴ Two efforts run two projects; one effort will run a standalone project. Time to maturity is two periods, so probability of success of a standalone project is $p(1)^2$. If $\tau V < p(1)^2 V$ (as assumed), then running a standalone project is worthwhile.

As liquidation value increases, a VC is more likely to liquidate projects. If a diversified VC liquidates one or both projects, its flexibility option evaporates. Then, the distinction between specialized and diversified VCs disappears, making their performance equal.

Next, let us introduce cross-sectional knowledge spillover (α).

Lemma 2 (Cross-sectional knowledge spillover): With cross-sectional knowledge spillover α , $E(\pi_{AA}) = \max\{2\tau, p(2)^2 + \tau, 2p(1+\alpha)^2\}V$. The relative performance of diversification increases with the marginal impact of rearranging VC effort ($p(2)-p(1)$) and decreases with the marginal impact of cross-sectional knowledge spillover ($p(1+\alpha)-p(1)$). The larger the liquidation value becomes, the more similar the performances of diversification and specialization.

Cross-sectional spillover improves the payoff from specialization. The larger the cross-sectional spillover, the greater the value of specialization. Thus, the choice between diversification and specialization depends on the relative strengths of the flexibility option of diversification and cross-sectional knowledge spillovers with specialization.

The choice of diversification or specialization reflects three possible cases at period two. First, if both projects fail, the value is zero. This applies to both diversification and specialization.

Second, if only one project goes up, the expected value is $p(2)V$. This is because the diversified VC is able to exercise the flexibility option, which is not possible in initial specialization. The survival of only one project generates endogenous specialization, in that the VC concentrates its efforts on developing that project. This situation demonstrates that specialization can result from the initial composition of a VC holding $\{A, A\}$ (initial specialization) or from flexibility that arises in later stages based on survival at earlier stages (endogenous specialization). Endogenous specialization arises from the flexibility option that is embedded in diversification.

Third, if both projects go up, the expected values are $2p(1+\alpha)V$ for specialized portfolios and $2p(1)V$ for diversified portfolios. At the next stage, the project succeeds with probability $p(\bullet)$ and the no-default value continues to be V . Thus, the value of two projects is $2p(\bullet)V$. Both

specialized and diversified VCs split their two units of effort equally across the two projects. Due to the cross-sectional knowledge spillover, the aggregate effort becomes $(1+\alpha)$ for specialized VCs and 1 per project for diversified VCs. Assuming positive knowledge spillover, specialization dominates diversification. Therefore, the choice between diversification and specialization depends on whether the second or the third case dominates; the second case (one project goes up) leads to diversification while the third (both projects go up) leads to specialization.

Lemma 3 (Performance shocks): Let us use following notations: $\{\varepsilon_A, \varepsilon_B\}$ are shocks to $\{A, B\}$. ε is the shocks to specialized VC. $p_M \equiv p(e_A + \varepsilon_A) = p(2 - e_A + \varepsilon_B)$, in which e_A solves the equation and $\{\varepsilon_1, \varepsilon_2\}$ are $\{p^{-1}(\tau/p(1))-1, p^{-1}(\tau/p(2))-2\}$ respectively. $F(\varepsilon)$ is the cumulative distribution function of ε , $\varepsilon_A, \varepsilon_B$. Then, we have:

$$\begin{aligned} E(\pi_{AA}) &= F(\varepsilon_2) \pi_{AA}^{(0,0)} + (1 - F(\varepsilon_1)) \pi_{AA}^{(1,1)} + (F(\varepsilon_1) - F(\varepsilon_2)) \pi_{AA}^{(2,0)}. \\ E(\pi_{AB}) &= F(\varepsilon_2)^2 \pi_{AB}^{(0,0)} + (1 - F(\varepsilon_1))^2 \pi_{AB}^{(1,1)} + (1 - F(\varepsilon_2))^2 - (1 - F(\varepsilon_1))^2 \pi_{AB}^{(2,0)}. \end{aligned}$$

We use following notations.

$$\begin{aligned} \pi_{AA}^{(0,0)} &\equiv 2V\tau = \pi_{AB}^{(0,0)}. \\ \pi_{AA}^{(1,1)} &\equiv 2E(p(1+\varepsilon)p(1) | \varepsilon \geq \varepsilon_1)V. \\ \pi_{AA}^{(2,0)} &\equiv E(p(2+\varepsilon)p(2)+\tau | \varepsilon_2 \leq \varepsilon < \varepsilon_1)V. \\ \pi_{AB}^{(1,1)} &\equiv 2E(p_M^2 p(1) + p_M(1 - p_M)p(2) | \min\{\varepsilon_A, \varepsilon_B\} > \varepsilon_1)V. \\ \pi_{AB}^{(2,0)} &\equiv E(p(2 + \max\{\varepsilon_A, \varepsilon_B\})p(2) + \tau | \max\{\varepsilon_A, \varepsilon_B\} < \varepsilon_2 \text{ and } \min\{\varepsilon_A, \varepsilon_B\} < \varepsilon_1)V. \end{aligned}$$

We will briefly walk through the calculations. Before period 1, at the investment decision stage, the VC knows the terminal value of surviving projects (V), the recovery rate if liquidated before the end period (τ), the degree of cross-sectional knowledge spillover (α), the cumulative distribution function of performance shocks ($F(\varepsilon)$), and the functional form of the probability of a continuation state ($p(\bullet)$).⁵ Given this information, the VC calculates the schedules of effort and liquidation that would be optimal for different values of performance shocks, which allows the VC to calculate the expected values of the specialization $\{A, A\}$ and diversification $\{A, B\}$ portfolios. The VC then selects the portfolio with the largest expected returns.

⁵ Note that knowledge of the cumulative distribution function and the continuation probability function forms implies knowledge of the pattern of possible future events rather than precise estimates of specific probabilities.

The initial optimization calculations determine subsequent allocations of effort, while each subsequent period reveals the viability of the projects. After the choice of specialization or diversification, a performance shock with some value occurs at the beginning of period 1. Following the performance shock, the VC uses the initial calculations to determine whether to liquidate one or both projects and to select the level of effort that reflects the realized performance shock.⁶ In turn, the value of the performance shock and selected effort determine the probability that unliquidated projects will survive and reach period 2. At the beginning of period 2, the VC reallocates effort to surviving projects, again based on the initial calculations for that realized state of the world. This level of effort in turn determines the probability that projects will survive to the end of period 2, which is the point at which surviving projects realize termination value V .⁷

We briefly address two of the simplifying assumptions, noting that they have limited impact on the results. First, we assume a performance shock in only the first period. A second shock would change our results as $\pi_{AA}=2p(1+\alpha+\varepsilon)p(1+\alpha+\varepsilon_{2nd})V$ without liquidation; this would not qualitatively change our results. We can make similar changes for other cases, although the notation would become more complex with no material change in results. Intuitively, a shock in the terminal period has little impact in our case because shocks are serially uncorrelated, players are risk neutral, and no link exists between the first- and second-period efforts when there is no serial knowledge spillover. Second, recall that we assume no disutility of effort (e.g., wages or foregone leisure). A way to address disutility of effort would be to increase the recovery rate so that the value after liquidation (τV) includes the value of saved efforts of the VC; doing so would not change the basic result.

⁶ Optimal effort will be (1,1) for unliquidated specialized portfolios; optimal effort for diversified portfolios can range from 0 to 2 for each project depending on the realized values of shocks and, in turn, the number of surviving projects.

⁷ The model allows liquidation at period 2 but this is never optimal without a shock at period 2. The prior assumption implies that at period 2 the expected return of a surviving project with one unit of effort is greater than the liquidation value; relaxing this assumption does not change our results qualitatively.

To consider the implications of Lemma 3 about specialization, the point $(\varepsilon_1, \varepsilon_2)$ at which a VC exercises the liquidation option decreases with knowledge spillovers (α) and increases with recovery rates (τ). The higher the knowledge spillover or the lower the recovery rate, the less likely that a VC will liquidate specialized funds.

When considering diversified portfolios, the expected returns from diverse projects depend on the performance shocks to the two different projects. If both shocks produce sufficiently low performance, then the VC will liquidate both projects immediately. If expected performance of one project after a performance shock is above the cutoff and performance of the other is below it, the VC will liquidate the project with the low performance and focus on the project with high performance in order to commit two units of effort to the superior project. If both performance shocks produce high results, the VC will manage both projects and divide its efforts so that the probabilities of success are the same for both. The expected return from operating two different projects is given by: $\pi_{AB} = 2(2p_M p(1) + p_M(1-p_M)p(2))V$; $p_M \equiv p(e_A + \varepsilon_A) = p(2 - e_A + \varepsilon_B)$, which exploits the property of $p(\bullet)$. Because the VC has two projects, if both succeed in the first period then it splits its effort in marginally equal ways in the second period. If only one project succeeds in the first period, then the VC will only invest its effort in the successful project. The VC receives the payoff, V , from project A if it is successful in both periods; the same is true for project B .

Similar to the previous benchmark model, we can measure the value of the flexibility option by comparing expected profits with and without the flexibility to adjust efforts. Without the flexibility option, a diversified VC commits one unit of effort in each project. If a project fails, effort in the project becomes valueless. When only one project survives in a diversified VC, the value of flexibility is $p(2 + \max\{\varepsilon_A, \varepsilon_B\})\Delta p$ in which $\Delta p \equiv p(2) - p(1)$. When both projects survive, the value of flexibility is $p_M(1 - p_M)\Delta p + p_C p(1)$, in which $p_C \equiv 2p_M - \sum_i p(1 + \varepsilon_i) > 0$. This calculation shows

that the value of flexibility increases as the marginal value of effort (Δp) increases.

An additional implication of Lemma 3 is that the expected effort by a VC is a varying non-monotonic function of the performance signal. To use the notation in Lemma 3, when $\varepsilon_2 \leq \varepsilon_A$ and $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$, the higher the performance signal of a venture (ε_A), the more a VC exerts effort for the venture. On the other hand, when $\varepsilon_2 \leq \varepsilon_A$ and $(\varepsilon_A + \varepsilon_B)/2 \geq \varepsilon_1$, the higher the performance signal of a venture (ε_A), the less the VC exerts effort for the venture. When $\varepsilon_A < \varepsilon_2$, the venture receives no effort. Lemma 4 states these results.

Lemma 4 (Non-monotonic function): A diversified VC's effort to a project is a varying non-monotonic function of performance shocks to projects⁸. Specifically,

- (1) When the shock to a project is low, a diversified VC's effort to the other project is a step function of the other project's shock taking low and high values.
- (2) When the shock to a project is moderate, the VC's effort to the other project takes low, maximal, and then decreasing values as the shock to the other project increases.
- (3) When the shock to a project is high, the VC's effort to the other project resembles an inverted U shape function of the shock to the other project: the effort takes low, increasing, and then decreasing values.

The analysis has implications about how the performance of the entrepreneurial firm will influence a VC's effort. When an entrepreneur faces a very bad situation, the VC will abandon the entrepreneur and invest no effort to assist the project. When the situation is moderate, the VC will work hard on behalf of the project. When the situation is very good, the VC will reduce effort if it can shift efforts to other projects. Thus, the relationship between project performance and the level of VC effort is non-monotonic, similar to an inverted-U shape.

The results of our model help assess the underlying causality in findings of prior empirical research. Jääskeläinen, Maula, and Seppä (2006) find that VC performance is an inverted U-shaped function of effort, which they explain as the result of informational and interpersonal aspects of VC effort. However, our study suggests a different causality than the earlier empirical work concludes.

⁸ Specialized projects create a non-monotonic function if p is concave and a VC has outside options for effort. As a shock increases, the marginal value of effort decreases, which induces the VC to deploy effort for outside options.

Jääskeläinen *et al.* argue that, beyond some inflection point, high effort causes decreasing performance; in contrast, we argue that high performance causes low effort (i.e., how much attention the VC's are paying to different projects). Clearly, further empirical research needs to consider the possibility of such endogeneity.

Results concerning changes in management are also relevant. Hellman and Puri (2002) argue that VCs need both control rights and the ability to guide top management. Contractual control rights, such as board seats and the power to replace the target firm's management, help attenuate agency problems between VCs and their target projects, while the VC's managerial efforts enhance the value of entrepreneurs who lack business expertise and social capital. In parallel, Boeker and Wiltbank (2005) find that changes in top management of entrepreneurial ventures commonly occur with either very low or very high growth. Combining these arguments, we can reason that VCs engage most actively in post-investment effort either at very low or very high firm growth: (1) management change occurs either at very low or very high growth (Boeker and Wiltbank, 2005); (2) management change is a key professionalization activity by VCs (Hellman and Puri, 2002); therefore, (3) VCs exert intensive post-investment effort at very low or high growth. This reasoning is in line with our findings. In our model, the performance shock (ε) influences the probability of success of a project and, therefore, determines the growth rate of a project.⁹

Let us summarize our results until now. VCs can choose to take either a specialized strategy by choosing two similar projects or a diversified strategy by choosing two differing projects. Similar projects allow the VCs to seek higher returns from their efforts because there is a spillover of the knowledge derived from the projects, while choosing diverse projects helps spread the value

⁹ The following logic underlies the point that higher p implies higher growth rate over periods. A two-period model yields $p_1 p_2 V$ in which p_1 and p_2 are the first and second period up probabilities. Let I_0 be an arbitrary initial investment amount. Then, the two-period annualized growth rate (g) needs to solve $(1+g)^2 = p_1 p_2 V / I_0$, such that $g = (p_1 p_2 V / I_0)^{1/2} - 1$. In sum, any factor that affects the probability of success also influences the growth rate.

of VC effort over independent projects. A VC has two units of effort to divide across the two projects. How much effort the VC invests in a particular project affects the likelihood that the project will succeed. Projects take two periods to develop, so the VC may exert up to two units of effort in each of two periods. If a project fails in either period, its termination value is 0. The VC's effort in differentiated projects will have no impact on each other, while similar projects have a spillover effect such that effort for one project will affect the other. Our model suggests that a VC's post-investment efforts for a project will be a non-monotonic function of the project's expected performance, where exogenous shocks influence expected performance, with the non-monotonic shape being strongest for diversified VCs. Moreover, the value of the flexibility option of a diversified VC increases as the marginal value of VC effort increases because the diversified VC can liquidate a low-performing venture and devote all effort to other venture.

Next, we introduce serial knowledge spillovers and multiple stages. To foreshadow the results, first, cross-sectional and serial knowledge spillovers are substitutes in determining the relative performance of specialized over diversified VCs. Both types of spillover create benefits for specialization. When they work jointly, however, each reduces the marginal benefit of the other because knowledge that spills from similar sources (the same firm one period ago or a current similar firm) becomes partially redundant. Second, projects with few decision-making stages favor specialization, while increasing the number of decision-making stages can favor either diversification or specialization, depending on the extent of cross-sectional knowledge spillover – diversification when spillovers are low, specialization when spillovers are high.

In the presence of serial knowledge spillover, VCs can use experience gained through their previous investments and efforts. Serial knowledge spillovers intensify the tradeoff between diversification and specialization. Specialization offers deep but volatile knowledge over time,

while diversification offers shallow but stable knowledge over time. The nuanced knowledge of a specialized VC is deep and accumulates quickly, but with narrow scope. Suppose a negative shock occurs: Then all the entrepreneurs in a specialized VC's portfolio will be in trouble. In this unfortunate situation, the VC's specialized knowledge, accumulated over time, loses value. By contrast, it is less likely that all entrepreneurs in a diversified VC's portfolio will fail simultaneously. The existence of serial knowledge spillover magnifies such effects over time.

To introduce serial knowledge spillover formally, we set the probabilities of success at the first and second stage as $\{p(e_{i0}), p(\rho_i e_{i0} + e_{i1})\}$. $\{e_{i0}, e_{i1}\}$ is the level of effort at the first and second stage for project i . ρ_i is the serial knowledge spillover parameter, which denotes how experience from the previous period influences the current period's performance. When $\rho=0$, the model reduces to the previous model. When $\rho>0$, the likelihood of success in a current stage increases as the VC expends more effort in the previous stage.

Let us introduce simplifying assumptions: $p(-\infty)=0$, $p(\infty)=1$, $p'(-\infty)=\infty$, and $p'(\infty)=0$. In turn: $p''(\bullet) \leq 0$ and $p'''(\bullet) \geq 0$. Lemma 5 outlines the expected profits of diversified and specialized VCs when serial knowledge spillover varies. If we change the simplifying assumption about $p(\bullet)$, the theorem can include parameters values for $p(-\infty)$, $p(\infty)$, $p'(-\infty)$, and $p'(\infty)$.

Lemma 5 (Serial knowledge spillover): Expected profits under specialization and diversification when serial knowledge spillover exists are: $E(\pi_{AA}) = \max\{2\tau, p(2)p(2(1+\rho))+\tau, 2p(1)p(1+\rho)\}V$ and $E(\pi_{AB}) = \max\{2\tau, p(2)p(2(1+\rho))+\tau, 2(p(1)^2p(1+\rho)+p(1)(1-p(1))p(2+\rho))\}V$. In addition, (1) $E(\pi_{AB}) \geq E(\pi_{AA})$, i.e., diversification weakly dominates specialization; (2) the larger the marginal impact of rearranging VC effort ($p(2+\rho)-p(1+\rho)$) on the expected profits, the larger the domination becomes; (3) the larger the serial knowledge spillover (ρ), the weaker the domination becomes; (4) the larger the liquidation value, the more similar the performance of diversification and specialization.

Lemma 5 shows that a VC's behavior changes only slightly with serial knowledge spillover. First, the larger the serial knowledge spillover becomes, the less likely a project is liquidated; that is, compared with the case without serial knowledge spillover, a VC is more likely to exercise the

option to continue projects. Hence, because serial knowledge spillover enhances the ongoing impact of effort, a VC can save projects that it would have liquidated without knowledge spillover. Second, a diversified VC becomes more likely to continue both projects as serial knowledge spillover increases. The intuition parallels the first: Increasing effort with serial knowledge spillover enables a diversified VC to rescue more projects. Third, the larger the serial knowledge spillover, the weaker the domination of diversification becomes. Larger serial spillover reduces the value of flexibility option ($p(2+\rho)-p(1+\rho)$) due to the concavity of probability function ($p'' \leq 0$). Intuitively, the more projects that are likely to survive, the less are chances to exercise the flexibility option, i.e., moving efforts from bad to good projects. The lower the value of flexibility option becomes, the less the diversification dominates specialization. Fourth, the impact of liquidation value is similar to cases without serial knowledge spillover. Corollary 2 shows that our results about performance shocks (lemma 4) hold in the presence of serial knowledge spillover.

Corollary 2 (Non-monotonic function with serial spillover): In the presence of serial knowledge spillover, a diversified VC's effort to a project is non-monotonic and varying function of performance shocks to projects. Specifically, Lemma 4 holds with serial knowledge spillover: (1) when the shock to a project is low, a diversified VC's effort to the other project is step function of the other project's shock taking minimal and maximal values; (2) when the shock to a project is moderate, the VC's effort to the other project takes minimal, maximal and then decreasing values as the shock to the other project increases; (3) when the shock to a project is high, the VC's effort to the other project resembles an inverted U-shape function of the shock to the other project, such that the effort takes minimal, increasing and then decreasing values.

Corollary 2 shows that a non-monotonic shape continues to hold when we introduce serial knowledge spillover, but that serial knowledge spillover modifies the shape. An important point is $\varepsilon_1 = p^{-1}(\tau/p(1+\rho))-1$, which is a decreasing function of the serial spillover (ρ) and an increasing function of recovery rate (τ). If the average of performance shock is greater than this point ($\varepsilon_A + \varepsilon_B \geq 2\varepsilon_1$), a diversified VC is likely to save both projects and allocate resource from the project with high shock to the one with low shock. Thus, the larger the serial spillover, the more likely a

VC's effort to a project is negatively related with the project's performance.

Intuitively, serial knowledge spillover amplifies the impact of effort. If a spillover is large, even minimal effort can be enough to reach the point at which the VC is indifferent about liquidating one project or continuing it. Once both projects pass the point of indifference, a VC needs to reallocate effort so as to subsidize the project with low-value performance shock until the VC balances the marginal values of effort in both projects. Hence, the larger the serial knowledge spillover, the more likely cross-subsidization in a diversified VC's portfolio occurs in allocating post investment efforts from high- to low-performing projects (see Scharfstein and Stein (2000) for a discussion of divisional rent-seeking).

Next, let us compare the performance of diversification and specialization in response to the changes in serial and cross-sectional knowledge spillovers.

Lemma 6 (Serial and cross-sectional knowledge spillover): Expected profits under specialization and diversification when both serial and cross-sectional knowledge spillover exist are: $E(\pi_{AA}) = \max\{2\tau, p(2)p(2(1+\rho))+\tau, 2p(1+\alpha)p((1+\alpha)(1+\rho))\}V$ and $E(\pi_{AB})$ is same to Lemma 5.

Corollary 3 (Diversification versus specialization): The relative performance of specialization decreases with the marginal impact of rearranging VC effort ($p(2+\alpha)-p(1+\alpha)$) and increases with the marginal impact of both cross-sectional and serial knowledge spillovers ($p((1+\alpha)(1+\rho))-p(1)$). In addition, the larger either serial (ρ) or cross-sectional (α) knowledge spillover, the more that cross-sectional and serial spillovers substitute for each other in determining the relative performance of specialization over diversification. The larger the liquidation value becomes, the more similar the performances of diversification and specialization.

Cross-sectional spillover also improves the relative performance of specialization. First, the marginal impact of rearranging VC effort ($p(2+\bullet)-p(1+\bullet)$), which applies to diversification, decreases with cross-sectional knowledge spillover. This increases the relative performance of specialization. Intuitively, the more likely that both projects will survive, the fewer disadvantages the lack of flexibility option implies. Second, the marginal impact of both cross-sectional and serial knowledge spillovers ($p((1+\alpha)(1+\rho))-p(1)$) increases with the value of cross-sectional spillover,

which applies to specialization. This also increases the relative performance of specialization.

In addition, serial spillover affects the value of the flexibility option that is part of the diversification choice. The value of flexibility declines as the amount of serial knowledge spillover increases. The larger the serial spillover, the less likely a VC will liquidate a project; i.e., the less likely that the VC benefits from flexibility by liquidating one project and reallocating its effort. This in turn implies that greater serial knowledge spillover reduces the relative value of diversification.

Corollary 4 restates this intuition.

Corollary 4 (Serial and cross-sectional substitutes): Cross-sectional and serial knowledge spillovers are substitutes for the relative performance of specialized over diversified VCs for the large values of either spillover. Each type of spillover reduces the positive impact of the other type of knowledge spillover for the relative performance of specialization over diversification.

We now extend the model to a multi-stage case in which VCs can undertake post-investment effort multiple times, which allows us to examine how cross-sectional knowledge spillovers influence early- versus late-stage financing decisions and/or choice of uncertain projects by specialized and diversified VCs. A VC that undertakes early-stage financing will need to engage in monitoring and intervention for longer periods and provide more units of effort than a VC with later-stage investment. In addition, early-stage financing faces lower probability of success, but generates higher cash flows from successful projects. Lemma 7 compares the performance of specialized and diversified VCs based on the number of decision stages (N) and the degree of cross-sectional knowledge spillover (α).

Lemma 7 (Cross-sectional knowledge spillover in multiple-stages): Let us assume $\rho=0$, $\varepsilon=0$, $\tau=0$, and discount factor $\beta<1$.¹⁰ If $\alpha<1$, there exists N^* such that the ratio of diversification and specialization performances (π_{AB}/π_{AA}) is an increasing function of N for $N>N^*$. If $\alpha>1$, the larger the number of decision stages, then the smaller the relative performance of diversification over specialization. Thus, cross-sectional knowledge spillovers attenuate the benefits for diversification that arise as the number of decision stages increases.

¹⁰ The assumptions make the model tractable; if we double the number of stages in the model, each decision stage needs to consider effort reallocation when one project survives and the other fails, which does not have an analytic solution.

Let us consider how the number of stages affects the impact of cross-sectional spillover for specialized VCs and the value of the flexibility option for diversified VCs. Cross-sectional knowledge spillovers increase as the stages increase because a VC exerts effort for a project at each stage; this effort spills into the other project in a specialized portfolio and accumulates over periods $(p(1+\alpha)^N)$. In parallel, the value of the flexibility option increases with the number of periods because a diversified VC has more opportunities to exercise flexibility as the number of stages increases $(2\beta^N p(1)(1-p(1))(p_{12}-p_{11}))$, in which $p_{12} \equiv (p(1)^{2N} - p(2)^N) / (p(2) - p(1)^2)$ and $p_{11} \equiv (p(1)^{2N} - p(1)^N) / (p(1) - p(1)^2)$. The more decision stages, the more valuable the flexibility option becomes; we call this effect the *accumulation of flexibility option*.

This set of influences has two implications for how the number of stages affects the relative value of diversification. First, if cross-sectional spillover is low, the relative value of diversification increases with the number of stages because the positive effect of flexibility (*accumulation of flexibility option*) dominates the spillover effects. Second, if cross-sectional spillover is high, the relative advantage of diversification declines with the number of stages, because the cross-sectional spillover effect dominates the accumulation of flexibility option. Let us regard cross-sectional knowledge spillover as small if $\alpha < 1$ and large if $\alpha \geq 1$. When cross-sectional knowledge spillover is small, diversified VCs will finance early-stage entrepreneurs and specialized VCs will support late-stage entrepreneurs; when cross-sectional knowledge spillover is large, however, specialized VCs will prefer funding earlier-stage projects. In sum, diversification dominates specialization when projects require many decision-making stages (e.g., early-staged financing) as long as cross-sectional knowledge spillover is low, while specialization gains on diversification for either projects with few decision stages or when cross-sectional knowledge spillover is high.

Empirical research helps provide confidence in the validity of the model, while the model

again assesses the underlying causality and extends the implications of the empirical patterns.

Wasserman's (2008) field research finds that VCs participating in later-stage financing are similar to banks and asset managers. He argues that, due to the nature of information, the VCs participating in early-stage financing tend to be more active in young ventures and need to provide tacit, intuitive, and indivisible knowledge, whereas VCs for later-stage ventures rely more on explicit and narrow expertise. If a VC acquires explicit and focused skills, it wants to apply them to ventures in order to exploit knowledge spillovers, but only in narrow types of projects or locations (specialization) where the skills apply. This result aligns with our proposition about knowledge spillovers, although Wasserman does not consider either cross-sectional or serial spillovers.

In addition, Wasserman shows that VCs for early/later stages perform better/worse if the ratio between senior and junior venture capitalists ('structural leverage') is high. VCs with few senior venture capitalists may have a narrow scope of knowledge about entrepreneurs or have specialized knowledge about only specific sectors. Wasserman argues that VCs with few seniors should focus on the later stages, which corresponds to our conjecture that specialized VCs benefit by engaging in projects that require fewer post-investment activities, at least unless cross-sectional knowledge spillover is large. Similarly, we propose that a diversified VC has better capability to finance early-stage projects, which aligns with Wasserman's finding that VCs with many senior venture capitalists fund early-stage entrepreneurs.

Our approach shows that the prior empirical findings reflect special aspects of a more encompassing underlying model. We demonstrate that cross-sectional knowledge spillover can influence which types of VCs will finance early- and late-stage projects. When the spillover is small, diversified VCs focus on early-stage projects with multiple decision stages and specialized VCs finance later-stage projects, as Wasserman predicts.

When cross-sectional knowledge spillover is large, however, a greater number of decision stages can shift portfolio emphasis to specialization, contradicting Wasserman. The number of stages also arises here. When the decision steps increase, early efforts have strong impact as a project goes on. This means that efforts at later stages become relatively less important than efforts at early stages. Diversified VCs create value by flexibly allocating resources at later stages, i.e., after observing project performance signals. Hence, diversified VCs gain less advantage from larger numbers of stages when serial knowledge spillovers are high. Corollary 5 states this intuition.

Corollary 5 (Cross-sectional knowledge spillover in multiple stages): Under low levels of cross-sectional knowledge spillover, projects with many stages (early-stage financing) create the greatest benefits for diversified VCs, while projects that require fewer decision stages (late-stage financing) suit specialized VCs. With high cross-sectional knowledge spillovers, projects with a larger number of decision stages favor specialized VCs.

To summarize the results, cross-sectional and serial knowledge spillovers are substitutes in determining the performance of specialization over diversification. Serial and cross-sectional knowledge spillovers create benefits for both specialization and diversification, with greatest benefits for specialization. However, the higher one of the knowledge spillovers becomes, the lower the benefits of the other form of spillover on the relative performance of specialization over diversification. Diversified VCs benefit relative to specialized VCs as the number of decision stages increases as long as spillovers are low; specialized VCs gain advantages from projects with more decision stages when cross-sectional knowledge spillover is high.

DISCUSSION

We develop a decision model for VC portfolio choice in which a VC chooses between fund specialization and diversification. Key factors in decision include potential knowledge spillovers among projects as well as the need to consider a VC's post-investment effort for monitoring and managing entrepreneurial ventures. The model reflects ideas from real options arguments.

Specialization by a VC fund reflects a call option with liquidation value as the exercise price.

Diversification reflects the compound of a call option and a flexibility option.

Implications

The model has several implications. First, the basic two-stage model suggests that a VC's post-investment effort for an entrepreneurial firm is a non-monotonic function of performance shocks to the firm, similar to an inverted-U shape. The VC calculates expected values for diversified and specialized portfolios and related optimal schedules of effort and liquidation, based on a small number of parameters: The value of knowledge spillovers across projects, project liquidation and termination values, the functional form of the cumulative distribution function of performance shocks, and the degree to which shocks and effort affect the probability of continuation. The actual shocks then determine the VC's choice of which projects to continue and which allocation of effort to select. When a shock causes the performance of an entrepreneurial firm to deteriorate significantly, a VC liquidates the investment. Within diversified portfolios with two surviving ventures, the VC invests less effort in a lower or higher performing venture, while investing maximum effort in a moderate performing venture.

Second, the larger the extent of serial or cross-sectional knowledge spillover, the greater the relative performance of specialization over diversification. Serial and cross-sectional knowledge spillovers substitute in their impact on the relative performance of specialization. The larger one type of the knowledge spillover becomes, the smaller the positive impact of the other type of spillover on the relative performance of specialization.

Third, the multiple-stage model finds that cross-sectional knowledge spillovers and the number of post-investment decisions influence the relative performance of specialization and diversification. If cross-sectional knowledge spillover is low, diversified VCs tend to participate

more in earlier-stage ventures. Specialized funds focus on later-stage projects if knowledge spillover is low, while gaining more benefits from earlier-stage investments as cross-sectional spillovers increase.

Comparison to other approaches

The model provides a more systematic approach to venture capitalist specialization and diversification than prior studies. Our model of VC organization provides plausible cohesive explanations for a dispersed set of results from the empirical literature on VC strategy and performance.

At the heart of the model is the careful consideration of the effect of knowledge spillovers, both within a given period and over time. Drawing on knowledge spillovers leads to explanations that do not require assuming risk aversion or differential skill mixes of VCs – two alternative explanations for VC specialization and diversification strategies. Norton and Tenenbaum (1993) find that information-sharing opportunities induce VCs to reduce diversification in terms of sectors, stages, and number of portfolio firms. They argue that the ability to control risks increases with VC specialization, due to the informational advantage. However, we focus on return rather than risk because VC investors can control the risks of ventures with their own portfolio diversification. Our approach and the risk-aversion argument have different implications for studying agency behavior between VCs and ventures. Risk aversion implies that observed diversification and specialization choice will be close to random if we control agency problems. We instead argue that even after controlling the agency problems, we will observe a systematic pattern of diversification and specialization as a function of flexibility and knowledge spillover.

Our model also differs from VC capability approaches. Manigart *et al.* (2002) apply a resource-based view in order to explain cross-country variations in the required rate of return in

venture capital investing. In turn, Schwienbacher (2008) argues that US venture capitalists have better capabilities than European VCs in using financial contracts, offering this as an explanation for why US venture capitalists often outperform European VCs. Rather than arguing that exogenously determined differentiated capabilities lead VCs to select specialization or diversification, our model helps explain how differentiated capabilities arise. We initially argue that the extent of cross-sectional knowledge spillover lead firms to choose specialization or diversification. As serial knowledge spillover and stages of financing enter following initial choices, VC capabilities for diversification and specialization will evolve endogenously. In addition, the model helps explain how VCs with specialization and diversification capabilities distribute across projects with different stages of financing, types of entrepreneurs, and uncertainty.

It is useful to compare and contrast our approach with other studies that involve knowledge spillovers. Agarwal *et al.* (2008) examine spillovers as a process mechanism for creative construction in entrepreneurship. The symbiotic relationships between individuals and their knowledge environments yield endogenously created entrepreneurial opportunities. Their emphasis, similar to ours, is on managing knowledge spillovers to improve performance. In their approach, the spillovers come out of the symbiotic relations, whereas the spillovers in our model arise from the serial and/or cross-sectional absorption of the VC's knowledge by the ventures themselves.

Spillovers and knowledge transfer also are central to Franco and Filson (2006), who focus on employee transfer as the mechanism for knowledge transfer between firms within an industry. They develop an agent-based model where employees *pay* to learn and then move to other firms in equilibrium. Their model is an industry-level analysis, while our analysis is a VC model with cross-sectional knowledge spillover across similar ventures or serial spillover over periods.

A cross-sectional study by Gompers *et al.* (2008) finds that specialist VCs outperform

generalists; they reason that generalists are not good at allocating funding across sectors or at selecting investments within sectors. This is consistent with a bounded rationality argument that the calculation routines involve a much larger decision space for generalists to consider. They also find that experience helps determine VC performance. Experience combined with learning leads to greater relevant knowledge. These results are consistent with our results, which consider cross-sectional and serial spillovers. Further, the Gompers *et al.* argument suggests that the VC organization is not important, because high-performing VCs are internally specialized. In our model, we do not invoke differential skills for specialized and diversified VCs – only that effort increases the probability of performance. However, we do include experiential learning as serial spillovers. In turn, venture performance and subsequent VC performance depend upon the strategy to specialize or diversify, rather than upon the differential skills of the VC. Gompers *et al.* (2008) also do not consider multi-period flexibility.

Our model also extends real options analysis, which traditionally focused on financial resources (e.g., Trigeorgis, 1993; Amram and Kulatilaka, 1999). Knowledge spillovers work differently than financial resources, both because they spread across projects and because they cannot be stored. First, a spillover can occur without using up an effort resource, i.e., assigning effort to one venture has positive impact on other similar ventures, but no impact on the others if they differ. Monetary resources do not have the same effect because money spent in one venture is not available for another venture whether the ventures are identical or not. The core point is that, when there are knowledge spillovers, limited VC effort has different implications than limited monetary resources. Second, VC effort involves the application of skills and knowledge that are not storable, largely fixed at a point in time. In short, knowledge spillovers together with limited VC effort provide a mechanism for explaining VC specialization and diversification strategies. Together

these comparisons indicate that spillovers and knowledge transfer, which are central to our model, are important mechanisms in VC organization and performance.

Limitations and extensions

The models have several limitations that provide avenues for future research. We examine VC funds with only two investments and with only perfectly correlated or completely independent ventures. We treat the value of VC effort as equivalent for all VCs, with differences in the marginal impact of effort arising only from the current level of success and effort of ventures. Future models also might relax several assumptions; for example, they might allow benefits for VCs if they do not expend effort or allow penalties for dispersing effort among diversified projects.

Future research also could extend the ideas about the optimal organization of VC funds. For instance, one could analyze how the specialized or diversified organization of VC funds influences the governance relationship between VCs and entrepreneurs. It would also be useful to tease out the causality in the relationship between venture performance and VC effort.

Overall, our model provides a focused approach to assessing how effort and knowledge spillovers affect a VC's choice of investment portfolios. We expect that our main results will hold under extended analysis. We believe that this study provides a useful base for such extensions.

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Table 1: Examples of Diversified and Specialized Venture Capital Funds

Diversified VC Fund	 http://www.kpcb.com	Kleiner Perkins Caufield & Byers, Menlo Park, CA <ul style="list-style-type: none"> • <i>Geographic diversification</i>: US, China, others. • <i>Industry portfolio</i>: Alternative energy, bio, health, Internet, and others.
	 http://www.sequoiacap.com	Sequoia Capital, Menlo Park, CA <ul style="list-style-type: none"> • <i>Geographic diversification</i>: US, China, India, Israel, others • <i>Industry portfolio</i>: Highly diversified portfolio covering seed, early stage, growth, public, and others.
Specialized VC Fund	 http://www.galen.com	Galen Partners, Stamford, CT <ul style="list-style-type: none"> • <i>Location</i>: US preferred • <i>Specialty</i>: Healthcare
	 http://www.emergencecap.com	Emergence Capital Partners, San Mateo, CA <ul style="list-style-type: none"> • <i>Location</i>: US preferred • <i>Specialty</i>: Early growth stage, technology-enabled services, \$1-\$10 million initial investment

Sources: Company web pages

Table 2: Comparing Our Model to Our Building Blocks

Dimensions in our model	Knowledge Spillover View of Strategic Entrepreneurship	Real Options Theory (ROT)
Heterogeneity of VC funds (dependent variable)	Partial: No detailed argument about types of heterogeneities and how they arise	No: ROT has not been applied to explain VC heterogeneity
Uncertainty	Yes: Uncertainty is an important variable in the spillover view	Yes: ROT has been used to model decision making under uncertainty
Spillovers	Yes: Knowledge spillover is a central concept for strategic entrepreneurship	Partial: Serial spillovers are analogous to learning options, but cross-sectional spillovers are not part of traditional real options arguments
Flexibility	No: VC flexibility across projects is not explicit in the spillover view	Yes: Flexibility options are a common form of real option
Managerial effort	Partial: Effort to create and find investment opportunities is an important concept, but without considering non-monotonic relationships between effort and performance	No: ROT focuses on capital investments and tangible resources.

APPENDIX A: PROOFS

Lemma 1 (Benchmark model: No knowledge spillovers or performance shocks): $2p(1) > p(2)$ holds due to concavity. Therefore, a VC prefers splitting its two units of effort equally over projects rather than concentrating. Both specialized and diversified VCs have three choices of allocating their efforts. First, they liquidate both projects to earn $2\tau V$. Second, they liquidate one and execute the other to earn $((\tau + p(2)^2)V$. The liquidated project generates τV . A VC injects two efforts to the surviving project for two periods to generate $p(2)^2 V$. Third, they can invest in both projects to generate $2p(1)^2 V$ for specialized VCs and $2(p(1)^3 + p(1)(1-p(1))p(2))V$ for diversified VCs. In the diversification case, a VC invests one unit of effort per project at period 1. The $2p(1)^3$ term occurs because, when both projects succeed simultaneously with the probability $p(1)^2$, the surviving two projects yield expected value $2p(1)V$ because each project succeeds with $p(1)$. $2p(1)((1-p(1))p(2)V$ arises because only one survives with probability $2p(1)(1-p(1))$. Then, a VC will invest two efforts in the surviving project to realize its expected value $p(2)V$. In the specialized case, a VC invests one effort per project as we model with the Leontief assumption. The expected payoff for the two periods is $2p(1)^2 V$ because the two projects succeed or fail together..

Corollary 1 (Diversification versus specialization): Let $k \equiv p(2)/p(1) > 1$. Then, $E(\pi_{AB})/E(\pi_{AA}) = p(1) + k(1-p(1))$, which is always positive and increases with k . k increases with $p(2)-p(1)$. When either project is liquidated, diversification and specialization produce the same payoff. When liquidation value (τV) is large, it is likely that the VC will liquidate one project.

Lemma 2 (Cross-sectional knowledge spillover): First, a specialized VC can liquidate both projects to earn $2\tau V$. Second, it can liquidate one and execute the other to earn $(\tau + p(2)^2)V$. A liquidated project generates τV . The VC injects two efforts to the survived one for two periods to generate $p(2)^2 V$. Third, it can invest in both projects to generate $2p(1+\alpha)^2 V$. The probability of joint success is $\min(p_1, p_2)$, in which $\{p_1, p_2\}$ are the probabilities of success in the first and second project. Due to this Leontief formulation, the VC sets $p_1 = p_2$, so that it invests equal amount of effort in both projects in the first and second periods. Due to cross-sectional knowledge spillover, one effort generates α effort in the other project. Thus, the expected profit becomes $2p(1+\alpha)^2 V$: one effort in each period in each project and spillover between projects in each period. When liquidation value is large, it is likely that one of projects is liquidated.

Lemma 3 (Performance shocks): The expected profit from specialization is $\pi_{AA}^{(0,0)}$, $\pi_{AA}^{(1,1)}$, and $\pi_{AA}^{(2,0)}$ when $\varepsilon < \varepsilon_2$, $\varepsilon \geq \varepsilon_1$ and $\varepsilon_2 \leq \varepsilon < \varepsilon_1$, respectively. $\pi_{AA}^{(0,0)}$ arises when a specialized VC cannot generate higher output with two efforts than with no effort (i.e., liquidation). This occurs when $p(2+\varepsilon)p(2)V < \tau V$ or $\varepsilon < \varepsilon_2$. $\pi_{AA}^{(1,1)}$ arises when a specialized VC can generate higher output with one effort than with no effort. This occurs when $p(1+\varepsilon)p(1)V > \tau V$ or $\varepsilon > \varepsilon_1$. $\pi_{AA}^{(2,0)}$ arises in other cases when a specialized VC can generate higher output with two efforts than with no effort, but cannot with one effort. This occurs when $p(2+\varepsilon)p(2)V < \tau V < p(1+\varepsilon)p(1)V$. Given the definition of $F(\bullet)$, the expected profit from specialization becomes $E(\pi_{AA}) = F(\varepsilon_2) \pi_{AA}^{(0,0)} + (1 - F(\varepsilon_1)) \pi_{AA}^{(1,1)} + (F(\varepsilon_1) - F(\varepsilon_2)) \pi_{AA}^{(2,0)}$. The expected profit from diversification is $\pi_{AB}^{(0,0)}$, $\pi_{AB}^{(1,1)}$, and $\pi_{AB}^{(2,0)}$ when $\max\{\varepsilon_A, \varepsilon_B\} < \varepsilon_2$, $\min\{\varepsilon_A, \varepsilon_B\} > \varepsilon_1$ and other cases, respectively. $\pi_{AA}^{(0,0)}$ arises when a specialized VC cannot generate higher output with two efforts than with no effort (i.e., liquidation). This occurs when $p(2+\varepsilon)p(2)V < \tau V$ for both projects or $\max\{\varepsilon_A, \varepsilon_B\} < \varepsilon_2$. $\pi_{AB}^{(1,1)}$ arises when a diversified VC can generate higher output with one effort than with no effort for both projects. This occurs when

$p(1+\varepsilon)p(1)V > \tau V$ for both projects or $\min\{\varepsilon_A, \varepsilon_B\} > \varepsilon_1$. $\pi_{AB}^{(2,0)}$ arises in other cases when a diversified VC can generate higher output with two efforts than with no effort, but cannot with one effort. This occurs when $\max\{\varepsilon_A, \varepsilon_B\} < \varepsilon_2$ and $\min\{\varepsilon_A, \varepsilon_B\} < \varepsilon_1$. Therefore, the expected profit from diversification is $E(\pi_{AB}) = F(\varepsilon_2)^2 \pi_{AB}^{(0,0)} + (1 - F(\varepsilon_1))^2 \pi_{AB}^{(1,1)} + (1 - F(\varepsilon_2))^2 (1 - F(\varepsilon_1))^2 \pi_{AB}^{(2,0)}$. $\{\varepsilon_1, \varepsilon_2\}$ are $\{p^{-1}(\tau/p(1))-1, p^{-1}(\tau/p(2))-2\}$ respectively; they are unique because $p^{-1}(\tau/p(x))-x$ is a monotonically decreasing function of x .

Lemma 4 (Non-monotonic function): Let ε_i be the shock between $-\infty$ and ∞ to the probability of success of diversified project $i = A, B$. Let $F(\bullet)$ be the cumulative density function of ε_i , and $f(\bullet)$ be probability density function. Project A receives a VC's effort as follows: $\{\varepsilon_1, \varepsilon_2\}$ are $\{p^{-1}(\tau/p(1))-1, p^{-1}(\tau/p(2))-2\}$ respectively.

- $\varepsilon_A < \varepsilon_2$: Project A receives no effort. Even if a diversified VC invests two efforts for two periods, A can generate only $p(2+\varepsilon_A)p(2)V < \tau V$, which is lower than liquidation value. Thus, the VC will commit two efforts to the other project or not depending on performance shock.
- $\varepsilon_2 \leq \varepsilon_A$ and $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$: In this case, a VC cannot save both projects due to $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$. It chooses the project with higher-value performance shock and discontinues the lower-value shock. Project A receives two efforts with conditional probability $F(\varepsilon_A \wedge 2\varepsilon_1 - \varepsilon_A)/F(2\varepsilon_1 - \varepsilon_A)$ and zero effort with probability $(1 - F(\varepsilon_A \wedge 2\varepsilon_1 - \varepsilon_A) + F(2\varepsilon_1 - \varepsilon_A))/F(2\varepsilon_1 - \varepsilon_A)$. The former is the probability that the shock to B is lower than that to A and $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$ conditional on $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$. The latter is the probability to A is lower than that to B and $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$ conditional on $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$.
- $\varepsilon_2 \leq \varepsilon_A$ and $(\varepsilon_A + \varepsilon_B)/2 \geq \varepsilon_1$: In this case, the VC can save both projects (due to $(\varepsilon_A + \varepsilon_B)/2 \geq \varepsilon_1$) and balance the probability marginally. The VC intends to invest $(2 + \varepsilon_A + \varepsilon_B)/2$ in both projects if possible. Thus, A receives $\max(\min(2, (2 - \varepsilon_A + \varepsilon_B)/2), 0)$ with conditional probability $f(\varepsilon_B)/(1 - F(2\varepsilon_1 - \varepsilon_A))$ and B receives $\max(\min(2, (2 + \varepsilon_A - \varepsilon_B)/2), 0)$.

The logic supports the proofs for (1) – (3): When $\varepsilon_A < \varepsilon_2$: the effort to project B is zero and two before and after $\varepsilon_B = \varepsilon_2$ respectively. When $\varepsilon_2 < \varepsilon_A < \varepsilon_1$, the effort to project B remains zero and jumps to two at $\varepsilon_A = \varepsilon_B$. Then, it remains two until $2\varepsilon_1 - \varepsilon_A$ and then decreases. When $\varepsilon_1 < \varepsilon_A$, the effort to project B remains zero until $2\varepsilon_1 - \varepsilon_A$, and then increases until $\varepsilon_A = \varepsilon_B$, and then decreases.

Lemma 5 (Serial knowledge spillover): $2p(x) > p(2x)$ holds due to concavity. Therefore, a VC prefers splitting its effort, spillover, and performance shocks equally over projects rather than concentrating. Both specialized and diversified VCs have three choices of allocating their efforts. First, they liquidate both projects to earn $2\tau V$. Second, they liquidate one and execute the other to earn $(p(2)p(2(1+\rho)) + \tau)V$. A liquidated project generates τV . A VC injects two efforts to the surviving projects for two periods to generate the probability $p(2)$ of continuing to next stage. In addition, two efforts in first period create a spillover effect in the second period as 2ρ in addition to two efforts in the second period. This generates probability of success in the next period of $p(2(1+\rho))$, resulting in the combined value of $(p(2)p(2(1+\rho)) + \tau)V$. Third, they can invest in both projects to generate $2p(1)p(1+\rho)V$ by specialized VCs and $2(p(1)^2 p(1+\rho) + p(1)(1-p(1))p(2+\rho))$ by diversified VCs. In the specialized case, a VC allocates one effort per project. One effort in the first period produces spillover by ρ in the second period. The expected payoff for the two period is $2p(1)p(1+\rho)V$ since two projects succeed or fail together. In the diversified case, a VC invests one unit of effort per project at period 1. Two projects succeed with probability $p(1)^2$, and one project succeeds with $2p(1)(1-p(1))$. When two projects succeed, the expected payoff is $2p(1+\rho)V$ because the payoff for each project is $p(1+\rho)V$ with one effort and spillover. When one project succeeds, the

expected payoff is $p(2+\rho)V$ because the VC invests two efforts for the surviving project and the spillover from the surviving project. Since $2(p(1)^2p(1+\rho)+p(1)(1-p(1))p(2+\rho))V > 2p(1)p(1+\rho)V$, diversification weakly dominates specialization. Also, since $p'(1+\rho) > p'(2+\rho)$, the larger the serial knowledge spillover, the weaker the domination of diversification becomes. Alternatively, $p(2+\rho)-p(1+\rho)$ decreases with ρ to reduce diversification domination as serial spillover increases. In addition, when liquidation value is large, it is likely that the VC will liquidate one project.

Corollary 2 (Non-monotonic function with serial spillover): Let ε_i be the shock between $-\infty$ and ∞ to the probability of success of diversified project $i = A, B$. Let $F(\bullet)$ be the cumulative density function of ε_i , and $f(\bullet)$ be probability density function. Project A receives the VC's effort as follows. $\{\varepsilon_1, \varepsilon_2\}$ are $\{p^{-1}(\tau/p(1+\rho))-1, p^{-1}(\tau/p(2(1+\rho)))-2\}$ respectively. (1) $\varepsilon_A < \varepsilon_2$: Project A receives no effort. Even if a diversified VC invests two efforts for two periods, A can generate only $p(2+\varepsilon_A)p(2(1+\rho))V < \tau V$, lower than liquidation value. (2) $\varepsilon_2 \leq \varepsilon_A$ and $(\varepsilon_A + \varepsilon_B)/2 < \varepsilon_1$: In this case, the VC cannot save both projects. Thus, it chooses the project with higher-value performance shock, and discontinues the project with the lower-value shock. Project A receives two efforts with conditional probability $F(\varepsilon_A \wedge 2\varepsilon_1 - \varepsilon_A)/F(2\varepsilon_1 - \varepsilon_A)$ and zero with probability $(1 - F(\varepsilon_A \wedge 2\varepsilon_1 - \varepsilon_A) + F(2\varepsilon_1 - \varepsilon_A))/F(2\varepsilon_1 - \varepsilon_A)$. (3) $\varepsilon_2 \leq \varepsilon_A$ and $(\varepsilon_A + \varepsilon_B)/2 \geq \varepsilon_1$: In this case, the VC can save both projects and balance the probability. The VC intends to invest $(2 + \varepsilon_A + \varepsilon_B)/2$ to both projects if possible. Thus, A receives $\max(\min(2, (2 - \varepsilon_A + \varepsilon_B)/2), 0)$ with conditional probability $f(\varepsilon_B)/(1 - F(2\varepsilon_1 - \varepsilon_A))$ and B receives $\max(\min(2, (2 + \varepsilon_A - \varepsilon_B)/2), 0)$. This implies the direct proofs for (1) – (3) similar to Lemma 4. When $\varepsilon_B < \varepsilon_2$: the effort to project A is zero and two before and after $\varepsilon_A = \varepsilon_2$ respectively. When $\varepsilon_2 < \varepsilon_B < \varepsilon_1$, the effort to project A remains zero and jumps to two at $\varepsilon_A = \varepsilon_B$; it remains two until $2\varepsilon_1 - \varepsilon_B$ and then decreases. When $\varepsilon_1 < \varepsilon_B$, the effort to project A remains zero until $2\varepsilon_1 - \varepsilon_B$, and then increases until $\varepsilon_A = \varepsilon_B$, and then decreases.

Lemma 6 (Serial and cross-sectional knowledge spillover): $E(\pi_{AB})$ is same to Lemma 5 because cross-sectional spillover does not apply to diversification. In case of specialization, first, a specialized VC can liquidate both projects to earn $2\tau V$. Second, it can liquidate one and execute the other to earn $(p(2)p(2(1+\rho))+\tau)V$. A liquidated project generates τV . The VC injects two efforts to the survived one for two periods. In addition, two efforts in first period generate a spillover effect in the second period as 2ρ . This generates $(p(2)p(2(1+\rho))+\tau)V$. Third, it can invest in both projects to generate $2p(1+\alpha)p((1+\alpha)(1+\rho))V$. The probability of joint success is $\min(p_1, p_2)$, in which $\{p_1, p_2\}$ are the probabilities of success in the first and second project. Due to this Leontief formulation, the VC sets $p_1 = p_2$, so that it allocates equal amount of effort in both projects in the first and second periods. Due to cross-sectional knowledge spillover, one effort generate α effort in the other project. This makes the probability of continuation into second period as $p(1+\alpha)$. In the second period, the VC allocates equal effort into both projects. Due to spillovers, this generates the effect of $(1+\alpha)$. In addition, there is serial spillover $\rho(1+\alpha)$ from the first period. In sum, the expected profit becomes $2p(1+\alpha)p((1+\alpha)(1+\rho))V$.

Corollary 3 (Diversification versus specialization): Let $k_1 \equiv p(2+\alpha)/p(1+\alpha)$ and $k_2 \equiv p((1+\alpha)(1+\rho))/p(1)$. Then, the ratio of third elements in $E(\pi_{AB})$ and $E(\pi_{AA})$ become $(p(1)+k_1(1-p(1)))/k_2$. Also, $\partial^2 k_2/(\partial \alpha \partial \rho) > 0$ for large values of α or ρ . Therefore cross-sectional and serial spillovers substitute for each other in determining the relative performance of specialization over diversification for large knowledge spillovers. When liquidation value is large, it is likely that the VC will liquidate one project. On liquidation, specialization and diversification generate the same

payoff.

Corollary 4 (Serial and cross-sectional substitutes): This proof restates Corollary 3 about $\partial^2 k_2 / (\partial \alpha \partial \rho) > 0$ for large values of α or ρ .

Lemma 7 (Cross-sectional knowledge spillover in multiple-stages): The profits in case of specialization and diversification, when $N \geq 2$, $\rho = 0$, and $\varepsilon = 0$, are respectively: $\pi_{AA}(N) = 2\beta^N V p (1 + \alpha)^N$, $\pi_{AB}(N) = 2\beta^N V (p(1)^{2N} - p_X(p(1)^{2N} - p(2)^N))$ $p_X \equiv p(1)(1 - p(1)) / (p(2) - p(1)^2)$. The value of the flexibility option is $2\beta^N p(1)(1 - p(1))(p_{12} - p_{11})$, in which $p_{12} \equiv (p(1)^{2N} - p(2)^N) / (p(2) - p(1)^2)$ and $p_{11} \equiv (p(1)^{2N} - p(1)^N) / (p(1) - p(1)^2)$. Then π_{AB} / π_{AA} increases with N for sufficiently large N and $\alpha < 1$. If $\alpha > 1$, π_{AB} / π_{AA} decreases with N . β does not affect the ratio if it is less than one.

Corollary 5 (Cross-sectional knowledge spillover in multiple stages): This proof restates Lemma 7. Under low levels of cross-sectional knowledge spillover ($\alpha < 1$), projects with many stages (early-stage financing) create the greatest benefits for diversified VCs because π_{AB} / π_{AA} is an increasing function of N for $N > N^*$. Inversely, projects that require fewer decision stages (late-stage financing) suit specialized VCs because π_{AA} / π_{AB} is a non-increasing function of N for $N < N^*$. In addition, high cross-sectional knowledge spillovers ($\alpha > 1$) make projects with larger numbers of decision stages favor specialized VCs because π_{AA} / π_{AB} is an increasing function of N for $N > N^*$.